

Розроблено комплексну математичну модель стану каналу багатоантенних систем радіозв'язку. Модель враховує: вплив навмисних завад та завмирань сигналу, кількість приймальних антен, ефект Доплера, коефіцієнт кореляції, швидкості та напрямки руху приймача та передавача, міжсимвольну інтерференцію, фазовий джитер та нахил констеляційної матриці. Моделювання стану каналу багатоантенних систем радіозв'язку проведено для кожного окремого антенного каналу, після чого на виході формується узагальнена оцінка. Розробка запропонованої комплексної математичної моделі обумовлена необхідністю підвищення точності опису стану каналу багатоантенних систем радіозв'язку з прийнятною обчислювальною складністю. Запропонована модель дозволяє підвищити точність опису стану каналу багатоантенних систем радіозв'язку за рахунок врахування додаткових дестабілізуючих факторів, тим самим підвищити точність оцінювання стану каналу. Хотілося б зазначити, що при цьому відмічається збільшення обчислювальної складності на рівні 5–7% за рахунок збільшення кількості показників, що оцінюються. Зазначену комплексну математичну модель доцільно використовувати в радіостанціях з програмованою архітектурою для підвищення їх заводо захищеності за рахунок підвищення точності оцінювання характеристик приймально-передавального тракту щодо стану каналу. Проведено вивчення кореляції між антенами багатоантенних систем радіозв'язку. Результати показують, що за наявності прямої видимості між приймачем та передавачем кореляція сигналу є високою і тому очікується невеликий приріст від використання декількох антен, а при відсутності умов прямої видимості кореляція сигналу низька

Ключові слова: засоби радіозв'язку, модель Джейкса, спектр Доплера, обчислювальна складність, констеляційна матриця, заводо захищеність

DEVELOPMENT OF A COMPLEX MATHEMATICAL MODEL OF THE STATE OF A CHANNEL OF MULTI-ANTENNA RADIO COMMUNICATION SYSTEMS

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1. Introduction

The MIMO (Multiple Input Multiple Output) technology has found practical use in many modern telecommuni-

cation systems. The MIMO technology is used in wireless LANs of the IEEE 802.11x standard, as well as WIMAX and LTE wireless networks, etc. [1–5]. The essence of the MIMO technology is similar to the method of diversity reception,

when several uncorrelated copies of the signal are created on the receiving side due to antenna diversity, polarization, frequency or time diversity of the signal.

Spatial multiplexing is implemented in MIMO radio systems: the data stream in the transmission is split into two or more sub-streams, each of which is transmitted and received via different antennas [1–10].

The noise immunity of multi-antenna radio communication systems is influenced by intentional noise and signal fading that arise during multipath propagation of radio waves. In order to provide a stable radio communication in conditions of active radio-electronic suppression and selective fading, the radio communication system should have information about signal and noise conditions in the channel.

The analysis of recent local conflicts has shown that they are the most common in cities. This indicates the increase in the requirements for radio communication stability, which is associated with a high density of buildings and high mobility of subscribers.

The increase of the requirements for radio communication in cities determines the consideration of more parameters that affect the quality of radio communication.

The mentioned causes the search for new scientific approaches to improve the accuracy of the description of the functioning of MIMO system channels.

All this confirms the relevance of the chosen research direction.

2. Literature review and problem statement

The feature of the multipath channel is its non-stationary nature due to the presence of constant changes in the signal propagation conditions in the channel, which leads to the distortion of the transmitted signal. In addition to distortions caused by the special nature of radio wave propagation, the transmitted signal may be affected by intentional and accidental interference.

In the article [3], a mathematical model of the MIMO system for an unmanned aerial vehicle is developed. The given mathematical model takes into account the space-time correlation function, the Doppler spectral power density, the level crossing rate and the average fading time. The disadvantage of the proposed model is the neglect of the intentional noise effect and phase characteristics of the channel.

In the article [4], a geometric model of the state of the channel of the MIMO system is developed. The proposed model is designed to take into account the movement of cars in the transmission of information through the channels of the MIMO system. In this model, the angle of incidence and the angle of deviation are variable parameters. However, the proposed model is based only on one ring of scatterers and does not take into account the effect of intentional interference and other destabilizing factors.

In the article [5], the development of a 3D mathematical model of the MIMO system channel for an unmanned aerial vehicle is carried out. The given mathematical model is based on the traditional geometric model and takes into account the space-time correlation function, the Doppler spectral power density, the level crossing rate and the average fading time. The disadvantage of the proposed model is the neglect of the intentional noise effect and phase characteristics of the channel.

In the article [6], the mathematical model of the MIMO system channel for unmanned aerial vehicles is proposed.

In the indicated mathematical model, the propagation medium is represented in the form of three-dimensional elliptic cylinders. The given model takes into account only spatial correlation of signals, which does not allow to obtain full information about the MIMO system channel.

The article [7] is devoted to the development of an integrated model describing the state of the channel of multi-antenna systems. The integrated model takes into account the basic characteristics of the radio wave propagation environment and antenna array parameters. The main parameters, taken into account in the model are signal attenuation, signal fading, signal correlation, cross-polarization coupling. However, the model does not take into account the effect of intentional interference, and also the indicated mathematical model does not work at a low signal/noise ratio.

In the article [8], a 3D mathematical model of the MIMO system is developed. The developed mathematical model is theoretically informative with a higher degree of freedom in height. However, this mathematical model is intended only for calculating the signal propagation path and does not take into account most of the known characteristics of the channel.

In the article [9], the authors consider the effect of mutual interference at the transmitter output on the performance of MIMO systems. The assessment of the effect of mutual interference is carried out only on the basis of the bit error probability, which prevents the development of effective multi-parametric measures of adaptation to the situation in the channel.

In the article [10], the authors consider the influence of relief characteristics on the performance of MIMO systems. The proposed model takes into account the Doppler shift, time correlation, polarization correlation, and the spatial-statistical correlation properties of fading. This mathematical model is limited only to the consideration of the signal propagation path without taking into account the influence of noise and channel characteristics of the MIMO system.

In the article [11], the authors proposed a channel model upward for MIMO systems. The proposed mathematical model is designed to calculate the instantaneous channel matrix, but it does not take into account the slope of the constellation matrix of the channel and signal propagation parameters.

In the article [12], the authors proposed a model of the MIMO system. The proposed mathematical model is designed to calculate the correlation matrix of spatial channels in a polarized homogeneous plane lattice, taking into account the bit error probability and spectral efficiency. The indicated mathematical model is not intended for work in complex conditions of the radio-electronic situation and does not take into account the phase characteristics of the channel.

In the article [13], the authors proposed a model of the MIMO system. The proposed mathematical model is designed to calculate the entropy capacity, channel ellipticity, and the mean square delay diversity. The given mathematical model is developed for the description of spatial characteristics of the channel, is not intended for the work in complex conditions of electronic environment and does not take into account the phase characteristics of the channel.

In the article [14], the authors proposed a model of the MIMO system. The proposed mathematical model is designed to calculate signal propagation parameters, namely, delay and arrival direction. The mathematical model is also designed to describe the spatial characteristics of the

channel, is not designed to work in complex conditions of the radio electronic environment and does not take into account the phase characteristics of the channel.

In the article [15], the authors proposed a model of the MIMO system. The proposed mathematical model is intended for the calculation of energy and spectral characteristics of the channel of the MIMO system. However, the model is adapted for operation in city conditions and is not intended for work in complex conditions of the radio-electronic situation and does not take into account the parameters of radio wave propagation.

In articles [16], the authors proposed a model of the MIMO system. The proposed mathematical model is intended for the calculation of energy and spectral characteristics of the system channel of multisite MIMO systems. However, the model is adapted for operation in the city and is not intended to work in difficult conditions of the radio electronic environment and does not take into account the parameters of radio wave propagation.

In articles [17, 18], the authors proposed a model of the MIMO system. The proposed mathematical model is designed to calculate losses on the propagation path for MIMO systems. However, the model is adapted for operation in city conditions and is not intended for work in difficult electronic conditions and does not take into account the parameters of phase characteristics of the channel.

In the article [19, 20], an analysis of the influence of intentional interference of additive and multiplicative nature on the functioning of the MIMO system is carried out. However, the authors of these works do not take into account the signal propagation loss and movement of network subscribers.

The known mathematical models do not take into account the effect of intentional interference, the slope of the constellation matrix of the channel, the inter-symbol interference, the phase jitter, the type of signal-code design and the effect of signal fading. Also, the known scientific papers do not accurately take into account the influence of the transmitter and receiver motion on the functioning of the MIMO system channel. These drawbacks can be eliminated by developing an integrated mathematical model of the MIMO system channel state, which would take into account a greater number of destabilizing factors and would have a moderate computational complexity.

3. The aim and objectives of the study

The aim of the study consists in developing a comprehensive mathematical model of multi-antenna radio communication systems that allows taking into account a greater number of destabilizing factors with moderate computational complexity.

To achieve the goal, the following objectives were set:

- to determine the list of destabilizing factors and substantiate the structure of the state model of the channel of multi-antenna radio communication systems;
- to check the adequacy and reliability of the proposed model of the MIMO system;
- to identify the advantages and disadvantages of the proposed model.

4. Determination of the list of destabilizing factors and substantiation of the structure of the state model of the channel of multi-antenna radio communication systems

Evaluation of efficiency and modeling of the proposed complex mathematical model of the MIMO system channel were carried out in the MathCad 14 software.

Verification of the adequacy and reliability of the MIMO system channel model developed in the work, was conducted by comparing with the results presented by the *Iterative Solutions* group [21]. The developments presented on this electronic resource allow to simulate the characteristics of noise immunity of almost all known mobile data transmission systems.

In general, the MIMO system has M_t transmitters (transmitting antennas) and M_r receivers (receiving antennas) (Fig. 1). The transmitted signals arrive to M_r receiving channels [1–10].

Consider the MIMO system $M_t \times M_r$, shown in Fig. 1. The high-speed data flow is divided into M_t independent sequences at $1/M_t$, which are then transmitted simultaneously from several antennas, respectively, using only $1/M_t$ of the primary frequency band.

The data flow converter at the transmitter end of the communication line converts the serial stream into parallel, and at the receiving terminal, it performs the inverse transformation. In the indicated mathematical model, the following types of intentional interference: barrage noise jamming and partial band noise jamming are considered. These types of intentional noise are represented as additive white Gaussian noise with different overlapping coefficients [2].

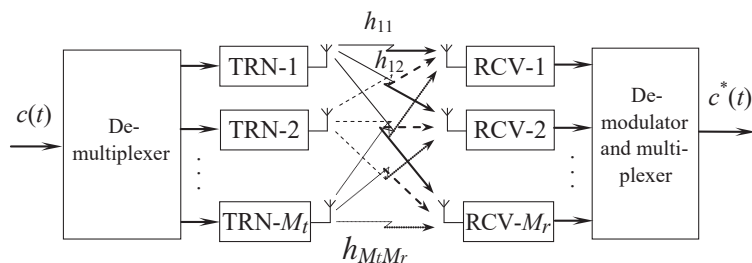


Fig. 1. Block diagram of the MIMO system

Problem statement.

Given: the parameters of the MIMO system and the communication channel $\Psi = \{\psi_i\}$, $i = 1, m$: M_t is the number of transmitting antennas, M_r is the number of receiving antennas, the type of modulation, P_c is the power of the useful signal, ΔF_c is the frequency band of the signal, h_c^2 is the signal/noise ratio (SNR), $H(t)$ is the transmission characteristic of the communication channel, R is the code speed.

Limitations and assumptions: modulation type is the phase modulation (PM- M), where M is the volume of the signal ensemble; the dimension of the signal ensemble $2 \leq M \leq 16$; type of the noise-immune code – a turbo code with the speed $0.2 \leq R \leq 0.75$; $M_t \leq 8, M_r \leq 8$, the speed of movement is 120 km/h; the frequency range of the transmitter is 5.8–6.0 GHz; types of intentional interference: barrage noise jamming and partial band noise jamming.

The MIMO multipath channel model can be represented in accordance with [5–9] as follows:

$$z_r(t) = \text{Re} \left[\sum_{H_{\min}}^{H_{\max}} (C_{l,\partial,r} \times \Psi_{r,c,l}) \times \sum_{\xi=1}^{\Xi} \sum_{n=1}^{N_l} \left[\kappa_{chn} u_x(t - (s-1)T - \tau_l) + \kappa_{slm} u_x(t - (s-1)T - \tau_l) + n(t) \right] \right], \quad 0 < t < \infty, \quad (1)$$

where κ_{slm} , κ_{chn} are random in-phase and quadrature components of the transmission coefficient in the l -th ray of the s -th transmitting element, Ξ is the total number of beams in the channel, τ_l is the delay time in the l -th ray, u_x are the known realizations of signals at the receiver input, $x=(0.1,\dots,M-1)$ is the sequence of information symbols, M is the position of the signal modulation, T is the transmission time of the signal elements, $n(t)$ is the intentional interference presented in the form of additive white Gaussian noise with different overlap coefficients of the spectrum, $C_{l,\partial,r}$ is the value of the signal-code construction point for the l -th ray in the symbol ∂ of the frame r ;

$$\Psi_{r,\partial,l}(t) = \begin{cases} \left(\frac{t - sT_\partial - \Xi T_\partial}{T_u} \right) & \text{for } (s + Nr)T_s \leq t \leq (\Xi_r + 1)T_\partial, \\ 0 & \text{for other occasions,} \end{cases} \quad (2)$$

$$l' = l - (H_{\max} + H_{\min})/2, \quad T_\partial = T_u + T_g, \quad (3)$$

where H_{\min} and H_{\max} are the minimum and maximum values of the MIMO channel (lower and upper limits); r is the MIMO frame number, T_u is the duration of the useful part of the MIMO frame, T_∂ is the MIMO frame length, f is the reference frequency of the MIMO transmitter.

Consider the process of transmitting one element of MIMO ($t \in [0, NT_\partial]$) [8, 10].

The signal $z^*(t)$ at the receiver input has the form:

$$z^*(t) = z(t) + n(t), \quad (4)$$

where $n(t)$ is the function that describes the noise in the channel, consisting of the useful signal $z(t)$, which can be represented as the relation:

$$C_{\partial,l}^* = C_{\partial,l} + n_{\partial,l}, \quad (5)$$

where $n_{\partial,l}$ is the component $n(t)$ superimposed on $C_{\partial,l}$ in the symbol ∂ of the MIMO channel l after the transformation.

Since $C_{\partial,l}^*$ is a complex number, its real and imaginary parts in (4) are conveniently presented as a matrix of channel components:

$$\begin{pmatrix} \text{Re}\{C_{\partial,l}^*\} \\ \text{Im}\{C_{\partial,l}^*\} \end{pmatrix} = \begin{pmatrix} \text{Re}\{C_{\partial,l}\} \\ \text{Im}\{C_{\partial,l}\} \end{pmatrix} + \begin{pmatrix} \text{Re}\{n_{\partial,l}\} \\ \text{Im}\{n_{\partial,l}\} \end{pmatrix}. \quad (6)$$

Since the linear transformation matrix is inverted, its geometric distortions are similar to the sequence of reflections, rotations, extensions, and shifts.

In the case of reflection is impossible, because the noise can not cause a transformation. The other three transformations correspond to phase shift, amplitude mismatch and quadrature error. Also in the typical channel, there are inter-symbol interference, phase jitter and Gaussian noise.

Let us consider each of the transformations:

$$\left(\text{Re}\{C_{\partial,l}\} \text{Im}\{C_{\partial,l}\} \right)^T \rightarrow \left(\text{Re}\{C_{\partial,l}^*\} \text{Im}\{C_{\partial,l}^*\} \right)^T$$

in the absence of $n_{\partial,l}$. The phase shift is a deterministic phase error, representing the rotation of the constellation diagram around the axis by the angle θ_{const} :

$$\begin{pmatrix} \text{Re}\{C_{\partial,l}^*\} \\ \text{Im}\{C_{\partial,l}^*\} \end{pmatrix} = \begin{pmatrix} \cos \theta_{\text{const}} & -\sin \theta_{\text{const}} \\ \sin \theta_{\text{const}} & \cos \theta_{\text{const}} \end{pmatrix} \begin{pmatrix} \text{Re}\{C_{\partial,l}\} \\ \text{Im}\{C_{\partial,l}\} \end{pmatrix}. \quad (7)$$

The inconsistency of amplitudes is realized by introducing the gain k_E for a real channel, different from the corresponding gain of the imaginary channel, namely:

$$\begin{pmatrix} \text{Re}\{C_{\partial,l}^*\} \\ \text{Im}\{C_{\partial,l}^*\} \end{pmatrix} = \begin{pmatrix} k_E & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \text{Re}\{C_{\partial,l}\} \\ \text{Im}\{C_{\partial,l}\} \end{pmatrix}. \quad (8)$$

The quadrature error is a result of multiplying by the matrix, which causes the inclination of the constellation diagram relative to the reference value:

$$\begin{pmatrix} \text{Re}\{C_{\partial,l}^*\} \\ \text{Im}\{C_{\partial,l}^*\} \end{pmatrix} = \begin{pmatrix} 1 & k_s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \text{Re}\{C_{\partial,l}\} \\ \text{Im}\{C_{\partial,l}\} \end{pmatrix}, \quad (9)$$

where k_s is the angle of inclination of the imaginary and actual component of the channel from the orthogonality.

Divide the component $n_{\partial,l}$ into two components, one of which (n_ϕ) is associated with inter-symbol interference in the symbol ∂S , another (n_i) is the additive white Gaussian noise in the MIMO channel. Given that the interference is caused by a false signal, which causes the shift of symbols in the constellation diagram. The interference is modeled by a false signal vector with the amplitude A and phase ϕ , which depends on the moment of measurement and the frequency difference between false and useful signals. Then the matrix $(\text{Re}\{n_{\partial,l}\} \text{Im}\{n_{\partial,l}\})^T$ will look like:

$$\begin{pmatrix} \text{Re}\{n_{\partial,l}\} \\ \text{Im}\{n_{\partial,l}\} \end{pmatrix} = \begin{pmatrix} A \cos \phi \\ A \sin \phi \end{pmatrix} + \begin{pmatrix} \text{Re}\{n_i\} \\ \text{Im}\{n_i\} \end{pmatrix}. \quad (10)$$

Phase jitter, in contrast to phase shift, inconsistency of amplitudes, and quadrature error, is a random error and causes the constellation diagram to rotate by the angle θ_i . This random error is a random variable with Gaussian distribution, zero mean and variance σ_i^2 , i. e. $\theta_i \sim G(0, \sigma_i^2)$. Then:

$$\begin{pmatrix} \text{Re}\{C_{\partial,l}^*\} \\ \text{Im}\{C_{\partial,l}^*\} \end{pmatrix} = \begin{pmatrix} \cos \theta_i & -\sin \theta_i \\ \sin \theta_i & \cos \theta_i \end{pmatrix} \begin{pmatrix} \text{Re}\{C_{\partial,l}\} \\ \text{Im}\{C_{\partial,l}\} \end{pmatrix}. \quad (11)$$

Summarizing (6), taking into account (9)–(13), we obtain:

$$\begin{pmatrix} \text{Re}\{C_{\partial,l}^*\} \\ \text{Im}\{C_{\partial,l}^*\} \end{pmatrix} = K \begin{pmatrix} \cos \theta_i & -\sin \theta_i \\ \sin \theta_i & \cos \theta_i \end{pmatrix} \begin{pmatrix} \cos \theta_{\text{const}} & -\sin \theta_{\text{const}} \\ \sin \theta_{\text{const}} & \cos \theta_{\text{const}} \end{pmatrix} \times \begin{pmatrix} k_E & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & k_s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \text{Re}\{C_{\partial,l}\} \\ \text{Im}\{C_{\partial,l}\} \end{pmatrix} + \begin{pmatrix} A \cos \phi \\ A \sin \phi \end{pmatrix} + \begin{pmatrix} \text{Re}\{n_i\} \\ \text{Im}\{n_i\} \end{pmatrix}. \quad (12)$$

The estimation of unknown parameters in (12) is based on an analysis of statistical moments of the adopted symbols $C_{\partial J}^*$.

Approximating the relation (12) for the random phase angles and the effect of intentional interference, we obtain the following mathematical expectation of the components $C_{\partial J}^*$ (modeling components):

$$\begin{aligned} M \left[\begin{array}{l} \text{Re}\{C_{\partial J}^*\} = \\ = M \left[K k_E \text{Re}\{C_{\partial J}\} - K(k_E k_s - \theta_{\text{const}}) \text{Im}\{C_{\partial J}\} \right], \\ \text{Im}\{C_{\partial J}^*\} = M \left[\begin{array}{l} K k_E \theta_{\text{const}} \text{Re}\{C_{\partial J}\} - \\ - K(k_E k_s - \theta_{\text{const}} + 1) \text{Im}\{C_{\partial J}\} \end{array} \right]. \end{array} \right. \end{aligned} \quad (13)$$

The phase jitter is determined by the successive calculation of the covariance of the two components of the accepted symbol $C_{\partial J}^*$:

$$\begin{aligned} \text{Cov}[\text{Re}\{C_{\partial J}^*\}, \text{Im}\{C_{\partial J}^*\}] &= \\ &= -K^2 k_E \sigma_i^2 \left(\text{Re}\{C_{\partial J}\} \text{Im}\{C_{\partial J}\} + [\text{Re}\{C_{\partial J}\}]^2 k_s \right). \end{aligned} \quad (14)$$

The expression (14) defines the phase jitter variance of the received signal σ_i^2 . The amplitude A of the interfering signal can be found by calculating the order moment $m_4[\text{Re}\{C_{\partial J}^*\}]$ from one of the components of the accepted symbol $C_{\partial J}^*$ and the squared variance $D[\text{Re}\{C_{\partial J}^*\}]$:

$$A = \left(8D[\text{Re}\{C_{\partial J}^*\}]^2 - \frac{8}{3} m_4[\text{Re}\{C_{\partial J}^*\}] \right)^{1/4}. \quad (15)$$

The effect of intentional interference can be by calculating the actual and imaginary part $C_{\partial J}^*$:

$$\begin{aligned} D[\text{Re}\{C_{\partial J}^*\}] &= K^2 [\text{Im}\{C_{\partial J}\}]^2 \sigma_i^2 + D[\text{Re}\{n_i\}] + \frac{1}{2} A, \\ D[\text{Im}\{C_{\partial J}^*\}] &= K^2 \left([\text{Re}\{C_{\partial J}\}]^2 + k_s^2 [\text{Im}\{C_{\partial J}\}]^2 + \right. \\ &\quad \left. + 2k_s \text{Re}\{C_{\partial J}\} \right) k_E^2 \sigma_i^2 + \\ &\quad + D[\text{Im}\{n_i\}] + \frac{1}{2} A. \end{aligned} \quad (16)$$

Fast and slow signal fading due to the propagation of radio waves and movement of network subscribers (time and frequency shifts of the signal) are the multiplicative

interference taken into account in the given mathematical model.

To obtain a realistic distribution channel model, it is necessary to make some assumptions regarding the wave propagation scenario.

Nowadays, many models have been developed to describe the scenario of radio wave propagation in the channels of the MIMO system. The main parameters that were taken into account during modeling were:

- height of transmitting and receiving antennas;
- position of terminals relative to antennas;
- Doppler spectrum, as well as other parameters for the calculation.

The well-known analysis of the Jakes model showed that the autocorrelation $R(\tau)$ and the Doppler power spectrum $P(f)$ of the channel are presented in [3]:

$$\begin{aligned} R(\tau) &= J_0(2\pi f_D \tau), \\ P(f) &= \begin{cases} \frac{1}{\pi f_D} \frac{1.5}{\sqrt{1-(f/f_D)^2}}, & |f| < f_D, \\ 0, & \text{otherwise,} \end{cases} \quad (17) \\ f_D &= \frac{v}{\lambda}, \end{aligned}$$

where f_D is the maximum Doppler shift, v is the relative velocity of the receiver relative to the transmitter, and J_0 is the zero-order Bessel function.

Models with two rings and three-dimensional channels perfectly describe urban areas with a large number of environments and are presented in Fig. 2.

In Fig. 2, the following notation is used: D is the distance between the transmitter and the receiver, β_i is the angle of reflection from the scatterer of the outer ellipse, a_i is the angle of reflection from the scatterer of the inner ellipse, v_T is the transmitter speed, v_R is the receiver speed, a_m is the major semiaxis of the outer ellipse, b_m is the minor semiaxis of the outer ellipse.

The proposed model is based on the elliptical channel model developed in [2]. This model was designed to simulate the up-link channel of a mobile subscriber to a base station channel in a cell where the base station is not high, and there may be a line of sight.

Similar conditions are very common for communication networks, with the possibility of self-organization. The number and position of objects surrounding the transceivers depend on the type of terrain. In this model, there is an assumption that the surrounding objects located between the transmitter and the receiver are placed evenly within the two ellipses.

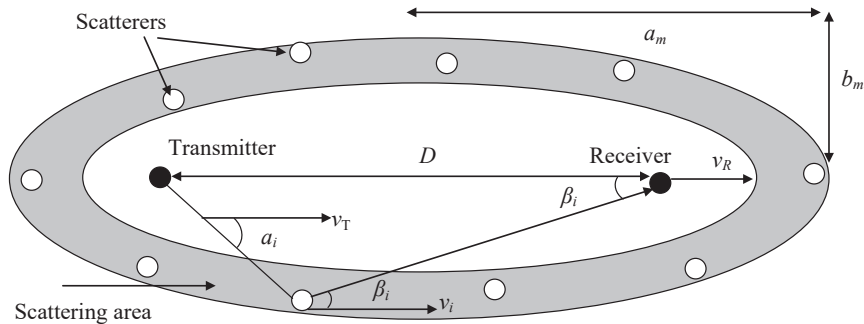


Fig. 2. Principle of taking into account the motion parameters of the transmitter and receiver in the mathematical model of the MIMO system

The parameters a_m and b_m of the outer ellipse are calculated from the delay propagation using the following equations [6], while those of the inner ellipse are determined by the subscriber's motion geometry.

$$\begin{aligned} \alpha_m &= \frac{c\tau_m}{2}, \\ b_m &= \frac{1}{2}\sqrt{c^2\tau_m^2 - D^2}, \\ \tau_m &= 3.244\sigma_t + \tau_0, \end{aligned} \tag{18}$$

where τ_m is the maximum signal delay time, σ_t is the signal delay time, τ_0 is the minimum signal delay time (line of sight between the transmitter and the receiver), D is the distance between the transmitter and the receiver, c is the wave propagation time. In this model, the minimum average delay time of signal propagation was adopted at the level of 103 nanoseconds, due to the fact that large signal delay values lead to lower antenna correlation.

Assume that the existence of foreign objects between the transmitter and the receiver leads to a block of the line of sight. When there is a line of sight between the transmitter and the receiver, the component reflected from the ground is added, if the distance between the transmitter and the receiver satisfies the following equation:

$$D \geq \frac{4\pi \cdot h_t \cdot h_r}{\lambda}, \tag{19}$$

where h_t and h_r are the heights of the antenna of the transmitter and receiver, respectively, and λ is the wavelength.

The right side of the expression (19) is the minimum distance for the first Fresnel zone, reflected from the earth's surface, and, thus, the effect of the signal reflected from the earth's surface can exist only if the expression (19) holds [11, 12]. The environment between the transmitter and the receiver is not considered fixed, but their speed is evenly distributed between 0 and the maximum limit.

It can easily be shown that the Doppler shift for any path (i) is given by the formulas (20) or (21) [13, 14]. Equation (21) follows (20), since the last term in (20) is much smaller than the first one.

Considering the elliptic model in Fig. 2, the maximum Doppler shift is no longer determined only by the relative velocity of the transmitter/receiver ($v_T - v_R$) as in the Jakes model, since the environment is not fixed [3, 14].

$$\begin{aligned} f_d(i) &= f \left[\left(1 + \frac{v_T - v_i}{c} \cdot \cos(\alpha_i) \right) \cdot \left(1 + \frac{v_i - v_R}{c} \cdot \cos(\beta_i) \right) \right] - f, \\ f_d(i) &= \frac{f}{c} [(v_T - v_i) \cdot \cos(\alpha_i) + (v_i - v_R) \cdot \cos(\beta_i)] + \\ &+ \frac{f}{c^2} (v_T - v_i)(v_i - v_R) \cos(\alpha_i) \cos(\beta_i), \end{aligned} \tag{20}$$

$$f_d(i) \approx \left[\frac{f}{c} (v_T - v_i) \cdot \cos(\alpha_i) + (v_i - v_R) \cdot \cos(\beta_i) \right]. \tag{21}$$

The channel function $h(t)$ at the time (t) can be represented by the expression:

$$h(t) = \sum_{i=0}^N g_i \cdot \exp \left(j \left[\frac{2\pi \cdot f_d \cdot d}{c} + \theta_i + \varphi_i \right] \right) \cdot u(t - t_i), \tag{22}$$

where g_i is the reflection coefficient, t_i and θ_i is the delay time of the excess distance and phase, respectively, φ_i is the random phase, N is the number of signal paths, $u(t)$ is the unit incremental function. The presence of a line of sight is provided with $i=0$.

5. Verification of the adequacy and reliability of the proposed model of the MIMO system

For the modeling, 10 scatterers were used. The maximum speed of the receiver was set at 120 km/h, with the transmitter moving at a speed of 90 km/h. The ratio of the line-of-sight component to any of the other scatterers equals k .

The spread of the delay is 103 nanoseconds, according to [9]. The distance (D) between the transmitter and the receiver is 1 km, and the height of the antenna is 1.5 m, the signal transmission rate is 5.9 GHz.

It is found that the amplitude distribution of the received signal with the help of the developed model corresponds to the Rayleigh distribution in the absence of a line of sight and Rice distribution when there is line of sight between the transmitter and the receiver. This is consistent with the statistics obtained in the measurements [11, 17]. The Rice distribution can be approximated by the Gaussian distribution in the presence of a stable line of sight [4].

The Doppler spectrum is presented in Fig. 4. Comparing the obtained values in Fig. 3 with the classical Jakes spectrum [3], it is found that the maximum Doppler shift exceeds the shift, which was proposed by Jakes. In the Doppler spectrum, the Jakes spectrum is limited to f_d , given in the expression (17), whereas in connection with the high mobility of subscribers, the spectrum goes beyond this value, as shown in Fig. 4. This effect appears in the autocorrelation function, which is a faster variation compared to the Jakes model. Both models give the same results if the scatterer speeds are zero. Similar conclusions were made in [21] with the help of practical measurements, which confirms the adequacy and reliability of the obtained mathematical expressions.

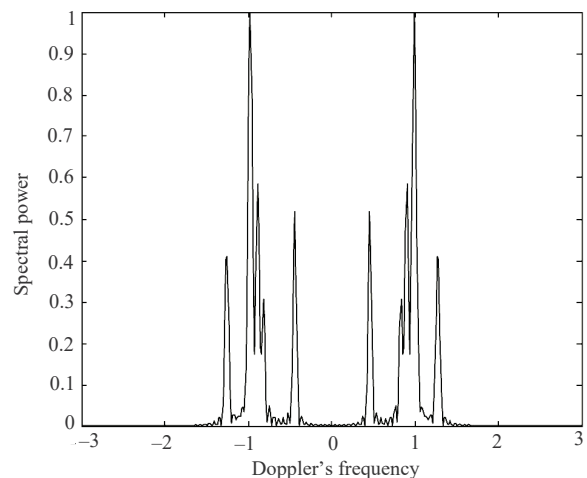


Fig. 3. Dependence of the spectral power of the Doppler spectrum on the Doppler's frequency (normalized to the Jakes maximum)

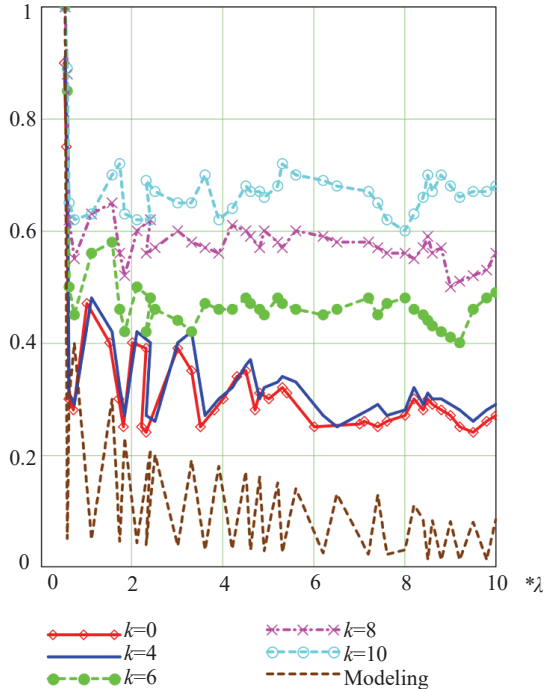


Fig. 4. Dependence of the correlation coefficient on the wavelength value

The correlation between two antennas (ρ_{ij}) can be calculated theoretically for the fades described by the Rayleigh distribution using the probability distribution of the arrival angle $p(\alpha)$ and the equation [23].

$$\rho_{ij} = \int_0^{2\pi} e^{j(2\pi/\lambda)d \cos(\varphi-\psi)} \cdot p(\varphi) \cdot d\varphi, \quad (23)$$

where d is the distance between the antennas and ψ is the ray angle. For mobile terminals, it is usually assumed that the medium (acting as scatterers) is evenly distributed in a circle around the terminal (Li model), which leads to the distribution of the AOA of the following equation [19]:

$$p(\varphi) = \begin{cases} \frac{1}{2\pi}, & 0 \leq \varphi \leq 2\pi, \\ 0, & \text{otherwise.} \end{cases} \quad (24)$$

Fig. 4 compares the correlation between antennas in the presence of a line of sight between the transmitter and the receiver and, in its absence, using an elliptic correlation model in the expression (23). As can be seen, the expression (23) gives an optimistic estimate of the correlation due to the assumption of a uniform distribution of correlation angles, which is realistic only in channels with many scatterers.

Exact mathematical expressions for calculating the bit error probability for phase-modulated (PM), quadrature amplitude-modulated (QAM) and hierarchically quadrature-modulated (HQM) signals are given in [20–23].

The average bit error probability while using PM- M , QAM- M , HQM- M for the MIMO system in the conditions of fluctuation noise, intentional noise, selective fading, and Doppler effect will be determined by the following expression:

$$P_B^L = \frac{P_{1B} + P_{2B} + \dots + P_{LB}}{L}, \quad (25)$$

where L is the number of channels in the MIMO system, P_{LB} is the average bit error probability in each channel of

the MIMO system, taking into account the effect of fluctuation noise, intentional noise, the type of signal modulation, selective fading, Doppler effect, intersymbol interference, phase jitter. Using the obtained analytical dependences, the noise immunity of the MIMO system is estimated for different values of transceiver and channel parameters. Dependences of the bit error probability on the signal/noise ratio for different numbers of antennas ($N=1, 2, 4$ and 8) are shown in Fig. 5.

The error probability decreases inversely to the signal/noise ratio in the step, equal to the number of receiving antennas, namely as $P_{er} \sim (h_0^2)^{-N}$. The results of calculating the bit error probability at different values of the correlation coefficient r are shown in Fig. 6.

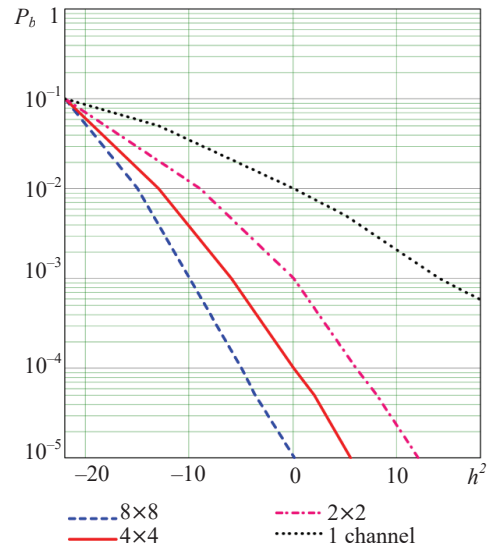


Fig. 5. Bit error probability depending on the signal/noise ratio for different numbers of receiving antennas ($N=1, 2, 4$ and 8)

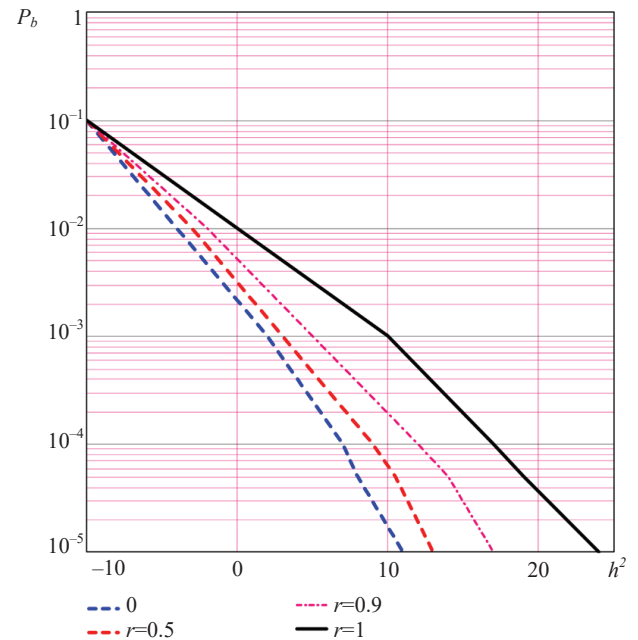


Fig. 6. Bit error probability depending on the signal/noise ratio in the fading channel for eight antennas at different values of the correlation coefficient r

It can be seen that when the correlation coefficient increases, the power required to provide a given error probability increases. However, this growth is uneven. Thus, the correlation of signals does not lead to significant losses if the correlation coefficient does not exceed ≈ 0.5 .

Fig. 7 shows the dependence of the bit error probability in the Rice channel on the average signal/noise ratio h^2 for different values of the power ratio of the static and Rayleigh components ($\zeta = -50; 0; 5; 10$ dB).

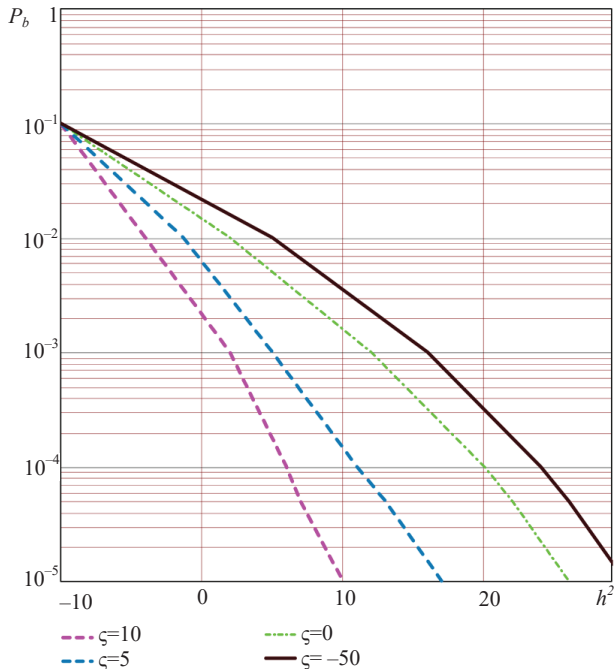


Fig. 7. Bit error probability depending on the signal/noise ratio for different values of ζ (dB)

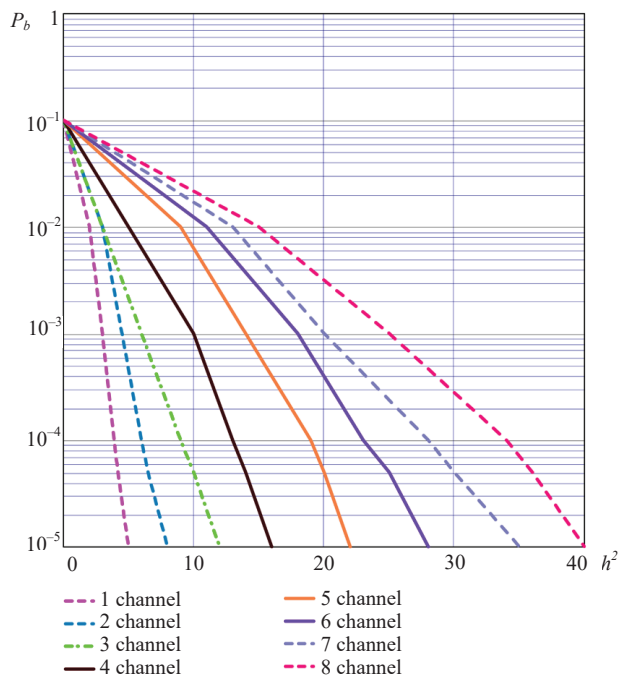


Fig. 8. Bit error probability in the own channel, depending on the signal/noise ratio while using the 8×8 MIMO scheme

It can be seen that with the same signal/noise ratio, the error probability decreases with an increase in the coefficient ζ with a decreasing Rayleigh component of signal fluctuations.

The given graphic dependencies allow us to analyze the noise immunity of multi-antenna radio communication systems and select rational values depending on the radio-electronic situation.

6. Discussion of the results of the development of an integrated mathematical model of the state of the channel of multi-antenna radio communication systems

The integrated mathematical model for assessing the state of the channel of multi-antenna radio communication systems is proposed. The simulation of the proposed method in the MathCad 14 software environment is carried out.

The main advantages of the proposed integrated assessment model are:

- unambiguity of the obtained estimation of the channel state;
- wide scope of use (radio communication and radar systems);
- the possibility of adaptation to the signaling situation in the channel;
- greater accuracy of the channel assessment;
- the possibility of synthesizing the optimal structure of the radio communication device.

The benefits of this model are due to the greater number of destabilizing factors compared to the known ones. The model takes into account the intentional interference of the additive and multiplicative nature, destabilizing factors caused by the mutual movement of transmitters and receivers. The effect of intentional interference, inclination of the constellation matrix of the channel, inter-symbol interference, phase jitter, type of signal-code construction and the effect of signal fading can be taken into account with the help of the expressions (1)–(16). With the expressions (17)–(21), it is possible to calculate the effect of motion of receivers and transmitters, the nature of the radio wave propagation area.

The disadvantages of the proposed complex mathematical model include a higher computational complexity compared with simpler mathematical models. This is due to the calculation of more indicators of the channel state.

This mathematical model is suitable for the use in radio stations with a programmable architecture, which operates in conditions of active radio-electronic suppression. While using the proposed model, there are the following restrictions:

- the minimum average delay time of signal propagation cannot be less than 103 nanoseconds;
- the frequency range of the transceiver should be in the range from 2 to 6 GHz;
- the speed of subscribers should not exceed 120 km/h;
- the use of positional signal-code structures;
- signal retransmission losses are not taken into account;
- the number of antenna channels from 2 to 32.

This complex mathematical model will allow:

- to identify the structure of the interference, its type and law of setting;
- to assess the state of the channel;
- to use efficient signal-code designs to ensure the noise immunity of channels;

- to ensure the efficient use of the radio frequency resource of programmable radio communication equipment;
- to increase the speed of estimation of communication channels;
- to develop measures aimed at increasing noise immunity.

The proposed integrated mathematical model should be used in developing software for modules (blocks) for evaluating promising radio communication equipment based on the open architecture interfaces of the SCA 2.2 version.

This study is the further development of research carried out by the authors, aimed at developing the methodological principles of operational management of radio resources of radio communication systems, published earlier [24, 25]. Directions of further research should be directed towards the reduction of computational expenses and taking into account the non-Gaussian noise influence.

7. Conclusions

1. A list of destabilizing factors are determined and analytical expressions are developed to take into account different types of intentional noise, selective signal fading, described by the Rice and Rayleigh law, Doppler effect, intersymbol interference, phase jitter, high mobility of objects (transmitter and receiver).

2. The adequacy and reliability of the obtained analytical expressions with the Jakes model are checked. In comparison, it is found that the Doppler power spectrum goes beyond the maximum Jakes frequency due to the motion

of the scatterers and the transmitter with the receiver. The research of the correlation between antennas is carried out. The results show that in the presence of line of sight between the receiver and the transmitter, the correlation is high and therefore a small increase is expected from the use of several antennas, and in the absence of a line of sight conditions, the correlation is low.

3. The proposed complex mathematical model of the channel state of multi-antenna radio communication systems is more accurate compared with the known ones, due to the consideration of a greater number of destabilizing factors, which in turn leads to an increase in computational complexity.

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