Еколого-економічні моделі (типу Леонтьєва-Форда) відіграють особливу роль в розв'язанні принципових проблем перспективного планування з врахуванням природокористування. На їх основі може бути реалізована задача обґрунтування величини затрат на охорону навколишнього середовища з врахуванням соціально-економічного ефекту та розподілу їх у територіально-галузевому розрізі. На основі запропонованої балансової моделі окреслено типові узагальнення («розширення») моделі, які, загалом, збільшують її розмірність, але не «випадають» з класу лінійних. Зокрема, досліджено вплив на зміни обсягів валових галузевих випусків в наслідок зміни структурних галузевих пропорцій, що відповідає зміні технологічного укладу функціонування еколого-економічної системи у галузевому розрізі.

З метою розв'язання поставленої задачі розвинуто застосування алгоритмів методу базисних матриць, оснащені технологією визначення розв'язків системи матричних лінійних рівнянь відповідно до змін та проведення узагальнень моделі. При цьому зміни можуть зазнавати окремі елементи чи група елементів, один чи група рядків (стовпців), в блоках підматриць матриці. Запропоновані алгоритми реалізовані для випадку змін матриці обмежень вихідної системи без перерахунку (заново).

Розглянуто різноманітні варіанти змін в моделі та їх вплив на новий розв'язок у випадку «збурення» в підматрицях матриці обмежень (групи елементів, що утворюють блок) моделі. Зокрема, при «включенні» («виключенні») нових блоків підматриць, тобто збільшенні (чи зменшенні) розмірності початкової матриці обмежень математичної моделі.

Такі моделі подаються лінійною системою, зокрема, системою лінійних алгебраїчних рівнянь (СЛАР).

Такий підхід відкриває можливість проводити направлені зміни в моделі з метою досягнення в подальшому бажаних пропорцій «корисної» та «шкідливої» компонент у структурі виробництва (як розв'язок задачі).

Подальший розвиток запропонованої теорії дозволяє перейти до вивчення питань агрегування балансової схеми «витрати-випуск», визначення певного коридору допустимих змін з метою досягнення цільового орієнтиру по обсягам галузевих випусків

Ключові слова: матричні системи, міжнародні екологічні угоди, метод базисних матриць, матричні еколого-економічні моделі

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ALGORITHMIZING THE METHODS OF BASIS MATRICES IN THE STUDY OF BALACE INTERSECTORAL ECOLOGICAL AND ECONOMIC MODELS

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1. Introduction

The management process with using models is considered as finding optimal solutions for analyzing the behavior of a real economic system without realizing an experiment with the system itself. Thus, the use of economic-mathematical modeling methods gives an opportunity to obtain the result not by an experiment, but to offer recommendations for further development based on the fundamental provisions of scientific analysis. Changing the model, rather than the real object during its study, makes it possible to quickly obtain all the necessary information, which reflects its internal relationships, qualitative characteristics, and quantitative parameters. Due to the application of mathematical methods in the study of economics, an adequate phenomenological theory of a large number of economic phenomena has been constructed. In addition, a large number of economic processes receive an explanation not only from the application of classical mathematical methods but also from the standpoints of a qualitative theory of differential equations, bifurcations of dynamic systems, fuzzy logic, neural networks, synergy etc. Significant progress in the interdisciplinary interaction of economics and mathematics make it possible to include social policy in the economic system, which aims to ensure consistency between economic development and social standards. A different aspect of such a policy is an ecological

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component, which at the present stage of development of civilization goes beyond national and territorial boundaries and becomes a global issue. Most researchers agree that in case of large-scale changes in the natural environment and climate, the economic system undergoes a direct impact. As an example, this primarily concerns nature operating branches, since such changes affect the quality of the resource base, which ultimately leads to changes in the economy in general.

In the theoretical and applied sense, it remains urgent to develop and apply mathematical algorithms to the analysis of ecological and economic interaction under various scenarios of the implementation of managerial decisions. At the same time, an important aspect of a new environmental policy is achievement of an optimal balance between the environmental and economic components. Such an interaction provides a possibility of a steady economy development in the context of reducing the negative impact on the natural environment.

At the same time, the processes of world globalization and the absence of boundaries for the processes of distribution of environmental problems put forward the urgent task of universalization of the elaborate mathematical algorithms. This implies the possibility of their successful application to various national economies, as well as scaling. The ecological component in such conditions should have the property of completeness – the inclusion of additional new types of restrictions.

Consideration of such changes in the process of modeling causes a scope expansion of the methods and algorithms to conduction of analysis of group refinements in the blocks (quadrants) of model matrix on the properties of linear system. We should emphasize the importance of studying the «imitation» of system properties as a result of an increase («extension») or a decrease in the dimension («narrowing») of the dimension matrix of restrictions (inclusion-exclusion of blocks of matrices, groups of rows or columns). Specifically, it is the evolution of solutions properties in course of such an improvement in the model of processes, in particular, the ecological-economic without solving the problem initially.

In the applied sense, an urgent aspect is development of algorithms for the analysis of ecological and economic interaction with various group changes in the model, in particular, during the construction and research of balance models and the study of the sectoral structure of the economy (the «cost-issue» method).

2. Literature review and problem statement

Despite numerous discussions and controversies over a number of scientific studies, the UN Framework Convention on Climate Change [1] has recognized the excessive concentration of greenhouse gases in the Earth's atmosphere as the main cause of global climate change. This became the basis for the adoption of the Kyoto Protocol in 1997 [2], which specified the conditions for emission reductions for each country. The ultimate goal of these agreements is to achieve stabilization of the greenhouse gases concentration in atmosphere at the level that would not tolerate an imminent man-made impact on the climate system.

A further step in the international community's response to the negative impacts of climate change on socio-economic development was the adoption of the Paris Accord (PU) on climate protection by the United Nations. Its main objective is to reduce greenhouse gas emissions globally [3].

Today, the study of implementation of the CP and PU provisions is a separate position in scientific research. Their content defines, first of all, the behavior of the country at the interstate level in the process of international co-operation. However, the successful implementation of international cooperation within the framework of international environmental agreements is possible only under the conditions of corresponding changes on the lower hierarchical levels of economic system. It is clear that there are some changes in certain ecological and economic elements, the formulation of laws that set quantitative and qualitative relationships between them at each of the selected levels, and as a result it leads to the need to choose another structure of the ecological and economic model.

Today, most of the studies about ecological and economic interaction in the process of implementation of the Paris Agreement relate primarily to the conceptual aspects of its implementation, the historical experience of implementation of certain mechanisms for the optimal implementation of environmental constraints. So, this work [4] underlines the importance of the Paris Agreement as the next step of world community in reducing greenhouse gas emissions and provides a solid historical background on the development of international cooperation on sustainable development from the creation of the UN Framework Convention on Climate Change. The Agreement interconnection with the Kyoto Protocol, the Marrakesh Summit and the Doha Amendment is discovered and its special role as an instrument in achieving the goal of reducing greenhouse gas emissions is outlined.

Given the fact that the most effective tool for solving environmental problems is economic encouragement, a number of authors explore theoretical-methodological principles for building a single ecological-economic complex within the framework of the Paris Agreement. In work [5], authors analyzed the connection of regional, national and international policy in future climate cooperation. Besides purely administrative mechanisms it is proposed to use market and economic mechanisms as the most effective means of encouragement to achieve a desired level of emissions, and their symbiosis can become a tool for reducing the costs of implementing environmental policies. The emphasis was on the necessity of implementing the minimum environmental restrictions on the first stage, development of national standards, metrics, reporting and verification rules. The next step might be to develop mechanisms for transferring emission quotas between the parties of the agreement, which will establish rules for interaction between government and private agents, and will facilitate the construction of different forms of communication between them, thus contributing to environmental integrity and cost effectiveness.

It should be noted that there is a series of works devoted to the adaptation of national economies to new environmental conditions among the other spectrum of scientific studies related to the implementation of provisions of the Paris Agreement. It is clear that each of them reflects the implementation of its characteristics and is often controversial. Today the leading players' economies in reducing greenhouse gas emissions such as economically industrialized countries, the Chinese People's Republic, India, and the Russian Federation has been studied deeply [6–10].

These studies are mainly conceptual and theoretical and do not make it possible to determine the specific quantitative values of the implementation of various mechanisms. However, their results make it possible us to proceed to studying the problem at the level of mathematical modeling. One of the commonly used approaches in this case is macroeconomic models based on Keynes' theory, for example, Oxford and Dri-Wefa models [11]. Most often they are used to investigate equilibrium in individual markets, cyclical transformations, convergence and stability, long-term growth and forecasting. However, such models do not use assumptions about the full employment of the primary production factors and perfect competition in all the markets, which impedes the possibility of its extrapolation to different types of national economies. In addition, the parameters of such macroeconomic models, as a rule, are estimated on the basis of time series, which may make significant inaccuracies in the predicted calculations.

The question of optimality of the distribution of responsibility under the Kyoto protocol based on the fuzzy set of apparatus and decision-making theory is investigated in [12]. One of the main shortcomings of most international agreements on environmental protection is the lack of specific mechanisms for their implementation, in the first place, the formalized rules of distribution of responsibility, in particular, financial, which can be determined with a certain amount of accuracy. the deal agents can take the appropriate co-operation decision based on this forecast. In the opinion of authors, it is advisable to consider fuzzy statements of distribution models of collective costs in the development of the proposed mechanisms of quotas allocations, since the parameters introduced during the formation of individual potential income of agents are empirical, and therefore inaccurate.

Interdisciplinary balance models, which explore the mutual influence of economy structure on environment are another class of ecological-economic models that make it possible us to investigate the balance of the general system. The interdisciplinary model of Leontiev-Ford and its generalization belong to this class of models [13]. Ecological-economic modeling according to the inter-sectoral balance-of-payments scheme make it possible to determine prices, balance finance industries and economic costs for pollution control, forecast the impact of changes in added value on prices, and also on production volumes on condition of implementation of one or another nature-conservation strategy.

Applied models of general equilibrium are constructed on such an ideology [14, 15]. Their substantial advantage is that they operate large amounts of statistical data of the interbranch balance or national accounts system, and thus make it possible to take into the fullest possible consideration the interbranch relations structures, as well as investigate sectoral and macroeconomic effects of economy. In the early 1990's, the applied general equilibrium models became a standard tool for analyzing the consequences of introducing various mechanisms of economic, social, energy or environmental policy. Such models are the most used abstractions, which make it possible to consider economic interrelations in the economic system with maximum completeness, based on a bulk of statistical data.

Considering the large dimension of these models, their disadvantage is the complexity of the implementation of simulation experiments. This complexity is due to the peculiarities of matrix structures that form a model. All this makes it possible to claim that it is expedient to choose a basic model of balance or Leontiev type to study the balanced ecological-economic interaction. The next step is to develop reliable methods for obtaining solutions in different types and scales of matrix structures, which will make it possible implementation of various scenario options for the implementation of environmental and economic policies.

Consideration of such a property in the process of modeling causes the expansion of usage scope of methods and algorithms of conducting the analysis of group changes in blocks (quadrants) of a model matrix on the properties of linear system, in particular, solutions in the process of improving the model processes or to be more specific ecological-economic [13].

In the applied sense, an urgent aspect is development of algorithms for the analysis of ecological and economic interaction (inter-sectoral model of Leontiev-Ford and its generalization) with various group changes in the model, which were initiated and developed during the construction and research of balance models and the study of the sectoral structure of the economy (the «cost-issue» method).

This implies the presence of a mathematical apparatus of taking into account the effect of changes (refinements) on the properties of a new model, of course, without the procedure of re-solving the problem again (initially). It should be noted that the models of ecological-economic processes (as linear systems) have block (cellular) structure – the quadrants of the matrix of constraints. In particular, the classical scheme of interbranch balance in the first quadrant contains interdisciplinary flows that correspond to functional-structural branch relationships.

It is known that for linear models, the most effective computational procedures and industrial implementations, for example, variants of the simplex-method [16, 17], are conventionally divided into methods applied to the direct problem and to the duality. The determining part of it is the exact Gauss method (complete exclusion) of the solution of linear algebraic equations system that is used for each iteration. In the Gauss method: the equivalent transformations of the initial model to the model of a simple structure (with a diagonal matrix of constraints) are directed; the volume of calculations on iterations decreases from iteration to iteration. It should be noted that such a transformation of the initial model somewhat limits the carrying out of construction accounting analysis with changes in the model.

The basis of the proposed method of artificial basis matrices [18] and its corresponding algorithms is the idea of a base matrix formed by linearly independent lines of matrix of constraints. The process of iterative «bridging» (replacing) the limits of auxiliary system with the relaxed constraints of the main system is being carried out. Detailed explanation of the method justification, its properties, the results of the computational experiment, comparisons with other well-known methods can be found in [19, 20]. The method of basic matrices (MBM) proposed in [18], «works» if it is the other way round, it conducts an iterative transformation of a model of a simple structure (with known properties) by successive inclusion of restrictions of the original system. That is, the process of «restoration» is carried out and consists in a transition to the original initial in accordance with the formulas of the method of communication elements in the iterations. As a result, we set the value of the rank, find the inverse matrix, the solution, control the conditionality, and so on.

The MBM algorithms naturally extend to the analysis of the influence of changes in the linear system in various components of the model (element, string, column, group of rows (columns), submatrix blocks of the matrix of constraints, etc. Partial studies on the analysis of the effect of such changes on the properties of linear system (without redistribution of the task) were carried out in [21, 22] – an element, a string, a column of the matrix of constraints.

Structurally, the analysis of such changes corrects the effect of changes in individual elements, rows in the columns of the technological matrices of model, as well as in the forming cells (blocks). This necessitates the development of the algorithms discussed in [21, 22], the development of an evaluation of the change effect in blocks of matrix structure on solution of equations system and a number of other problems.

3. The aim and objectives of the study

The aim of this study is to develop methods and algorithms for analyzing the properties of processes presented by linear models. It is believed that such models may undergo various improvements, changes and refinements during the study. In particular, the aim is to study the ecology-economic model of «cost-issue» type of Leontiev-Ford, which consider the costs of implementing the restrictions under the Paris Agreement and various options for improvements.

To achieve the goal, the following tasks were set – to estimate expenses incurred in implementing the restrictions under the Paris Agreement on the basis of the cost-issue balance sheet:

– to explore the possibility of including additional economic and environmental constraints and new factors in the model, as well as changes in classical initial assumptions about the technological structure;

– to develop consideration of the influence of various «scenarios» of typical changes for the construction of basic algorithms in the matrix of constraints of the ecological-economic model (element, string, column, group of rows or columns, block of elements in the submatrix of restrictions);

– to offer an approach to constructing algorithms for analyzing the effects of changes in elements of matrix of constraints for modeling processes involving the inclusion (exclusion) of new matrix blocks, «expansion» (or «narrowing») of the dimension of the initial matrix of mathematical model constraints.

4. Development of the basic concept of modeling the ecological-economic process

It is known that:

– the majority of investigated processes by their nature are nonlinear, that is, they do not have an adequate mathematical representation in the class of linear models (matrix structures);

– the most effective computing probes and industrial implementations, for example, the simplex-method [16, 17] were developed for the linear models;

– the linear model becomes more adequate in the introduction of nonlinear dependencies in the elements of model. The model is linear again when it is acquired by parameters of the model of specific values;

– important parameters of the complex process during the simulation («changes») and the simplifications are reflected («pass») in the values of the elements of matrix structure, in particular, in the individual elements, rows, columns, and blocks (matrixes) of the matrix of restrictions, etc.;

– simulation can be considered as a series of successive refinements («changes») of a certain group of elements (submatrix or block) of matrix of constraints or «narrowing» («extension»), reducing (increasing) the dimension of the matrix of constraints. Thus, it is a certain iterative process of solving a sequence of interconnected tasks undergoing change.

An important role in modeling the processes of economy and ecology (in solving the problems of nature use) relies on balance ecological-economic models of Leontiev type [13]. Within such models, it is possible to optimally combine groups of industrial and environmental productions, as well as their interconnections. In this case, it is proposed to take into account the costs of implementing emission limitations of greenhouse gases in the structure of the main industries. Structural ecological-economic models (according to [21, 22]) can be meaningfully interpreted as cellular (block). In this matrix, certain relationships can be traced between the blocks. And mathematically such models are presented by linear systems.

By the criteria of simplicity, visibility, structuring as the basic balance ecological and economic model (for improvement) we shall consider the matrix model of Leontiev-Ford:

$$x_1 = A_{11}x_1 + A_{12}x_2 + y_1,$$

$$x_2 = A_{21}x_1 + A_{22}x_2 - y_2. \tag{1}$$

Equations of model (1) reflect the balance of material and auxiliary (ecological) production. In system (1):

– $x_1 = (x_1^1, x_2^1, \ldots, x_n^1)^{\mathrm{T}}$ – vector-column of production volumes;

– $x_2 = (x_1^2, x_2^2, \ldots, x_m^2)^{\mathrm{T}}$ – vector-column of volumes of destroyed pollutants;

– $y_1 = (y_1^1, y_2^1, \ldots, y_n^1)^{\mathrm{T}}$ – vector column of volumes of the final product;

– $y_2 = (y_1^2, y_2^2, \ldots, y_m^2)^{\mathrm{T}}$ – vector-column of volumes of permanent pollution;

– $A_{11} = (a_{ij}^{11})_1^n$ – square matrix of coefficients of direct costs of products i for the production unit of production j;

– $A_{12} = (a_{ig}^{12})_{i,g=1}^{n,m}$ – rectangular product cost matrix i per unit of destruction of pollutants g;

– $A_{21} = (a_{kj}^{21})_{k,j=1}^{m,n}$ – rectangular emission matrix k per unit of manufactured product j;

– $A_{22} = (a_{kg}^{22})_1^m$ – square emission matrix k per unit of destruction of pollutants g.

In work [13], it was proposed to take into account the costs of implementing the emission limitations of greenhouse gases in the structure of the main production sectors in the form of:

$$\begin{cases} x_1 = A_{11}x_1 + A_{12}x_2 + Cy_2 + y_1, \\ x_2 = A_{21}x_1 + A_{22}x_2 - y_2, \end{cases} \tag{2}$$

where Cy_2 are the costs related to greenhouse gas emissions (costs for maintenance of greenhouse gas emissions, in particular, it is the fee for emission permits); $C = (c_{ig}^{12})_{i,g=1}^{n,m}$ – rectangular product cost matrix i per unit of pollutant emissions g.

We can outline some typical variations of changes and refinements for the model (2):

A. Influence on the properties of the linear changes system (without solving the problem) – element, line, column of the matrix of constraints. It was investigated in [21, 22].

B. Balance of material and auxiliary (ecological) production including the structure of change will be reflected in the result of the modification of model «expansion» of the equation model production (1).

For example, in the system (1) $x_1=(x_1^1,x_2^1,\ldots,x_n^1)^{\mathrm{T}}$ – the vector column of production volumes may take the form:

$$\bar{x}_1=(x_1,\tilde{x}_1)=\left(\underbrace{x_1^1,x_2^1,\ldots,x_n^1}_{x_1},\ \underbrace{x_{n+1}^1\ldots,x_{n+p}^1}_{\tilde{x}_1}\right)^{\mathrm{T}},$$

here $\tilde{x}_1$ components of «expansion» («wave» from above) in the initial vector x_1; $x_2=(x_1^2,x_2^2,\ldots,x_m^2)^{\mathrm{T}}$ vector column of volumes of destroyed pollutants can get the following form:

$$\bar{x}_2=(x_2,\tilde{x}_2)=\left(\underbrace{x_1^2,x_2^2,\ldots,x_m^2}_{x_2},\ \underbrace{x_{m+1}^2\ldots,x_{m+q}^2}_{\tilde{x}_2}\right)^{\mathrm{T}}.$$

According to this, $y_1=(y_1^1,y_2^1,\ldots,y_n^1)^{\mathrm{T}}$ is the vector column of the volumes of final product takes such a form:

$$\bar{y}_1=(y_1,\tilde{y}_1)=\left(\underbrace{y_1^1,y_2^1,\ldots,y_n^1}_{y_1},\ \underbrace{y_{n+1}^1\ldots,y_{n+p}^1}_{\tilde{y}_1}\right)^{\mathrm{T}};$$

a $y_2=(y_1^2,y_2^2,\ldots,y_m^2)^{\mathrm{T}}$ is the vector-column of volumes of permanent pollution:

$$\bar{y}_2=(y_2,\tilde{y}_2)=\left(\underbrace{y_1^2,y_2^2,\ldots,y_m^2}_{y_2},\ \underbrace{y_{m+1}^2\ldots,y_{m+q}^2}_{\tilde{y}_2}\right)^{\mathrm{T}}.$$

$\bar{A}_{11}=(\bar{a}_{ij}^{11})_1^{n+p}$ – square matrix of coefficients of direct costs of products *i* for the production of one unit *j*.

$\bar{A}_{12}=(\bar{a}_{ig}^{12})_{i,g=1}^{n+p,m+q}$ – rectangular product cost matrix *i* per unit of destruction of pollutants *g*.

$\bar{A}_{21}=(\bar{a}_{kj}^{21})_{k,j=1}^{m+q,n+p}$ – rectangular emission matrix *k* per unit of manufactured product *j*.

$\bar{A}_{22}=(\bar{a}_{kg}^{22})_1^{m+q}$ – square emission matrix *k* per unit of destruction of pollutants *g*.

Then, the costs of implementing the emission limitations of greenhouse gases in the «extended» structure of the main industries (according to (2)) will take the form:

$$\begin{cases}\bar{x}_1=\bar{A}_{11}\bar{x}_1+\bar{A}_{12}\bar{x}_2+\bar{C}\bar{y}_2+\bar{y}_1,\\ \bar{x}_2=\bar{A}_{21}\bar{x}_1+\bar{A}_{22}\bar{x}_2-\bar{y}_2,\end{cases}$$

where $\bar{C}=(\bar{c}_{ig}^{12})_{i,g=1}^{n+p,m+q}$ is a rectangular product cost matrix *i* per unit of pollutant emissions *g*.

It is easy to make sure that the «extended» model has a cellular structure, with the structural blocks (1) and (2) naturally «moving» to it. This trend is also traced in case of «narrowing» the model (on the contrary).

C. «Inclusion», along with economic and environmental factors in the model of the other (or «exclusion»), modify the model by including (excluding) the corresponding blocks of subnets (cells), that is, the «extension» («narrowing») of the model. Naturally, the complication (simplification) of the model correlates with the increase (decrease) of the dimension of constraints matrix.

D. «Overlap» of additional restrictions (two-way restrictions on variables, condition of their additionality) on the components of vectors: volumes of production, volumes of destroyed pollutants, volumes of final products of the form, volumes of non-degraded contamination causes the modification of the model (complication), but does not output it beyond linearity.

The outlined typical variants of the modifications of the linear model indicate the need to equip the appropriate algorithms for studying such models (with the cell structure) with the ability to take this feature into account.

5. Technologies of analysis of «group» changes influence in the ecological-economic model based on algorithm of basis matrices method

The method of basic matrices (in the following MBM) used in this study is a method of simplex methods type [18], which is aimed precisely at the analysis and solving of the problems set above. It should be noted that these tasks are primarily the analysis of linear systems (in particular, SLAR), and are basic (according to [16–18]) when conducting more complex studies and generalizations.

According to [18], the submatrix A_b, consisting of m linearly independent linear-normals (i_1, i_2,..., i_m) of constraints, will be called the artificial basis, and the solutions u_0 of the corresponding system of equations $A_b u=C^0$, where $C^0=(c_{i_1},c_{i_2},\ldots,c_{i_m})^{\mathrm{T}}$ the artificial basis.

Let: e_{ri} – elements of the matrix A_b^{-1}, inversed to A_b; $u_0=(u_{01}, u_{01}, \ldots, u_{0m})$ – basic solution; $\alpha_r=(\alpha_{r1}, \alpha_{r2}, \ldots, \alpha_{rm})$ is the vector of development of the vector-normal limit $\alpha_r u\le c_r$ by the lines of the basis matrix A_b; $\Delta_r=\alpha_r u_0-c_r$ is nonconnection of the r limitation (1) at the vertex. All the entered elements in the new basis matrix $\bar{A}_b$, different from A_b with one line will be denoted by a line from above.

In accordance with Theorems 1 [18], the relation between the coefficients of development of limit normals, the elements of inverse matrices, the basic solutions, and the non-constraints of restrictions in two adjacent basic matrices is established. On the basis of them, a scheme for determining the rank of system (1) and the solution of the system of equations, successive changes in the basis matrices and the corresponding artificial solutions can be constructed (Algorithm 1 [18]).

Initial studies on the analysis of the effects of changes on the properties of the linear system (without solving the problem) were carried out in [21, 22] – an element, a string, a column of the matrix of constraints. In the future, the algorithms 1 and 2 proposed in [21, 22] will be developed to analyze the influence of typical changes in the matrix structure blocks on the solution of the system of equations, and the «expansion» («narrowing») of the matrix of restrictions by inclusion (exclusion) of the blocks.

6. Investigation of the properties of matrix structures at various variations of the elements in the matrix

According to the presentation, the first block of equations of the proposed model (2) reflects the economic balance that is the distribution of sectoral gross output to the production consumption of the main and auxiliary industries, the final consumption of the main production and the costs associated with the fulfillment of obligations under the Paris Agreement.

The second block of equations (2) reflects the physical balance of greenhouse gases, as the sum of emissions caused by the activities of the main and auxiliary industries, and their unchanged volumes.

The economic content of the model variables requires consideration of their inalienable meanings. This is closely related to the question of the productivity of balance models, which make it possible to speak about the real functioning of a production system capable of providing intermediate consumption, a positive volume of the final product and compliance with the established limits on greenhouse gas emissions.

According to [22] we specify a model in the form:

$$Au = C, \tag{3}$$

where

$$A = \begin{pmatrix} E_1 - A_{11} & -A_{12} \\ -A_{21} & E_2 - A_{22} \end{pmatrix},$$

$$u = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = (u_1, u_2, \ldots, u_m)^{\mathrm{T}}$$

– $Au = C$ is a dimensional vector, x_1, x_2 is a «sub vector» u,

$$C = \begin{pmatrix} E_1 & C \\ 0 & -E_2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix},$$

E_1, E_2 – block unit matrices of corresponding dimension, 0 – block zero matrix, Cy_2 – costs are related to greenhouse gas emissions. That is the cost of servicing greenhouse gas emissions. In particular, this is the fee for emission allowances, and $C = (c_{ig}^{12})_{i,g=1}^{n,m}$ – a rectangular product cost matrix per unit of emissions of the pollutant g.

We will also consider the system modified (in the elements of the matrices A_{11}, A_{12}, A_{21}, A_{22} and C) in relation to the system of linear algebraic equations of the form:

$$\bar{A}u = \bar{C}. \tag{4}$$

The following basic algorithms variants of analysis of changes in the constraints matrix of the linear system are distinguished:

A. Analysis of the effect of changes in the lines of the constraint matrix.

Let the solution u_0 and the inverse matrix A_σ^{-1} of the SLR type (3) of the main system be initially known. For definiteness we consider that in the system of «changes» there are lines $i+1, i+2, \ldots, i+i_0$ in the form:

$$\bar{a}_{i+s} u = \bar{c}_{i+s}, \quad s = \overline{1, i_0},$$

where

$$\bar{a}_{i+s} = a_{i+s} + a'_{i+s}, \quad \bar{c}_{i+s} = c_{i+s} + c'_{i+s}, \quad s = \overline{1, i_0}.$$

Based on the relations established in [21], we investigate the properties of SLAR (4) with a square matrix of constraints with perturbations in the elements of the «group» of strings. We will conduct a sequential (ordinal) replacement of the rows of basis matrices respectively perturbed and calculate (change) elements of the method in iterations. An important condition during the iterations is to preserve the non-degeneracy of the matrix of task limitations.

Algorithm 1.

Preparatory step. Let it be known that $s = 1$, u_0, A_σ^{-1}, where s is an iterative counter, i_0 is the number of perturbed lines in the block of restrictions matrix SLR (4).

Step 1. We provide $k = i + s$. We find development on the matrix lines A_σ^{-1} of the normal limit $\bar{a}_k u = \bar{c}_k$ (modified restriction $(a_k + a'_k)u = c_k + c'_k$) based on the ratio:

$$\bar{\alpha}_k = (\bar{\alpha}_{k1}, \bar{\alpha}_{k2}, \ldots, \bar{\alpha}_{km}) = \bar{a}_k \times A_b^{-1} = (a_k + a'_k) \times A_b^{-1},$$

where

$$\bar{\alpha}_{kk} = a_k e_k + a'_k e_k = 1 + a'_k e_k, \quad \bar{\alpha}_{ki} = a'_k e_i, \quad A_\sigma^{-1}, \quad i = \overline{1, m},$$

where e_k, e_i, A_σ^{-1}, $i = \overline{1,m}$ are columns of the matrix A_σ^{-1}. For example, $e_k = (A_b^{-1})_k$ is the k-th column of the inverse matrix.

Step 2. We find the disorder of perturbed restriction:

$$\bar{\Delta}_k = (a_k + a'_k)u_0 - (c_k + c'_k) =$$
$$= (a_k u_0 - c_k) + (a'_k u_0 - c'_k) = \Delta_k + \Delta'_k = \Delta'_k.$$

Step 3. Check the condition of the reference $\bar{\alpha}_{kk} \neq 0$ during the operation of replacing the string k with the normal k-th limitation $\bar{a}_k u = \bar{c}_k$ (in this case $k = i + s$). We find:

$$\lambda = -\frac{\bar{\Delta}_k}{\bar{\alpha}_{kk}} \text{ and } \bar{e}_k = \lambda \times e_k.$$

Step 4. Form a new solution $\bar{u}_0 = u_0 + \bar{e}_k$.

Step 5. Find the columns of the new inverse matrix $\bar{A}_\sigma^{-1}$:

$$\bar{e}_k = \frac{e_k}{\bar{\alpha}_{kk}}, \quad \bar{e}_i = e_i - \frac{e_k}{\bar{\alpha}_{kk}} \times \bar{\alpha}_{ki}, \quad A_\sigma^{-1}, \quad i = \overline{1, m}.$$

Consider that $u_0^{(k)} = \bar{u}_0$, $A_\sigma^{(k)-1} = \bar{A}_\sigma^{-1}$.

Step 6. When $s \neq i_0$ place $u_0 = u_0^{(k)}$, $A_\sigma^{-1} = A_\sigma^{(k)-1}$, $s = s+1$, proceed to step 1, otherwise, the next step.

The final step. It is conducting a comparative characteristic of the properties of the «modified» and the main SLR.

We find the solution of the «changed» problem (without first solving) according to the algorithm of i_0-inertia.

B. Changes in the individual elements of the matrix of constraints.

The algorithm discussed above can be applied to the analysis of the effect of «point», that is, in individual elements of a line of constraints matrix or the vector of normal changes.

We will investigate the properties of elements of the method during transition from SLR with unbroken strings (3) to SLAR (4) with perturbed strings.

For certainty, we have a string of (3) matrices of the A form:

$$a_{i_0} = (a_{i_0 1}, a_{i_0 2}, \ldots, a_{i_0 m}),$$

which was perturbed in the components $r_1, r_2, \ldots, r_s$ in the form:

$$a'_{i_0} = (0, \ldots 0, a_{i_0 r_1}, 0, \ldots 0, a_{i_0 r_2}, 0, \ldots 0, a_{i_0 r_s}, 0, \ldots, 0).$$

In general, the perturbed line can be represented in the form $\bar{a}_{i_0} = a_{i_0} + a'_{i_0}$. Note that the «unbroken» vector of the development of a string a_{i_0} by the lines of the basis matrix A_b has the following structure:

$$\alpha_{i_0} = \Big(\underbrace{0}_{1}, \ldots, 0, \underbrace{1}_{i_0}, 0, \ldots, \underbrace{0}_{m} \Big).$$

Consequence 1. The following relations are true for the modified components of the development vector:

$$\bar{\alpha}_{i_0} = \left(a_{i_0} + a'_{i_0}\right)A^{-1} = a_{i_0}A^{-1} + a'_{i_0}A^{-1} = \alpha_{i_0} + a'_{i_0}A^{-1} =$$

$$= \begin{pmatrix} \sum_{j=1}^{s} \underbrace{a'_{i_0 r_j} e_{r_j 1}}_{1}, \sum_{j=1}^{s} \underbrace{a'_{i_0 r_j} e_{r_j 2}}_{2}, \ldots, \sum_{j=1}^{s} \underbrace{a'_{i_0 r_j} e_{r_j i_0 - 1}}_{i_0 - 1}, \sum_{j=1}^{s} \underbrace{a'_{i_0 r_j} e_{r_j i_0}}_{i_0}, \\ \sum_{j=1}^{s} \underbrace{a'_{i_0 r_j} e_{r_j i_0 + 1}}_{i_0 + 1}, \ldots, \sum_{j=1}^{s} \underbrace{a'_{i_0 r_j} e_{r_j m}}_{m} \end{pmatrix},$$

$$\Delta_{i_0}\left(u_0\right) = \bar{a}_{i_0} u_0 - c_{i_0} =$$

$$= a_{i_0} u_0 - c_{i_0} + a'_{i_0} u_0 = 0 + a'_{i_0} u_0 = \sum_{j=1}^{s} a_{i_0 r_j} u_{0 r_j}.$$

It is easy to make sure that the formulas are simplified during computations are done. One can take into account the specificity of changes in the model when algorithm 1 that is above is working.

On the basis of [21], it is possible to specify the calculations of all elements of the interbranch balance model in the transition to a new inverse basis matrix and the solution of SLR (4) whose matrix of restrictions has been changed in one of the rows.

Consequence 2. Transition from the unperturbed system of SLAR (3) to the «perturbed» system (4) in a plural of lines is performed sequentially in terms of the number of «perturbed» lines of iteration for replacing the rows a_{i_0} of the base matrix A with perturbed lines $\bar{a}_{i_0} = a_{i_0} + a'_{i_0}$, $i_0 \in I_0$. The specificity of perturbations in the individual elements of the constraint matrix line is taken into account under the process of converting the elements of method. In particular, the fulfillment of the condition of linear independence of the matrix of constraints is controlled (according to algorithm 1).

C. Analysis of the influence of changes in the columns of matrix of constraints.

In the process of modeling, changes and refinements may occur as individual elements, lines of matrix of constraints, as well as columns. In [22] we studied the influence of the change of k-th column of the constraints matrix A in the form of $\bar{A}_k = A_k + A'_k$ on the solution u_0, where $A_k = \left(a_{1k}, a_{2k}, \ldots, a_{mk}\right)^{\mathrm{T}}$, $A_k = \left(a'_{1k}, a'_{2k}, \ldots, a'_{mk}\right)^{\mathrm{T}}$ (the form of perturbation in (3)) and an auxiliary dual pair of linear programming was constructed for the system.

In particular, in [22] it is determined:

1. Connections between direct and dual tasks lead to bundles of perturbations of rows and matrix columns. In particular, the change of solutions (3) for perturbation of the columns of matrix in the form $\bar{A}_k = A_k + A'_k$ coincides with the changes of the decomposition coefficients of a normal C^{T} vector when the line k of the transposed matrix A^{T} is changed in the form $\bar{A}_k^{\mathrm{T}} = A_k^{\mathrm{T}} + A_k'^{\mathrm{T}}$ according to the scheme of basis matrices to the correspondingly constructed dual problem.

2. As a result of the ties of direct and dual tasks we get a change of the elements of the inverse matrix A_b^{-1} (3) when we replace a k-th column with a matrix column in the form $\bar{A}_k = A_k + A'_k$. A match is established up to transpose with changes of the inverse matrix $(A_b^{\mathrm{T}})^{-1}$ when the matrix A_b^{T} line k is changed in the form $\bar{A}_k^{\mathrm{T}} = A_k^{\mathrm{T}} + A_k'^{\mathrm{T}}$ according to the scheme of the basis matrices of the dual problem method.

3. The component connection of the solution vectors u_0 and $\bar{u}_0$ undergoing the replacement of k-th column of constraints matrix A_k by a column $\bar{A}_k = A_k + A'_k$ is described by relation [22]:

$$\bar{u}_{0k} = \frac{u_{0k}}{1 + \left(A_b^{-1}\right)_k \times A'_k} = \frac{u_{0k}}{\bar{L}_{kk}},\ i = k,$$

$$\bar{u}_{0i} = u_{0i} - \frac{u_{0k}}{1 + \left(A_b^{-1}\right)_k \times A'_k} \times \left[\left(A_b^{-1}\right)_i \times A'_k\right] =$$

$$= u_{0i} - \frac{u_{0k}}{\bar{L}_{kk}} \times \bar{L}_{ki},\ i \neq k,$$

and the condition for preservation of the solution non-degeneracy is the fulfillment of the condition:

$$1 + \left(A_b^{-1}\right)_k \times A'_k \neq 0.$$

Since

$$\bar{L}_k = \left(\bar{L}_{k1}, \bar{L}_{k2}, \ldots, \bar{L}_{km}\right) = A_b^{-1} \times \bar{A}_k = A_b^{-1} \times \left(A_k + A'_k\right).$$

So

$$\bar{L}_{kk} = 1 + L'_{kk} = 1 + \left(A_b^{-1}\right)_k \times A'_k,$$

$$\bar{L}_{ki} = L'_{ki} = \left(A_b^{-1}\right)_i \times A_k,$$

A_σ^{-1}, $i = \overline{1, m}$, $\left(A_b^{-1}\right)_k$ – k-th line of the matrix.

4. Since $\bar{u}_0 = u_0 - (u_{0k}/\bar{L}_{kk}) \times L'_k$ is a representation of the equation of line in a parametric form, where u_0 is the initial vector, L'_k is the vector of normal, $-u_{0k}/\bar{L}_{kk}$ is the value of the displacement parameter along the vector L'_k (from u_0). The value of the vector component L'_k and $-u_{0k}/\bar{L}_{kk}$ forms the perturbation value in the column with the number k.

The technology of influencing the changes of a single column on the SLR solution [22] can be extended to an analysis of the effect of column group changes by constructing an appropriate iterative procedure for considering the effect of changes in each column.

Let the solution and the inverse matrix A_σ^{-1} of the SLR type (3) of the main system be initially known. For definiteness, we assume that in the system (3) changes are experienced by columns $k = i+1, i+2, \ldots, i+i_0$, in the form of $\bar{A}_k = A_k + A'_k$, that is:

$$k = i + s,\ s = \overline{1, i_0},$$

where i_0 is a number of columns of constraints matrix that have changed.

Based on the established relationships, one can construct an algorithm for investigating the properties of the SLAR (4) with a square matrix of constraints with changes in the elements (by conducting a directed substitution of the columns of the basis matrices accordance with the changed ones).

Algorithm 2.

Preparatory step. Let u_0, A_σ^{-1} be known. Assume $s = 1$, is the iterative counter, where s is the inverse matrix of SLAR (3).

Step 1. We provide $k = i + s$ and carry out the substitution of the k-th column of the restrictions matrix A_σ^{-1} A_k by the column $\bar{A}_k = A_k + A'_k$. Let us find the vector:

$$\bar{L}_k = \left(\bar{L}_{k1}, \bar{L}_{k2}, \ldots, \bar{L}_{km}\right) = A_b^{-1} \times \bar{A}_k = A_b^{-1} \times \left(A_k + A'_k\right),$$

where

$$\bar{L}_{kk} = 1 + L'_{kk} = 1 + \left(A_b^{-1}\right)_k \times A'_k,$$

$$\bar{L}_{ki} = L'_{ki} = \left(A_b^{-1}\right)_i \times A'_k,$$

A_σ^{-1}, $i = \overline{1,m}$, $\left(A_b^{-1}\right)_k$ – k-th line of the matrix.

Step 2. When the condition $\bar{L}_{kk} \neq 0$ is fulfilled, the condition for the non-degeneracy of the matrix of constraints is fulfilled. We form a new solution:

$$\bar{u}_{0k} = \frac{u_{0k}}{1+\left(A_b^{-1}\right)_k \times A'_k} = \frac{u_{0k}}{\bar{L}_{kk}}, \quad i = k,$$

$$\bar{u}_{0i} = u_{0i} - \frac{u_{0k}}{1+\left(A_b^{-1}\right)_k \times A'_k} \times \left[\left(A_b^{-1}\right)_i \times A'_k\right] =$$

$$= u_{0i} - \frac{u_{0k}}{\bar{L}_{kk}} \times \bar{L}_{ki}, \; i \neq k.$$

Step 3. Finding the elements of inverse matrix A_σ^{-1} in the result of the column's A_k replacement by the column $\bar{A}_k$:

$$\bar{e}_k = \frac{e_k}{\bar{L}_{kk}}, \quad \bar{e}_i = e_i - \frac{e_k}{\bar{L}_{kk}} \times \bar{L}_{ki}, \quad i = \overline{1,m}, \quad A_\sigma^{-1},$$

where the column k, e_k of the inverse matrix has the form $e_k = \left(e_{1k}, e_{2k}, \ldots, e_{mk}\right)^{\mathrm{T}}$, $k \in I$, in accordance with [22].

Step 4. We give $u_0^{(i+s)} = \bar{u}_0$, $A_\sigma^{(i+s)-1} = \bar{A}_\sigma^{-1}$, k, $A_\sigma^{-1} = A_\sigma^{(i+s)-1}$.

Step 5. Assume $s = s+1$ so when $s \neq i_0$ we have $s = s+1$ and move on to the Step 1, otherwise, to the next step.

The final step. Conducting a comparative characteristic of the properties of perturbed and basic SLR.

According to the algorithm, there is a solution of the modified problem (without first redefining) after i_0 iterations.

D. Changes in the blocks of the matrix of restrictions SLR.

It is known that mathematical models of the study of processes of various nature, in particular, ecological-economic, submitted by the Leontiev-Ford model, contain structural features in the form of a block-cell matrix of constraints. Of course, the changes in one block of the matrix of constraints affect the resulting properties of the model.

We introduce into consideration a block matrix A and A^{-1} (with known properties) in the form:

$$A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1q} \\ A_{21} & A_{22} & \cdots & A_{2q} \\ \cdot & \cdot & \cdots & \cdot \\ A_{p1} & A_{p2} & \cdots & A_{pq} \end{pmatrix},$$

$$A^{-1} = \begin{pmatrix} E_{11} & E_{12} & \cdots & E_{1q} \\ E_{21} & E_{22} & \cdots & E_{2q} \\ \cdot & \cdot & \cdots & \cdot \\ E_{p1} & E_{p2} & \cdots & E_{pq} \end{pmatrix},$$

which contain $p \times q$ «sub matrixes» of the matrix A and its corresponding inverse.

If $I = \{1,2,\ldots,m\}$ are the numbers of the columns of the matrix A, then we will assume:

$$I = \left\{ \underbrace{1,2,\ldots,q_1}_{I_1}, \underbrace{q_1+1,2,\ldots,q_2}_{I_2}, \underbrace{q_2+1,2,\ldots,q_3}_{I_3}, \ldots, \underbrace{q_{q-1}+1,2,\ldots,q_q}_{I_q} \right\},$$

so that $I_1, I_2, \ldots, I_q$ is the subset of the partition of index numbers set I and $Q = \{1,2,\ldots,q\}$ are the numbers of the blocks of the partition.

If $J = \{1,2,\ldots,n\}$ are the numbers of matrix A lines, then, similarly to the above mentioned, one can submit the following partition for the lines:

$$J = \left\{ \begin{matrix} \underbrace{1,2,\ldots,p_1}_{J_1}, \underbrace{p_1+1,2,\ldots,p_2}_{J_2}, \underbrace{p_2+1,2,\ldots,p_3}_{J_3}, \ldots, \\ \underbrace{p_{q-1}+1,2,\ldots,p_q}_{J_p} \end{matrix} \right\},$$

$J_1, J_2, \ldots, J_q$ are subsets of the partition J, $P = \{1,2,\ldots,p\}$ are the numbers of the partition blocks.

For example, we will investigate the relationships between elements of the method, in particular, the inverse matrices and solutions in case of perturbation in the block, A_{ij}, where $i \in P$, $j \in Q$, $A_{ij} \subset A$.

According to the implemented changes, we can write that:

$$u = \left(u_1, u_2, \ldots, u_m\right) = \left(u^{(1)}, u^{(2)}, \ldots, u^{(q)}\right),$$

$$u^{(1)} = \left(u_1, u_2, \ldots, u_{q1}\right), \quad I_1 = \{1,2,\ldots,q_1\},$$

$$u^{(2)} = \left(u_{q_1+1}, u_{q_1+2}, \ldots, u_{q2}\right),$$

$$I_2 = \{q_1+1, q_1+2, \ldots, q_2\},$$

$$u^{(j)} = \left(u^{(j)}_{q_{j-1}+1}, u^{(j)}_2, \ldots, u^{(j)}_{qj}\right),$$

$$I_j = \{q_{j-1}+1, q_{g-1}+2, \ldots, q_j\},$$

$j \in Q$ – sub-vectors to vector of variables $u = \left(u_1, u_2, \ldots, u_m\right)$.

Accordingly:

$$u_0 = \left(u_{01}, u_{02}, \ldots, u_{0m}\right) = \left(u_0^{(1)}, u_0^{(2)}, \ldots, u_0^{(q)}\right),$$

where $u_0^{(j)} \subset u_0$, $j \in Q$ – sub-vectors to vector of variables u_0,

$$a_r = \left(a_{r1}, a_{r2}, \ldots, a_{rm}\right) = \left(a_r^{(1)}, a_r^{(2)}, \ldots, a_r^{(q)}\right),$$

$$r \in J, \quad r \in J_i, \quad i \in P,$$

where

$$u^{(j)} = \left(u^{(j)}_{q_{j-1}+1}, u^{(j)}_2, \ldots, u^{(j)}_{qj}\right) \subset u,$$

$j \in Q$ – sub-vector vector of variables $u = \left(u_1, u_2, \ldots, u_m\right)$.

Then:

$$a_r u - c_r = a_{r1}u_1 + a_{r2}u_2 + \ldots + a_{rm}u_m - c_r =$$
$$= a_r^{(1)}u^{(1)} + a_r^{(2)}u^{(2)} + \ldots + a_r^{(q)}u^{(q)} - c_r,$$

$$r \in J_i, \quad r \in J, \quad i \in P,$$

For a non-constraint limit we can write:

$$\Delta_r = a_{r1}u_{01} + a_{r2}u_{02} + \ldots + a_{rm}u_{0m} - c_r =$$
$$= a_r^{(1)}u_0^{(1)} + a_r^{(2)}u_0^{(2)} + \ldots + a_r^{(q)}u_0^{(q)} - c_r,$$

$$r \in J_i, \quad r \in J, \quad i \in P,$$

and when perturbed:

$$\bar{\Delta}_r = \bar{a}_r u_0 - c_r = (a_r + a'_r)u_0 - c_r = a_r u_0 - c_r + a'_r u_0 = a'^{(j)}_r u^{(j)}_0,$$

$$r \in J_i, \;\; r \in J, \;\; i \in P.$$

For a column k, e_k of the inverse matrix of the form:

$$e_k = \begin{pmatrix} \underbrace{e_{1k}, e_{2k}, \dots, e_{q_1 k}}_{I_1}, \underbrace{e_{q_1+1k}, e_{q_1+2k}, \dots, e_{q_2 k}}_{I_2}, \dots, \\ \underbrace{e_{q_q+1k}, e_{q_q+2k}, \dots, e_{q_{q-1}+q_q k}}_{I_q} \end{pmatrix}^{\mathrm{T}} =$$

$$= \left(e^{(1)}_k, e^{(2)}_k, \dots, e^{(q)}_k\right)^{\mathrm{T}}, \; k \in I,$$

$$\alpha_{rk} = a_r e_k = a^{(1)}_r e^{(1)}_k + a^{(2)}_r e^{(2)}_k + \dots + a^{(q)}_r e^{(q)}_k,$$

$$r \in J_i, \;\; r \in J, \;\; i \in P.$$

Consequence 3. $A_{ij} \subset A$ elements of the method undergo simplifications during the calculation, when elements of the block A_{ij}, where $i \in P$, $j \in Q$, are perturbed, in particular:

$$\bar{\alpha}_{rk} = (a_r + a'_r)e_k = a_r e_k + a'_r e_k = \alpha_{rk} + a'^{(j)}_r e^{(j)}_k,$$

$$r \in J, \;\; r \in J_i, \;\; i \in P, \;\; j \in Q, \;\; k \in I;$$

$$\bar{\Delta}_r = \bar{a}_r u_0 - c_r = a^{(j)}_r u^{(j)}_0 - c_r,$$

$$r \in J, \;\; r \in J_i, \;\; j \in Q.$$

Consequence 4. In case of perturbation in elements of several blocks of the line r, $r \in J$, A_{ij}, where $i \in P$, $j \in Q$, $A_{ij} \subset A$ elements of the method undergo simplifications during the calculation, in particular:

$$\bar{\alpha}_{rk} = (a_r + a'_r)e_k = a_r e_k + a'_r e_k = \alpha_{rk} + \sum_{j \in Q_0} a'^{(j)}_r e^{(j)}_k,$$

$$r \in J, \;\; r \in J_i, \;\; i \in P, \;\; k \in I;$$

$$\bar{\Delta}_r = \bar{a}_r u_0 - c_r = 0 + \sum_{j \in Q_0} a'^{(j)}_r u^{(j)}_0,$$

$$r \in J, \;\; r \in J_i.$$

Consequence 5. The resulting effect of perturbations in the block A_{ij}, where $i \in P$, $j \in Q$, $A_{ij} \subset A$ is determined after the iterations of the MBM by the number of rows $r \in J_i$.

Elements of the method undergo simplifications (discrepancies, relative estimates, and line schedules by lines of the base matrix according to algorithm 1) during computation.

It is easy to make sure that the formalization of representation of constraints matrix (as a block structure) clearly indicates the order of the application of algorithms 1 or 2 (their combination).

E. »Extension» and «narrowing» of the matrix of SLR constraints.

It is known that mathematical models of the study of processes of various nature, in particular, ecological-economic, are constantly being improved. And it is natural to predict that the representation of the Leontiev-Ford model may contain structural «improvements» in the form of inclusion of new blocks of subnets. That is, the initial matrix can become part of a «larger» matrix of constraints («stacking»). Of course, such changes in one block of matrix constraints also affect the resulting properties of the model.

We take into consideration subsidiary block-cell matrices structurally similar to A and A^{-1}:

$$A_0 = \begin{pmatrix} A_{11} & A_{11} & \dots & A_{1q} & 0 \\ A_{21} & A_{22} & \dots & A_{2q} & 0 \\ . & . & \dots & . & 0 \\ A_{p1} & A_{p1} & \dots & A_{pq} & 0 \\ 0 & 0 & \dots & 0 & I_{p+1q+1} \end{pmatrix},$$

$$E_0 = \begin{pmatrix} E_{11} & E_{11} & \dots & E_{1q} & 0 \\ E_{21} & E_{22} & \dots & E_{2q} & 0 \\ . & . & \dots & . & 0 \\ E_{p1} & E_{p1} & \dots & E_{pq} & 0 \\ 0 & 0 & \dots & 0 & I_{p+1q+1} \end{pmatrix},$$

where I_{p+1q+1} is a single-diagonal matrix, A_0 and E_0 contain respectively A and A^{-1} (entered earlier as cellular).

According to a well-known cellular square matrix, you can set the properties of the ointment $\bar{A}$, $A \subset \bar{A}$ of the form:

$$A = \begin{pmatrix} A_{11} & A_{12} & \dots & A_{1q} \\ A_{21} & A_{22} & \dots & A_{2q} \\ . & . & \dots & . \\ A_{p1} & A_{p2} & \dots & A_{pq} \end{pmatrix},$$

$$\bar{A} = \begin{pmatrix} A_{11} & A_{12} & \dots & A_{1q} \\ A_{21} & A_{22} & \dots & A_{2q} \\ . & . & \dots & . \\ A_{p+11} & A_{p+12} & \dots & A_{p+1q+1} \end{pmatrix},$$

which contains respectively $p \times q$, $(p+1)\times(q+1)$ «sub-matrix», that is, we have added a «column» of the sub-matrix blocks and the «string» of the sub-matrix blocks and the corresponding SLAR.

It is easy to make sure that:

– matrices are A_0 and E_0 direct and inverse matrices;

– if the matrix A_0, is formed only by diagonal cellular matrices $A_{11}, A_{22}, \dots, A_{pp}$, $p = q$, then the inverse of them will form diagonal cell matrices $E_{11}, E_{22}, \dots, E_{pp}$, of the inverse matrix E_0;

– the establishment of properties $\bar{A}$ («extended» matrix) in the assumption of known A and A^{-1} is based on the consistent application of algorithms 2 (in columns) and 1 (in rows), respectively;

– form the auxiliary matrices A_0 and E_0;

– the iterative transition from A_0 with the inverse E_0 to $\bar{A}, \bar{A}^{-1}$ is done by successive substitution («inclusion») of the «columns» of submatrix blocks and «lines» of blocks («extension»);

– the elements of the method, in particular, changes in the solution and in the inverse matrix are listed;

– on the contrary the establishment of properties A and A^{-1} according to known $\bar{A}, \bar{A}^{-1}$ (the «narrowing» of the matrix) is based on the «inverse» sequential application of the algorithm 2 (in columns) and the algorithm 1 (in rows), respectively. The transition to the auxiliary A_0 and the inverse E_0, that includes A and A^{-1} is carried out by successive replacement («exclusion») of the columns and lines with the transition to the structure A_0 and E_0. A and A^{-1} single out» at the next stage.

We track changes in the solution and the inverse matrix during the iterations.

F. Incorporation of restrictions on components of model vectors.

The «overlapping» of the two-sided constraints on the variables (hyper parallelepiped variables) necessitates the use of the MBM scheme [17] in case of the inclusion of additional constraints into the model, that is, the analysis of the compatible solvability of SLR and SLAN (systems of linear algebraic inequalities).

7. Discussion of peculiarities of matrix structures analysis by algorithms of the method of basic matrices

In general, it seems expedient to apply the ideology of the simplex-method while analyzing the effects of changes in the linear model. As far as this method already partially includes the possibility of considering group changes during the input-output (line or column) in the basis of iterations depending on the implementation method [16–18].

It should be noted that the method of basic matrices also provides the ability to take into account the effect of changes in both the string and the column in the variants of algorithm [21, 22], which naturally extends to a group of rows and columns.

Considering the influence of changes (refinements) on the properties of the new model (of course, without the procedure of re-solving the task again (initially)) is an important component of method or algorithm. The internal structure of the MBM organically fits into the analysis of the impact of changes as the next iteration of the method. In the method we «proceed» sequentially from the iteration to the iteration from the simple «known» model to the given, and then to the modified. For example, in the Gauss method we «proceed» from a given model to a model of a simple structure. And then there is a new task. That is, the features of the method manifest themselves in different tasks in different ways.

The blockiness (cellularity) of the constraint matrix structure of the model and the content contained in them caused the development of algorithms of inclusion and group changes not only in rows and columns, but also in separate sub-vectors or a group of elements that are constituent lines or columns.

MBM's versatility as the «solver» of SLAR, SLAN provides additional opportunities for improving the model by including restrictions on variables, and so on. The MBM contains *i* generalizations for the analysis of LPP (linear programming tasks) and SNMP (non-linear nonlinear models).

Essentially, the well-known models are being improved and complicated. For example, from Leontiev economic model to the eco-economic model of Leontiev-Ford, there are also known [13] *i* improvements of the last one. The construction of new algorithms for analysis (taking into account, for example, the «expansion» of the model) in the process of improving, incorporating new components of analysis, etc., is introduced in the method of basic matrices.

Increasing the size of the matrix of constraints in the model necessitates analyzing the effects of such changes. In particular, the verification of the non-degeneracy condition of the matrix of constraints. The MBM has incorporated the control of the non-degeneracy of constraints matrix, as well as the possibility of restoring non-degeneracy by directed changes in the matrix of constraints.

It is desirable to «lean» on the achievement of the previous stage or model in the simulation. There is a property of «continuity» of analysis of the influence of changes in the transition from the model «less» to «bigger» and vice versa in the method of basic matrices.

8. Conclusions

1. It is proposed to take into account the cost of fulfilling the restrictions under the Paris Agreement on the basis of the cost-issue balance sheet. Typical generalizations («extensions») of the model are outlined, which, in general, increase its dimension, but do not fall out of the class of linear ones. Variants of modifications and improvements of the model can be investigated on the basis of algorithms 1 and 2. As a result model simplification («narrowing») can be carried out.

2. The paper substantiates the basic algorithms for consideration the influence of various «scenarios» of typical changes (improvements) in the matrix of constraints of ecological-economic model (element, row, column, group of rows or columns, block of elements in the sub-matrix of constraints). These algorithms are extended to the analysis of «extensions» and «narrowing» of the matrix of constraints. By their very nature they are the iterative procedures for refining the known solution of unbroken system by carrying out some amount of homogeneous iterations of the MBM.

3. Further research is advisable to carry out including additional economic and environmental constraints and new factors, constraints on vectors components, classical assumptions about technological structure.

4. The proposed MBM analysis technology of the effects of changes in the elements of constraints matrix can be improved for the simulation of processes involving inclusion of (exclusion of) new matrix blocks, expansion of (or narrowing) the dimension of initial matrix of mathematical model constraints.

The question about the directional changes in the model (with the achievement of the given restrictions on the properties of the solutions) arises now.

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