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При розробці технологічних процесів отримання холоднопресованих спечених деталей малої пористості особлива увага приділяється механізму зміни щільності. На практиці порошкової металургії дуже часто використовують багатокомпонентні шихти, що складаються як з пластичних металів, так з погано стискаємих включень і з'єднань. До такої шихти в рівній мірі можна віднести і шихту, що складається з порошку заліза, чавуну і скла. У цій шихті перший компонент (основа) – це пластичне залізо, а два інших – чавун і скло – пружні компоненти. Викликає певний інтерес, яке ущільнення може бути отримано в такому випадку і які результуючі рівняння можна застосувати для оцінки механіки ущільнення такої порошкової шихти.

Пропонуються результуючі рівняння ущільнення пористих порошкових тіл із залізо-чавун-скло. Наведено аналіз ізотропного, жорсткопластичного зміцнюючого матеріалу типу залізо-чавун-скло. При ущільненні такого матеріалу швидкість дисипації енергії (тиск пресування) визначається швидкостями зміни обсягу тіла і його формозміни. Показано, що відмінність стискаємих (чавун і скло) і пластичних (залізо) матеріалів, що ущільнюються, формує особливі механічні властивості матриці. Отже, гідростатичний тиск може впливати на формозміну тіла, а дотичні напруження – на зміну обсягу. Отримані результати математичного підходу для отримання результуючих рівнянь ущільнення пружнопластичного середовища показали шлях побудови теорії пластичності тіла, що ущільнюється, при якому виключається необхідність врахування виду поверхні навантаження. При обліку поверхні навантаження неможливо отримання універсальних рівнянь з ущільнення пористого пружнопластичного середовища. Показано, що для застосування класичного формулювання моделі пружнопластичного тіла, що ущільнюється, необхідно вважати, що поверхня навантаження опукла-замкнута

Ключові слова: поверхня навантаження, пористе тіло, результуючі рівняння, залізо-чавун-скло, швидкість деформації, інваріант тензора, ізотропний матеріал

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COMPACTION OF POROUS POWDER BODY CONSISTING OF THE ELASTIC-PLASTIC MEDIUM

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1. Introduction

A consistent phenomenological description of the processes of formation of powders and porous bodies of the elastic-plastic medium as the most important element eliminates the choice of governing or rheological equations. For sintering and hot pressing, thanks to the works [1, 2], some clarity in understanding of this issue has been achieved, while for

cold molding processes characterized by plastic flow, there is no consensus about the type of governing equations. In this regard, the formation of general restrictions imposed on such equations, based on the current concepts of irreversible thermodynamics and continuum mechanics, is relevant. In this case, an approach to constructing a theory of plasticity should be used, based on setting the properties of the dissipative function [3–6].

2. Literature review and problem statement

In [2, 3], physical justification is considered and mathematical evaluation of the elastic-plastic deformation of powder compressed materials is performed. In these works, it is shown that the solution of the boundary-value problem of the process of cold compaction of the elastic-plastic medium does not depend on the type of loading surface. It is determined by dependences of axial and lateral pressure on porosity. However, the author does not consider the fact that each pressing scheme can correspond to a certain character of bending of layers parallel to the base plane prior to pressing.

The phenomenological approach to the processes of compaction of the powder medium proposed in [4] takes into account the case of forming blanks of bushing type. However, this is the simplest type of blank, on which analytical dependencies are obtained.

The question of building a mathematical model of blanks of a more complex shape remains open. In this case, it is necessary to consider not only the mechanics of compaction of the plastic component, but also interaction and joint deformation of both the plastic and elastic component, taking into account the peculiarities of the loading surface.

The solution of this problem in this context is considered in [7–9]. However, they do not take into account the participation of several elastic components (cast iron and glass), along with the plastic component of the medium, with different characteristics and behavior during deformation.

Therefore, there is the problem of form change during compaction of the isotropic, rigid-plastic hardening powder medium in which energy transfer rate (pressing pressure) depends on the rate of volume and form change.

3. The aim and objectives of the study

The aim of the work is to obtain the resulting equations describing the compaction of the porous powder body consisting of the elastic-plastic medium.

To achieve this aim, the following objectives are set:

- to formulate general restrictions on governing equations, based on current concepts of irreversible thermodynamics and continuum mechanics;

- to choose an approach to the construction of the elastic-plastic medium and justify the type of mathematical model of the isotropic, rigidly plastic hardening material, taking into account the rates of volume and form change of the body.

4. Obtaining the resulting equations on compaction of a porous powder body

4.1. Selection and justification of restrictions on governing equations

The isotropic, rigid-plastic hardening material such as “iron-cast iron-glass”, the energy dissipation rate of which is determined by the rates of volume and form change is introduced into consideration.

The last two parameters are, respectively, the first invariant of the strain rate tensor ℓ_{ij} and the second invariant of its deviator, and, therefore, expressed through ℓ_{ij}

$$\ell = \ell_{ij} \cdot \delta_{ij}, \quad \gamma = \left(\ell_{ij} - \frac{1}{3} \ell \delta_{ij} \right) \left(\ell_{ij} - \frac{1}{3} \ell \delta_{ij} \right).$$

In the future, hydrostatic pressure and shear stress intensity, which are, respectively, the first invariant of the stress tensor σ_{ij} and the second invariant of its deviator, will also play an important role. They are connected with σ_{ij} by the relations

$$p = \frac{1}{3} \sigma_{ij} \delta_{ij}, \quad \tau^2 = (\sigma_{ij} - p \delta_{ij}) (\sigma_{ij} - p \delta_{ij}).$$

4.2. Choosing an approach to the construction of the elastic-plastic medium based on setting the properties of the dissipative function

We take the following definition of an elastic-plastic body: the dissipative function D is homogeneous, of the first degree by ℓ_{ij} ; the same function serves as a potential for the stress tensor

$$\sigma_{ij} = \frac{dD}{d\ell_{ij}}. \tag{1}$$

We consider materials for which the von Mises yield criterion is valid in the following form

$$(P - P_1)\ell + (\tau - \tau_1)\gamma = 0, \tag{2}$$

where P , τ are the stresses corresponding to the kinematic parameters ℓ and γ ; p_1 , τ_1 are any other stresses.

For isotropic material, the properties of which are specified above, the function D allows for ℓ , γ , the current porosity θ , as well as the parameters χ_k , characterizing the state of the powder particle material or the porous body framework as arguments.

By the Euler’s theorem on homogeneous functions, the postulate on uniformity D by ℓ and γ is expressed by the equation

$$\frac{dD}{d\ell} \cdot \ell + \frac{dD}{d\gamma} \cdot \gamma = D. \tag{3}$$

Let us proceed to the analysis of the corollaries of the postulate (1). From (1), taking into account what was said about the arguments of the function D , the following equation can be obtained:

$$\sigma_{ij} = \frac{dD}{d\ell} \cdot \frac{d\ell}{d\ell_{ij}} + \frac{dD}{d\gamma} \cdot \frac{d\gamma}{d\ell_{ij}}. \tag{4}$$

Since by definition

$$\frac{d\ell}{d\ell_{ij}} = \delta_{ij}, \quad \frac{d\gamma}{d\ell_{ij}} = \frac{1}{\gamma} \left(\ell_{ij} - \frac{1}{3} \ell \delta_{ij} \right), \tag{5}$$

the relation (4) can be given the following form

$$\sigma_{ij} = \frac{dD}{d\ell} \cdot \delta_{ij} + \frac{1}{\gamma} \frac{dD}{d\gamma} \left(\ell_{ij} - \frac{1}{3} \ell \delta_{ij} \right). \tag{6}$$

In this case, the tensor identity is used

$$\sigma_{ij} = p \delta_{ij} + (\sigma_j - p \delta_{ij}).$$

We find a simple relationship between p , τ and ℓ , γ

$$p = \frac{dD}{d\ell}, \quad \tau = \frac{dD}{d\gamma}. \tag{7}$$

From (7) and (8), the equation follows

$$\ell_{ij} - \frac{1}{3} \ell \delta_{ij} = \frac{\gamma}{\tau} (\sigma_{ij} - p \delta_{ij}), \tag{8}$$

expressing the fact of similarity of deviators and the classical theory of plasticity similar to the Levy-Mises equation. This equation characterizes tensor properties of compacted materials that satisfy the above postulates. Along with it, there should be equations characterizing scalar properties of the medium.

In order to obtain them, on the basis of (7) and (3), we simplify the expression for

$$D = p\ell + \tau\gamma. \tag{9}$$

Assuming throughout what follows that p and τ are rather smooth functions ℓ and γ , we differentiate both sides of the equation (9) first with respect to ℓ , and then with respect to γ . Then, using (3) and (7), we get

$$\frac{dP}{d\ell} + \frac{dP}{d\gamma} \gamma = 0; \quad \frac{d\tau}{d\ell} \cdot \ell + \frac{d\tau}{d\gamma} \gamma = 0. \tag{10}$$

The last two expressions can be considered as a system of first-order partial differential equations with respect to p and τ . Direct check shows that its general solution is

$$p = p(s, \theta, \chi_k); \quad \tau = \tau(s, \theta, \chi_k), \tag{11}$$

where $s = \ell / \gamma$.

The equations obtained characterize the scalar properties of compacted materials.

Equations (8) and (11) are a complete system of governing equations of the plastically compacted body. If its tensor properties do not differ from those for viscous and plastically incompressible materials, scalar properties possess a known originality. In order to emphasize it, we present the equations characterizing the scalar properties of a viscous porous body.

$$p = \frac{4}{3} \eta_0 \frac{(1-\theta)}{\theta} \ell, \quad \tau = \eta_0 (1-\theta)^2 \gamma, \tag{12}$$

these equations are uniquely solvable with respect to ℓ and γ . At the same time, from equations (12) it is impossible to determine ℓ and γ , p and τ depend on their relationship.

Thus, there is a cross-effect of various invariants of stress and strain rates on each other. This phenomenon is not characteristic of classical models of viscous and elastic bodies, naturally inherent in the model of a plastically compacted body.

The specified formal difference between viscous compressed and plastically compacted materials predetermines special mechanical properties of the latter: hydrostatic pressure can affect the form change, and shear stresses – volume change. This property should be associated with the dilatancy effect [7], characteristic of compacted materials.

It turns out further that the functions entering the equation (11) are not arbitrary and must satisfy a certain relation. Indeed, from (7) it follows that

$$\frac{dP}{d\gamma} = \frac{d\tau}{d\ell}. \tag{13}$$

Given (11), we find

$$\frac{dP}{d\gamma} = \frac{dP}{ds} \cdot \frac{ds}{d\gamma} = -\frac{dP}{ds} \cdot \frac{s}{\gamma},$$

$$\frac{d\tau}{d\ell} = \frac{d\tau}{ds} \cdot \frac{ds}{d\ell} = \frac{d\tau}{ds} \cdot \frac{1}{\gamma}.$$

Substituting the stresses found into equality (13), we finally get

$$\frac{dP}{ds} S + \frac{d\tau}{ds} = 0. \tag{14}$$

Thus, whatever the governing equations (11), the functions p and τ entering them must satisfy (14).

Let us proceed to the analysis of the corollaries of the postulate expressed by inequality (2). To do this, we consider two different stress states, characterized by stresses p_1, τ_1 and p_2, τ_2 .

Let the first statically admissible stress state correspond to the kinematic field, characterized by the value of the parameter $s=s_1$, and the second – to $s=s_2$. Then, according to (2), there is a pair of inequalities

$$(p_1 - p_2)s_1 + (\tau_1 - \tau_2) \geq 0;$$

$$(p_2 - p_1)s_2 + (\tau_2 - \tau_1) \geq 0,$$

adding them up, we get

$$(p_1 - p_2)(s_1 - s_2) \geq 0.$$

The latter inequality holds for any value of s , therefore the function p is a monotonically non-decreasing function.

Using this result, it is also possible to obtain additional information regarding τ . It follows from the monotony of p that $dp/ds \geq 0$. Therefore, on the basis of (14) and by virtue of the assumption of smoothness of the function p and τ

$$\frac{d\tau}{ds} \geq 0; \quad s \leq 0; \quad \frac{d\tau}{ds} = 0; \quad s = 0; \quad \frac{d\tau}{ds} \leq 0; \quad s \geq 0.$$

The resulting inequalities show that with negative values of s , τ increases, and with $s=0$ it reaches a maximum, and with positive s – decreases. Note, however, that τ is non-negative by definition. Therefore, the graph is “bell-shaped”, and with an unlimited increase $|s|$ τ asymmetrically tends to zero (Fig. 1, *a*). The last of these circumstances imposes known limitations on the form of the function p : its graph will have two horizontal asymptotes (Fig. 1, *b*). The functions p and τ are bounded.

The tensor properties of this model are characterized by the equation (8).

Note that such a formulation of the governing equations is close to the traditional models of a viscous and elastic body, in which the scalar equations are formulated as stress – strain rate or stress – strain ratios. It should be noted that the special provision of the theory of plasticity is justified by the fact that it was intensively developed in relation to incompressible media. Related to this is the importance of such specific concepts as yield stress, loading surface, and others.

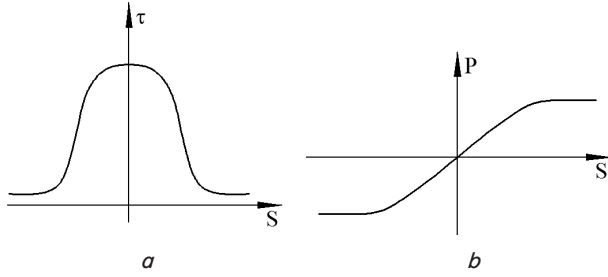


Fig. 1. Dependence of the intensity of shear stresses and average pressure on the ratio of rates of compaction and form change of the elastic-plastic porous body:
a – shear stresses *b* – average pressure

5. Discussion of the results of constructing equations of compaction of the porous powder body consisting of the elastic-plastic medium

The result obtained in this work indicates the way to construct a theory of plasticity of a compacted body, which eliminates the need to formulate the type of loading surface.

At the same time, the ideas developed here do not contradict the classical ones and moreover, confirm them. To see this, we use monotonicity and smoothness of the function *p* and the solution of the first of the equations (11) with respect to *S*. Substituting the result into the second expression (11), we obtain the equation

$$\tau = \tau(P, \theta, \chi_k). \tag{15}$$

Its characteristic difference is that it does not contain deformations. Since the values of *p* and τ correspond to plastic flow, we have the function

$$f(P, \tau, \theta, \chi_k) \equiv \tau - \tau(P, \theta, \chi_k) = 0. \tag{16}$$

It can be identified with the loading function, and the surface corresponding to it in the stress tensor space – with the loading surface. The same name is saved for the graph of the equation (15) in the coordinates of *p*, τ . The properties of this surface with fixed θ and χ_k are defined using inequality (2). Then, according to (15), this inequality can be written as

$$\tau(p_1) - \tau(p_2) + (p_1 - p_2)s_1 < 0.$$

Given (14), we have the equality

$$\frac{d\tau}{ds} = \frac{d\tau}{dp} \cdot \frac{dP}{ds}, \quad S = \frac{d\tau}{dP}. \tag{17}$$

As a result of substitutions (17) into the last inequality, by means of obvious transformations we find

$$\tau(P_2) \leq \tau(P_1) + \left. \frac{d\tau}{dP} \right|_{P=P_1} \cdot (P_2 - P_1).$$

The resulting inequality shows that the graph of the function $\tau(p)$ lies under the tangent. Consequently, the loading surface is convex. Since the functions *p* and τ are bounded, we can conclude that this surface is also closed.

In order to determine the relationship between the surface of loading and the strain rate tensor, we use the fact

that, due to the independence of the loading function *f* of *s*, there is the inequality

$$\frac{df}{ds} \equiv \frac{df}{dP} \cdot \frac{dP}{ds} + \frac{dP}{d\tau} \cdot \frac{d\tau}{ds} = 0.$$

Given (14), as well as the definition $S = \ell / \gamma$, we get

$$\frac{df}{dP} \gamma = \frac{df}{d\tau} \cdot \ell. \tag{18}$$

This result allows for a simple geometric interpretation. The pair of numbers ℓ , γ can be considered as vector components in the *p*, τ plane. Then equality (18) shows that this vector is collinear to the normal *K* of the surface *f* or is orthogonal to it.

Equality (18) acquires a more general meaning, if we use its parametric representation

$$\ell = \mu \frac{df}{dp}, \quad \gamma = \mu \frac{df}{d\tau}.$$

Then, using (8) by simple transformations, we obtain the equation

$$\ell_{ij} = \mu \left(\frac{df}{dp} \delta_{ij} + \frac{1}{\tau} \frac{df}{d\tau} (\sigma_{ij} - p\delta_{ij}) \right) \equiv \mu \frac{df}{d\tau_{ij}}, \tag{19}$$

showing that in the stress tensor space, the strain rate tensor is “orthogonal” to the loading surface, that is, the associated law is true [3–6].

Thus, the classical formulation of the model of the elastic-plastic compacted body is a consequence of the previously developed representations. In this case, the loading surface (16) must be closed, convex, and the function *f* must satisfy the equation (18). Expressions (16) and (18) are the scalar governing equations. Tensor properties remain the same.

Note, however, that the traditional form of the model, associated with the known arbitrariness of the choice of loading surface, leads to the equations (11) only with a strictly convex surface. This can be seen on the example of the Drucker-Prager model (the generalized Coulomb-Mohr plasticity condition) [7], where *p* and τ are bound by the linear equation

$$\tau + \alpha p + c = 0.$$

The law associated with such a condition of plasticity leads to the corollary

$$\alpha\gamma - \ell = 0.$$

This corollary, together with the previous relation, does not allow one to uniquely determine *p* and τ .

The most common example of a strictly convex loading surface is spheroid [8–11], whose equation in the *p* – τ axes is

$$\frac{p^2}{\psi(\theta)} + \frac{\tau^2}{\psi(\theta)} = k^2,$$

where $\psi(\theta)$, $\phi(\theta)$ are the porosity functions; *k* is the value associated with the yield stress of the base metal.

The flow rule associated with this surface (19) leads to the equation

$$\frac{p}{\Psi} \gamma = \frac{\tau}{\phi} \ell.$$

The last two equations can be solved with respect to p and τ

$$p = \frac{\frac{\Psi}{\phi} \cdot S}{\sqrt{1 + \frac{\Psi}{\phi} \cdot S^2}} \sqrt{\phi k}, \quad \tau = \frac{1}{\sqrt{1 + \frac{\Psi}{\phi} \cdot S^2}} \sqrt{\phi k}. \quad (20)$$

Thus, the scalar governing equations of the models [8–11] can be represented as (11), while the functions p and τ satisfy the equation (14) and have the properties stated above.

The equation (14) can be used to specify models of plastically compressed media. For example, if for some reason the function $p=p(S, \theta, \chi_k)$ is known, the function τ can be determined from the relation, which is a corollary of (14).

$$\tau = \tau(0, \theta, \chi_k) - sp + \int_0^s p dS, \quad (21)$$

where $\tau(0, \theta, \chi_k)$ is the value proportional to the frequent shear yield stress.

All this suggests that during the deformation of the elastic-plastic medium, two equations must be solved. The first equation determines the influence of hydrostatic pressure on the continuum mechanics, and the second considers strict convexity of the surface of the flow rule of isotropic rigid-plastic material.

However, the limitation of this study should be noted, due to the fact that the results obtained do not take into account the properties of the components of the elastic-plastic medium sufficiently. They are of a general nature, which can be attributed to shortcomings at this stage of research. For this reason further it is necessary to consider a model that allows taking into account the properties of each component (strength, ductility, rigidity, etc.) of the elastic-plastic medium.

6. Conclusions

1. In the case of elastic-plastic deformation of compacted materials, hydrostatic pressure can influence shear deformations, and shear stresses can lead to a change in volume.

2. Along with the previously mentioned form of loading, there is a form of the governing equations expressed by the equation of the loading surface

$$f(p, \tau, \theta, \chi_k) \equiv \tau - \tau(p, \theta, \chi_k) = 0$$

and the relation

$$\frac{dF}{dP} \cdot \gamma = \frac{df}{d\tau} \cdot \ell.$$

In the case of a strictly convex loading surface, these two forms are equivalent.

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