

Запропоновано та апробовано на реальних даних методичний підхід до визначення оптимальних параметрів стохастичної системи управління запасами в умовах економічно обґрунтованого дефіциту, який дозволяє оперативно реагувати на коливання попиту і динаміку змін стану запасів. Цей підхід пов'язує, зокрема, витрати від дефіциту та відповідний рівень сервісу зі значеннями страхового запасу, партії постачання та порогового рівня запасів. Методика застосовна до нормального закону розподілу споживання запасів та часу доставки.

Визначення оптимальних параметрів системи управління запасів в умовах нестабільного споживання та поповнення дозволяє запобігти завищенню обсягів товарно-матеріальних запасів на складах виробничих і торговельних підприємств. Одночасно зменшується потреба в обіговому капіталі (вартість інвестицій в запаси) та величина сукупної доданої вартості в ланцюгах постачання товарів, знижуються ризики утворення неліквідних запасів.

В результаті проведеного дослідження встановлено причинно-наслідковий зв'язок між рівнем обслуговування споживачів, рівнем дефіциту, партією постачання та обсягом страхового запасу. Доведено, що процедури визначення параметрів стохастичної системи управління запасами має передувати визначення економічно доцільного рівня дефіциту, який визначатиме оптимальний рівень обслуговування клієнтів та оптимальний розмір страхового запасу за критерієм сукупних витрат.

Запропонований підхід дозволяє обробити накопичені статистичні дані про динаміку змін стану запасів в реальному режимі часу з використанням статистичних методів та знайти компроміс між розміром страхового запасу, рівнем дефіциту та рівнем обслуговування споживачів за критерієм мінімуму сукупних витрат

Ключові слова: страховий запас, стохастичні моделі управління запасами, рівень дефіциту, рівень обслуговування споживачів

DETERMINATION OF PARAMETERS OF THE STOCHASTIC INVENTORY MANAGEMENT SYSTEM IN THE CONDITIONS OF ECONOMICALLY-BASED SHORTAGE

L. Savchenko

PhD, Associate Professor*

E-mail: lidia_savchenko@ukr.net

M. Grygorak

PhD, Associate Professor,

Head of Department*

E-mail: m_grigorak@ukr.net

*Department of Logistics

National Aviation University

Kosmonavta Komarova ave., 1, Kyiv,

Ukraine, 03058

1. Introduction

The problem of inventory optimization is topical for any enterprise, and effective management of replenishment and consumption processes has a decisive influence on the financial position and competitiveness of the company, as well as the quality of customer satisfaction. That is why the newest concepts of inventory management are nascent on the joint financial and production management, logistics and marketing, operations research and mathematical statistics.

The traditional approach to inventory management at domestic enterprises was to increase the volume of inventory and avoid shortages. However, the active use of logistics methods and technologies has changed the economic essence of stocks – from assets to liabilities, as well as management philosophy in terms of the optimal level of client service and acceleration of commodity movement in supply chains. From a Japanese management point of view, the stock should be associated with the cost, and it should be considered as a buffer that smooths supply irregularities, which restock, and consumption, use of stock characteristics. Improvement of information support, differentiation of supply sources and methods of transportation create preconditions for a significant reduction of the stock level without loss of quality of customer service, which helps to decrease total expenses of the enterprise.

It is determined in the work [1] that effective inventory management allows the enterprise to meet or exceed consumer expectations, creating such reserves of each product, which maximize net profit. This means that it is necessary to determine the optimal inventory size, which is necessary to meet the needs of consumers and would provide the maximum return of funds invested in the goods. These two components do not contradict but complement each other. In other words, in terms of effective inventory management, it is necessary to determine such an optimal inventory level, which minimizes the cost of the inventory control system, and, accordingly, maximizes profit. The total cost criterion is based, on the one hand, on the value of operating costs for the creation and holding of stocks, and on the other – potential losses due to the shortage of goods because of reduced sales, degradation of the quality of consumer service or increased operating costs for urgent delivery of goods.

It should be noted that in the scientific literature, there are different approaches to building inventory management strategies. On the one hand, the widespread technology JIT, which binds inventory with the planned volume of consumption, helps to spread the strategy of stock minimization, sometimes to zero levels [2]. On the other hand, the variability of the environment and uncertainty of consumption conditions determine the need to create safety stock, and then we can say about the strategy of stocks optimization [3].

Exactly the second approach actualizes the need to develop mathematical instruments to determine economically defined inventory levels in terms of minimizing the total costs associated with the creation and support of stocks. The quality of the construction of the corresponding system affects, on the one hand, the level of customer satisfaction with the level of service, and on the other hand – the level of costs of the entire logistics chain. Therefore, it is necessary to constantly seek a compromise between enough inventory level in points of sale, level of allowable shortage and absence of inventory excess in the supply chain. This requires the creation of such an inventory management system, which would be able to most accurately and quickly react to changes in the external environment with the unchanging maintenance of quality parameters and customer service speed.

2. Literature review and problem statement

The theory of optimal inventory management was born in the 19th century, when in scientific writings [4, 5] the analytical models of calculating the optimal order quantity under the conditions of deterministic demand were elaborated. A distinctive feature was a certain idealization of supply and stable replenishment conditions.

The new stage of the inventory management theory relates to the rapid development of logistics, which some scientists reference as a science of control of the stock movement [6, 7]. Some authors, in particular [8, 9], separately investigated the processes of formation and optimization of safety stocks in the conditions of the uncertainty of demand. The authors of [10] developed the so-called stochastic quasi-gradient method of solving the problems of convex stochastic programming. Modifications of this method were considered in the works [11, 12].

At the same time, the use of the developed mathematical methods in business is limited, on the one hand, by the existence of assumptions and simplification of real conditions, and on the other – the difficulties of obtaining the source data for calculations [13, 14].

It should be noted that the researchers have paid significant attention to the economic essence of inventories and their influence on the financial results of the enterprises. The author of [1] claims that effective inventory management allows companies to better meet the needs of consumers and thus achieve high ROI in warehousing.

On the other hand, globalization of supply chains and attempts to satisfy the needs of consumers perfectly led to the creation of considerable excess stocks in the channels of distribution of produced goods, which was named the “bullwhip effect” [15]. That is why scientific researches [16, 17] emphasizes the need to assess the risk of stock accumulation and the erroneous development of some methods of calculating the size of safety stock aimed to avoid the deficit.

A planned shortage or delay of supply sometimes helps to lower the overall cost of the system. In [18], the author presented a model of economic production quantity with planned backorders and proved its economic efficiency. The author [19] used the analytic geometry and algebra for inventory management models with the shortage. Two types of the costs of breach of obligations were used: one – linear, others – fixed.

The work [20], which investigates the enlarged range of inventory control system costs, including costs of damage to goods, costs of extraordinary supply in case of shortage, loss from lost sales is interesting.

The authors of [21] developed a model of managing single-product inventory, considering a partial shortage. It is assumed that in case of shortage, a part of clients will not wait for the goods to come but will go to the competitor, which would affect the general expenditures. In [22], the shortage is viewed as a valid, set value. The authors developed a model with time-dependent demand and storage costs for building optimal stock replenishment policies. Some authors, for example, [26], determined the optimum shortage for specific objects (in [26] – poultry farms), which determined the use of specific costs groups and additional partial criteria.

Thus, the analysis of modern literature on planning the inventory shortage testifies that the inclusion of appropriate costs in the general model allows considering not only the direct expenses of the system but also lost sales and costs of the unplanned supply. The expansion of the range of expenses can significantly affect the value of inventory management system parameters.

The research in the area of stochastic inventory management systems is represented by a wide range of works. In particular, the authors of [24] developed an optimizing model for supply chain, considering the uncertain demand, production cost, distribution of transportation costs, shortage losses and other parameters.

Due to global research [25], it is possible to estimate the share of the authors, who paid attention to the problems of stock shortage, as well as stochastic models of inventory management in 2008–2018. The authors examined 56 publications that were included in 32 journals of ScienceDirect and Scopus. Approximately 15 % of the works related to stochastic models of inventory management, about 12 % touched the issues of economically viable shortage. None of the works contained, on the one hand, the determination of economically viable shortage, and on the other – the determination of stochastic inventory management system parameters considering the optimum level of service.

Thus, a review of scientific literature and problems related to stock optimization at different enterprises showed a high degree of development of appropriate mathematical methods and models. On the other hand, there is a low level of practical use due to the need to account for many factors influencing the formation and holding of inventory in exact conditions. The problem of determination of inventory management system parameters under the conditions of uncertainty and economically justified deficit by the criterion of minimization of the general expenses of the enterprise requires a scientific basis and methodological support.

3. The aim and objectives of the study

The aim of the study is to develop a methodical approach to calculating the parameters of the stochastic inventory management system in conditions of economically based shortage by means of statistical methods.

In accordance with the stated aim, the following objectives of the study are defined:

- to develop a procedure that would allow linking deterministic and stochastic conditions of the inventory control

system, in particular, costs of shortage, the value of safety stock and order quantity;

- to prove the expediency of the cost-effective level of inventory shortage and to associate it with the size of safety stock and other parameters of the inventory management system with a normal distribution of parameters;
- on the basis of compromise between the levels of shortage, safety stock and client service to develop the method of calculating the parameters of the stochastic inventory management system, to estimate its sensitivity to input parameters.

4. Materials and methods of the study

4. 1. The method of determining the parameters of stochastic inventory management system in conditions of economically based shortage

The main input parameter of the inventory management system is demand. In real conditions, demand often has a random pattern. In addition, it should be considered that replenishment of stocks (supply) also occurs with certain time fluctuations. Of course, it is possible to work with averaged values of demand and time of delivery, but in some cases, neglect of the stochastic nature of consumption and replenishment of inventory may result in incorrect calculations and, consequently, increase of logistic costs.

According to [6, 13, 15, 26], the main parameters of the inventory management system are the maximum desired stock, the order quantity, the size of safety stock, reorder point. However, we believe that an economically based shortage should also be present among the parameters of the inventory management system.

To build an effective inventory management system in stochastic conditions, the algorithm is proposed (Fig. 1).

Consider each of the stages of this algorithm in more detail.

1. Removal of abnormal values.

A gross error or outliers is an error of a certain measurement included in a series of measurements, which for these conditions differs dramatically from other results of this series [27].

In logistics, outliers can be results related to the emergence of conditions that do not have to repeat in the future – stop of production equipment, car accidents, theft of warehouse or transport supplies, supplier strike, etc. If the probability of such events in the future is extremely low, observations that are registered due to the above events must be removed from a series of data as gross errors, as they “differ dramatically from other results” (according to the definition).

Possible criteria for checking the values for the presence of gross errors are shown in Table 1.

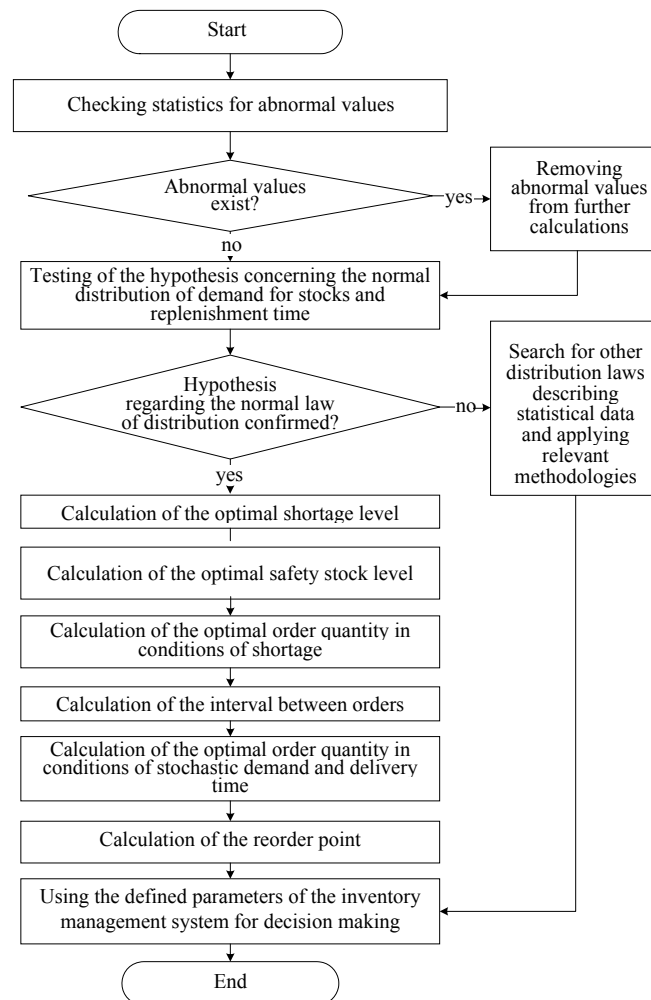


Fig. 1. Algorithm of defining the parameters of the stochastic inventory management system in conditions of economically justified shortage

Table 1
Criteria for checking the statistical values for outliers

Criterion name	The number of measurements for which the criterion is applied	The condition of availability of a gross error	Features of use
Dixon (Variation criterion)	$n \leq 10$	$\frac{x_n - x_{n-1}}{x_n - x_1} > Z_q$	The results of measurements are ranked by growth
Romanovsky	$n \leq 20$	$\frac{ \bar{x} - x_i }{S} \geq \beta_q$	–
“Three Sigma”	$n > 20 - 50$	$ \bar{x} - x_i > 3\sigma$	–
Chauvenet	$n \leq 10$	$n=3: \bar{x} - x_i > 1,6S;$ $n=6: \bar{x} - x_i > 1,7S;$ $n=8: \bar{x} - x_i > 0,9S;$ $n=10: \bar{x} - x_i > 2S$	Determined depending on the number of measurements
Grubbs	$n < 50$	$\frac{ \bar{x} - x_i }{S} \geq v_p$	–
Irwin		$\frac{x_{n+1} - x_n}{\sigma} > \theta_p$	The results of measurements are ranked by growth or in descending order. From the received series, two largest or two smallest values are chosen and the criterion is calculated

If an outlier is detected, the value must be excluded, and the numerical characteristics are recalculated.

For subsequent calculations, the value of the sample mean and sample standard deviation is needed. The sample mean:

$$\bar{x} = \frac{\sum_{i=1}^N x_i n_i}{N} = \frac{\sum_{i=1}^n x_i f_i}{n} \tag{1}$$

where x_i – the middle of the i -th interval; n_i – the number of observations in the i -th interval; N – the total number of statistical observations; n – the total number of intervals; f_i – the frequency at which the values fall in the i -th interval.

Formulae for calculating dispersion and standard deviation are chosen depending on the number of observations (Table 2).

Table 2
Formulas for dispersion and standard deviation

Number of observations	Formula for calculating the dispersion	Formula for calculating the standard deviation
More than 30	$D = \frac{\sum_{i=1}^n n_i (x_i - \bar{x})^2}{n}$	$\sigma = \sqrt{D}$
Less than 30	$S^2 = \frac{n}{n-1} \cdot D$	$S = \sqrt{S^2}$

2. Verification of the hypothesis concerning the normal distribution of demand and time of delivery.

The problem of identification of the distribution law describing statistical data is an extremely important stage of forecasting and planning.

A statistical hypothesis is any assumption concerning frequency function, probabilities (probability density function) or quantitative characteristics of numerical series [28].

To verify the hypothesis of the normal distribution of sales and delivery times, it is proposed to apply the Pearson chi-squared test. To do this, we must calculate the theoretical values of the probability of getting to each of the intervals. This is assisted by the Laplace transform (probability density function of the normal distribution).

The probability p_i for the normal distribution of a random variable X is calculated according to the formula [27]:

$$p_i = \Phi(x_1) - \Phi(x_2) = \Phi\left(\frac{x_{li} - \bar{x}}{\sigma}\right) - \Phi\left(\frac{x_{ri} - \bar{x}}{\sigma}\right), \tag{2}$$

where x_{li} and x_{ri} – the lower and upper bounds of the intervals, respectively.

The values of the Laplace function for the positive values of the argument x ($0 \leq x \leq 5$) are given in [29]. For the values $x > 5$, $\Phi(x) = 0.5$ is taken. For negative x values, the same table is used, given that the Laplace function is odd, i. e. $\Phi(-x) = -\Phi(x)$.

The value of the Pearson’s criterion or Chi-squared criterion (χ^2) is calculated by the formula [28]:

$$\chi^2 = \sum_{i=1}^n \frac{(f_{ei} - f_{ti})^2}{f_{ti}}, \tag{3}$$

where n – the number of groups into which empirical data are divided; f_{ei} – the frequency observed in the i -th group; f_{ti} – theoretical frequency.

For the χ^2 distribution, a table (such as in [30]) is created, where the critical value of the χ^2 acceptance criterion for the selected significance level α and degrees of freedom is specified.

The hypothesis of the normal distribution is accepted when the actual χ^2 value does not exceed the critical table one (Fig. 2).

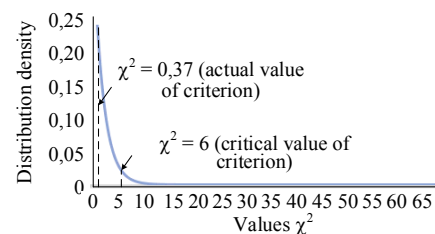


Fig. 2. Illustration of deciding on a hypothesis about the normal distribution using the χ^2 criterion graph (for degrees of freedom $k=2$)

Table values show the right border of the χ^2 distribution graph, which means that it does not confirm the hypothesis with the significance level α .

You can do without tabular values and use the function of the MS Excel 2010 CHISQ.INV.RT program (0.05; 2). As the first argument here is the significance level, as the second – the number of degrees of freedom. The value cal-

culated by the MS Excel 2010 program is usually somewhat more exact than the tabular one.

To assess the correctness of the hypothesis concerning the belonging to the normal distribution, it is recommended to calculate an additional p-level or p-value (probability to obtain such or even greater value of the criterion in the equity of the null hypothesis) [31].

3. Calculation of the optimal level of shortage (if the hypothesis for the normal distribution is accepted).

If the hypothesis of belonging of statistical data to the normal distribution is confirmed, it is possible to use the formula for the optimum level of shortage and tabular data for normal distribution.

According to [26], the optimal level of shortage:

$$S = \frac{c_h}{c_h + c_{def}}, \quad (4)$$

where C_h – the cost of storing a unit of goods per 1 day; C_{def} – losses from the shortage of a unit of goods per 1 day.

As for the shortage losses, there are certain problems with determining the value of these expenditures [14]. There can be quite a lot of approaches. Consider a few of them:

1) if the client who wants to buy a product and is not able to do it because of the lack of goods in stock, purchases this product in another company. In this case, the shortage losses correspond to the value of lost profit from the sale of goods, which is in lack;

2) if the client who cannot buy a product because of its absence, is offered a discount with which he will be able to obtain a product if he waits for arriving of the new shipment. In this case, the shortage losses correspond to the value, which reduces the profit from the production unit through the discount;

3) if for satisfaction of the client and fast delivery of the missing goods, blitz-orders with an increased cost of delivery are done. Then the additional cost of unplanned supply is distributed between all units of the shipment, so this will be shortage losses;

4) one option can be a combination of others. For example, ordering the blitz-supply and simultaneously offering a discount, to make customer be waiting for the product and do not apply to the company-competitor. Then the shortage losses are summed up.

4. Calculation of the amount of safety stock (considering the optimal level of shortage).

If the demand (sales, need) and time of delivery (the time of order fulfillment) obey the normal distribution law, the calculation of the safety stock requires a table value Z (the number of entailing no deficit standard deviations) [32].

The number of entailing no deficit standard deviations and service level are connected by the NORMSINV function of the MS Excel 2010 program. In order to find the number of entailing no deficit standard deviations at a certain service level, it is necessary to specify the =NORMSINV function (service level).

The probability of shortage (in this technique – the optimum level of shortage) is defined as

$$d=1-L, \quad (5)$$

where L – service level [33].

Considering the Z values and parameters of the distribution of consumption volume and delivery time, the safety stock is calculated according to the formula:

$$Z_s = z \cdot \sqrt{\bar{t} \cdot \sigma_s^2 + \bar{S}^2 \sigma_t^2}, \quad (6)$$

where Z_s – the value of safety stock, un.; z – the number of entailing no deficit standard deviations; \bar{t}_n – average execution time of the order, days; s_s – standard deviation of demand, un./day; \bar{S} – average consumption, un./day; s_t – standard deviation of delivery time, days [6].

5. Calculation of the amount of optimal order quantity considering the possibility of shortage.

The model of inventory management with the possibility of shortage provides the calculation of the optimal order quantity by the formula

$$Q_{opt} = \sqrt{\frac{2DC_s}{C_h}} \cdot \sqrt{\frac{C_h + C_{def}}{C_{def}}}, \quad (7)$$

where D – demand for goods during a certain period of time, un.; C_s – the cost of order delivery, which does not depend on the size of the order, UAH/un.; C_h – the cost of storing a unit of goods during a certain period of time, UAH/un.; C_{def} – the cost of shortage of a unit of goods during a certain period of time, UAH/un. [1].

6. Calculation of the interval between orders.

To calculate the optimal interval between orders, first the number of deliveries per year should be determined:

$$K=D/Q_{opt}. \quad (8)$$

The interval between orders is calculated by dividing the number of days in the period by the number of deliveries for that period:

$$I=T/K. \quad (9)$$

7. Calculation of the optimal order quantity in stochastic conditions.

The order quantity can be calculated as follows:

$$Q'_i = S \cdot (t_i + \bar{t}) + Z \cdot \sqrt{(t_i + \bar{t}) \cdot \sigma_s^2 + \bar{S}^2 \sigma_t^2} - Z_{Ti} - Z_{ii}, \quad (10)$$

where Q'_i – the quantity of the order i , un.; t_i – time interval between orders, days; Z_{Ti} – the level of the current inventory when issuing an order i , un.; Z_{ii} – the quantity of the stocks on the way not received at the moment of issuing an order i , un. [34].

If the inventory level at the time of the order is zero and there are no stocks on the way, the order quantity can be obtained by the formula [34]:

$$Q'_i = S \cdot (t_i + \bar{t}) + Z \cdot \sqrt{(t_i + \bar{t}) \cdot \sigma_s^2 + \bar{S}^2 \sigma_t^2}. \quad (11)$$

8. Calculation of the threshold level of the inventory management system.

The threshold level of inventory determines the level at which it is necessary to make the next order for replenishment of stocks. In stochastic conditions of consumption and receiving orders, the threshold level is determined by the formula [34]:

$$Q_{ROP} = \bar{S}t + Z \cdot \sqrt{t\sigma_s^2 + \bar{S}^2\sigma_t^2} = \bar{S}t + Z_s. \tag{12}$$

4. 2. Checking the algorithm on real data, sensitivity analysis of the model

Consider the application of the above methods on a practical example. Let there be sales statistics for 12 months of the year, as well as statistics of delivery time of goods (Tables 3, 4).

Table 3

Statistics of sales volume

Month number	Sales volume, t
1	14
2	12
3	13
4	15
5	11
6	13
7	14
8	13
9	12
10	15
11	13
12	14

Table 4

Statistics of delivery time

Delivery number	Time of delivery, days
1	5
2	6
3	5
4	7
5	5
6	4
7	5
8	6
9	5
10	5
11	6
12	6

1. Extraction of abnormal values from the statistics.

Since the number of our statistical values is 12, we can use the criteria of Romanovsky or Grubbs. The formulas view to apply these criteria (Table 1) is the same, differing only in tabular data, which are compared with the left part of the condition (inequality).

Because the number of values presented in the statistics is less than 30 ($n=5<30$), to calculate the standard deviation, use the “corrected” formula (Table 2).

Calculated values of the sample mean (1) and sample corrected standard deviation (Table 2): $x = 13.2$ un., $\sigma = 0.972$ un.

Now you can calculate the values of the left part of the condition for the anomalies of values (Table 5).

Compare the obtained values of the left part of the condition for the abnormal values with the table ones (Table 6).

As we can see, all the values of the left part of the condition concerning anomalies (Table 5) are smaller than any value of the Romanovsky criterion (Table 6). This means that with a probability of at least 99%, all statistics are not abnormal or gross values. According to the Grubbs criterion

(Table 6), the last value of the criterion that corresponds to 99 % of non-membership with the abnormal value, is violated for the sales volume in the month No. 5 ($2.26 > 2.229$). If we accept the need for 99 % of the absence of abnormal values in the statistics, the sales number of the 5th month must be removed from statistics. The sample mean and standard deviation must be recalculated. We believe that the confidence probability of 0.98 will be enough to test the anomalies of the statistical series. This would mean that no statistics values have been removed as abnormal.

Table 5

Calculation of the values of the left side of the condition for values abnormality

Month number	Sales volume, t	Value of the left part of the condition for the anomalies of values $\frac{ \bar{x} - x_i }{S}$
1	14	0.823045
2	12	1.234568
3	13	0.205761
4	15	1.851852
5	11	2.263374
6	13	0.205761
7	14	0.823045
8	13	0.205761
9	12	1.234568
10	15	1.851852
11	13	0.205761
12	14	0.823045

Table 6

Critical values of the Romanovsky and Grubbs criteria (β_q and v_p)

Confidence probability, p	Number of values $n=12$	
	Romanovsky criterion	Grubbs criterion
0.9	2.75	2.663
0.95	2.66	2.519
0.98	2.52	2.387
0.99	2.39	2.229

2. Verification of the hypothesis concerning the normal distribution law.

Let us calculate the required number of groups by the formula $k=1+3.322 \cdot \lg(12)=4.6$. As the rounding of the integers in the calculations is carried out in a greater direction, $k=5$.

Find the intervals of sales volumes for each of 5 periods.

For this purpose, in the monthly statistics (Table 3) we find the largest and smallest value: $x_{\min}=11$, $x_{\max}=15$.

If you calculate with the MS Excel 2010 program, you can use the MAX and MIN functions.

The width of each of the five intervals will be 0.8 units ($w=0.8$). Thus, any value of demand between 11 and 11.8 units is included in the 1st interval. According to Table 3, it is the value of the month No. 5.

For the 2nd interval, the left border begins with all numbers that are greater than the right border of the 1st interval, that is, after 11.8.

The right border of the 2nd interval is calculated similarly: $x_{i2}=x_{i1}+w=11.8+0.8=12.6$.

Similarly, the boundaries of other intervals are determined.

Then the frequency of hitting values in each of the five intervals must be calculated (the last column of Table 7).

Table 7

Specifying the number of observations in each interval and frequency

Interval number, i	Lower border, x_{i1}	Upper border, x_{i2}	Number of values, n_i	Month number	Frequency, $f_i = n_i/n$
1	11	11.8	1	5	0.083333
2	11.8	12.6	2	2, 9	0.166667
3	12.6	13.4	4	3, 6, 8, 11	0.333333
4	13.4	14.2	3	1, 7, 12	0.25
5	14.2	15.0	2	4, 10	0.166667

The procedure for calculating the Pearson criterion components is given in Table 8. Let us explain how to get columns 5 and 6. We find the value of $\varphi(x_1)=\varphi(-2.26)$. Since $\varphi(-x)=-\varphi(x)$, we find the values in the table corresponding to the value of x_1 modulo: $\varphi(2.26)=0.4881$. So, the value of $\varphi(-2.26)=-\varphi(2.26)=-0.4881$. Similarly, we find the value of the Laplace function for all other arguments x_1 and x_2 .

All the components of the χ^2 formula are calculated in Table 8, column 9. The total value of the χ^2 criterion is 0.37.

Because the number of intervals is $n=5$, the number degrees of freedom equals $k=5-3=2$.

The calculated value of χ^2 equals 0.37, tabular is 6.0, and $0.37 < 6.0$, so the hypothesis of the normal distribution is accepted.

The test of the hypothesis for the normal distribution of delivery time is carried out similarly. Here are just the results. Distribution parameters of delivery time: sample

mean – 4.67 days, sample corrected standard deviation – 1.03 days. The calculated value of χ^2 equals 1.84, table – 6.0. Consequently, $1.84 < 6.0$, and the hypothesis of the normal distribution is accepted.

3. Calculation of the optimal shortage level.

Set the data about the inventory management system. Let the product cost is 100 UAH, and the storage cost is approximately 50 % of its cost per year, therefore, the annual cost of the storage of a unit of goods will be $100 \cdot 0.5 = 50$ UAH, a day – $50/365 = 0.137$ UAH.

The goods are sold at the price of 150 UAH, the profit from the sale is 50 UAH. Calculate the optimal parameters of the inventory management system by the following scenarios:

1. If in the absence of goods, the client refuses to expect the supply and goes to the competitor, the shortage losses are $C_{def} = 50$ UAH.

2. In the second version, we can offer the client a discount of 5 % of the goods price in case he agrees to wait for the next delivery, i. e. the sale price will be $150 \cdot 0.95 = 142.5$ UAH and profit will decrease by $50 - 42.5 = 7.5$ UAH. Thus, in this option $C_{def} = 7.5$ UAH.

3. In the case of a decision on extra delivery, the cost of goods is increased by 2 UAH. Then the profit from the sale of goods is reduced by 2 UAH, i. e. $C_{def} = 2$ UAH.

4. In this option, the client is offered to wait for the next delivery and given a 5 % discount. Thus, there is a combination of options 2 and 3. The shortage losses will be $7.5 + 2 = 9.5$ UAH.

The value of the optimal shortage level for the above four scenarios is shown in Table 9.

It can be seen that the worst option is the client's leave to a competitor company, in this case, the acceptable shortage level is only 0.3 %. The maximum shortage level is valid in case of extra supply – 6.4 %. However, this case is very sensitive to the unit cost of the blitz-delivery, that is, additional costs per unit of goods delivered.

Table 8

Calculation of theoretical frequencies and Pearson criterion

Interval number, i	Number of observations, n_i	The first argument in the Laplace function, $x_1 = (x_{i1} - x) / \sigma$	The second argument in the Laplace function, $x_2 = (x_{i2} - x) / \sigma$	Laplace function first value, $\varphi(x_1)$	Laplace function second value, $\varphi(x_2)$	The probability of falling in the i -th interval, $p_i = \varphi(x_2) - \varphi(x_1)$	Theoretical frequency, $n \cdot p_i$	Pearson criterion components, K_i
1	1.00	-2.26	-1.44	-0.4881	-0.4251	0.06	0.76	0.08
2	2.00	-1.44	-0.62	-0.4251	-0.2324	0.19	2.31	0.04
3	4.00	-0.62	0.21	-0.2324	0.0832	0.32	3.79	0.01
4	3.00	0.21	1.03	0.0832	0.3485	0.27	3.18	0.01
5	2.00	1.03	1.85	0.3485	0.4678	0.12	1.43	0.23
Total	12.00	-3.09	1.03	-0.71	0.24	0.96	11.47	0.37

Table 9

Results of calculating the optimal parameters of the inventory management system with stochastic demand and delivery time

Parameter	Formula for calculation	Un.	Scenario number			
			1	2	3	4
Optimal shortage level	(5)	%	0.3	1.8	6.4	1.4
Optimal safety stock level	(6)	un.	1.27	0.95	0.70	1.01
Optimal order quantity under deterministic conditions	(7)	un.	35.71	35.99	36.87	35.92
Optimal order quantity under stochastic conditions	(11)	un.	39.90	39.52	39.23	39.60
Optimum threshold level	(12)	un.	3.33	3.01	2.76	3.07

4. Calculation of the safety stock level (considering the optimal shortage level).

Table values of non-deficiency standard deviations Z for the four values of the optimal shortage level are equal to 2.76; 2.06; 1.52; 2.2.

When the MS Excel 2010 NORMSINV function is used to obtain the tabular values, we set the given service level in unit fractions as an argument.

To calculate the safety stock value, we recall the statistical parameters of demand and delivery time:

1) for demand: sample mean – 13.2 units per month, i. e. 0.44 units per day, corrected standard deviation – 0.0324 units per day;

2) for delivery time: sample mean – 4.67 days, corrected standard deviation is 1.03 days.

The value of the optimum safety stock level for the above four scenarios is shown in Table 9. Safety stock quantity differs significantly for different scenarios – from 0.7 to 1.27 units (almost twice).

5. Calculation of the optimal order quantity considering the possibility of shortage.

Let us present the initial data included in the formula:

– annual demand – 159 un.;

– delivery costs – 200 UAH;

– costs for storing one unit of goods during a year – 50 UAH;

– losses from shortage of one unit of goods during a year:

1 scenario – $50 \cdot 365 = 18,250$; 2 scenario – $7.5 \cdot 365 = 2737.5$;

3 scenario – $2 \cdot 365 = 730$; 4 scenario – $9.5 \cdot 365 = 3467.5$ UAH.

The value of the optimal order quantity for the above four scenarios can be seen in Table 9. The values are not very different for different shortage losses. This is caused by the known statement on the stability of the Wilson model. For the following calculations, it will be enough to take a certain average value of the optimal order quantity – 36 units.

6. Calculation of the interval between orders.

To calculate the optimal interval between the orders, the number of deliveries per year must be determined by the formula (8): $K=4.42$.

According to (9), the interval between orders is calculated as dividing 365 days in the period by the amount of supply during this period (4.42): $I=82.6$ days.

7. Calculation of the optimal order quantity in the stochastic conditions.

The results of previous calculations are used to determine the optimal order quantity in the stochastic conditions (Table 9).

It can be seen that the values of the optimal order quantity considering the stochastic consumption and delivery frequency differ slightly depending on the number of non-deficient standard deviation (2.76, 2.06, 1.52, 2.2) and varies from 39.23 to 39.90 un. Thus, we can claim that the shortage losses weakly affect the order quantity both for differential and stochastic parameters.

8. Calculation of the threshold level (ROP – Reorder Point) of the inventory management system.

Let us calculate the value of the threshold level, at which you should make an order for replenishment of stocks (Table 9). It is seen that the value of the threshold level varies depending on the amount of safety stock from 2.76 to 3.33 un.

5. Discussion of the methodology and prospects of its application

Thus, the proposed algorithm allows linking the deterministic and stochastic conditions of the inventory management system.

Shortage costs and correlation with the storage costs set by the scenario approach must determine the most expedient shortage level in these conditions. And, therefore, the optimal service level, which, in turn, determines the volume of safety stock.

It can be argued that the volume of the optimal order quantity is weakly dependent on the different values of shortage costs, which is associated with the stability of the Wilson model.

The optimal value of the threshold level or the re-order point directly depends on the amount of the safety stock: the higher the value of the optimum safety stock, the higher the value of the threshold level. Thus, the increase of the service level leads to an increase of all horizontal levels of the inventory control system – safety, threshold level and maximal level of stocks.

The strong side of the study is the development and testing of the algorithmic procedure for the determination of inventory management system parameters in the conditions of uncertainty, which can react promptly to the dynamics of changes in consumer demand and inventory status. This allowed combining the deterministic methods of determining the optimal order quantity and stochastic methods of calculating the volume of safety stock.

Economically expedient level of shortage determines the optimal level of customer service and the optimum amount of safety stock by the criterion of total costs.

The restriction of this method is the assumption that the intensity of consumption and time of replenishment of stocks as random quantities have normal distribution law.

The possibilities for further research is testing of the proposed methodological approach to inventory management with a seasonal pattern.

The threats to the results of the study are that in a rapidly changing market environment the company is forced to constantly change the inventory management strategy, not always guided by the formalized criteria. Targeting the non-deficiency of stocks increases the risks of creating excess inventories and thus increases the irrational losses of funds and enterprise resources.

From a practical point of view, the proposed approach allows avoiding the creation of excess inventory in supply chains and, accordingly, reducing storage costs, the need for the working capital and the risks of accumulation of non-liquid stocks.

6. Conclusions

1. The algorithmic procedure of finding the parameters of the stochastic inventory management system, which provides the calculation of economically expedient shortage level, the optimal amount of safety stock, connected with customer service level is developed, by the criterion of the total costs. Shortage costs and correlation with the storage cost set by the scenario approach determine the most expedient shortage level in the real conditions of creation and storage of inventory.

2. The advisability of calculating the parameters of the inventory management system with a normal distribution of

demand and delivery time on the basis of economically justified shortage level is justified. Thus, exactly the estimated shortage level should determine the optimal service level, not vice versa. In fact, the ratio, which determines the optimal shortage level, takes into account all components of the system costs, and, therefore, finds the shortage level according to the criterion of total costs, which include in particular the costs of the insufficient service level.

3. Based on the found compromise between shortage level, safety stock, and client service level, the parameters of the stochastic inventory management system – optimal order quantity, reorder point and the threshold level of stocks, which consider the level of the economically reasonable shortage and the optimal safety stock were calculated. This approach allows forecasting the status of the stock and reacting promptly to any changes in consumer demand or goods delivery terms.

References

1. Shraybfeder Dzh. Effektivnoe upravlenie zapasami. Moscow: Al'pina Biznes Buks, 2008. 304 p.
2. Franco C. E., Rubha S. An overview about JIT (just-in-time) – inventory management system // International Journal of Research – Granthaalayah. 2017. Vol. 5, Issue 4. P. 14–18. URL: <https://zenodo.org/record/569368#.XGPOY9IzbIV>
3. An Optimization Model for Inventory System and the Algorithm for the Optimal Inventory Costs Based on Supply-Demand Balance / Jiang Q., Xing W., Hou R., Zhou B. // Mathematical Problems in Engineering. 2015. Vol. 2015. P. 1–11. doi: <https://doi.org/10.1155/2015/508074>
4. Harris F. W. How Many Parts to Make at Once // Operations Research. 1990. Vol. 38, Issue 6. P. 947–950. doi: <https://doi.org/10.1287/opre.38.6.947>
5. Wilson R. H. A Scientific Routine for Stock Control // Harvard Business Review. 1934. Vol. 13, Issue 1. P. 116–128.
6. Bauersoks Donald Dzh. Logistika: integriruvannaya cep' postavok. Moscow: ZAO «Olimp-Biznes», 2008. 640 p.
7. Sergeev V. I. Upravlenie kachestvom logisticheskogo servisa // Logistika segodnya. 2010. Issue 1. P. 10–16.
8. Vozhzhov A. P., Lunyakov O. V., Vozhzhov S. P. Safety stock determination with application of the Poisson processes to incoming and outgoing flows // Ekonomika i upravlenie: teoriya i praktika. 2015. Issue 1. P. 30–35. URL: <https://elibrary.ru/item.asp?id=25096500>
9. Atamanchuk Yu. S., Pasenchenko Yu. A. Modeling of enterprise's inventory management with allowance for uncertain demand // Aktualni problemy ekonomiky ta upravlinnia: zbirnyk naukovykh prats molodykh vchenykh. 2016. Issue 10. URL: <http://ela.kpi.ua/jspui/handle/123456789/22508>
10. Ermol'ev Yu. M. O nekotorykh metodakh stohasticheskoy optimizatsii // Kibernetika. 1966. Issue 6. P. 96–98.
11. Ermoliev Y. M., Norkin V. I. On nonsmooth and discontinuous problems of stochastic systems optimization // European Journal of Operational Research. 1997. Vol. 101, Issue 2. P. 230–244. doi: [https://doi.org/10.1016/s0377-2217\(96\)00395-5](https://doi.org/10.1016/s0377-2217(96)00395-5)
12. Adaptivnye algoritmy stohasticheskoy optimizatsii i teorii igr / Yu. M. Ermol'ev (Ed.). Moscow: Nauka, 1990. 184 p.
13. Sterligova A. N. Upravlenie zapasami v cepyakh postavok: ucheb. Moscow: Infra-M, 2008. 430 p.
14. Savchenko L. V. Vytraty vid defitsytu v systemakh upravlinnia zapasamy // Rynok posluh kompleksnykh transportnykh system ta prykladni problemy lohistyky: 14 mizhnarodna naukovo-praktychna konferentsiya. Kyiv, 2012. P. 107–109.
15. Dai J., Li S., Peng S. Analysis on Causes and Countermeasures of Bullwhip Effect // MATEC Web of Conferences. 2017. Vol. 100. P. 05018. doi: <https://doi.org/10.1051/mateconf/201710005018>
16. Inventory management and the bullwhip effect during the 2007–2009 recession: evidence from the manufacturing sector / Doolley K. J., Yan T., Mohan S., Gopalakrishnan M. // Journal of Supply Chain Management. 2010. Vol. 46, Issue 1. P. 12–18. doi: <https://doi.org/10.1111/j.1745-493x.2009.03183.x>
17. Abbou R., Moussaoui C., Loiseau J. J. Effects of inventory control on bullwhip in logistic systems under demand and lead time uncertainties // IFAC-PapersOnLine. 2015. Vol. 48, Issue 3. P. 266–271. doi: <https://doi.org/10.1016/j.ifacol.2015.06.092>
18. Cárdenas-Barrón L. E. Economic production quantity with rework process at a single-stage manufacturing system with planned backorders // Computers & Industrial Engineering. 2009. Vol. 57, Issue 3. P. 1105–1113. doi: <https://doi.org/10.1016/j.cie.2009.04.020>
19. Cárdenas-Barrón L. E. The derivation of EOQ/EPQ inventory models with two backorders costs using analytic geometry and algebra // Applied Mathematical Modelling. 2011. Vol. 35, Issue 5. P. 2394–2407. doi: <https://doi.org/10.1016/j.apm.2010.11.053>
20. Preservation of deteriorating seasonal products with stock-dependent consumption rate and shortages // Journal of Industrial and Management Optimization. 2016. Vol. 12, Issue 4. P. 11–11. doi: <https://doi.org/10.3934/jimo.2016011>
21. Analysis of an EOQ inventory model with partial backordering and non-linear unit holding cost / San-José L. A., Sicilia J., García-Laguna J. // Omega. 2015. Vol. 54. P. 147–157. doi: <https://doi.org/10.1016/j.omega.2015.01.007>
22. Pervin M., Roy S. K., Weber G.-W. Analysis of inventory control model with shortage under time-dependent demand and time-varying holding cost including stochastic deterioration // Annals of Operations Research. 2016. Vol. 260, Issue 1-2. P. 437–460. doi: <https://doi.org/10.1007/s10479-016-2355-5>
23. Nobil A. H., Sedigh A. H. A., Cárdenas-Barrón L. E. A Generalized Economic Order Quantity Inventory Model with Shortage: Case Study of a Poultry Farmer // Arabian Journal for Science and Engineering. 2018. Vol. 44, Issue 3. P. 2653–2663. doi: <https://doi.org/10.1007/s13369-018-3322-z>
24. Zhang X., Huang S., Wan Z. Optimal pricing and ordering in global supply chain management with constraints under random demand // Applied Mathematical Modelling. 2016. Vol. 40, Issue 23-24. P. 10105–10130. doi: <https://doi.org/10.1016/j.apm.2016.06.054>
25. Momeni Z., Azizi A. Current Order and Inventory Models in Manufacturing Environments: A Review from 2008 to 2018 // International Journal of Online Engineering (iJOE). 2018. Vol. 14, Issue 06. P. 223. doi: <https://doi.org/10.3991/ijoe.v14i06.8055>
26. Grigor'ev M. N., Dolgov A. P., Uvarov S. A. Logistika. Prodvinutiy kurs. Moscow: Iz-vo Yurayt, 2016. 472 p.
27. Ancyferov S. S., Rusanov K. E., Afanas'ev M. S. Obrabotka rezul'tatov izmereniy: ucheb. pos. Moscow: Ikar, 2014. 228 p.

28. Allen W. R. Testing statistical hypotheses, by E. L. Lehmann. John Wiley and Sons, Inc., New York, 1959 // Naval Research Logistics Quarterly. 1961. Vol. 8, Issue 2. P. 201–201. doi: <https://doi.org/10.1002/nav.3800080209>
29. Tablica zacheniy funktsii Laplasa. URL: <https://math.semestr.ru/corel/table-laplas.php>
30. Kriticheskie tochki raspredeleniya X2. URL: <https://math.semestr.ru/group/xixi.php>
31. Statistical tests, P values, confidence intervals, and power: a guide to misinterpretations / Greenland S., Senn S. J., Rothman K. J., Carlin J. B., Poole C., Goodman S. N., Altman D. G. // European Journal of Epidemiology. 2016. Vol. 31, Issue 4. P. 337–350. doi: <https://doi.org/10.1007/s10654-016-0149-3>
32. Bakhruzhyn V. Ye. Metody analizu danykh: navch. pos. Zaporizhzhia: KPU, 2011. 268 p.
33. LaLonde B. J., Cooper M. C., Noordewier T. G. Customer Service: A Management Perspective. Oak Brook, III: The Council of Logistics Management. 1988. 640 p.
34. Radionov A. R., Radionov R. A. Logistika. Normirovanie sbytovykh zapasov i oborotnykh sredstv predpriyatiya. Moscow: Delo, 2002. 16 p.

Розглянуто підходи до побудови багаторівневої ієрархічної системи показників оцінки якісного стану ділянки автомобільної дороги. Розроблено модель вагомостей параметрів та характеристик автомобільних доріг як дорожньо-транспортного активу у складі інформаційно-управлінської системи грошової оцінки. Проведення досліджень обумовлене необхідністю розробки уніфікованої кваліметричної моделі грошової оцінки ділянки автомобільної дороги. В результаті проаналізовано параметри та конструктивні елементи автомобільних доріг, що дозволило розглянути об'єкт оцінки – ділянку дороги з точки зору методології побудови складних багаторівневих ієрархічних систем. На основі запропонованого підходу побудовано повну кваліметричну модель для вартісної оцінки автомобільної дороги, яка ґрунтується на методі декомпозиції властивостей об'єкта та включає максимальну кількість можливих параметрів, характеристик, конструктивних елементів, показників транспортно-експлуатаційного стану.

Запропоновано механізм обґрунтування вагомостей параметрів та характеристик об'єкта оцінки – ділянки автомобільної дороги шляхом раціонального поєднання вартісного, експертного методів та методу еквівалентних співвідношень. Адекватність запропонованої моделі підтверджується аналізом стандартних похибок, які складають біля 5 %, що суттєво не впливає на достовірність результатів аналітичних розрахунків.

Результати проведених досліджень слугуватимуть подальшим розвитком інформаційно-управлінської системи грошової оцінки дорожньо-транспортного комплексу, метою функціонування якої є обґрунтування раціональних управлінських рішень щодо дорожніх активів в цілому та автомобільних доріг зокрема.

Розроблена модель вагомостей параметрів автомобільних доріг може бути використана підприємствами та організаціями дорожньої галузі та може бути підґрунтям для прийняття ефективних обґрунтованих управлінських рішень щодо дорожньо-транспортних активів

Ключові слова: вагомість параметрів, автомобільні дороги, рівень якісного стану, кваліметрія, ієрархічна модель, інформаційно-управлінська система

UDC 625.7/8:330.522:330.43

DOI: 10.15587/1729-4061.2019.156519

DEVELOPMENT OF A MODEL OF THE WEIGHT OF MOTOR ROADS PARAMETERS AS PART OF THE INFORMATION AND MANAGEMENT SYSTEM OF MONETARY EVALUATION

O. Slavinska

Doctor of Technical Sciences, Professor*

V. Stozhka

PhD

Ltd. GRANBUD LEADER

Lesi Ukrainky Blvd., 34, Kyiv, Ukraine, 01133

A. Kharchenko

PhD, Associate Professor*

E-mail: anna-x3@ukr.net

A. Bubela

PhD, Associate Professor*

A. Kvatadze

Junior Researcher

Department of Regulatory and

Engineering Support of Road Works

M. P. Shulgin State Road Research

Institute State Enterprise

Peremohy ave., 57, Kyiv, Ukraine, 03113

*Department of Transport Construction

and Property Management

National Transport University

Omelianovycha-Pavlenka str., 1, Kyiv, Ukraine, 01010

1. Introduction

Development of the transport infrastructure of any country has a key impact on the economy in general. In

particular, the World Bank annually develops a program in the form of manuals and reports [1] with the involvement of private investors to designing, funding, implementation and management of infrastructure projects.