

*Із виконаного аналізу відомих капілярних витратомірів малих потоків газів випливає перспективність побудови первинних вимірювальних перетворювачів витратомірів з лінійним вихідним сигналом. Завдяки стабільності розмірів прохідних каналів скляних капілярів такі витратоміри можуть мати високі метрологічні характеристики. У зв'язку із цим досліджено капіляр як чутливий елемент первинних перетворювачів витратомірів малих потоків газу.*

*Досліджено різні схеми капілярних первинних перетворювачів засобів для вимірювання малих витрат газів. Виконані дослідження забезпечують вибір оптимальної схеми первинного вимірювального перетворювача витрати за діапазоном вимірювання, а також кількості і розмірів прохідних каналів капілярів. Так наприклад, витратомір, побудований на основі пакета капілярів, має суттєво ширший діапазон вимірювання порівняно з іншими схемами.*

*Одержані аналітичні залежності, які забезпечують проектування одно капілярних, пакетних і мостових перетворювачів витратомірів. Наведені порівняльні характеристики вказаних первинних вимірювальних перетворювачів. Розроблені алгоритми розрахунку розмірів прохідних каналів капілярів перетворювачів з лінійним вихідним сигналом.*

*Оцінено вплив температури та барометричного тиску на відхилення статичної характеристики перетворювача. Встановлено, що капілярна мостова схема, на відміну інших, забезпечує часткову компенсацію впливу зовнішніх факторів.*

*Розроблений і досліджений капілярний витратомір кисню, побудований за мостовою вимірювальною схемою з лінійною функцією перетворення для системи автоматизації процесу виробництва заготовок волоконних світловодів. Верхня межа вимірювання витратоміра є на рівні 54 л/год., а його основна відносна похибка становить 0.8 %*

*Ключові слова: пакет капілярів, мостова капілярна схема, лінійність функції перетворення, малі витрати газу*

# DESIGN OF LINEAR CAPILLARY MEASURING TRANSDUCERS FOR LOW GAS FLOW RATES

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## 1. Introduction

A growing need to improve the accuracy of measurement and stabilization of low ( $0.1...10^3$  l/h) and micro consumption ( $<0.1$  l/h) of gas flows is predetermined by the development of modern technologies [1, 2]. Examples of such needs can be found in the production of fiber lightguides, as well as in vacuum equipment and medicine [3, 4]. Addressing these tasks is also important for the continuous preparation of gas mixtures with a predefined composition, specifically for the checking mixtures used by tools for analytic control [2, 5].

Measurements of low and micro flow rate of gases involve different methods and instruments, but the ones that are commonly used are rotameters and heat flow meters [6–8]. The error of such tools for the measurement of low flow rate are at the level of 1–3 %, that is too large for modern technological processes. There is a pressing need for measurements both in the ranges of low and micro flow rate with errors that do not exceed 1 % [5, 9]. There are even greater prospects,

in terms of accuracy, for the flowmeters of variable pressure differential, because, as studies have shown, known tools do not take into consideration the complete impact of all factors on the conversion function of a flowmeter.

Among the variable pressure flowmeters, the most widely used are the flowmeters whose primary measuring transducers are devices for a flow narrowing, specifically in the form of diaphragms, nozzles, and capillary tubes [5, 6]. The principle of operation of such flowmeters is based on the dependence of pressure drop at narrowing on the flow rate of gas. Among the various narrowing devices, the most promising are glass capillary tubes with a cylindrical flow-through channel with a laminar gas flow [10–12]. The capillaries can provide high sensitivity and linearity of the output signal in the specified ranges, the stability and reproducibility of measurement results, as well as the possibility to compensate for (reduce) the influence of interference.

The advantages of capillary flowmeters include a relative simplicity of the design, as well as high sensitivity, which

makes it possible to apply them for the measurements of both low and micro flow rate of gases. In order to provide for the advantages of capillary flowmeters at calculation, design, and construction, it is necessary to take into consideration the patterns in a gas flow along a capillary channel through the appropriate selection of its dimensions, as well as the respective circuitry of capillaries' junction. Thus, it is a relevant task to investigate different circuits of capillary flowmeters, and to parametrically optimize the structural dimensions of capillaries in order to improve the metrological characteristics.

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## 2. Literature review and problem statement

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In a general case, the relationship between a pressure difference at a capillary and the mass gas flow rate is described by a square dependence [8]. Given this, capillary flowmeters have a nonlinear consumption characteristic, thereby requiring individual calibration of a flowmeter.

The measurements of low flow rate of gases employ thermal capillary flowmeters, proposed to be applied to measure the low flow rate of natural gas [6]. In such a flowmeter, the capillary tube is connected to a thermal flow rate transducer in parallel to the pipeline along which much of the gas flows. However, such flowmeters have a large inertiality and a low measurement accuracy (error at the level of 2 %).

Paper [12] described a micro flowmeter that operates based on the method of variable pressure drop, it is built using a metallic capillary with a differential pressure gauge connected to its ends. The accuracy of the flowmeter is ensured by the thermal stabilization and adjustment of influence from external factors, as well as by a calibrating bench with an error limit at the level of 0.3 %. However, such a flowmeter is characterized by the non-linearity of conversion function, while its measurement range is limited by the Re number.

Paper [13] reminded of manufacturers of laminar flowmeters with thermocompensation because, according to the Hagen-Poiseuille law, a deviation of pressure and temperature from the assigned values causes additional errors due to a change in the density and viscosity of gas. Such flowmeters ensure the measurement of small flows of gases with an error at the level of 1 %. However, the proposed structures of flowmeters are quite complex in fabrication and do not provide for the complete compensation. In order to achieve high accuracy of capillary flowmeters, it is necessary to maintain the constant values of viscosity  $\mu$  and density  $\rho$  of gas. This is achieved through the stabilization of temperature  $T$  and gas pressure. In addition, designing a primary converter for the flowmeter requires the calibration of geometrical dimensions of the pass-through channel in a capillary. Because the real value of the diameter is typically different from the rated value, which causes significant errors in the calculation of flow rate, then such flowmeters are calibrated individually at a workplace and with respect to the conditions for measurement. In addition, for the case of a nonlinear conversion function the scale of a flowmeter must be calibrated over the entire range of measurement.

The preparation of gas mixtures involves the measurement, assignment, and stabilization of low flow rate of gas components using a set of throttles with different diameters of the pass-through channels [5]. However, the metrological characteristics of such devices remain at the level of known

capillary flowmeters. For the same purpose, the preparation of mixtures containing micro concentrations of components employs the laminar micro flowmeters that are calibrated using a gravimetric method and a chromatographer, which implies considerable cost of resources [14].

In order to measure viscosity, the work uses a capillary pressure divider, which, under conditions of constant parameters in a working medium, could be applied to measure low flow rate [15].

Therefore, we consider it a promising task to design the high-precision capillary measuring transducers for low flow rate of gases with linear conversion functions. The linearity of the conversion function is ensured through the calibration of a flowmeter only at one point of its scale (for the upper limit of measuring range), which thereby improves the accuracy of measurement.

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## 3. The aim and objectives of the study

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The aim of this study is to design and investigate capillary primary measuring transducers for low gas flow rate with the improved metrological characteristics: the linearity of the output signal, partial compensation for external influences, and an expanded measurement range.

To accomplish the aim, the following tasks have been set:

- to investigate the circuits of capillary flowmeters in order to identify capabilities for the linearization of a starting signal from a primary transducer, for expanding the ranges of measurement, and for compensating for the effect of external factors;
- based on the performed research, to design, as an example, a flowmeter of oxygen with the upper limit of measurement of 54 l/h in order to control the production of workpieces for fiber lightguides.

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## 4. Studying and modeling the capillary as a sensing element of the primary measuring transducer

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It is expedient to use, as sensing elements for the primary measuring transducers of low gas flow rate, glass capillary tubes, which are conventionally used in thermometry. Such capillaries are made with a diameter  $d$  of the pass-through cylindrical channel from 0.05 to 2 mm and a length  $l$  from 150 to 650 mm [16]. The stability of characteristics of such sensing items is predetermined by resistance against chemical, thermal, and mechanical influences.

A flow rate characteristic (FC) of a capillary takes the form  $G=f(\Delta P, P_w, d, l, T, \mu)$ , where, in addition to known,  $G$  is the mass flow rate;  $\Delta P=P_v-P_w$  is the pressure drop at a capillary;  $P_v, P_w$  are the absolute input and output pressures. In order to describe FC, different analytical expressions are used [11, 17]. Discrepancies between the results of determining a flow rate based on different expressions could amount to 66 %.

A mathematical model of the capillary was refined based on experimental research into consumption characteristics of capillaries with the pass-through channels' length  $l \in [10; 300]$  mm and diameter  $d \in [0.09; 0.3]$  mm.

We performed our study using an installation (Fig. 1), which includes stabilizer-pressure setter 1, model manometer 2, thermometer 3, barometer 4, and film flowmeter 5. The limit of the standard relative error for a film flowmeter is 0.25 %.

The study was performed for a varying pressure in the dosed gas at capillary  $CE$  ranging from 10 to 160 kPa and an atmospheric pressure at its output.

The same installation was employed for calibrating the designed flowmeters. Nodes  $a$  and  $b$  are intended to connect the examined objects.

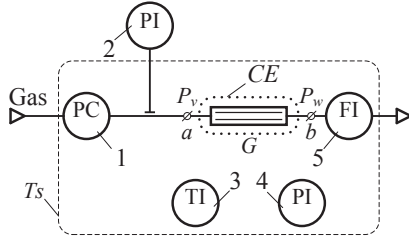


Fig. 1. Functional diagram of the flow rate measuring installation for studying the capillaries and capillary gas flowmeters

An analytical expression that describes FC of glass capillaries, derived with respect to the approximation of experimental data, takes the following form:

$$G = 4\pi\mu l \xi^{-1} \times \left( \left[ \left( 1 + \xi d^4 l^{-2} (512 R_g T \mu^2)^{-1} \Delta_p (\Delta_p + 2P_w) \right)^{1/2} - 1 \right] \right), \quad (1)$$

where, in addition to known,  $R_g$  is the gas constant;  $(T) = a_1 T + a_0$  is the linear approximation of the dynamic viscosity from temperature;  $\xi = 2.7$  is the coefficient of end effects.

A divergence between the experiment and the calculation, performed using equation (1), is at the level of 1 %.

In general, FC of the capillary is nonlinear. However, the main factors that predetermine this nonlinearity act in opposite directions. Specifically, the loss of pressure at the ends of a capillary deflect FC from a linear one downwards (negative curvature), while gas compression causes its upward deflection (positive curvature). Therefore, FC of a capillary, depending on the ratio of the specified factors, could demonstrate a curvature of different signs, or be linear under condition of compensation for the above factors. Fig. 2 shows FC (in relative coordinates) of glass capillaries with different  $l = \{30; 65; 290\}$  mm and  $d = 0.24$  mm, at the dosage of air and atmospheric pressure at the outlet from the capillary. The coordinate axes have dimensionless magnitudes  $q = G/G_{\max}$  and  $p = \Delta_p/\Delta_{p\max}$ , marked along them, where  $G_{\max}$  and  $\Delta_{p\max}$  are the maximum values for flow rate  $G$  and pressure drop  $\Delta_p$  at the capillary. Fig. 2 shows that a short capillary, for which a predominant influence is exerted by the end effects, has a negative curvature of FC (curve 1). For the long capillary, the influence of gas compression produces a dominating effect, which is why its FC acquires a positive curvature (curve 3). At a certain ratio of  $d$  to  $l$  in the capillary, FC is linear (curve 2).

Based on equation (1), we derived the condition for the FC linearity of capillaries in the following form

$$d^2 = \mu \left[ \xi / (512 R_g T) \right]^{-1/2} P_w^{-1} l. \quad (2)$$

It follows from dependence (2) that the linearity of FC for the capillary, which meters a certain type of gas ( $\mu, R_g$ ), could be ensured only at constant absolute temperature  $T$  of the metered gas and pressure  $P_w$  at the outlet from the capillary.

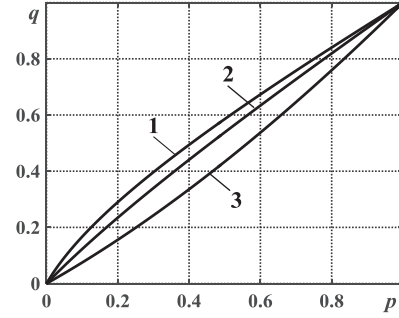


Fig. 2. Flow rate characteristics of capillaries in relative coordinates

Application of glass capillaries opens the prospect for constructing the primary measuring transducers of flowmeters for low gas flow rate with high metrological characteristics owing to the stability of FC and a possibility to obtain a linear conversion function.

### 5. Capillary transducers with a linear source signal

A primary measuring transducer (PMT) of capillary flowmeters, depending on the measurement range, can be constructed based on different circuits [11, 18]. PMT on a single capillary  $CE$  is shown in Fig. 3, *a*. Fig. 3, *b* shows PMT based on the package  $Pc$  (parallel connection of  $n$  identical capillaries  $CE_1, \dots, CE_n$ ). PMT based on the disbalanced bridge circuit (gas-dynamic capillary bridge GDB) is shown in Fig. 3, *c*.

In the circuits from Fig. 3, nodes  $a$  and  $b$  are connected to the pressure gauge  $PDI$ , which measures the difference in pressures  $\Delta_{pab}$ .

Fig. 3, *a* shows a circuit for the single-capillary primary measuring transducer. In it, the pressure gauge determines a flow rate. The pressure gauge is connected to the ends of the capillary (points  $a, b$ ).

Fig. 3, *b* shows a circuit for the package primary measuring transducer. The package represents a parallel connection of  $n$  capillaries, whose inputs and outputs are connected to the pressure gauge's chambers (points  $a, b$ ).

Fig. 3, *c* shows a bridge circuit for the primary transducer of flow rate, whose measuring diagonal is connected to a pressure gauge. The structural dimensions of the bridge's capillaries are chosen so that the throttle pressures  $P_{1a}$  and  $P_{1b}$  change linearly depending on the flow rate of gas that passes the bridge. The result of such a structure is that the pressure drop in the measuring diagonal changes linearly depending on the measured flow rate.

In a general case, conversion function  $\Delta_{pab} = f(G)$  for all capillary PMT of flow rate is nonlinear. As regards instrumentation, preference is given to PMT with a linear conversion function  $\Delta_{pab} = k_G G = k_c k_p G$ , where  $k_G$  is the flow rate coefficient of the respective capillary PMT;  $k_c, k_p$  are the structural and parametric complexes, respectively, of the transducer and metered gas.

Values for the upper limits  $G_{\max}$  in the measurement range of each capillary PMT, shown in Fig. 3, are linked via ratio  $G_{\max}^{(CE)} < G_{\max}^{(GDB)} < G_{\max}^{(Pc)}$ , and corresponding maximum pressure differentials  $-\Delta_{pab\max}^{(CE)} = \Delta_{pab\max}^{(Pc)} > \Delta_{pab\max}^{(GDB)}$ .

Maximum value for the measured flow rate is limited by the upper boundary of flow laminarity ( $Re_{lim} = 2320$ ).

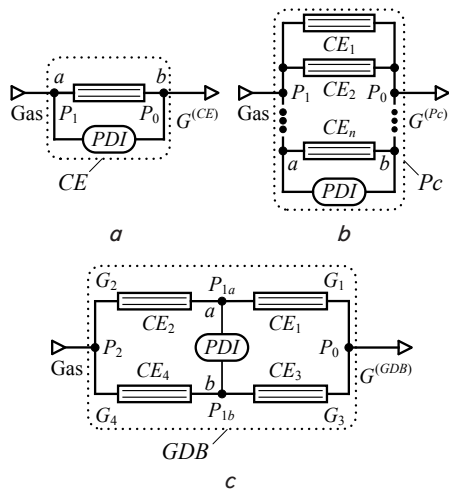


Fig. 3. Main circuits of capillary flow rate PMT based on: a – single capillary; b – package of capillaries; c – capillary disbalanced bridge

We show the features of design for the specified PMT below, as well as give the derived values for upper limits of measured flow rate and maximum changes in pressure.

**5. 1. A single-capillary PMT**

The necessary condition for obtaining a linear conversion function for PMT based on a single capillary is ensuring the ratio of its dimensions ( $d, l$ ), represented by dependence (2). Length of the capillaries is limited by the requirements to dimensions of execution for both PMT and a flowmeter in general; the recommended length  $l \in [5; 150]$  mm.

Dimensions ( $d, l$ ) for a linear capillary PMT when measuring the flow rate  $G^{(CE)}$  of gas are determined from the system:

$$\begin{cases} d \geq 4(\pi \mu Re_{lim})^{-1} G^{(CE)}; \\ l = \mu^{-1} [\xi / (512 R_g T)]^{1/2} P_0 d^2. \end{cases} \quad (3)$$

In case of exceeding the calculated value  $l$  for acceptable length ( $\leq 150$  mm), PMT must be constructed as a package of capillaries.

An analysis of equations (1) and (2) reveals that the maximum flow rate could be ensured for the accepted maximum length  $l_{max}=150$  mm. In order to determine the maximum measured mass flow rate  $G_{max}^{(CE)}$  through the capillary and the corresponding maximum pressure drop  $\Delta P_{ab,max}^{(CE)}$ , the following system of equations is applied:

$$\begin{cases} d = [\mu (512 R_g T / \xi)^{1/2} P_0^{-1} l_{max}]^{1/2}; \\ G_{max}^{(CE)} = \pi \mu Re_{lim} d / 4; \\ \Delta P_{ab,max}^{(CE)} = k_G^{(CE)} G_{max}^{(CE)}, \end{cases} \quad (4)$$

where

$$k_G^{(CE)} = k_c^{(CE)} k_p^{(CE)} = d^{-2} (32 \pi^{-2} \xi R_g T)^{1/2}.$$

Corresponding values for volumetric flow rates  $Q_{max}^{(CE)}$  are reduced to normal conditions ( $T_n=293.15$  K;  $P_n=101.325$  kPa) using the following dependence

$$Q_{max}^{(CE)} = G_{max}^{(CE)} / \rho_0 = G_{max}^{(CE)} P_n^{-1} R_g T_n.$$

Table 1 gives diameters  $d$  of linear capillaries  $CE$  with length  $l_{max}$ , determined from system (4), which meter the maximum flow rate  $G_{max}^{(CE)}$  (for  $\Delta P_{ab,max}^{(CE)}$ ) of a certain type of gas ( $\mu, R_g$ ). The project is carried out for absolute source pressure  $P_0=130$  kPa and temperature  $T=313$  K.

Based on Table 1, one selects a capillary and an appropriate pressure gauge under the above-specified conditions of measurement.

A PMT conversion function takes the form

$$\Delta P_{ub} = k_G^{(CE)} G^{(CE)}, \quad (5)$$

where  $k_G^{(CE)}$  is the coefficient of a linear capillary flow rate.

Equation (5) shows that the conversion function of such PMT is linear; the conversion coefficient is defined by the values for two complexes – structural  $k_c$  and parametric  $k_p$ . Complex  $k_c$  depends on geometrical dimensions of the pass-through channel in a capillary, and  $k_p$  – on the type of gas and temperature.

Table 1

Parameters for single-capillary PMT of gas flowmeters for the boundary values of measured flow rate

Gas	Air	N <sub>2</sub>	O <sub>2</sub>	Ar	He	H <sub>2</sub>	CO <sub>2</sub>	NH <sub>3</sub>	CH <sub>4</sub>
$d, \text{ mm}$	0.301	0.298	0.311	0.326	0.511	0.407	0.245	0.258	0.271
$\Delta P_{p,max}^{(CE)}, \text{ kPa}$	102.32	101.29	105.5	104.67	173.57	138.12	83.30	87.59	92.04
$G_{max}^{(CE)} \cdot 10^5, \text{ kg/s}$	1.049	1.001	1.187	1.319	1.903	0.681	0.698	0.505	0.568
$Q_{max}^{(CE)}, \text{ l/h}$	31	31	33	29	412	292	14	26	31

**5. 2. Package capillary transducer**

If there is a need to measure larger consumption than  $G_{max}^{(CE)}$ , that is, flow rate  $Re > 2320$ , instead of a single capillary one uses the package (Fig. 3, b).

In order to design capillaries in a package, it is necessary to first determine the dimensions ( $l; d$ ) for a linear capillary from system (3), in this case, one derives a value for the unacceptable length ( $l > l_{max}$ ) of a capillary. Such a capillary is replaced with a package of  $n$  identical capillaries of permissible length  $l_n \leq l_{max}$  and appropriate diameter  $d_n$  so that  $G^{(Pc)} = n G_n^{(CE)}$  is ensured. To this end, one derives from the first equation of system (4) the diameter  $d_n$  for a capillary in a package with acceptable length  $l_n$ . The second equation is used for determining the maximum flow rate  $G_{n,max}^{(CE)}$  through each capillary in the package at a maximum permissible pressure drop

$$\Delta P_{ub,max} = k_G^{(CE)} G_{n,max}^{(CE)},$$

where

$$k_G^{(CE)} = d_n^{-2} (32 \pi^{-2} \xi R_g T)^{1/2}.$$

The minimum number  $n$  of capillaries in the package is derived from dependence

$$n = [G^{(Pc)} / G_{n,max}^{(CE)}] + 1, \quad (6)$$

where the expression in square brackets defines the whole part of the relation.

Since the number  $n$  of capillaries in the package, calculated from dependence (6), yields a somewhat higher flow rate at pressure drop  $\Delta_{P_{ab, \max}}$ , the drop can be reduced

$$\Delta_{P_{ab}} = \Delta_{P_{ab, \max}} G^{(Pc)} / (n G_n^{(CE)}). \quad (7)$$

*Example.* It is required to determine number  $n$  and dimensions ( $d_n$ ;  $l_n$ ) for the pass-through channels of linear capillaries in the package in order to measure the flow rate of oxygen in the range with upper limit  $Q_{\max}=160$  l/h at a value for output pressure of  $P_0=130$  kPa and temperature  $T=313$  K in a capillary PMT.

By using dependence  $G_{\max}=Q_{\max}P_0(R_gT)^{-1}$ , one determines the mass flow rate  $G_{\max}=7.1007 \cdot 10^{-5}$  kg/s of oxygen that corresponds to the assigned volumetric  $Q_{\max}$ . One calculates the capillary's dimensions using system (3) and derives  $d=1.826$  mm and  $l=5.175$  m, which is unacceptable given the structural limitations.

One accepts the length of the capillary in the package to be  $l=150$  mm and, applying system (4), one determines the diameter  $d_n=0.3109$  mm, flow rate  $G_{n \max}=1.2088 \cdot 10^{-5}$  kg/s, and pressure drop  $\Delta_{P_{n \max}}=105.51$  kPa. The minimum number of capillaries in the package determined from dependence (6) equals  $n=6$ . The pressure drop, recalculated using expression (7), is equal to  $\Delta_{P_{ab}}=103.29$  kPa.

### 5. 3. A bridge capillary transducer

A bridge circuit (Fig. 3, c), employed as a basis of PMT, ensures a 2-time greater range of measured flow rate than a single capillary and a 2-time less than when using a package with the same number of capillaries as in the bridge. A special feature of this scheme is that the maximum pressure drop  $\Delta_{P_{ab, \max}}$  in its measuring diagonals can be assigned by 1–3 orders of magnitude less than for a package or a single capillary. In addition, such a circuit provides for a partial compensation for a change in barometric pressure and temperature.

In a general case, the output signal  $\Delta_{P_{ab}}$  in the measuring diagonal GDM is nonlinear. In order to obtain the linear conversion function of the scheme, it is necessary to ensure a certain ratio of capillaries' dimensions in the circuit.

To this end, we proposed a procedure for equivalent transformations of the bridge circuit to a single-capillary element.

The main diagram of the disbalanced capillary GDM can be considered as a parallel connection of two two-element pressure dividers  $DPr_1$  and  $DPr_2$ . The first divider  $DPr_1$  includes two capillaries  $CE_1$  and  $CE_2$  with respective dimensions ( $d_1$ ;  $l_1$ ) and ( $d_2$ ;  $l_2$ ), the second –  $DPr_2$  –  $CE_3$  and  $CE_4$  with dimensions ( $d_3$ ;  $l_3$ ) and ( $d_4$ ;  $l_4$ ). Divider  $DPr_1$  is replaced with an equivalent capillary  $CE_1^{(e)}$  with dimensions  $d_1^{(e)}$  and  $l_1^{(e)}$ , and  $DPr_2$ , respectively,  $CE_2^{(e)}$ , with dimensions  $d_2^{(e)}$  and  $l_2^{(e)}$ . These two capillaries  $CE_1^{(e)}$  and  $CE_2^{(e)}$  form a parallel connection (package) of capillaries, which in turn can be replaced with a single equivalent capillary  $CE^{(e)}$  with dimensions  $d^{(e)}$  and  $l^{(e)}$ .

In order to ensure the linearity of output signal  $\Delta_{P_{ab}}$  of GDM depending on a flow rate, it is proposed to make all capillaries linear. In this case, both dividers  $DPr_1$  and  $DPr_2$  are linear with inter-throttle pressures  $P_{1a}$  and  $P_{1b}$  in the respective dividers changing linearly depending on pressure  $P_2$ , and hence on flow rates  $G_1$  and  $G_2$ . The inter-throttle pressures  $P_{1a}$  and  $P_{1b}$  in dividers are determined from dependences [19]:

$$\begin{cases} P_{1a} = \lambda_1(1-\lambda_1)^{-1}(P_2 - P_0) + P_0; \\ P_{1b} = \lambda_2(1-\lambda_2)^{-1}(P_2 - P_0) + P_0. \end{cases} \quad (8)$$

$\Delta_{P_{ab}}$  can then be represented in the form:

$$\Delta_{P_{ab}} = P_{1a} - P_{1b} = k_\lambda \Delta_P, \quad (9)$$

where

$$\lambda_i = l_{2i} / l_{2i-1}; \quad \delta_i = d_{2i} / d_{2i-1}; \quad \delta_i^4 = \lambda_i^2 / (1-2\lambda_i), \quad i = 1, 2;$$

$$\Delta_P = P_2 - P_0; \quad k_\lambda = [(1-\lambda_1)(1-\lambda_2)]^{-1}(\lambda_1 - \lambda_2)\Delta_P.$$

Below is the sequence for deriving a dependence of pressure drop  $\Delta_{P_{ab}}^{(GDB)}$  in the measuring diagonal of the bridge on flow rate  $G$  taking into account the above-specified equivalent transformations.

The replacement of two-capillary ( $n=2$ ) linear pressure dividers  $DPr_1$  and  $DPr_2$  with respective equivalent linear capillaries  $CE_1^{(e)}$  and  $CE_2^{(e)}$  is based on the dependences obtained by authors in [20]. Thus, for the *first and second* branches of the bridge dimensions ( $d_j^{(e)}$ ;  $l_j^{(e)}$ ,  $j=1,2$ ) of equivalent capillaries  $CE_1^{(e)}$  and  $CE_2^{(e)}$  are determined using expressions:

$$d_j^{(e)} = \left[ \sum_{i=1}^n d_i^{-4} \right]^{-1/4}; \quad l_j^{(e)} = [d_j^{(e)}]^2 \sqrt{\xi X} P_0; \quad j = 1, 2. \quad (10)$$

The expression to replace the parallel connection of capillaries  $CE_1^{(e)}$  and  $CE_2^{(e)}$  with the equivalent linear capillary  $CE^{(e)}$  ( $d^{(e)}$ ;  $l^{(e)}$ ) takes the form:

$$\begin{aligned} (d^{(e)})^2 &= \sum_{j=1}^n (d_j^{(e)})^2 = \sum_{j=1}^n \left[ \sum_{i=1}^n d_{i+2(j-1)}^{-4} \right]^{-1/2}; \\ l^{(e)} &= (d^{(e)})^2 \sqrt{\xi X} P_0. \end{aligned} \quad (11)$$

Therefore, the GDM conversion function, obtained based on dependences (9) and (11), takes the form:

$$\Delta_{P_{ab}} = k_G^{(GDB)} G = k_c^{(GDB)} k_p^{(GDB)} G, \quad (12)$$

where

$$\begin{aligned} k_c^{(GDB)} &= [(1-\lambda_1)(1-\lambda_2)]^{-1}(\lambda_1 - \lambda_2) / (d^{(e)})^2; \\ k_p^{(GDB)} &= (32\xi R_g T)^{1/2} / \pi. \end{aligned}$$

(12) shows that a pressure drop  $\Delta_{P_{ab}}$  varies linearly depending on the flow rate and is defined by gas parameters (complex  $k_p^{(GDB)}$ ) and structural dimensions of the bridge capillaries (complex  $k_c^{(GDB)}$ ).

Based on the assigned values for parameters of circuit  $G_{\max}$ ,  $\Delta_{P_{\max}}$ ,  $P_0$ ,  $T$ , the type of gas, one calculates dimensions ( $d$ ;  $l$ ) for capillaries and parameters ( $G$ ;  $\Delta_P$ ) for the flow of gas in the following order  $CE_1 \Rightarrow CE_2 \Rightarrow CE_3 \Rightarrow CE_4$ .

*The algorithm for calculating the parameters of a bridge PMT*

1. Based on structural considerations, choose length  $l_1 \leq l_{\max}$ .

2. Applying dependence (2), determine diameter  $d_1$  for the pass-through channel in capillary  $CE_1$ .

3. Based on the selected branch, accept flow rate  $G_1 = G_{max}/2$ .

4. Determine pressure drop  $\Delta_{p_1}$  from formula (5).

5. Assign  $d_2$  (for example,  $d_2 = d_1$ ) and determine  $\delta_1 = d_2/d_1$ . An option is to assign length  $l_2$  and determine  $\lambda_1 = l_2/l_1$ .

6. Based on expression  $\delta_1^4 = \lambda_1^2 (1 - 2\lambda_1)^{-1}$  (given in dependence (9)), determine

$$\lambda_1 = (\delta_1^2 + \delta_1)^{0.5} - \delta_1.$$

An option is to determine  $\delta_1$  for known  $\lambda_1$ .

7. Based on the first equation from system (8), determine  $P_2$ .

8. Determine pressure drop  $\Delta_{p_3} = \Delta_{p_1} + \Delta_{p_{max}}$  at capillary  $CE_3$ .

9. Based on formula (5), determine  $d_3$  given known  $\Delta_{p_3}$  and  $G_3 = G_{max}/2$ .

10. For  $d_3$ , based on formula (2), derive length  $l_3$ .

11. Given known  $P_2$ ,  $\Delta_{p_3}$  and  $P_0$ , based on the second equation in system (8), determine

$$\lambda_2 = \Delta_{p_3} / (P_2 - P_0 + \Delta_{p_3}),$$

where  $\lambda_2 = l_4/l_3$ .

12. Based on the derived  $\lambda_2$ , determine length  $l_4 = \lambda_2 l_3$  for capillary  $CE_4$ .

13. By analogy to point 6, determine  $\delta_2 = d_4/d_3$  and then diameter  $d_4 = \delta_2 d_3$ .

Our study has shown that the designed bridge PMT, developed in accordance with the presented algorithm, ensures the linearity of its conversion function.

#### 5. 4. Selection of PMT circuit

Based on the research that we carried out, one can select the optimal hardware implementation of PMT depending on the upper limit of the range of measured flow rate for common gases (Table 1). Table 2 gives maximum values for the measured flow rates of CO<sub>2</sub> and He in the examined circuits; for the rest of gases, values of flow rate are intermediate. In addition, Table 2 gives an appropriate number of capillaries to ensure the specified flow rate.

Table 2

Comparative characteristics of PMT

PMT circuit	Maximal flow rate, l/h	Number of capillaries, pcs.
Capillary	14...412	1
Bridge	28...824	4
Package	140...4,120	10 (expedient)

Thus, Table 2 makes it possible to select the optimal circuit depending on the value for the measured flow rate.

#### 6. Influence of non-informative parameters on the flow rate PMT

Because all the examined PMT are based on the capillary, then, as follows from dependence (1), the most essential factors of influence are the absolute temperature of PMT and the starting gas pressure. Therefore, we analyzed errors

in measurements of flow rate caused by influence of the specified factors using PMT based on a single capillary, a package, and a gas-dynamic bridge.

A change in pressure at the output from PMT is predetermined by two factors – a change in barometric pressure and the characteristics of an absolute pressure stabilizer itself. Temperature also affects the starting signal of the circuit mostly by two channels – because of a change in viscosity and density of gas. Our study has been performed for the specified PMT using various gases (Table 1) in a range of change in temperature  $T = 313 \pm 0.5$  K and absolute pressure at the output from the transducer  $P_0 = 130 \pm 0.15$  kPa.

The study has shown that the effect of temperature on the output signal of a single-capillary PMT is at the level of 0.6 %/K, while that of change in pressure at the output from the circuit is 1 %/kPa.

The effects of the specified factors for a package of capillaries are the same as those for a single capillary.

We investigated the bridge PMT using a system of equations

$$\begin{cases} G_1 = G_2; \\ G_3 = G_4; \\ G = G_1 + G_3; \\ \Delta_{pab} = P_{1a} - P_{1b}, \end{cases} \quad (13)$$

where

$$G_i = A_i \left( [1 + Y_i B_i]^{1/2} - 1 \right); \quad A_i = 4\pi \mu l_i \xi^{-1};$$

$$Y_i = K_i X; \quad K_i = \xi d_i^4 l_i^{-2}; \quad i = \overline{1,4};$$

$$X = (512 R_g T \mu^2)^{-1}; \quad B_1 = P_{1a}^2 - P_0^2; \quad B_2 = P_2^2 - P_{1a}^2;$$

$$B_3 = P_{1b}^2 - P_0^2; \quad B_4 = P_2^2 - P_{1b}^2.$$

The study has shown that the effect of temperature on the output signal from a PMT bridge is at the level of 0.5 %/K, indicating partial compensation for the temperature compared to PMT based on a single capillary and a package. Effect of pressure at the output of all circuits is practically the same; not exceeding 0.7 %/kPa.

#### 7. A gas-dynamic capillary flowmeter of low flow rate of oxygen

Tools for the automated measurement of low flow rate of oxygen are used to prepare gas-vapor mixtures of halides with oxygen in the process of manufacturing quartz workpieces for the single-mode fiber lightguides [9]. This technological process requires the measurement of oxygen flow rate up to 54 l/h. In order to measure such a flow rate, we have developed a capillary flowmeter of oxygen with a linear conversion function, whose main circuit is shown in Fig. 4.

PMT of the flowmeter is built in line with the circuit in Fig. 3, c, based on capillaries whose pass-through channels' dimensions are determined employing the constructed algorithm, described in chapter 5.3 in this paper. Such a structure ensures the linearity of the output signal from the flowmeter. In order to eliminate the influence of temperature of the environment, all elements of the flowmeter, except for a temperature gauge  $TE$ , are placed in thermostat  $TS$ , the temperature in which is maintained at a level of  $313 \pm 1$  K. A deviation of

the stabilized temperature from the assigned temperature is compensated for by the unidirectional action of temperature on the bridge dividers. To eliminate the effect of barometric pressure and the user load, at the output of the bridge there is an absolute pressure stabilizer  $Sp$ . The output diagonal of the bridge is connected at nodes  $a$  and  $b$  to pressure gauge  $PDI$  with the unified current signal, which is transmitted to the microprocessor device  $MCU$  (microcontroller Arduino Uno).  $MCU$  in combination with temperature sensors  $TE$  and absolute pressure sensors  $PE$  computes and displays the current value for a flow rate in a convenient user-friendly form (flow rate, mass/volumetric), and controls the enabling/disabling of all valves.

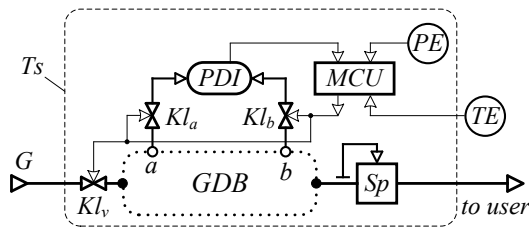


Fig. 4. Main circuit of the capillary bridge oxygen flowmeter

In order to measure a pressure drop, the output diagonal of the bridge employs the sensor XPCL04DTC (Honeywell) with a measuring range of 0...1.0 kPa. Pressure  $P_0$  at the output from the circuit is maintained at the level of 130 kPa via the absolute pressure stabilizer CAD-307. In order to measure atmospheric pressure, we applied the sensor 185PC15AT, temperature – DS18B20. The valves are devices of the type A322-0C2-U72 made by Camozzi. A system of temperature stabilization at  $313\text{ K} \pm 1$  (not shown in Fig. 4) includes two coolers, a heating element (a nichrome spiral) and a temperature sensor. Operation of the thermal stabilization system is controlled by  $MCU$ , which is the microcontroller Arduino Uno.

Calculation of dimensions for capillaries of the bridge scheme with a linear output signal in the measuring diagonal was performed in line with the above proposed algorithm. Dimensions of the pass-through channels in capillaries, in mm:

$$d_1=0.311, l_1=150; d_2=0.311, l_2=62.13;$$

$$d_3=0.309, l_3=148.58; d_4=0.312, l_4=61.89.$$

Taking into account an error of the film flowmeter (0.25 %) and a metrological margin, the limit of the standard relative error for a capillary flowmeter is at the level of 0.8 %.

## 8. Discussion of results of studying the capillary measuring transducers of low gas flow rate

We have proposed and investigated several variants for constructing a flow rate PMT. The advantages of the proposed PMT include the linear dependence of the output signal on the measured gas flow rate, the extended measuring range, partial compensation for the influence of non-informative parameters, and a larger choice of pressure gauges with different measurement ranges. Thus, in particular, a package of capillaries ensures a several-time wider measurement range while the bridge scheme demonstrates a two-fold increase in comparison with the single-capillary

flowmeters. In addition, the primary transducers themselves are made of glass, which provides high stability of their consumption characteristics, that is, they are almost not subject to corrosion, deformation, they are chemically resistant in contrast to those used that are made from metal, specifically described in paper [12]. The disadvantage of metallic capillaries is also a large temperature coefficient of expansion and a high probability of change in its characteristics when installing/dismantling and testing.

The linearity of conversion function in the developed PMT of flowmeters provides for improved accuracy in the flow rate measurement, it makes the calibration easier although does not eliminate the need for it. To ensure the linearity of the output signal, it is proposed to apply an absolute pressure stabilizer, which, although costly and has a bulky design, substantially reduces the impact of change in pressure at the output from PMT circuit and barometric pressure.

The study conducted makes it possible to select the optimal hardware implementation of PMT, the number of capillaries applied, and a range of measurements by a pressure gauge. In this case, the choice depends mainly on the upper limit of the range of the measured flow rate. Thus, for example, for the case of excessive length ( $>150$  mm) of the throttle in a single-capillary flowmeter, it is necessary to use a bridge circuit or a package of capillaries. In addition, the bridge circuit of PMT ensures partial compensation for the influence of non-informative parameters, because both branches of the bridge are identical in terms of their characteristics. An additional expansion of the range that a bridge measures is possible through the use of packages in the shoulders of the bridge instead of separate capillaries, which requires a separate study.

The primary measuring transducers of low gas flow rate, proposed by authors, operate based on the same method as described in paper [12] – a method of variable pressure drop. Therefore, the error of all such flowmeters depends on a decrease in the impact of external factors, as well as the errors from the calibrated bench. From this point of view, the capillary flowmeters, designed in this work, and those already known are similar. However, the proposed flowmeter compensates for the impact of external factors in the primary transducer itself, as well as the linear conversion function, which, in combination, ensures a better measurement accuracy compared to analogs, and demonstrate a much wider range of measurement.

The designed flowmeters, owing to their advantages specified above, have better metrological and operating characteristics, specifically, they ensure a lower measuring error ( $0.8 < 0.9\%$ ), high stability of characteristics, a wider range of measurement, as well as high resistance to aggressive environments.

Application of the developed flowmeters is promising for constructing other high-precision gas-dynamics tools, in particular, in order to assign low flow rate of gases, to obtain mixtures of different gases, as well as to test the flowmeters.

## 9. Conclusions

1. Capillary flow meters, which are constructed based on individual capillaries, have a measuring range limited by the Reynolds criterion and requirements to their dimensions.

The flowmeters, which are built based on a bridge circuit, have a greater range of measurement, provide for a partial compensation for external influences, however, given the random selection of structural dimensions for capillaries, they have a nonlinear conversion function.

2. Selection of the optimal PMT circuit depends on a range of measurement and a pressure drop at a pressure gauge. Thus, for example, a flowmeter, which is built based

on the package, provides for a significantly wider range. The designed flowmeters have a linear conversion function that improves the metrological characteristics.

3. The standard measurement error demonstrated by the developed flowmeter of low flows of oxygen is  $\pm 0.8\%$ , owing to the film flowmeter applied for its calibration (error of  $0.25\%$ ), as well as the stabilization of key factors of influence (pressures and temperature).

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