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# MATHEMATICAL MODEL OF BLADES BUNDLE WITH DAMPING CONNECTIONS


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***Розглянуто особливості досить ефективного способу заглушення небажаних вібрацій турбінних лопаток шляхом введення додаткових демпфірувальних елементів до їх зв'язок. Для детального вияву процесів демпфірування в Інституті термомеханіки Академії наук Чеської Республіки було підготовлено та проведено динамічні випробування пакета з п'яти турбінних лопаток. Експериментальним дослідженням передувало математичне моделювання процесу системи з п'яти приєднаних мас***

***Ключові слова: вібрація лопаток, демпфірувальні елементи, гума, експеримент, математична модель***

***Рассмотрены особенности достаточно эффективного способа подавления нежелательных вибраций турбинных лопаток путем введения дополнительных демпфирующих элементов в их связке. Для детального выявления процессов демпфирования в Институте термомеханики ASCR были подготовлены и проведены динамические испытания пакета из пяти турбинных лопаток. Экспериментальным исследованиям предшествовало математическое моделирование процесса системы из пяти присоединенных масс***

***Ключевые слова: вибрация лопаток, демпфирующие элементы, резина, эксперимент, математическая модель***


## 1. Introduction

The complex dynamic research of a turbo-machine blades and bladed disks becomes very important with the increase demands on higher power and longer life of steam and gas turbines. The Institute of Thermomechanics ASCR works for many years on the theoretical, numerical and experimental research of means for reduction of undesirable vibrations of blades The main attention has been concentrated on using blade damping heads, connected either by friction contact or by means of various type of inserted damping elements. Laboratory models of rotating bladed disc, the various combinations of blades bundle without rotation, analytical and numerical (FEM) methods were used to solve complex dynamic problems including non-linear stiffness and damping characteristics, contact pressures, micro-slips, impacts, etc. [1–4].

Many of these problems were solved also in the frame of scientific cooperation between Academy of Sciences of Czech Republic and National Academy of Sciences Ukraine during the last three years.

A small part of these works is presented in this paper. It is oriented on the dynamic study of five-blades-model with the blades heads connected by inserted special rubber damping elements.

## 2. Vibrations of blades bundle – experimental model

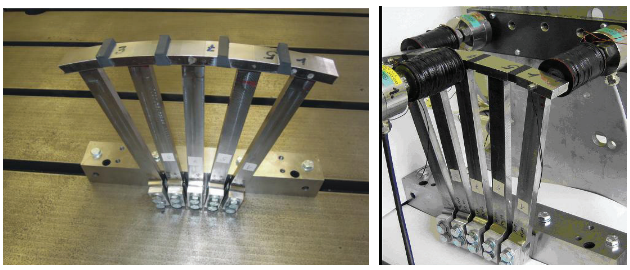
The experimental set consists of five models of blades with shroud heads rigidly fastened to a steel plate basement, Fig. 1a. and are excited by electro-magnets seen in Fig. 1b.

Inserted damping elements made of special rubber VITON known for its resistance against high temperatures (up to $T$=220$^0$C) and against many aggressive chemicals and oils. Mechanical properties of this material depend strongly on the temperature and frequency and therefore the mathematical modeling of dynamic properties of systems containing these elements need a special treatment. The dependences of its complex Young modulus $E^* = E_{Re} + iE_{Im}$ is described by the analytical formula in [4]:

$$E^* = 10.49*(1+0.1i)+15.88*(1+ \\ +0.06i)(if\alpha T)^{0.64} / (1+0.0063(if\alpha T)^{0.64} \quad (1)$$

where $\alpha T = 10\wedge(-20(T-4.4)/(134.5+T)$.

Placement of the prismatic VITON elements in the blade shroud presumes the shear deformation. Their geometric parameters are: base area $A$=0.00025 m$^2$ and height $h$=0.012 m. The complex shear stiffness $K_s^*$ of such elements is ascertained from eq. (1) at the assumption of volume incompressibility: $K_s^*(f,T) = E^*(f,T)A/3h$.



a b

Fig. 1. a - Experimental model; b - Model with electromagnets

Stiffness $k(f,T)$ and linear damping coefficient $b(f,T)$ of the viscous-elastic damping element can be derived from complex shear stiffness $K_s^*$. However both these parameters depend on frequency and temperature. The graphical representation of these dependences in the temperature range $T\in(20, 120)$ °C and in frequency range $f\in(100, 300)$ Hz are depicted in Fig. 2 and 3.

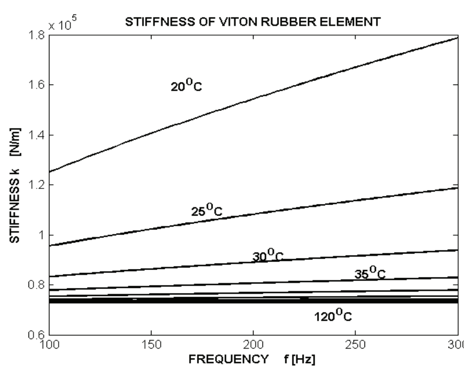



Fig. 2. Stiffness versus frequency

The properties of this viscous-elastic damping element are very variable near to the room temperature, but they stabilize at the highest temperature (≈120 °C) on the approximately constant stiffness $k$ ≈72800 N/m and damping coefficient $b$ [Ns/m] variable with frequency $f$ according to the hyperbolic law $b*f$ ≈ const. The linear regression function of rubber element stiffness at 20 °C is

$$k_t = 102300 + 250f \ \ [\text{N/m, Hz}] \quad (2a)$$

and at 100 °C $k_t = 72850 + 0.033f$ [N/m, Hz]. (2b)

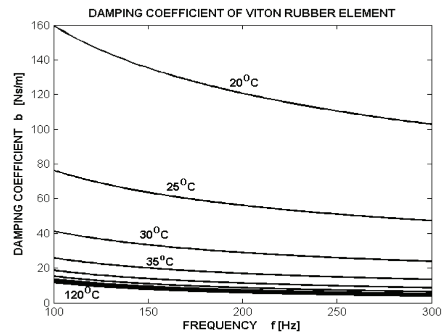



Fig. 3. Damping versus frequency

Experimental system in Fig. 1 can be modelled by a system shown in Fig. 4, where the blades are replaced by 1 DOF systems, the eigenfrequencies of which corresponds to the first bending eigenfrequencies of real blades. The torsion eigenfrequencies of these blades are supposed to be much higher than the bending ones and therefore the torsion vibrations of blades are not taken into account.

The dynamic measurements physical model began at room temperature. As the measurements at higher temperature will be more complicated, the preliminary analytical and numerical solution of simplified mathematical model with temperature 100 °C has to be carried out. The first stage of this study is in the presented paper.

Stiffness at 100 °C can be approximately described by (2b) and damping coefficient by

$$b_1 = 12.972 - 0.0316f \ [\text{Ns/m,Hz}]. \quad (3)$$

## 3. Mathematical model of five blades bundle

Scheme of dynamic computational model corresponding to the physical five blades bundle is shown in Fig. 4.

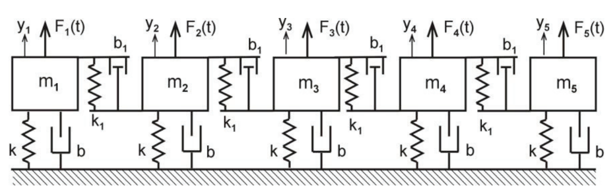



Fig. 4. Dynamic computational model of blades bundle

Damping of all separated steel blades is modelled by viscous damping with low coefficients $b$=0.4 Ns/m. According to the first bending eigenfrequency of real blade $f$ = 120.88 Hz, the reduced mass $m$=0.182 kg, and stiffness $k$=105000 N/m were ascertained.

The VITON rubber element properties at 100 °C are given by Voigt–Kelvin model with parameters (2b) and (3). Masses $m$ are loaded by the synchronized harmonic forces

$$F_i(t) = F_{0i}\cos(\omega t), \ i = 1,..5,$$

where the amplitudes $F_{0i}$ can be of various values. The excitation frequency ω varies linear with time through the whole eigenfrequency spectrum of system. Differential equations of motion by given vector of force amplitudes $\boldsymbol{F} = [F_{01}, F_{02}, F_{03}, F_{04}, F_{05}]'$ are:

$$M\ddot{Y} + B\dot{Y} + KY = F\cos(\omega t) \tag{4}$$

where

$$M = \begin{bmatrix} m & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & m \end{bmatrix},$$

$$K = \begin{bmatrix} k+k_1 & -k_1 & 0 & 0 & 0 \\ -k_1 & k+2k_1 & -k_1 & 0 & 0 \\ 0 & -k_1 & k+2k_1 & -k_1 & 0 \\ 0 & 0 & -k_1 & k+2k_1 & -k_1 \\ 0 & 0 & 0 & -k_1 & k+k_1 \end{bmatrix} \tag{4a}$$

The matrix $B$ has the same structure as $K$ with the exchange $k_1, k \Leftrightarrow b_1, b$.

Due to the frequency and temperature dependent coefficients $k_1(T,f)$, $b_1(T,f)$, also both matrices $K(T,f)$, $B(T,f)$ are non-constant quantities. Neglecting external and damping forces, the equations for free vibrations are in the matrix description

$$M\ddot{Y} + K(T,f)Y = 0\,. \tag{5}$$

Because the stiffness $k_1$ of inserted elements is at 100 °C moderately variable with frequency, the five blades system eigenfrequencies must be determined by means of modified programme *eig* in Matlab with consideration to (2b) that gives

$$\left[\Omega_1, \Omega_2, \Omega_3, \Omega_4, \Omega_5\right] = \left[120.88,\ 135.96,\ 169.19,\ 202.88,\ 226.48\right]\ \text{Hz}. \tag{6}$$

Corresponding modes of these eigenfrequencies are plotted in Fig. 5. From the forms of these modes can be estimated without any calculation that the higher is the eigenfrequency, the higher is also the damping of the corresponding mode.

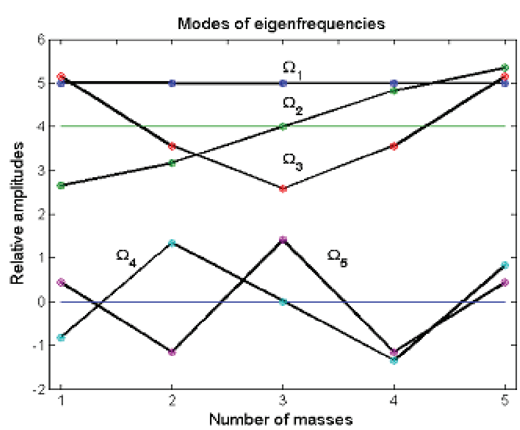



Fig. 5. Modes of free vibrations

## 4. Distortion of response curve at external sweep excitation

The simplest way how to gain the response curves of multi-degrees dynamical system is to calculate the response of mathematical model on the sweeping excitations. However obtained response curves are a little distorted against the stationary ones, but the quickness of solution and mainly the possibility of solution of the strongly non-linear systems is very often decisive. Analysis of influence of sweep velocity on the response's distortion has shown that sweep acceleration 0.2 [rad/s$^2$] causes only 3% decrease of resonance peak and 0.03% change of relative resonance frequency $\Delta\Omega/\Omega$ and can be therefore applied in the further solution.

## 5. Forced vibrations – stiffness and damping is frequency dependent

Experience with modes vibration of bladed disk in steam turbine show that the length of waves (both standing or running waves) of bladed system always include several blades and therefore the most important modes are those belonging to the low eigenfrequencies $\Omega_1, \Omega_2, \Omega_3$ in Fig. 5.

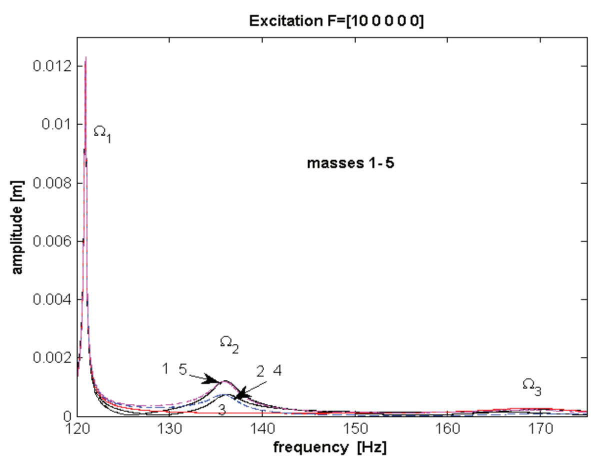



Fig. 6. Responses on one excitation force

The response curves of 5-blades bundle depend however also on the form of excitation force vector $\boldsymbol{F} = [F_{01}, F_{02}, F_{03}, F_{04}, F_{05}]$. If the force vector consists of only one force $F_{01} = 10$ N, (i.e. $\boldsymbol{F} = [10,0,0,0,0]$ N), then the response curves calculated at sweep passing through the first three resonance zones are plotted in Fig. 6. The first mode belongs to the umbrella type of disk and has been rarely excited. The inserted damping elements are not deformed and therefore this resonance is very low damped. The second and third resonance peaks are due to the strong damping of inserted elements and inappropriate excitation very put dawn. For detail analysis of these resonances, the force vector $\boldsymbol{F}$ with appropriate components has to be used.

Two harmonic forces acting in opposite sense on the side masses 1, 5 excite second resonance (Fig. 7) with the one node in the point 3. Three external forces acting on mass 1 (10N), mass 3 (-20N) and on mass 5 (10N) excite third resonance (Fig. 8) with two nodes mode. Although the number of external forces increases, the maximal amplitudes decrease.

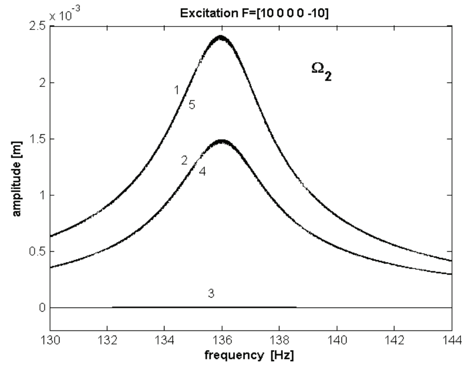



Fig. 7. Responses on two excitation forces

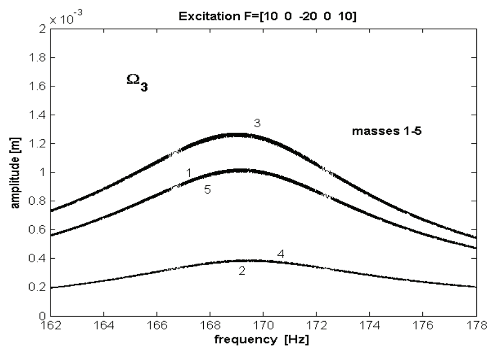



Fig. 8. Responses on three forces

## 6. Conclusion

Analysis of dynamic behaviour of numerical models of five blades bundle with inserted VITON-rubber-damping elements having frequency dependent properties is presented. The main attention was given to the response curves in the second and third resonance. It was shown that the level of resonance peaks depends on the type of vector of exciting force amplitudes and that application of viscous-elastic damping elements is advantageous for suppressing of higher resonance. Comparison with the damping properties of dry friction contacts in the same five blades bundle will be shown at presentation.

Elaborated method of solution and obtained results create basic theoretical background for evaluation of measurement on laboratory experimental bladed disk set and for evaluation of effectiveness of the friction element on suppression of forced vibration of blades.

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