Описана механічна модель одномасової вібромашини з поворотно-коливальним рухом платформи і віброзбудником у вигляді пасивного автобалансира. Платформа може коливатися навколо нерухомої осі. На платформу встановлений багатошаровий, багатороликовий або багатомаятниковий автобалансир. Вісь обертання автобалансира паралельна осі повороту платформи. Автобалансир обертається відносно платформи з постійною кутовою швидкістю. На корпусі автобалансира встановлений дебалансний вантаж для збудження швидких коливань платформи з частотою обертання автобалансира. Передбачалося, що кулі або ролики перекачуються по бігових доріжках всередині корпусу автобалансира без відриву і ковзання. Відносному руху вантажів перешкоджають Ньютонівські сили в'язкого опору. При нормально працюючому автобалансирі вантажі (маятники, кулі, ролики) не можуть наздогнати корпус і застряють на резонансній частоті коливань платформи. Цим збуджуються повільні резонансні коливання платформи. Таким чином, автобалансир використовується для збудження двочастотних вібрацій.

Із застосуванням рівнянь Лагранжа II роду виведені диференціальні рівняння руху вібромашини. Було встановлено, що в разі кульового і роликового автобалансира диференціальні рівняння руху вібромашини подібні (збігаються з точністю до позначень), а в разі маятникового автобаласира - відрізняються за формою.

Диференціальні рівняння руху вібромашини записані для випадку однакових вантажів.

Побудовані моделі застосовні як для аналітичного дослідження динаміки відповідних вібромашин, так і для проведення обчислювальних експериментів.

В аналітичних дослідженнях моделі призначені для пошуку усталених режимів руху вібромашини, визначення умови їх існування і стійкості

Ключові слова: інерційний віброзбудник, двочастотні вібрації, резонансна вібромашина, автобалансир, інерційна вібромашина

UDC 622.74: 621.928.235
DOI: 10.15587/1729-4061.2018.150339

MOTION EQUATIONS OF THE SINGLE-MASS VIBRATORY MACHINE WITH A ROTARY-OSCILLATORY MOTION OF THE PLATFORM AND A VIBRATION EXCITER IN THE FORM OF A PASSIVE AUTO-BALANCER

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1. Introduction

Among such vibratory machines as screeners, vibratory tables, vibratory conveyers, vibratory mills, etc., the multi-frequency- and resonance machines are very promising.

Multi-frequency vibratory machines demonstrate better performance [1] while resonance vibratory machines are the most energy-efficient [2]. That is why it is a relevant task to design such vibratory machines that would combine the advantages of multi-frequency and resonance vibratory machines [3].

The most promising techniques for exciting resonance vibrations are based on the phenomenon of a parametric resonance [4–8] or the Sommerfeld effect [9–23]. The resonance mode in such vibratory machines is resistant to a change in the mass of a vibration platform.

To induce resonance vibrations, it was proposed to apply passive auto-balancers of the ball-, roller-, pendulum-types [12]. Work of the method is based on the Sommerfeld effect [9]: under certain conditions, loads in an auto-balancer cannot catch up with the rotor and get stuck at the resonance frequency of platform oscillations, exciting thereby resonance oscillations [12–23]. Fixing the unbalanced mass at an auto-balancer's casing makes it possible, in this case, to induce faster vibrations – at the rotor frequency [12, 19–23].

The feasibility of the technique has not been theoretically investigated up to now for the case when the platform is set into rotary-oscillatory motion. The necessary stage of theoretical research is to describe a mechanical model of the vibratory machine and to derive differential equations of motion.

2. Literature review and problem statement

There are several techniques to excite resonance oscillations. The simplest one is inertial, under which the unbalanced mass rotates at the resonance frequency of platform oscillations [2]. A drawback of the method is that at a change in the mass of a platform the vibratory machine significantly deviates from the resonance mode of operation.

A common current technique is based on the application of electromechanical vibration exciters [3]. However, such vibration exciters require a sophisticated control system in order to respond to a current change in the mass and resonance frequency of platform oscillations.

Less sensitive to a change in the mass of a vibration platform is the technique for exciting resonance oscillations of the platform based on a parametric resonance phenomenon. A vibration exciter consists of one or more pendulums [4], or several rolling bodies (balls, rollers), placed in circular sectors [5]. The pendulums are mounted onto the axis, parallel to the longitudinal axis of the rotor, while centers of the circular sectors are at a certain distance from the longitudinal axis of the rotor. The technique is applicable for the single-mass [4–7] and [8] multi-mass vibratory machines, for the case of vibratory platforms on the isotropic [4, 6] and anisotropic [5, 7, 8] supports. The technique mostly exploits the main parametric resonance [7]. And though the system adjusts to a change in the mass of a platform, the range of shaft rotation speed, at which the parametric resonance is achieved, is small.

There are techniques to excite resonance oscillations of the vibratory platform that employ the Sommerfeld effect [9].

In [10], a pendulum is mounted onto the electric motor shaft. The rated shaft rotation frequency is slightly less than the resonance frequency of the vibratory platform oscillations. Owing to the Sommerfeld effect, the pendulum gets stuck at the resonance frequency of platform oscillations. The disadvantage of the method is that the electrical circuit of the motor in this case is overloaded. In [11], instead of a small engine, a wind wheel is applied, combined with an unbalanced mass. The wheel is fed a stream of compressed air. The wheel gradually accelerates to the resonance frequency of platform oscillations. The shortcoming of the method is the low performance coefficient of the system "compressed air – wind wheel – vibratory platform".

Paper [12] proposed to induce dual-frequency resonance vibrations using a passive auto-balancer. The technique is based on the same Sommerfeld effect that is employed in studies [10, 11]. Only, instead of a small engine, the authors exploit small forces of viscous resistance to the relative motion of loads (pendulums, balls, rollers). Additionally, as a second vibration exciter, the unbalanced mass at an auto-balancer's casing is applied. It should be noted that the small resistance forces initially accelerate the loads. The energy from the motor, in a much larger amount, is then transferred to the loads, similar to the way a gymnast twists hoops around the body.

Hypothetically, it is assumed that the vibration exciter in the form of a passive auto-balancer is applicable for single-, two-, three-mass vibratory machines with a different motion kinematics of vibratory platforms [12].

In this regard, there is an issue to substantiate the feasibility of a new technique to excite the two-frequency vibrations.

We shall consider several studies whose results can be indirectly employed in order to substantiate the technique.

A phenomenon of the loads that get stuck in an auto-balancer, which is undesirable, was theoretically investigated when studying the process of rotor balancing by the ball-type and pendular-type auto-balancers [13–18]. The most significant, new theoretical results were reported in:

[13] – within a spatial model of the rotor on isotropic supports, balanced by the pendulum-type auto-balancer;

[14] – within a flat model of the rotor on isotropic supports, balanced by the ball-type auto-balancer;

[15] – within a spatial model of the rotor on isotropic supports, balanced by the ball-type auto-balancer;

[16] – within a spatial model of the rotor on isotropic supports, balanced by two dual-pendulum auto-balancers;

[17] – within a flat model of the rotor on anisotropic supports, balanced by the ball-type auto-balancer.

These models lack the platform, there is only the rotor, mounted on flexible supports. That is why results from these studies are not directly applicable to substantiate the efficiency of the new method of vibration excitation. The papers found that balls or pendulums may get stuck at one of the resonance velocities of rotor rotation. This suggests that in the presence of platforms the loads can get stuck at the resonance frequencies of platform oscillations.

Similarly, in studies [13–16], the rotor is placed on isotropic supports, and in [17] – on anisotropic. However, the results obtained do not provide an answer to the question of whether the loads may get stuck given the high anisotropy of supports (rigidity in one of the directions tends to infinity).

Paper [18] investigated the balls that get stuck within a flat model of the rotor with the ball-type auto-balancer, in which the rotor is mounted on isotropic supports, which rest against a movable foundation. The research results could be applied to substantiate the feasibility of the technique when vibrations are induced in the dual-mass vibratory machine. To this end, it is necessary to treat the movable foundation as a vibratory platform.

Theoretically, the feasibility of the new technique was investigated for the single-mass vibratory machine with a rectilinear translational motion of the platform [19–21]. The following stages of research were adopted:

– equations of motion were derived [19];

– possible frequencies at which loads get stuck and respective motion modes were identified [20];

– the stability of motion modes [21] defined in [20] was examined.

In accordance with the results reported in papers [19–21]:

– differential equations of motion (for the single-mass, dual-mass, or three-mass) of the vibratory machine are reduced to the form independent of the type of an auto-balancer;

– loads in the auto-balancer get stuck at the resonance frequency of platform oscillations;

– in this case, despite the high anisotropy of supports, there occur the almost perfect dual-frequency vibrations of the platform.

The feasibility of the technique for the case of a single-mass vibratory machine with a rotary-oscillatory motion of the platform was tested by a 3D simulation of the dynamics of a vibratory machine [22] and when performing a field experiment [23]. The results obtained demonstrate that:

– the new technique of vibration excitation is feasible;

– despite the strong asymmetry of supports, the auto-balancer excites the almost perfect dual-frequency vibrations.

However, the design of such vibratory machines requires a theoretical investigation into their dynamics, as is the case in [19–21]. The necessary stage of such a research is the construction of a mechanical-mathematical model of the vibratory machine.

3. The aim and objectives of the study

The aim of this study is to construct mechanical-mathematical models of the single-mass vibratory machine with rotary-oscillatory motion of the platform and a vibration exciter in the form of a passive auto-balancer.

To accomplish the aim, the following tasks have been set:

– to describe a mechanical model of the single-mass vibratory machine with rotary-oscillatory motion of the platform and a vibration exciter in the form of a passive auto-balancer of the ball-, roller-, or pendulum-type;

– to derive differential equations of motion.

4. Methods for constructing mechanical-mathematical models of the single-mass vibratory machine with a rotary-oscillatory motion of the vibratory platform

In order to build mechanical models of the vibratory machines, provisions from the theory of vibratory machines [2, 3] are applied, as well as provisions from the theory of rotary machines with passive auto-balancers [14].

To derive the differential equations of motion, the Lagrangian equations of the second kind in the following form are employed [24]:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j}-\frac{\partial T}{\partial q_j}+\frac{\partial V}{\partial q_j}+\frac{\partial D}{\partial \dot{q}_j}=0, \quad /j=\overline{1,f}/, \qquad (1)$$

where t is the time; T is the kinetic, V is the potential energy of the system; D is the dissipative function; q_j is the generalized coordinate, $\dot{q}_j$ is the generalized velocity number j; f is the number of degrees of freedom for a mechanical system.

5. Results of construction of mechanical-mathematical models of the single-mass vibratory machine with a rotary-oscillatory motion of the vibratory platform

5. 1. Description of a mechanical model of the vibratory machine

A vibratory machine (Fig. 1, *a*) has a vibratory platform of mass M. The vibration platform rests on an elastic-viscous support with a coefficient of rigidity k_p and a viscosity coefficient b_p. The vibratory platform can rotate around a fixed axis that passes point O. The angle of rotation is denoted through ψ (Fig. 1, *b*). The platform hosts a vibration exciter in the form of the ball-, roller- (Fig. 1, *c*) or pendulum-type (Fig. 1, *d*) auto-balancer. An unbalanced mass of weight m_0 is attached to the casing of the auto-balancer. The distance from the center of mass of the unbalanced load to the center of the auto-balancer's casing (point K) is R_0. The auto-balancer's casing rotates relative to the platform at a constant angular rotation velocity ω. Position of the unbalanced mass relative to the platform is defined by angle ωt, where t is the time.

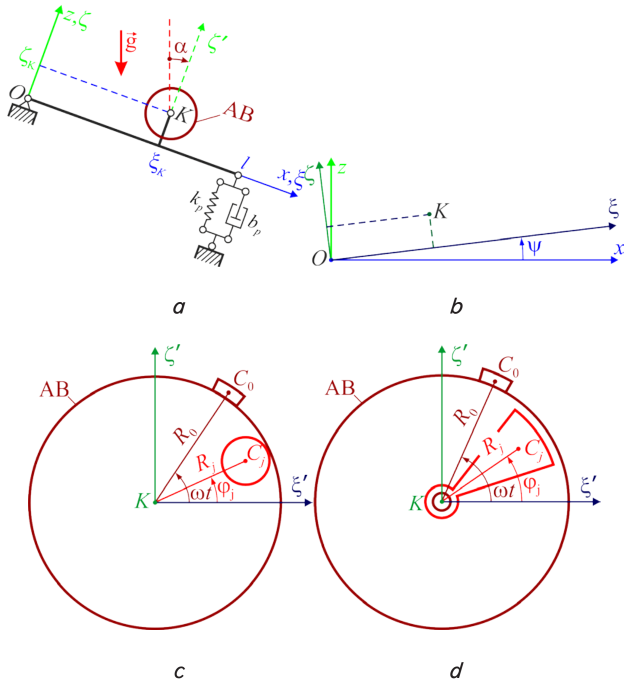

Fig. 1. Mechanical model of the single-mass vibratory machine with a rotary-oscillatory motion of the platform and a vibration exciter in the form of a passive auto-balancer: *a* — circuit of the vibratory machine; *b* — kinematics of the vibratory platform motion; *c* — kinematics of the unbalanced mass and balls (rollers) motion; *d* — motion kinematics of the unbalanced mass and pendulums

The passive auto-balancer consists of N loads. Load number j has mass m_j; its center of mass (point C_j) is at a distance R_j from the center of the auto-balancer's casing. Position of the load relative to the platform is defined by angle $/j=\overline{1,N}/$.

The central moment of inertia of the load about axis (including the unbalanced mass)

$$I_{C,j}=m_j\rho_j^2, \quad /j=\overline{0,N}/, \qquad (2)$$

where ρ_j is the radius of gyration of the load.

In turn, for a ball or a roller of radius r_j, respectively:

$$\rho_j=\sqrt{\frac{2}{5}}r_j,$$

$$\rho_j=\sqrt{\frac{1}{2}}r_j, \quad /j=\overline{1,N}/. \qquad (3)$$

Balls or rollers roll along the rolling tracks without interruption and slip.

The angular velocity of rotation of the pendulum or the unbalanced load around the center of mass

$$\omega_j=\dot{\psi}+\dot{\varphi}_j, \quad /j=\overline{0,N}/. \qquad (4)$$

The angular velocity of rotation of a ball or a roller around the center of mass

$$\omega_j = \dot{\psi} + \dot{\varphi}_j + \frac{R_j + r_j}{r_j}(\omega - \dot{\varphi}_j) =$$
$$= \dot{\psi} + \omega\left(\frac{R_j}{r_j} + 1\right) - \dot{\varphi}_j \frac{R_j}{r_j}, \qquad / j = \overline{1,N} /. \tag{5}$$

Note that motion of the system is described by generalized coordinates ψ, φ_j, $/ j = \overline{1,N} /$ and it has f=N+1 degrees of freedom.

Hereafter, for the sake of brevity, we shall term a differential equation of motion, derived from the Lagrangian equations of the second kind for the generalized coordinate ψ, the motion equation of the platform. Similarly, the equation that is obtained for the generalized coordinate φ_j shall be termed the motion equation of the load (number j).

5. 2. Derivation of the differential motion equations of the vibratory machine

5. 2. 1. Kinetic energy of the system

The kinetic energy of the platform and the auto-balancer's casing

$$T_O = \frac{1}{2} I_O \dot{\psi}^2, \tag{6}$$

where I_O is the moment of inertia of the platform, together with the auto-balancer's casing (without the unbalanced load on the casing) about axis.

Coordinates of the center of mass of the load (including the unbalanced) relative to the movable axes ξ, ζ:

$$\xi_{C,j} = \xi_K + R_j \cos\varphi_j, \;\; \zeta_{C,j} = \zeta_K + R_j \sin\varphi_j, \;\; / j = \overline{0,N} /, \tag{7}$$

where

$$\varphi_0 = \omega t, \;\; \dot{\varphi}_0 = \omega, \;\; \ddot{\varphi}_0 = 0. \tag{8}$$

Coordinates of the center of mass of the load relative to the fixed axes x, z

$$x_{C,j} = \xi_{C,j}\cos\psi - \zeta_{C,j}\sin\psi =$$
$$= (\xi_K + R_j\cos\varphi_j)\cos\psi - (\zeta_K + R_j \sin\varphi_j)\sin\psi,$$

$$z_{C,j} = \xi_{C,j}\sin\psi + \zeta_{C,j}\cos\psi =$$
$$= (\xi_K + R_j\cos\varphi_j)\sin\psi + (\zeta_K + R_j\sin\varphi_j)\cos\psi,$$

$$/ j = \overline{0,N} /. \tag{9}$$

Projections of velocity of the center of mass of the load upon the fixed axes x, z:

$$\dot{x}_{C,j} = \dot{\xi}_{C,j}\cos\psi - \xi_{C,j}\dot{\psi}\sin\psi -$$
$$-\dot{\zeta}_{C,j}\sin\psi - \zeta_{C,j}\dot{\psi}\cos\psi,$$

$$\dot{z}_{C,j} = \dot{\xi}_{C,j}\sin\psi + \xi_{C,j}\dot{\psi}\cos\psi +$$
$$+\dot{\zeta}_{C,j}\cos\psi - \zeta_{C,j}\dot{\psi}\sin\psi, \qquad / j = \overline{0,N} /,$$

where

$$\dot{\xi}_{C,j} = -R_j\dot{\varphi}_j\sin\varphi_j, \;\; \dot{\zeta}_{C,j} = R_j\dot{\varphi}_j\cos\varphi_j, \;\; / j = \overline{0,N} /.$$

In the expanded form

$$\dot{x}_{C,j} = -R_j\dot{\varphi}_j\sin\varphi_j\cos\psi - (\xi_K + R_j\cos\varphi_j)\dot{\psi}\sin\psi -$$
$$-R_j\dot{\varphi}_j\cos\varphi_j\sin\psi - (\zeta_K + R_j\sin\varphi_j)\dot{\psi}\cos\psi,$$

$$\dot{z}_{C,j} = -R_j\dot{\varphi}_j\sin\varphi_j\sin\psi + (\xi_K + R_j\cos\varphi_j)\dot{\psi}\cos\psi +$$
$$+R_j\dot{\varphi}_j\cos\varphi_j\cos\psi - (\zeta_K + R_j\sin\varphi_j)\dot{\psi}\sin\psi,$$

$$/ j = \overline{0,N} /.$$

Square of velocity of the center of mass of the load:

$$v_{C,j}^2 = \dot{x}_{C,j}^2 + \dot{z}_{C,j}^2 = R_j^2(\dot{\psi} + \dot{\varphi}_j)^2 + d_K^2\dot{\psi}^2 +$$
$$+2R_j\dot{\psi}(\dot{\psi} + \dot{\varphi}_j)(\xi_K\cos\varphi_j + \zeta_K\sin\varphi_j),$$

$$/ j = \overline{0,N} /, \tag{10}$$

where

$$d_K = \sqrt{\xi_K^2 + \zeta_K^2}$$

is the distance from the axis of rotation of the platform to point K.

According to the Koenig theorem [24], kinetic energy of the load (including the unbalanced):

$$T_j = \frac{1}{2} m_j v_{C,j}^2 + \frac{1}{2} I_{C,j}\omega_j^2, \;\; / j = \overline{0,N} /, \tag{11}$$

where ω_j is the angular velocity of rotation of the load around the center of mass.

Given (2), (4), (10), kinetic energy of the pendulum or unbalanced load

$$T_j = \frac{1}{2} m_j [R_j^2(\dot{\psi} + \dot{\varphi}_j)^2 + d_K^2\dot{\psi}^2 +$$
$$+2R_j\dot{\psi}(\dot{\psi} + \dot{\varphi}_j)(\xi_K\cos\varphi_j + \zeta_K\sin\varphi_j)] +$$
$$+\frac{1}{2} m_j \rho_j^2(\dot{\psi} + \dot{\varphi}_j)^2 = \frac{1}{2} m_j [d_K^2\dot{\psi}^2 + 2R_j\dot{\psi}(\dot{\psi} + \dot{\varphi}_j) \times$$
$$\times(\xi_K\cos\varphi_j + \zeta_K\sin\varphi_j) + (\rho_j^2 + R_j^2)(\dot{\psi} + \dot{\varphi}_j)^2]. \tag{12}$$

Given (2), (5), (10), kinetic energy of the ball or roller:

$$T_j = \frac{1}{2} m_j [R_j^2(\dot{\psi} + \dot{\varphi}_j)^2 + d_K^2\dot{\psi}^2 +$$
$$+2R_j\dot{\psi}(\dot{\psi} + \dot{\varphi}_j)(\xi_K\cos\varphi_j + \zeta_K\sin\varphi_j)] +$$
$$+\frac{1}{2} m_j \rho_j^2 \left[\dot{\psi} + \omega\left(\frac{R_j}{r_j} + 1\right) - \dot{\varphi}_j\frac{R_j}{r_j}\right]^2 =$$
$$= \frac{1}{2} m_j \left\{ \begin{array}{l} R_j^2(\dot{\psi} + \dot{\varphi}_j)^2 + d_K^2\dot{\psi}^2 + 2R_j\dot{\psi}(\dot{\psi} + \dot{\varphi}_j) \times \\ \times(\xi_K\cos\varphi_j + \zeta_K\sin\varphi_j) + \\ +\rho_j^2\left[\dot{\psi} + \omega\left(\frac{R_j}{r_j} + 1\right) - \dot{\varphi}_j\frac{R_j}{r_j}\right]^2 \end{array} \right\},$$

$$/ j = \overline{1,N} /. \tag{13}$$

Kinetic energy of the system:

– for the case of a pendulum-type auto-balancer

$$T = T_P + \sum_{j=1}^{N} T_j + T_0 = \frac{1}{2} I_O \dot{\psi}^2 +$$
$$+\frac{1}{2}\sum_{j=1}^{N} m_j [d_K^2 \dot{\psi}^2 + 2R_j \dot{\psi}(\dot{\psi} + \dot{\varphi}_j)(\xi_K \cos\varphi_j + \zeta_K \sin\varphi_j) +$$
$$+(\rho_j^2 + R_j^2)(\dot{\psi} + \dot{\varphi}_j)^2] + \frac{1}{2} m_0 [d_K^2 \dot{\psi}^2 + 2R_0 \dot{\psi}(\dot{\psi} + \omega) \times$$
$$\times(\xi_K \cos\omega t + \zeta_K \sin\omega t) + (\rho_0^2 + R_0^2)(\dot{\psi} + \omega)^2]; \quad (14)$$

– for the case of a ball- or a roller-type auto-balancer

$$T = T_P + \sum_{j=1}^{N} T_j + T_0 =$$
$$= \frac{1}{2} I_O \dot{\psi}^2 + \frac{1}{2}\sum_{j=1}^{N} m_j \left\{ \begin{array}{l} R_j^2 (\dot{\psi} + \dot{\varphi}_j)^2 + d_K^2 \dot{\psi}^2 + \\ +2R_j \dot{\psi}(\dot{\psi} + \dot{\varphi}_j)(\xi_K \cos\varphi_j + \zeta_K \sin\varphi_j) + \\ +\rho_j^2 \left[\dot{\psi} + \omega\left(\dfrac{R_j}{r_j} + 1 \right) - \dot{\varphi}_j \dfrac{R_j}{r_j} \right]^2 \end{array} \right\} +$$
$$+\frac{1}{2} m_0 [d_K^2 \dot{\psi}^2 + 2R_0 \dot{\psi}(\dot{\psi} + \omega)(\xi_K \cos\omega t + \zeta_K \sin\omega t) +$$
$$+(\rho_0^2 + R_0^2)(\dot{\psi} + \omega)^2]. \quad (15)$$

5. 2. 2. Potential energy and dissipative function of the system

Potential energy that corresponds to the platform:

$$V_p = k_p \frac{l^2 \psi^2}{2} + Mgh_C = k_p \frac{l^2 \psi^2}{2} +$$
$$+Mg[\xi_C \sin(\psi - \alpha) + \zeta_C \cos(\psi - \alpha)].$$

Potential energy that corresponds to load number j:

$$V_j = m_j g h_{C,j} = m_j g[(\xi_K + R_j \cos\varphi_j)\sin(\psi - \alpha) +$$
$$+(\zeta_K + R_j \sin\varphi_j)\cos(\psi - \alpha)].$$

Potential energy of the system

$$V = V_p + \sum_{j=1}^{N} V_j + V_0 = k_p \frac{l^2 \psi^2}{2} +$$
$$+Mg[\xi_C \sin(\psi - \alpha) + \zeta_C \cos(\psi - \alpha)] +$$
$$+\sum_{j=1}^{N} m_j g[(\xi_K + R_j \cos\varphi_j)\sin(\psi - \alpha) +$$
$$+(\zeta_K + R_j \sin\varphi_j)\cos(\psi - \alpha)] +$$
$$+m_0 g[(\xi_K + R_0 \cos\omega t)\sin(\psi - \alpha) +$$
$$+(\zeta_K + R_0 \sin\omega t)\cos(\psi - \alpha)]. \quad (16)$$

Dissipative function that corresponds to the platform

$$D_p = \frac{1}{2} b_p v_L^2 = \frac{1}{2} b_p l^2 \dot{\psi}^2.$$

Dissipative function that corresponds to load number j:

$$D_j = \frac{1}{2} b_j \left(v_j^{(r)} \right)^2 = \frac{1}{2} b_j R_j^2 (\dot{\varphi}_j - \omega)^2,$$

$$/\, j = \overline{1,N}\, /.$$

Dissipative function of the system

$$D = D_p + \sum_{j=1}^{N} D_j =$$
$$= \frac{1}{2} b_p l^2 \dot{\psi}^2 + \frac{1}{2}\sum_{j=1}^{N} b_j R_j^2 (\dot{\varphi}_j - \omega)^2,$$

$$/\, j = \overline{1,N}\, /. \quad (17)$$

5. 2. 3. Differential motion equation of the platform

Components generated by the kinetic energy in the motion equation of the platform

For the case of a pendulum-type auto-balancer:

$$\frac{\partial T}{\partial \dot{\psi}} = I_O \dot{\psi} + \sum_{j=1}^{N} m_j [d_K^2 \dot{\psi} + R_j (2\dot{\psi} + \dot{\varphi}_j) \times$$
$$\times(\xi_K \cos\varphi_j + \zeta_K \sin\varphi_j) +$$
$$+(\rho_j^2 + R_j^2)(\dot{\psi} + \dot{\varphi}_j)] +$$
$$+m_0 [d_K^2 \dot{\psi} + R_0 (2\dot{\psi} + \omega) \times$$
$$\times(\xi_K \cos\omega t + \zeta_K \sin\omega t) +$$
$$+(\rho_0^2 + R_0^2)(\dot{\psi} + \omega)] =$$
$$= \left\{ I_O + \sum_{j=1}^{N} \begin{array}{l} m_j [d_K^2 + R_j^2 + \rho_j^2 + \\ +2R_j (\xi_K \cos\varphi_j + \zeta_K \sin\varphi_j)] \end{array} \right\} \dot{\psi} +$$
$$+\sum_{j=1}^{N} m_j [R_j^2 + \rho_j^2 + R_j (\xi_K \cos\varphi_j + \zeta_K \sin\varphi_j)] \dot{\varphi}_j +$$
$$+m_0 [d_K^2 + R_0^2 + \rho_0^2 +$$
$$+2R_0 (\xi_K \cos\omega t + \zeta_K \sin\omega t)] \dot{\psi} +$$
$$+m_0 [R_0^2 + \rho_0^2 + R_0 (\xi_K \cos\omega t + \zeta_K \sin\omega t)] \omega;$$

$$\frac{\partial T}{\partial \psi} = 0;$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{\psi}} - \frac{\partial T}{\partial \psi} = \frac{d}{dt}\frac{\partial T}{\partial \dot{\psi}} =$$
$$= \left\{ I_O + \sum_{j=1}^{N} \begin{array}{l} m_j [d_K^2 + R_j^2 + \rho_j^2 + \\ +2R_j (\xi_K \cos\varphi_j + \zeta_K \sin\varphi_j)] \end{array} \right\} \ddot{\psi} +$$
$$+2\dot{\psi} \sum_{j=1}^{N} m_j R_j (-\xi_K \sin\varphi_j + \zeta_K \cos\varphi_j) \dot{\varphi}_j +$$
$$+\sum_{j=1}^{N} m_j [R_j^2 + \rho_j^2 + R_j (\xi_K \cos\varphi_j + \zeta_K \sin\varphi_j)] \ddot{\varphi}_j +$$
$$+\sum_{j=1}^{N} m_j R_j (-\xi_K \sin\varphi_j + \zeta_K \cos\varphi_j) \dot{\varphi}_j^2 +$$
$$+m_0 [d_K^2 + R_0^2 + \rho_0^2 +$$
$$+2R_0 (\xi_K \cos\omega t + \zeta_K \sin\omega t)] \ddot{\psi} +$$
$$+2\dot{\psi} m_0 R_0 (-\xi_K \sin\omega t + \zeta_K \cos\omega t) \omega +$$
$$+m_0 R_0 (-\xi_K \sin\omega t + \zeta_K \cos\omega t) \omega^2. \quad (18)$$

For the case of a ball- or a roller-type auto-balancer:

$$\frac{\partial T}{\partial \dot\psi} = I_O\dot\psi +$$
$$+\sum_{j=1}^{N} m_j \left\{ \begin{array}{l} R_j^2(\dot\psi+\dot\varphi_j)+d_K^2\dot\psi+ \\ +R_j(2\dot\psi+\dot\varphi_j)(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)+ \\ +\rho_j^2\left[\dot\psi+\omega\left(\dfrac{R_j}{r_j}+1\right)-\dot\varphi_j\dfrac{R_j}{r_j}\right] \end{array} \right\}+$$
$$+m_0[d_K^2\dot\psi+R_0(2\dot\psi+\omega)(\xi_K\cos\omega t+\zeta_K\sin\omega t)+$$
$$+(\rho_0^2+R_0^2)(\dot\psi+\omega)]=$$
$$=\left\{I_O+\sum_{j=1}^{N}\begin{array}{l} m_j[d_K^2+R_j^2+\rho_j^2+ \\ +2R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\end{array}\right\}\dot\psi+$$
$$+\sum_{j=1}^{N} m_j\left[R_j^2-\rho_j^2\frac{R_j}{r_j}+R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)\right]\dot\varphi_j+$$
$$+m_0[d_K^2+R_0^2+\rho_0^2+2R_0(\xi_K\cos\omega t+\zeta_K\sin\omega t)]\dot\psi+$$
$$+m_0R_0\omega(\xi_K\cos\omega t+\zeta_K\sin\omega t)+$$
$$+m_0(R_0^2+\rho_0^2)\omega+\omega\sum_{j=1}^{N}m_j\rho_j^2\left(\frac{R_j}{r_j}+1\right).$$

$$\frac{\partial T}{\partial \psi}=0;$$

$$\frac{d}{dt}\frac{\partial T}{\partial\dot\psi}-\frac{\partial T}{\partial\psi}=\frac{d}{dt}\frac{\partial T}{\partial\dot\psi}=$$
$$=\left\{I_O+\sum_{j=1}^{N}\begin{array}{l} m_j[d_K^2+R_j^2+\rho_j^2+ \\ +2R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\end{array}\right\}\ddot\psi+$$
$$+2\dot\psi\sum_{j=1}^{N}m_jR_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\varphi_j+$$
$$+\sum_{j=1}^{N} m_j\left[R_j^2-\rho_j^2\frac{R_j}{r_j}+R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)\right]\ddot\varphi_j+$$
$$+\sum_{j=1}^{N}m_jR_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\varphi_j^2+$$
$$+m_0[d_K^2+R_0^2+\rho_0^2+2R_0(\xi_K\cos\omega t+\zeta_K\sin\omega t)]\ddot\psi+$$
$$+2\dot\psi m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega+$$
$$+m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega^2. \quad (19)$$

Components generated by the potential energy and dissipative function in the motion equation of the platform:

$$\frac{\partial D}{\partial\dot\psi}+\frac{\partial V}{\partial\psi}=b_pl^2\dot\psi+k_pl^2\psi+$$
$$+Mg[\xi_C\cos(\psi-\alpha)-\zeta_C\sin(\psi-\alpha)]+$$
$$+\sum_{j=1}^{N}m_jg[(\xi_K+R_j\cos\varphi_j)\cos(\psi-\alpha)-$$
$$-(\zeta_K+R_j\sin\varphi_j)\sin(\psi-\alpha)]+$$
$$+m_0g[(\xi_K+R_0\cos\omega t)\cos(\psi-\alpha)-$$
$$-(\zeta_K+R_0\sin\omega t)\sin(\psi-\alpha)]. \quad (20)$$

Differential motion equation of the platform:

– for the case of a pendulum-type auto-balancer

$$\left\{I_O+\sum_{j=1}^{N}m_j[d_K^2+R_j^2+\rho_j^2+2R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\right\}\ddot\psi+$$
$$+2\dot\psi\sum_{j=1}^{N}m_jR_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\varphi_j+$$
$$+\sum_{j=1}^{N}m_j[R_j^2+\rho_j^2+R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\ddot\varphi_j+$$
$$+\sum_{j=1}^{N}m_jR_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\varphi_j^2+$$
$$+m_0[d_K^2+R_0^2+\rho_0^2+2R_0(\xi_K\cos\omega t+\zeta_K\sin\omega t)]\ddot\psi+$$
$$+2\dot\psi m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega+$$
$$+m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega^2+b_pl^2\dot\psi+$$
$$+k_pl^2\psi+Mg[\xi_C\cos(\psi-\alpha)-\zeta_C\sin(\psi-\alpha)]+$$
$$+\sum_{j=1}^{N}m_jg[(\xi_K+R_j\cos\varphi_j)\cos(\psi-\alpha)-$$
$$-(\zeta_K+R_j\sin\varphi_j)\sin(\psi-\alpha)]+$$
$$+m_0g[(\xi_K+R_0\cos\omega t)\cos(\psi-\alpha)-$$
$$-(\zeta_K+R_0\sin\omega t)\sin(\psi-\alpha)]; \quad (21)$$

– for the case of a ball- or a roller-type auto-balancer

$$\left\{I_O+\sum_{j=1}^{N}m_j[d_K^2+R_j^2+\rho_j^2+2R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\right\}\ddot\psi+$$
$$+2\dot\psi\sum_{j=1}^{N}m_jR_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\varphi_j+$$
$$+\sum_{j=1}^{N} m_j\left[R_j^2-\rho_j^2\frac{R_j}{r_j}+R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)\right]\ddot\varphi_j+$$
$$+\sum_{j=1}^{N}m_jR_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\varphi_j^2+$$
$$+m_0[d_K^2+R_0^2+\rho_0^2+2R_0(\xi_K\cos\omega t+\zeta_K\sin\omega t)]\ddot\psi+$$
$$+2\dot\psi m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega+$$
$$+m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega^2+b_pl^2\dot\psi+k_pl^2\psi+$$
$$+Mg[\xi_C\cos(\psi-\alpha)-\zeta_C\sin(\psi-\alpha)]+$$
$$+\sum_{j=1}^{N}m_jg[(\xi_K+R_j\cos\varphi_j)\cos(\psi-\alpha)-$$
$$-(\zeta_K+R_j\sin\varphi_j)\sin(\psi-\alpha)]+$$
$$+m_0g[(\xi_K+R_0\cos\omega t)\cos(\psi-\alpha)-$$
$$-(\zeta_K+R_0\sin\omega t)\sin(\psi-\alpha)]. \quad (22)$$

5. 2. 4. Differential motion equations of the load

Components generated by the kinetic energy for pendulum number *j*:

$$\frac{\partial T}{\partial\dot\varphi_j}=m_j[R_j\dot\psi(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)+$$
$$+(\rho_j^2+R_j^2)(\dot\psi+\dot\varphi_j)]=$$
$$=m_j\{(\rho_j^2+R_j^2)\dot\varphi_j+$$
$$+[\rho_j^2+R_j^2+R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\dot\psi\};$$

$$\frac{d}{dt}\frac{\partial T}{\partial\dot\varphi_j}=m_j\{(\rho_j^2+R_j^2)\ddot\varphi_j+[\rho_j^2+R_j^2+R_j(\xi_K\cos\varphi_j+$$
$$+\zeta_K\sin\varphi_j)]\ddot\psi+R_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\psi\dot\varphi_j\};$$

$$\frac{\partial T}{\partial\varphi_j}=m_jR_j\dot\psi(\dot\psi+\dot\varphi_j)(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)=$$
$$=m_j[R_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\psi^2+$$
$$+R_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\psi\dot\varphi_j];$$

$$\frac{d}{dt}\frac{\partial T}{\partial\dot\varphi_j}-\frac{\partial T}{\partial\varphi_j}=m_j\{(\rho_j^2+R_j^2)\ddot\varphi_j+[\rho_j^2+R_j^2+$$
$$+R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\ddot\psi-R_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\psi^2\}. \quad (23)$$

Components generated by the kinetic energy for ball or roller number j:

$$\frac{\partial T}{\partial\dot\varphi_j}=m_j\left\{\begin{array}{l}R_j^2(\dot\psi+\dot\varphi_j)+\\+R_j\dot\psi(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)-\\-\dfrac{R_j}{r_j}\rho_j^2\left[\dot\psi+\omega\left(\dfrac{R_j}{r_j}+1\right)-\dot\varphi_j\dfrac{R_j}{r_j}\right]\end{array}\right\}=$$
$$=m_jR_j\left\{\begin{array}{l}R_j\left(1+\dfrac{\rho_j^2}{r_j^2}\right)\dot\varphi_j+\\+\left[R_j-\dfrac{\rho_j^2}{r_j}+(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)\right]\dot\psi-\\-\dfrac{\rho_j^2}{r_j}\left(\dfrac{R_j}{r_j}+1\right)\omega\end{array}\right\};$$

$$\frac{d}{dt}\frac{\partial T}{\partial\dot\varphi_j}=m_jR_j\left\{\begin{array}{l}R_j\left(1+\dfrac{\rho_j^2}{r_j^2}\right)\ddot\varphi_j+\\+\left[R_j-\dfrac{\rho_j^2}{r_j}+(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)\right]\ddot\psi+\\+(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\psi\dot\varphi_j\end{array}\right\};$$

$$\frac{\partial T}{\partial\varphi_j}=m_jR_j\dot\psi(\dot\psi+\dot\varphi_j)(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)=$$
$$=m_jR_j[(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\psi^2+$$
$$+(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\psi\dot\varphi_j];$$

$$\frac{d}{dt}\frac{\partial T}{\partial\dot\varphi_j}-\frac{\partial T}{\partial\varphi_j}=$$
$$=m_jR_j\left\{\begin{array}{l}R_j\left(1+\dfrac{\rho_j^2}{r_j^2}\right)\ddot\varphi_j+\\+\left[R_j-\dfrac{\rho_j^2}{r_j}+(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)\right]\ddot\psi-\\-(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\psi^2\end{array}\right\}. \quad (24)$$

Components generated by the potential energy and dissipative function for load number j:

$$\frac{\partial D}{\partial\dot\varphi_j}+\frac{\partial V}{\partial\varphi_j}=b_jR_j^2(\dot\varphi_j-\omega)+m_jgR_j[-\sin(\psi-\alpha)\sin\varphi_j+$$
$$+\cos(\psi-\alpha)\cos\varphi_j]. \quad (25)$$

Differential motion equations:

– for pendulums

$$m_j\{(\rho_j^2+R_j^2)\ddot\varphi_j+[\rho_j^2+R_j^2+R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\ddot\psi-$$
$$-R_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\psi^2\}+$$
$$+b_jR_j^2(\dot\varphi_j-\omega)+m_jgR_j[-\sin(\psi-\alpha)\sin\varphi_j+$$
$$+\cos(\psi-\alpha)\cos\varphi_j]=0,\ /\ j=\overline{1,N}/; \quad (26)$$

– for rollers or balls

$$m_jR_j\left\{\begin{array}{l}R_j\left(1+\dfrac{\rho_j^2}{r_j^2}\right)\ddot\varphi_j+\\+\left[R_j-\dfrac{\rho_j^2}{r_j}+(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)\right]\ddot\psi-\\-(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\psi^2\end{array}\right\}+$$
$$+b_jR_j^2(\dot\varphi_j-\omega)+m_jgR_j[-\sin(\psi-\alpha)\sin\varphi_j+$$
$$+\cos(\psi-\alpha)\cos\varphi_j]=0,$$

$$/\ j=\overline{1,N}\ /. \quad (27)$$

5. 3. Differential motion equations of the vibratory machine in the general and particular cases

5. 3. 1. General case

We shall place, in the differential motion equations, the unknown components to the left part, and the known ones – to the right part. We obtain the following differential motion equations of the vibratory machine:

– for the case of a pendulum-type auto-balancer

$$\left\{I_O+\sum_{j=1}^{N}m_j[d_K^2+R_j^2+\rho_j^2+2R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\right\}\ddot\psi+$$
$$+b_pl^2\dot\psi+k_pl^2\psi+2\dot\psi\sum_{j=1}^{N}m_jR_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\varphi_j+$$
$$+\sum_{j=1}^{N}m_j[R_j^2+\rho_j^2+R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\ddot\varphi_j+$$
$$+\sum_{j=1}^{N}m_jR_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot\varphi_j^2+$$
$$+m_0[d_K^2+R_0^2+\rho_0^2+2R_0(\xi_K\cos\omega t+\zeta_K\sin\omega t)]\ddot\psi+$$
$$+2m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega\dot\psi+$$
$$+Mg[\xi_C\cos(\psi-\alpha)-\zeta_C\sin(\psi-\alpha)]+$$
$$+\sum_{j=1}^{N}m_jg[(\xi_K+R_j\cos\varphi_j)\cos(\psi-\alpha)-$$
$$-(\zeta_K+R_j\sin\varphi_j)\sin(\psi-\alpha)]+$$
$$+m_0g[(\xi_K+R_0\cos\omega t)\cos(\psi-\alpha)-$$
$$-(\zeta_K+R_0\sin\omega t)\sin(\psi-\alpha)]=$$
$$+-m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega^2,$$

$$
\begin{aligned}
& m_j(R_j^2+\rho_j^2)\ddot{\varphi}_j+b_jR_j^2(\dot{\varphi}_j-\omega)+\\
& +m_jgR_j[-\sin(\psi-\alpha)\sin\varphi_j+\cos(\psi-\alpha)\cos\varphi_j]+\\
& +m_j\{[R_j^2+\rho_j^2+R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\ddot{\psi}-\\
& -R_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot{\psi}^2\}=0,
\end{aligned}
$$

$$/\, j=\overline{1,N}\,/; \qquad (28)$$

– for the case of a ball- or a roller-type auto-balancer

$$
\begin{aligned}
& \left\{I_O+\sum_{j=1}^{N}m_j[d_K^2+R_j^2+\rho_j^2+2R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\right\}\ddot{\psi}+\\
& +b_pl^2\dot{\psi}+k_pl^2\psi+2\dot{\psi}\sum_{j=1}^{N}m_jR_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot{\varphi}_j+\\
& +\sum_{j=1}^{N}m_j\left[R_j^2-\rho_j^2\frac{R_j}{r_j}+R_j(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)\right]\ddot{\varphi}_j+\\
& +\sum_{j=1}^{N}m_jR_j(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot{\varphi}_j^2+\\
& +m_0[d_K^2+R_0^2+\rho_0^2+2R_0(\xi_K\cos\omega t+\zeta_K\sin\omega t)]\ddot{\psi}+\\
& +2m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega\dot{\psi}+\\
& +Mg[\xi_C\cos(\psi-\alpha)-\zeta_C\sin(\psi-\alpha)]+\\
& +\sum_{j=1}^{N}m_jg[(\xi_K+R_j\cos\varphi_j)\cos(\psi-\alpha)-\\
& -(\zeta_K+R_j\sin\varphi_j)\sin(\psi-\alpha)]+\\
& +m_0g[(\xi_K+R_0\cos\omega t)\cos(\psi-\alpha)-\\
& -(\zeta_K+R_0\sin\omega t)\sin(\psi-\alpha)]=\\
& =-m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega^2,
\end{aligned}
$$

$$
\begin{aligned}
& m_jR_j^2\left(1+\frac{\rho_j^2}{r_j^2}\right)\ddot{\varphi}_j+b_jR_j^2(\dot{\varphi}_j-\omega)+\\
& +m_jgR_j[-\sin(\psi-\alpha)\sin\varphi_j+\cos(\psi-\alpha)\cos\varphi_j]+\\
& +m_jR_j\left\{\begin{aligned}&\left[R_j-\frac{\rho_j^2}{r_j}+(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)\right]\ddot{\psi}-\\ &-(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot{\psi}^2\end{aligned}\right\}=0,
\end{aligned}
$$

$$/\, j=\overline{1,N}\,/. \qquad (29)$$

Expressions (28) and (29) show that the differential motion equations of the vibratory machine:

– for the case of a ball- or a roller-type auto-balancer, they coincide with accuracy to the sign;

– for the case of a pendulum-type auto-balancer, they differ from the differential motion equations for the cases of a ball- or a roller-type auto-balancer.

5. 3. 2. The case of identical loads

For the case of identical loads

$$m_j=m,\quad R_j=R,\quad \rho_j=\rho,\quad b_j=b,\quad /\, j=\overline{1,N}\,/, \qquad (30)$$

and differential motion equations of the vibratory machine take the form:

– for the case of a pendulum-type auto-balancer

$$
\begin{aligned}
& \left\{I_O+m\sum_{j=1}^{N}[d_K^2+R^2+\rho^2+2R(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\right\}\ddot{\psi}+\\
& +b_pl^2\dot{\psi}+k_pl^2\psi+2mR\dot{\psi}\sum_{j=1}^{N}(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot{\varphi}_j+\\
& +m\sum_{j=1}^{N}[R^2+\rho^2+R(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\ddot{\varphi}_j+\\
& +mR\sum_{j=1}^{N}(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot{\varphi}_j^2+\\
& +m_0[d_K^2+R_0^2+\rho_0^2+2R_0(\xi_K\cos\omega t+\zeta_K\sin\omega t)]\ddot{\psi}+\\
& +2m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega\dot{\psi}+\\
& +Mg[\xi_C\cos(\psi-\alpha)-\zeta_C\sin(\psi-\alpha)]+\\
& mg\sum_{j=1}^{N}[(\xi_K+R\cos\varphi_j)\cos(\psi-\alpha)-\\
& -(\zeta_K+R\sin\varphi_j)\sin(\psi-\alpha)]+m_0g[(\xi_K+R_0\cos\omega t)\cos(\psi-\alpha)-\\
& -(\zeta_K+R_0\sin\omega t)\sin(\psi-\alpha)]=-m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega^2,
\end{aligned}
$$

$$
\begin{aligned}
& m(R^2+\rho^2)\ddot{\varphi}_j+bR^2(\dot{\varphi}_j-\omega)+\\
& +mgR[-\sin(\psi-\alpha)\sin\varphi_j+\cos(\psi-\alpha)\cos\varphi_j]+\\
& +m\{[R^2+\rho^2+R(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\ddot{\psi}-\\
& -R(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot{\psi}^2\}=0,
\end{aligned}
$$

$$/\, j=\overline{1,N}\,/; \qquad (31)$$

– for the case of a ball-type or a roller-type auto-balancer

$$
\begin{aligned}
& \left\{I_O+m\sum_{j=1}^{N}[d_K^2+R^2+\rho^2+2R(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)]\right\}\ddot{\psi}+\\
& +b_pl^2\dot{\psi}+k_pl^2\psi+2mR\dot{\psi}\sum_{j=1}^{N}(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot{\varphi}_j+\\
& +m\sum_{j=1}^{N}\left[R^2-\rho^2\frac{R}{r}+R(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)\right]\ddot{\varphi}_j+\\
& +mR\sum_{j=1}^{N}(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot{\varphi}_j^2+\\
& +m_0[d_K^2+R_0^2+\rho_0^2+2R_0(\xi_K\cos\omega t+\zeta_K\sin\omega t)]\ddot{\psi}+\\
& +2m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega\dot{\psi}+\\
& +Mg[\xi_C\cos(\psi-\alpha)-\zeta_C\sin(\psi-\alpha)]+\\
& +mg\sum_{j=1}^{N}[(\xi_K+R\cos\varphi_j)\cos(\psi-\alpha)-\\
& -(\zeta_K+R\sin\varphi_j)\sin(\psi-\alpha)]+m_0g[(\xi_K+R_0\cos\omega t)\cos(\psi-\alpha)-\\
& -(\zeta_K+R_0\sin\omega t)\sin(\psi-\alpha)]=-m_0R_0(-\xi_K\sin\omega t+\zeta_K\cos\omega t)\omega^2,
\end{aligned}
$$

$$
\begin{aligned}
& mR^2\left(1+\frac{\rho^2}{r^2}\right)\ddot{\varphi}_j+bR^2(\dot{\varphi}_j-\omega)+\\
& +mgR[-\sin(\psi-\alpha)\sin\varphi_j+\cos(\psi-\alpha)\cos\varphi_j]+\\
& +mR\left\{\begin{aligned}&\left[R-\frac{\rho^2}{r}+(\xi_K\cos\varphi_j+\zeta_K\sin\varphi_j)\right]\ddot{\psi}-\\ &-(-\xi_K\sin\varphi_j+\zeta_K\cos\varphi_j)\dot{\psi}^2\end{aligned}\right\}=0,
\end{aligned}
$$

$$/\, j=\overline{1,N}\,/. \qquad (32)$$

6. Discussion of research results

The study conducted has allowed us to construct a model of the single-mass vibratory machine with a rotary-oscillatory motion of the platform and a vibration exciter in the form of a ball-, a roller- or a pendulum-type auto-balancer.

When deriving the differential motion equations of the vibratory machine, we considered the effect exerted by balls or rollers that roll along the rolling tracks. Owing to the effect and the rotary-oscillatory motion of the platform, the differential motion equations of the vibratory machine, for the case of:

– a ball- or a roller-type auto-balancer, are similar (coincide with accuracy to the sign);

– a pendulum-type auto-balancer, differ in their form from the differential equations for the case of a ball- or a roller-type auto-balancer.

It should be noted that the differential motion equations of the load directly include the platform's angle of rotation.

Note that for the case of translational motion of the platform [19]:

– differential motion equations of the vibratory machine are reduced to the form that is independent of the type of an auto-balancer;

– the differential motion equations of the load do not directly include the platform coordinate; they contain only its second time derivative.

This indicates a significant difference between the case of a rotary-oscillatory motion of the platform and the case of translational motion of the platform.

The derived differential equations of motion could be applied both for an analytical study into dynamics of the vibratory machine and for conducting computational experiments. This is their advantage. Note that the results reported in papers [22, 23]:

– suggest the form taken by approximate solutions to the differential motion equations of the system, which could be used when searching for them;

– could be applied to test the adequacy of the constructed physical-mathematical model of the vibratory machine (after finding the two-frequency modes of motion).

It should be noted that the constructed model does not take into consideration:

– a possibility of encounters among loads;

– the influence of a processed material on the dynamics of motion of the vibratory machine.

A first disadvantage of the model is typical for the analytical theory of passive auto-balancers [13–18]. It could be partially eliminated by adding the appropriate constraints to the angular coordinates of loads. However, that would greatly complicate the model and would make it impossible to conduct analytical studies. A second disadvantage is often found in the theory of vibratory machines [1–8]. Its elimination requires a greater reconsideration of the model.

In the future we plan to search for the two-frequency motion modes of the vibratory machine, to determine the conditions for their existence and stability.

7. Conclusions

1. We have constructed a model of the single-mass vibratory machine with a rotary-oscillatory motion of the platform and a vibration exciter in the form of a ball-, a roller-, or a pendulum-type auto-balancer. The model takes into consideration the effect exerted by the balls or rollers that roll along the rolling tracks.

2. We have derived the differential motion equations of the vibratory machine and established their following special features:

– differential motion equations of the vibratory machine, for the case of a ball- or a roller-type auto-balancer, are identical (coincide with accuracy to the sign);

– differential motion equations of the vibratory machine, for the case of a pendulum-type auto-balancer, differ in form from the differential equations for the case of a ball-type or a roller-type auto-balancer;

– differential motion equations of the load directly include the platform's angle of rotation.

The equations derived are applicable both in order to study analytically the dynamics of an appropriate vibratory machine and to carry out computational experiments.

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