

# DERIVATION OF ENGINEERING FORMULAS IN ORDER TO CALCULATE ENERGY-POWER PARAMETERS AND A SHAPE CHANGE IN A SEMI-FINISHED PRODUCT IN THE PROCESS OF COMBINED EXTRUSION

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Досліджено можливості використання та вбудовуваності кінематичних трапецеїдальних модулів з криволінійною границею різної форми. На основі енергетичного методу отримані узагальнені формули для розрахунку потужності сил деформування всередині осьового трапецеїдального кінематичного модуля. Відокремлені різні види вибору функцій, що описують криволінійну границю осьового трапецеїдального модуля. Проаналізовано можливість використання відомих прийомів лінеаризації підінтегральних залежностей для розрахунку потужності сил деформування в разі неможливості отримання даної величини у вигляді аналітичної функції. Запропоновано шляхи отримання інженерних формул розрахунку складових наведеного тиску всередині осьового трапецеїдального кінематичного модуля. На основі енергетичного методу отримані формули для розрахунку поетапного формозмінення напівфабрикату за допущення  $\dot{\gamma}_{rz} = 0$  в межах осьового трапецеїдального кінематичного модуля.

Здійснено моделювання процесу комбінованого видавлювання порожнистих деталей з фланцем та встановлено закономірності формоутворення від геометричних параметрів. Отримано дані про поетапне формозмінення напівфабрикату в процесі деформування. Проведено порівняльний аналіз розрахункових схем для прямолінійного трапецеїдального кінематичного модуля та з криволінійною границею за допущення  $\dot{\gamma}_{rz} = 0$  в межах розглянутого модуля.

Підтверджено, що освітлені шляхи отримання інженерних формул та запропонований на їх основі алгоритм розрахунку процесів комбінованого видавлювання спрощують розробку технологічних рекомендацій. Це стосується як визначення силового режиму видавлювання, так і попередньої оцінки формозмінення напівфабрикату з можливістю керування витіканням металу в процесі деформування

**Ключові слова:** комбіноване видавлювання, кінематичний модуль, енергетичний метод, лінеаризація функцій, процес деформування

## 1. Introduction

The development of branches of modern machine building is inseparably related to the development of new resource-saving technologies enabling the production of high-quality products with the lowest indicators of production labor intensity and the highest coefficient of utilization of metal. In this regard, the effective resource-saving methods of metal treatment by pressure based on cold plastic deformation are becoming increasingly important [1]. We will note that the processes of cold volume forging (CVF) by extrusion enable

obtaining the billets with precise dimensions and high surface quality.

However, in addition to the listed merits, there are a series of problems related to metal plasticity under conditions of a complex stressed-strained state and the processes of its strengthening [2, 3]. Long-term development of CVF processes is related to developing and mastering the processes of combined extrusion [4]. A combination of radial and longitudinal extrusion opens great prospects for ensuring high complexity of obtained parts and lowering force parameters [5].

However, it should be borne in mind that the processes of combined extrusion with some degrees of freedom of metal flow occur in a self-regulating mode. Thus, it requires preliminary assessment of not only energy-power parameters of the deformation process, but also a step-by-step change in the shape of a semi-finished product. However, at present there are objective difficulties related to determining the optimal kinematic parameter (metal flow rate) and obtaining the formulas of reduced pressure in the analytical form. This causes difficulties in obtaining engineering formulas of increments of a semi-finished product and, consequently, hinders preliminary evaluation of the rationality of using the processes of combined extrusion, ensuring the required dimensions of a part. The search for new ways of obtaining engineering formulas for calculation of both energy-power parameters and changing the shape of a billet during deformation will allow solving the problems of predicting the increments of a semi-finished product. This, in turn, will compensate for the lack of recommendations on the use of processes of combined extrusion during production.

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## 2. Literature review and problem statement

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An analysis of data from the scientific literature reveals that the main problem of studying the processes of combined extrusion with several degrees of metal flow is to determine the optimal kinematic parameters of the deformation process [6–13]. This, in turn, makes it difficult to obtain the information about the stagewise and limit change in the shape of a billet [6] and leads to complexities in the development of the technological mode. Paper [7] presented an analysis of the influence of geometrical parameters, such as the radius of matrix curvature, the height of a gap and friction conditions during the process of direct-reverse-radial extrusion. In this case, the finite element method in ABAQUS software was used, however, the analytical dependences of power parameters and increments of a billet were not obtained. Article [8] is dedicated to the study of the process of combined reverse-direct extrusion of a pipe billet. The influence of geometric parameters of a tool and a part, as well as of friction conditions, on extrusion force was analyzed. Research into energy-power parameters of the process and a change in the shape of a part is limited to modeling in Deform-2D. Thus, the issues related to obtaining the calculation formulas of magnitudes of phased increments of a semi-finished product remained unresolved, which does not make it possible to predict a change in the shape of a part in the deformation process.

In paper [9], the approaches and methodology of construction of different kinematically possible velocity fields (KPVF) and the ways of obtaining analytical solutions of deformation forces are developed. The family of functions  $T=T(r, M)$  of parameter  $M$  one that is responsible for the shape of the sloped boundary is used as one of the options of description of the curvilinear boundary of the trapezoidal modules. However, when using the data of kinematic modules, the magnitudes of increments of a semi-finished product in an explicit form were not received and demand optimization by parameter  $M$ . Article [10] explored the process of combined reverse-direct extrusion by the top evaluation method using arbitrarily oriented triangular elements, combined with the method of finite elements (UBET). Experimental data are compared with the results of mode-

ling both by extrusion force, and by a change in the shape of a semi-finished product. The influence of the geometry of a tool and a billet on deformation force was analyzed and the insignificant influence of friction conditions was found. However, the magnitudes of optimal kinematic parameters and increments of a semi-finished product were not received in the form of finished engineering formulas. In paper [11], the analysis of the process of combined radial-reverse extrusion in a conical matrix for the second intermediate stage of the process was performed. The height  $h^*$  of the metal flow separation boundary to the cup wall and the flange area was taken as a geometric variable parameter. In this case, the optimal value of magnitude  $h^*$  and of full power in the explicit form were not obtained and are found numerically. Thus, it is difficult to predict a stagewise change in a shape of a billet. In paper [12], the process of the combined radial-reverse extrusion with a disjointed and joined deformation site was explored and the limits of using the proposed computational schemes were established. In addition, for the schemes with an axial trapezoidal module with a sloped curvilinear boundary of flow separation, the magnitudes of increments of a semi-finished product and reduced pressure in the explicit form were not obtained. In paper [13], the diagram of the areas that are critical in terms of formation of a defect in the form of shrinkage in the bottom part during combined extrusion of parts with a flange was proposed. However, these studies did not imply receiving computation formulas to determine the increments of a semi-finished product during deformation.

Thus, the studies into the processes of combined extrusion with several degrees of flowing were not fully covered. The main problems of construction of solutions in the explicit form that is suitable for subsequent calculations should include the difficulties associated with the use of kinematic modules with curvilinear boundary of flow separation and the optimization of kinematic parameters of the deformation process. In this regard, it is necessary to search for the ways to create more complete and accurate calculation schemes of the shape change prediction. The created developments and recommendations will contribute to widening the possibilities of combined extrusion processes and to expanding their use during production.

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## 3. The aim and objectives of the study

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The aim of this study is to find effective ways to obtain engineering formulas and the algorithm for calculation of energy-power parameters and prediction of the change in a shape of a semi-finished product in the combined extrusion processes with several degrees of freedom of flow based on the method of kinematic modules.

To accomplish the aim, the following tasks have been set:

- to analyze the possibilities of using the known techniques for linearization of integrand dependences in order to calculate the power of deformation forces;
- to demonstrate the ways to simplify the function of deformation intensity and to obtain the dependences of power of deformation forces in the analytical form;
- to perform comparative analysis of dependences and reduced pressure increments of a semi-finished product obtained using various functions, describing the curvilinear border of the axial trapezoidal module, as well as to identify conditions of using the developed KPVF;

– to develop an algorithm for calculation of energy-power parameters and prediction of a change in the shape of a semi-finished product based on the proposed KPVF in the processes of combined extrusion.

**4. Specific features in deriving engineering formulas for calculating energy-power parameters when modeling the processes of combined extrusion**

We will note that the dimensions and the configuration of a deformation site play a decisive role in the construction of KPVF, and as a result, lead to the simplification or complication of the mathematical calculation apparatus. The use of the elementary rectangular modules does not cause complications in subsequent calculations due to their good embeddability in more complex schemes. However, they are impossible to be used to describe the flow of material in kinematic elements with curvilinear boundaries, imitating the shape of the contact surfaces of a tool or the surface of material flow separation within a part itself. For the axisymmetric processes, KPVF that meet these requirements for modules with sloping and curvilinear boundaries were examined in papers [14, 15].

These modules are discussed in the cylindrical coordinate system  $r, \theta, z$  considering axial symmetry and equality to zero of the circling velocity component  $v_\theta=0$ . Kinematically possible velocities are assigned based on previously conducted experimental research and analysis of the characteristics of metal flow.

In the general case, the KPVF constructed to describe the flow of billet material should meet the kinematically boundary condition in velocities (KBC) and the conditions of incompressibility and continuity.

In the general case, accepting appropriate assumptions about the form of dependence of component  $v_z=v_z(z, r, C_i)$ , taking into consideration the condition of volume constancy in the differential form, after term-by-term integration, it is possible to obtain the expression for function  $v_r$ :

$$v_r = -\frac{1}{r} \left[ \int \frac{\partial v_z(z, r, C_i)}{\partial z} r dr + C_{i+1} \right]. \tag{1}$$

Values  $C_1, \dots, C_{i+1}$  are found based of the boundary conditions corresponding to given KPVF and continuity condition at the boundaries between the modules. Using this procedure, the KPVF for different varieties of modules can be obtained. However, it is not always possible to obtain the analytic expressions when calculating integrals determining the power of plastic deformation, friction and shear forces, due to the need to integrate cumbersome functions, including irrational functions. This applies especially to the use of modules with curvilinear boundaries, often used in the calculation of the combined extrusion processes.

One of the ways to obtain engineering formulas for calculation of extrusion processes is to use different ways of linearization of integrand functions simplifying subsequent integration during the calculation of power. Primarily, this refers to calculations of power of plastic deformation forces, which contains the expression of the intensity of deformation rate:

$$\dot{\epsilon}_i = \frac{\sqrt{2}}{3} \sqrt{(\dot{\epsilon}_z - \dot{\epsilon}_r)^2 + (\dot{\epsilon}_z - \dot{\epsilon}_\theta)^2 + (\dot{\epsilon}_r - \dot{\epsilon}_\theta)^2 + \frac{3}{2} \dot{\gamma}_{rz}^2}, \tag{2}$$

where

$$\dot{\epsilon}_z = \frac{\partial v_z}{\partial z}, \quad \dot{\epsilon}_r = \frac{\partial v_r}{\partial r}, \quad \dot{\epsilon}_\theta = \frac{v_r}{r}, \quad \dot{\gamma}_{rz} = \frac{\partial v_z}{\partial r} + \frac{\partial v_r}{\partial z}.$$

Linearization of the expression, which includes two arbitrary functions  $M_1$  and  $M_2$  with the relative error of not more than 6 %, is often used [15]:

$$\sqrt{M_1^2 + M_2^2} \approx |M_1| + 0.4 \cdot |M_2| \text{ at } |M_1| > |M_2|. \tag{3}$$

In some cases, a linearization variation in the following form is used [16]:

$$\sqrt{M_1^2 + M_2^2} \approx |M_1| + 0.5 \cdot \frac{M_2^2}{|M_1|} \text{ at } |M_1| > |M_2|. \tag{4}$$

If it is possible to determine the largest of the components of rates of relative deformations at the selected KPVF within a module, it is appropriate to apply linearized dependences, as shown in paper [15]:

$$\dot{\epsilon}_i = 1.08 |\dot{\epsilon}_{\max}|, \tag{5}$$

where  $\dot{\epsilon}_{\max}$  is the maximum component of deformation rate by absolute magnitude.

In the general case at axisymmetric process, described in cylindrical coordinates  $r, \theta, z$ , it is possible to accept with relative error of up to 10 %:

$$\sqrt{3} \dot{\epsilon}_i = \begin{cases} |\xi| + 0.4 \cdot |\dot{\gamma}_{rz}| & \text{at } |\xi| > |\dot{\gamma}_{rz}|; \\ 0.4 \cdot |\xi| + |\dot{\gamma}_{rz}| & \text{at } |\xi| < |\dot{\gamma}_{rz}|, \end{cases} \tag{6}$$

where

$$\frac{\sqrt{3}}{2} \xi = \begin{cases} \dot{\epsilon}_r - \dot{\epsilon}_\theta & \text{at } |\dot{\epsilon}_r - \dot{\epsilon}_\theta| > |\dot{\epsilon}_r - \dot{\epsilon}_z|; \\ \dot{\epsilon}_r - \dot{\epsilon}_z & \text{at } |\dot{\epsilon}_r - \dot{\epsilon}_z| > |\dot{\epsilon}_r - \dot{\epsilon}_\theta|; \text{ if } (\dot{\epsilon}_r - \dot{\epsilon}_\theta)(\dot{\epsilon}_r - \dot{\epsilon}_z) > 0; \\ \dot{\epsilon}_r - \dot{\epsilon}_\theta & \text{at } (\dot{\epsilon}_r - \dot{\epsilon}_\theta)(\dot{\epsilon}_r - \dot{\epsilon}_z) < 0. \end{cases}$$

If it is impossible to use the methods of linearization of integrand functions that were considered above, it is possible to obtain the approximated estimate of integrals of irrational functions, taking the form:

$$I = \iiint_V \dot{\epsilon}_i dV = \iiint_V \sqrt{F_1^2 + \dots + F_n^2} dV, \tag{7}$$

where  $\dot{\epsilon}_i$  is the intensity of deformation rates in elementary volume  $dV$ .

The integrals of form (7) are often impossible to be obtained in elementary functions, therefore approximate integration with the two-way accuracy estimation or cubature formulas are used [17]. The powers of shear and friction forces with the account of KPVF for modules with curvilinear boundaries can also require appropriate simplifications and transformations.

The variant to overcome these difficulties is the search for functions (or a whole family of functions), describing a sloped curved boundary of the kinematic module allowing us to simplify considerably the received powers of deformation, friction and shear forces. Subsequent finding the

optimal kinematic parameters of the process enabling to obtain the estimation of a phased change of the shape of a semi-finished product in the form of finished formulas is one of the main objectives of the research. At the same time, broadening the base of kinematic modules with various shapes of a curvilinear boundary and evaluation of the effectiveness of their embedding in the designed calculation schemes of the processes will allow solving the set problems, including those of prediction of a change in the shape of a semi-finished product.

**5. Deriving engineering formulas for the calculation of power of deformation forces for the axial trapezoidal kinematic module**

Consider the widely used axial trapezoidal module with a different choice of the type of the sloped border (Fig. 1).

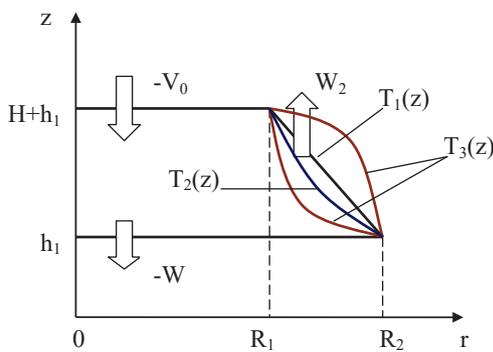


Fig. 1. Axial trapezoidal kinematic module

Let us consider KPVF in the generalized form:

$$\begin{cases} V_z = -V_0 \cdot \frac{R_1^2}{T^2(z)} + W_2 \cdot \left(1 - \frac{R_1^2}{T^2(z)}\right), \\ V_r = -\frac{r \cdot R_1^2}{T^3(z)} \cdot T'(z) \cdot (V_0 + W_2). \end{cases} \quad (8)$$

The value of  $\dot{\epsilon}_z, \dot{\epsilon}_r, \dot{\epsilon}_\theta, \dot{\gamma}_{rz}$  and of intensity of deformation rate  $\dot{\epsilon}_i$  for the given kinematic module:

$$\begin{cases} \dot{\epsilon}_1 = 2 \frac{C_1}{T^3(z)} \cdot T'(z), \quad \dot{\epsilon}_{r1} = -\frac{C_1}{T^3(z)} \cdot T'(z), \\ \dot{\epsilon}_\theta = -\frac{C_1}{T^3(z)} \cdot T'(z), \quad C_1 = (V_0 + W_2)R_1^2, \\ \dot{\gamma}_{rz} = -\frac{C_1}{T^4(z)} \left( T''(z)T(z) - 3[T'(z)]^2 \right) \cdot r, \\ \dot{\epsilon}_i = \frac{C_1}{\sqrt{3}T^3(z)} \sqrt{12[T'(z)]^2 + r^2 \left( \frac{T''(z)T(z) - 3[T'(z)]^2}{T(z)} \right)^2}. \end{cases} \quad (9)$$

**5.1. Analysis of the application of techniques for linearization of integrand dependences to calculate the power of deformation forces**

Let us consider particular cases of the given axial kinematic module, accepting various types of functions of a sloped boundary in the obtained generalized KPVF (8) and components (9).

Using the rectilinear sloped border of flow separation in the simplest case, we obtain ratios [9]:

$$T_1(z) = k \cdot (z - (H + h_1)) + R_1, \quad k = T_1'(z) = \frac{R_1 - R_2}{H}. \quad (10)$$

The expression of intensity of deformation rate  $\dot{\epsilon}_i$  for the given kinematic module takes the form:

$$\dot{\epsilon}_i = (V_0 + W_2) \sqrt{4 \cdot \left[ \frac{k \cdot R_1^2}{T^3(z)} \right]^2 + 3 \cdot r^2 \cdot \left[ \frac{k^2 \cdot R_1^2}{T^4(z)} \right]^2}. \quad (11)$$

The resulting expression (14) is rather cumbersome. To verify the possibility of using the above formulas of linearization (1)–(3) in this case, it is sufficient to solve the problem of comparative analysis of functions within a given kinematic module:

$$\begin{cases} M_1 = M_1(r, z) = 2 \cdot \frac{R_1^2}{T^3(z)} \cdot |k|, \\ M_2 = M_2(r, z) = \sqrt{3} \cdot \frac{R_1^2}{T^4(z)} \cdot k^2 \cdot r. \end{cases} \quad (12)$$

Turning to geometrical parameters of the process, referred to  $R_2$ , we will construct surfaces  $M_1(r, z)$  and  $M_2(r, z)$  (Fig. 2).

To verify the possibility of using linearization in the form of (4), it is sufficient to solve the problem of comparative analysis of functions  $\xi(r, z)$  and  $\dot{\gamma}_{rz}(r, z)$  within the given kinematic module after accepting  $\frac{\sqrt{3}}{2} \xi(r, z) = \dot{\epsilon}_r - \dot{\epsilon}_z$ . Proceeding to geometric parameters of the process, referring to  $R_2$ , we will construct surfaces  $\xi(r, z)$  and  $\gamma(r, z) = \dot{\gamma}_{rz}(r, z)$  (Fig. 3).

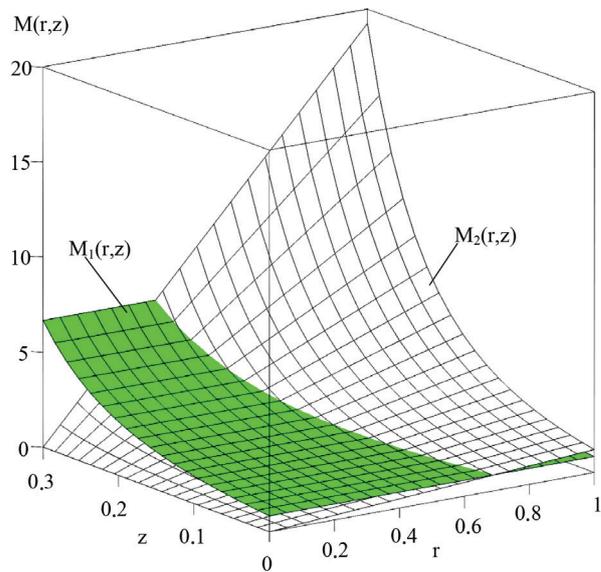


Fig. 2. Comparative analysis of components of intensity of deformation rates  $\dot{\epsilon}_i$  at  $\bar{R}_1 = 0.6, \bar{h}_1 = 0.1, \bar{H} = 0.2$

For these parameters of the process, condition  $|M_1| > |M_2|$  for using the above formulas of linearization (1)–(3) is not met over the entire region of the kinematic module. Similarly, the use of linearization in the form of (4) also leads to the complexities of determining the components that satisfy

condition  $|\xi| > |\dot{\gamma}_{rz}|$  throughout the whole volume of the axial trapezoidal module. Therefore, this method for simplification of integrand expressions to calculate the power of deformation forces is ineffective. The intensity of deformation rates  $\dot{\epsilon}_i$  for the kinematic module in its original form for the rectilinear border enabled us to find the power of deformation forces and the magnitude of reduced pressure  $\bar{p}_1$  in the analytical form [12].

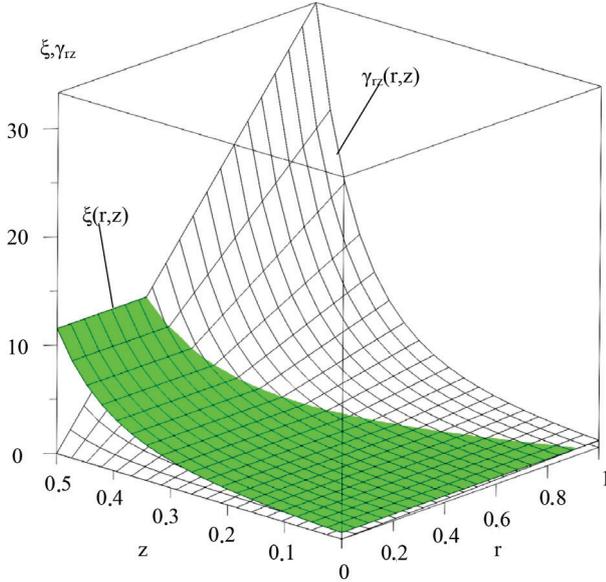


Fig. 3. Comparative analysis of components of intensity of deformation rates  $\dot{\epsilon}_i$  at  $\bar{R}_1=0.5$ ,  $\bar{h}_1=0.2$ ,  $\bar{H}=0.3$

**5. 2. Techniques to derive the power of deformation forces in analytical form**

The transition to a curvilinear sloped axial border of the trapezoidal kinematic module leads to the complication of the mathematical apparatus. Linearization of integrand expressions of deformation intensity  $\dot{\epsilon}_i$  is difficult because of impossibility of an unambiguous selection of functions that satisfy condition  $|M_1| > |M_2|$  within the entire volume of the given kinematic module. The possibility of obtaining the power of deformation forces in analytical form for arbitrarily selected function  $T=T(z)$ , passing through points  $(R_1, H+h_1)$  and  $(R_2, h_1)$  becomes much more complicated.

Let us select the kind of a boundary in the form of function  $T=T_2(z)$ , which satisfies differential equation:

$$T''(z) \cdot T(z) - 3 \cdot [T'(z)]^2 = 0. \tag{13}$$

Taking into consideration the general solution of this equation and boundary conditions  $T_2(H+h_1)=R_1$  and  $T_2(h_1)=R_2$ , we finally obtain:

$$T_2(z) = \frac{1}{\sqrt{C_1 \cdot z + C_2}}, \tag{14}$$

where

$$C_1 = (R_2^2 - R_1^2) / (R_1^2 \cdot R_2^2 \cdot H), \quad C_2 = 1/R_2^2 - C_1 \cdot h_1.$$

Selection of function (14) as a sloped boundary allows us to considerably simplify the expression of deformation

intensity and to use for subsequent calculations the function that takes the form:

$$\dot{\epsilon}_i = -\frac{2 \cdot R_1^2}{T^3(z)} \cdot T'(z) \cdot (V_0 + W_2). \tag{15}$$

Thus, there is no need to use linearization of integrand expressions to calculate the power of deformation forces. Using (15), we obtain the power of deformation forces in the form:

$$N_{\partial 2} = \pi \sigma_s \cdot R_1^2 \cdot (V_0 + W_2) \times \left[ C_1 \cdot R_1^2 \cdot H + 2 \cdot \ln \frac{R_2}{R_1} - (C_2 + C_1 \cdot h_1) \cdot (R_2^2 - R_1^2) \right]. \tag{16}$$

A comparative analysis of intensity of deformation rates  $\dot{\epsilon}_i$  (Fig. 4) for rectilinear  $T=T_1(z)$  and curvilinear  $T=T_2(z)$  sloped border revealed that it was impossible to point out clearly the priority of choosing one of them. In this case, for this axial trapezoidal kinematic module, engineering formulas for calculation of the power of deformation forces were obtained both in the case of rectilinear and curvilinear sloped border. However, the choice of curvilinear sloped border  $T=T_2(z)$  meets the assumption  $\dot{\gamma}_{rz} = 0$ , the validity of which requires compulsory verification during the calculations of not only power parameter, but also of a change in the shape of specific process of combined extrusion.

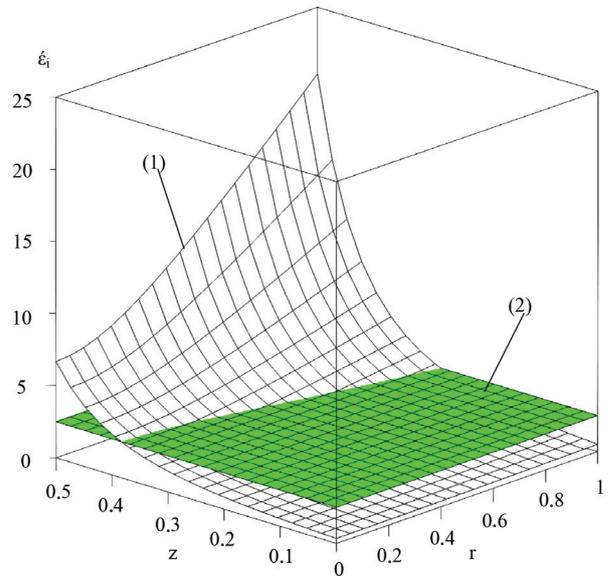


Fig. 4. Comparative analysis of magnitude  $\dot{\epsilon}_i$  for rectilinear sloped border (1) and curvilinear sloped border (2) at  $\bar{R}_1=0.5$ ,  $\bar{h}_1=0.2$ ,  $\bar{H}=0.3$

Accepting in the general expression KPVF (8) the variant of substitution of the expression of shear deformation with function  $T=T_3(z)$  that is more «convenient» in terms of the subsequent use, we obtain known expressions [9]:

$$T_3(z) = A \cdot [B - C \cdot e^{2M(z-h_1)}]^{-1/2}, \tag{17}$$

where

$$A = R_1 R_2 \sqrt{1 - e^{2MH}}, \quad B = R_2^2 - R_1^2 e^{2MH}, \quad C = R_2^2 - R_1^2.$$

Similarly to the previous scheme, the components for the calculation of energy-power parameters of the process were obtained [9]. Linearization of integrand expressions of deformation intensity  $\dot{\epsilon}_i$  is also complicated because of the impossibility of an unambiguous choice of the functions that satisfy conditions (1)–(3) within the entire volume of the given kinematic module. Selection of the family of functions of kind (17) made it possible to take into consideration the influence of shear deformations, but significantly complicated obtaining the magnitude of power of deformation forces.

**5. 3. Comparative analysis and identification of conditions for using the developed KPVF**

The axial trapezoidal module with a rectilinear sloped boundary  $T=T_1(z)$  of the separation of modules 1 and 3 was used in modeling the process of the combined radial-reverse extrusion (Fig. 5) of parts with a flange and showed efficiency of using at  $2h_1R_2 / (R_2^2 - R_1^2) < 1$ . Calculation formulas for determining the magnitude of reduced pressure and increment in dimensions of a semi-finished product were obtained [12]. The use of a curvilinear sloped border in the form  $T=T_3(z)$  in modeling process of the combined radial-reverse extrusion of parts with a flange showed the effectiveness of using at  $2h_1R_2 / (R_2^2 - R_1^2) > 1$ . However, the optimization of parameter  $M \in (-\infty, 0) \cup (0, +\infty)$  is complicated and requires approximate or numerical calculations, which prevents obtaining increments of dimensions of a semi-finished product in the explicit form.

That is why the study of embeddability of module 1 with a sloped boundary  $T=T_2(z)$  as more convenient in subsequent calculations of the combined extrusion process is relevant. Special attention will be paid to the possibility of obtaining explicitly and the verification of conformity of the magnitude of increments of dimensions of a semi-finished product in the process of combined radial-reverse extrusion with the real one.

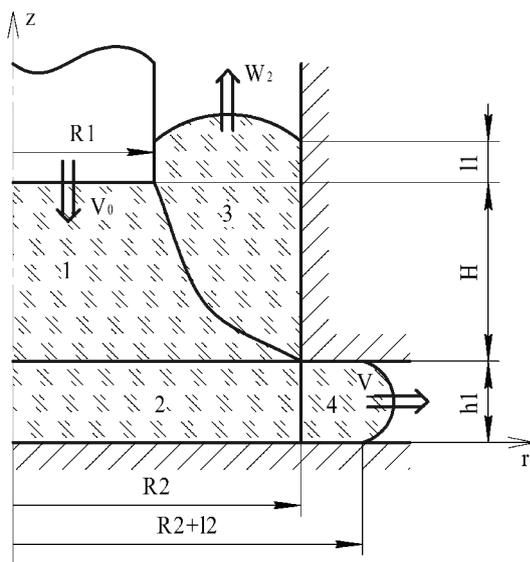


Fig. 5. Calculation scheme OOD-1.2 of the process of combined radial-reverse extrusion [12]

Calculation of the components of powers of forces of deformation, cutting and friction at the given form of the sloped border is straightforward. The optimal value of the rate of metal flow in the vertical direction, taking into consideration

the condition that the power of shear forces between kinematic modules 1 and 2 is equal to zero, takes the form:

$$W_2 = \frac{R_1^2 \cdot H - h_1 \cdot (R_2^2 - R_1^2)}{(R_2^2 - R_1^2) \cdot (H + h_1)} \cdot V_0. \tag{18}$$

Taking into consideration ratio (18), by integrating in section  $[0, Hx]$ , we obtained the engineering formulas of calculation of the increment of a semi-finished product in vertical  $\Delta l_1 \uparrow$  and radial  $\Delta l_2 \rightarrow$  directions in the course of the deformation process at the initial height of billet  $H_0$ :

$$\Delta l_1 \uparrow = \frac{1}{R_2^2 - R_1^2} \cdot \left( R_1^2 \cdot \Delta Hx + h_1 \cdot R_2^2 \cdot \ln \left| 1 - \frac{\Delta Hx}{H_0} \right| \right), \tag{19}$$

$$\Delta l_2 \rightarrow = \sqrt{\frac{R_1^2 \cdot \Delta Hx - (R_2^2 - R_1^2) \cdot l_1 \uparrow}{h_1} + R_2^2 - R_1^2}.$$

Taking into consideration the found optimal value of the rate of metal flow in the vertical direction, we obtained the magnitude for the reduced pressure:

$$\bar{p}_2 = \left[ \begin{aligned} & (1 + \bar{W}_2) \cdot \left[ C_1 \cdot R_1^2 \cdot H + 2 \cdot \ln \frac{R_2}{R_1} - \right. \\ & \left. - (C_2 + C_1 \cdot h_1) \cdot (R_2^2 - R_1^2) + \right. \\ & \left. + \frac{2}{\sqrt{3}} \cdot \ln \left( \frac{R_2}{R_1} \right) + \frac{C_1^2}{12\sqrt{3}} \cdot (R_2^6 - R_1^6) \right] + \\ & + \bar{W}_1 \cdot \left( \frac{R_2^2}{R_1^2} \cdot \left( 1 + \frac{2}{\sqrt{3}} \cdot \ln \left( \frac{R_2 + l_2}{R_2} \right) \right) + \frac{R_2 \cdot h_1}{\sqrt{3} \cdot R_1^2} \right) + \\ & + \frac{2}{3\sqrt{3}} \cdot \mu_s \cdot \left[ \begin{aligned} & (1 + \bar{W}_2) \cdot \left( C_1 \cdot R_1^3 + 6 \cdot \frac{l_1 + \Delta Hx}{R_1} \right) + \\ & + 6 \cdot \bar{W}_2 \cdot \frac{R_2 \cdot (H + l_1)}{R_1^2} + \\ & + \bar{W}_1 \cdot \left( \frac{R_2^3}{R_1^2 \cdot h_1} + 6 \cdot \frac{R_2^2 \cdot l_2}{R_1^2 \cdot h_1} \right) \end{aligned} \right] \end{aligned} \right], \tag{20}$$

where

$$C_1 = \frac{R_2^2 - R_1^2}{R_1^2 \cdot R_2^2 \cdot H}, \quad C_2 = \frac{1}{R_2^2} - C_1 \cdot h_1,$$

$$\bar{W}_2 = \frac{R_1^2 \cdot H - h_1 \cdot (R_2^2 - R_1^2)}{(R_2^2 - R_1^2) \cdot (H + h_1)}, \quad \bar{W}_1 = \frac{R_1^2 - \bar{W}_2 \cdot (R_2^2 - R_1^2)}{R_2^2}.$$

The character of increments in a semi-finished product during the process for rectilinear  $T=T_1(z)$  and curvilinear  $T=T_2(z)$  sloping boundary is identical. However, a curvilinear sloped boundary of flow separation is characterized by lower values of magnitude  $\Delta l_1 \uparrow$  and higher values  $\Delta l_2 \rightarrow$  compared to the use of the kinematic module with a rectilinear boundary of flow separation. Comparative analysis of increments of dimensions of a semi-finished product was performed for (1):  $R_1=10.5$  mm,  $R_2=14$  mm,  $h_1=5$  mm,  $H_0=14$  mm and (2):  $R_1=10.5$  mm,  $R_2=14$  mm,  $h_1=3$  mm,  $H_0=14$  mm, based on the proposed theoretical assessments with point data obtained by modeling in Qform2/3d (Fig. 6). The point data

of the increment of a semi-finished product in the vertical direction, obtained by modeling in Qform2/3d are presented below (Fig. 7). Both schemes of the process demonstrate the overstatement of theoretically derived data. However, the deviation of magnitude  $\Delta l_1 \uparrow$  throughout the entire deformation process for  $T=T_2(z)$  indicates the smallest deviation compared to  $T=T_1(z)$  from the point data obtained by modeling in Qform2/3d. We performed comparative analysis of the magnitude of reduced pressures for  $R_1=10.5$  mm,  $R_2=14$  mm,  $H_0=14$  mm at  $\mu_s=0$  and  $H_x=0$  for the calculation schemes at different magnitudes of the height of a flange (Fig. 8). Value  $\bar{p}_2$  of the scheme with a curvilinear sloped boundary  $T=T_2(z)$  has lower values at admissible heights of a flange in comparison with the obtained magnitude  $\bar{p}_1$  based on the scheme with a rectilinear sloped boundary  $T=T_1(z)$ .

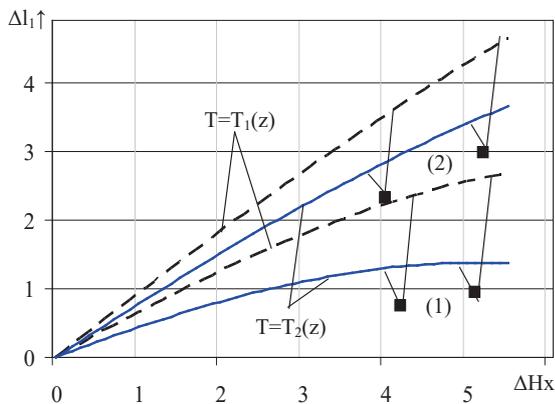


Fig. 6. Comparative analysis of increments of dimensions of a semi-finished product at  $T=T_1(z)$  and  $T=T_2(z)$

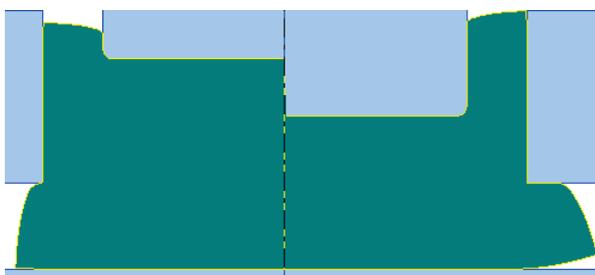


Fig. 7. Point data of increment of semi-finished product in the vertical direction, received by modeling in Qform2/3d

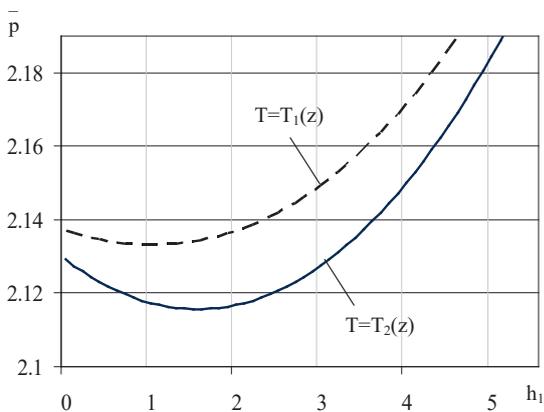


Fig. 8. Comparative analysis of reduced pressure at  $T=T_1(z)$  and  $T=T_2(z)$

A similar pattern is also observed for modeling of the process at  $R_1=10.5$  mm,  $R_2=18$  mm,  $H_0=20$  mm. The smallest deviation of increments of a semi-finished product in the vertical direction from those obtained in Qform2/3d corresponds to the calculation scheme with a rectilinear boundary of flow separation (Fig. 9, a). In this case, the lowest value of reduced pressure for admissible values of flange height  $h_1$ , including those at  $h_1=5$  mm, (corresponds to the given process of deformation) was also obtained for the scheme with the line of flow separation in the form of  $T=T_1(z)$  (Fig. 9, b).

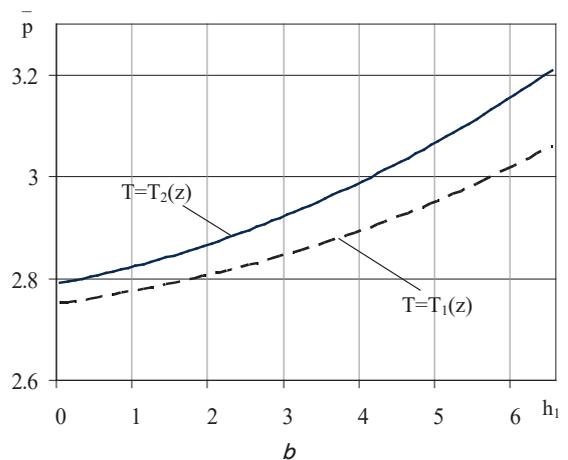
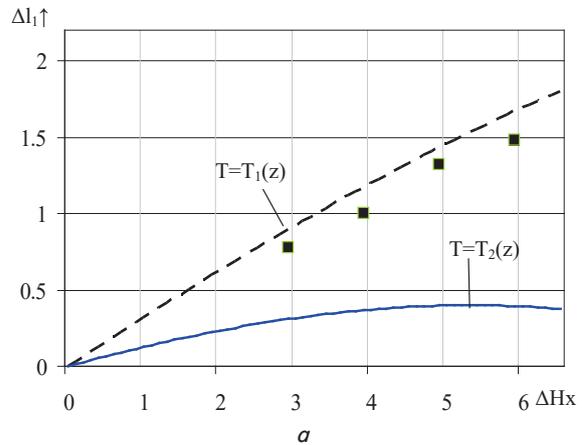


Fig. 9. Comparative analysis of at  $T=T_1(z)$  and  $T=T_2(z)$ :  
 a – of increments of a semi-finished product;  
 b – of reduced pressure

Thus, the smallest value of energy-power parameters of the deformation process acts as the criterion for selection of the appropriate calculation scheme and formulas for calculation of increments of semi-finished product in the process of deformation. Determining the boundaries of using the proposed calculation schemes with the possibility to obtain the data on a change in the shape of a semi-finished product in the form of ready formulas is an urgent task in this case. Based on the ease of comparing the obtained engineering formulas  $\bar{p}_1$  [12] and the proposed formula  $\bar{p}_2$ , condition of the preferable use of the calculation scheme with  $T=T_2(z)$  takes the form  $\bar{p}_2 < \bar{p}_1$ . Thus, admission  $\dot{\gamma}_{rz}=0$  is correct and the given kinematic module with a curvilinear boundary in the form of  $T=T_2(z)$  can be recommended for the use in the framework of the calculation scheme of the combined radial-reverse extrusion.

#### 5.4. Development of an algorithm to calculate the processes of combined extrusion based on the proposed KPVF

An important stage of designing the technological process of combined extrusion with several degrees of flow freedom is preliminary assessment of the final change in the shape of a billet. Availability of engineering formulas for the computation of increments of a semi-finished product and forces in the course of the deformation process will contribute to decreasing the cost and time of design and construction preparation of production.

Based on the energy method of the upper assessment, it is possible to construct a vast number of calculation schemes that correspond to various technological parameters of the process. The algorithm of calculation of processes of combined extrusion based on KPVF consists of the following stages.

The first stage includes an analysis of metal flow inside a billet. This will allow determining the rationality of using the proposed module in the general form as an axial kinematic module. Availability of characteristics of a sloped boundary of the developed kinematic module (convexity or concavity, straightness) will enable determining the type of curve  $T=T(z)$ .

The second stage of calculation of the components of reduced pressure during selecting a sloped boundary in the form of  $T=T_2(z)$  is based on the use of power of deformation forces in the form of (16). However, if the adjacent module 2 is identical to the calculation scheme OOD-1.2, the optimal value of the rate of metal flow in the vertical direction  $W_2$  is determined by ratio (18). If module 2 is different from the one above, but  $V_r > 0$ , the optimal value of magnitude  $W_2$  can be found taking into consideration that the power of shear forces between kinematic modules 1 and 2 is equal to zero.

The third stage of calculation of the increments of a semi-finished product in the vertical direction  $\Delta l_1 \uparrow$  includes integration of the optimal value of kinematic parameter  $W_2$  on section  $[0, Hx]$ .

If the character of the sloped boundary of the axial kinematic module is ambiguous, it is necessary to use all of the enumerated types of curve  $T=T(z)$  and comparative analysis of the obtained reduced pressures. The criterion for selection of the preferable calculation scheme will be the lowest value of magnitude  $\bar{p}$ .

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#### 6. Discussion of results of studying the processes of combined extrusion based on the method of kinematic modules

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The research proved the ineffectiveness of linearization of integrand dependences for calculation of the power of deformation forces inside the axial trapezoidal kinematic module. It is associated with a complex pattern of flow inside kinematic modules with several boundaries of output velocities. To overcome these difficulties, the ways of simplification of the intensity of deformation rates based on the selection of the «convenient» function of the boundary of flow separation. In this case, the use of assumption  $\dot{\gamma}_{rz} = 0$  in determining the type of function  $T=T_2(z)$ , describing a sloped curvilinear boundary of the axial kinematic module made it possible to simplify the intensity of deformation rates. This made it possible to obtain the magnitudes of reduced pressure in the analytical form  $\bar{p}_2$  and optimize the rate of

metal flow in the vertical direction. As a result, we managed to obtain the engineering formulas of increments of a semi-finished product in the course of the deformation process. They can be used based of the proposed algorithm for calculating the processes of combined extrusion with some degrees of flow freedom. The stages of calculation include analysis of metal flow inside a billet, the rational selection of the developed KPVF and the use of the obtained magnitudes of power of deformation forces and the optimal value of kinematic parameter  $W_2$ .

Thus, obtaining the data on final dimensions of a semi-finished product does not require the use of commercial software products of modeling of the process. The restrictions of using the developed calculation scheme with the existence of the boundary of flow separation  $T=T_2(z)$  were detected for the process of combined radial-reverse extrusion of hollow parts with a flange. It is recommended to use the condition of the least value of reduced pressure as the selection criteria for the developed analytical solutions and those known earlier.

The aim of further research is to develop new kinematic modules with curvilinear boundaries and study their embeddability calculation schemes of combined extrusion. This will make it possible to identify the boundaries of their rational use for the prediction of shape formation in various processes of combined extrusion with several degrees of flow freedom.

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#### 7. Conclusions

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1. The possibilities of using known techniques for linearization of integrand dependences for calculation of the power of deformation forces were analyzed. We showed the ineffectiveness of their use for the axial trapezoidal kinematic module with a sloped boundary due to the complexity of determining the components, satisfying the required conditions throughout the entire volume of the module.

2. The ways of simplification of the function of deformation intensity using assumption  $\dot{\gamma}_{rz} = 0$  for the axial kinematic module with a curvilinear sloping boundary were proposed. On its basis, the dependences of the power of deformation forces in the analytical form were obtained, which greatly simplifies the further assessment of energy-power parameters of the deformation process.

3. The comparative analysis of dependences and reduced pressure and increments of a semi-finished product obtained with the use of various functions, describing a curvilinear boundary of the axial trapezoidal module was performed. This made it possible to prove the validity of using assumption  $\dot{\gamma}_{rz} = 0$  for the axial trapezoidal kinematic module with the boundary in the form of curve  $T=T_2(z)$ . The conditions of using the developed KPVF for boundary  $T=T_2(z)$  in the form of inequality  $\bar{p}_2 < \bar{p}_1$  compared to the previously used rectilinear boundary  $T=T_1(z)$  were determined.

4. The algorithm of calculation of energy-power parameters and of prediction of a change in the shape of a semi-finished product in the processes of combined extrusion based on the proposed KPVF was developed. Thus, we have proven that the developed kinematic module could be effectively embedded into general calculation schemes for combined extrusion.

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