

Судна на мілководній рейдовій стоянці є під впливом складної системи хвиль. Ця система – результат дифракції на судні хвилювання, що набігає з моря. При переході на мілководдя тривимірні хвилі стають двовимірними. Довжини хвиль зменшуються, періоди зберігаються. Гребені хвиль стають вище й загострюються. Навколо судна на рейдовій стоянці виникає зона трансформації, де хвилювання знову стає тривимірним. Визначення характеристик трансформованого судном хвилювання важливо для проведення робіт з ліквідації аварійних розливів нафти. Буксири, нафтосміттєзбірники, бонопостановники повинні працювати в будь-якій точці навколо аварійного судна, зокрема з боку набігання. Тому розміри зони трансформації хвилювання й висоти хвиль у цій зоні визначають безпеку експлуатації допоміжних суден. Існуюче рішення лінійної дифракційної задачі повинне бути перероблене для використання нелінійної теорії хвиль.

Наведене рівняння хвильового профілю в заданих точках спостереження навколо нерухомого подовженого судна на значному мілководді. Рівняння отримане з виразу для потенціалу швидкостей дифрагованого хвильового руху, викликаного набіганням косих регулярних хвиль кінцевої амплітуди. Характеристики хвиль, що набігають, визначені по теорії Стокса п'ятого порядку. Нелінійну задачу перетворено до комбінації п'яти лінійних. Рішення виконано методом зрощуваних асимптотичних розв'язків (ЗАР).

По отриманих формулах виконані розрахунки хвильових профілів у заданих точках навколо судна в задані моменти часу. Варіюються глибина акваторії, крутість хвиль, курсовий кут хвилювання. Наведені приклади профілів нелінійних і лінійних хвиль у площині перетину судна. Показані подібність і відмінності лінійних і нелінійних хвиль навколо судна на мілководній рейдовій стоянці

Ключові слова: дифракція нелінійних хвиль на судні, метод зрощуваних асимптотичних розв'язків, значне мілководдя

DETERMINING THE CHARACTERISTICS OF DIFFRACTED SEA WAVES OF FINITE AMPLITUDE AROUND A VESSEL IN CONSIDERABLE SHALLOW WATERS

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Received date 06.06.2019

Accepted date 06.07.2019

Published date 28.08.2019

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1. Introduction

Problems on the hydrodynamics of sea waves and vessels employ a transition from linear theories and ratios to the non-linear ones. Such a transition is justified for wind waves with a maximum steepness in deep water, for long waves in considerable shallow waters, and in the zone of wave destruction.

A significant part of the Ukraine's sector of the Black Sea water area is relatively shallow. In the coastal zone where ports are located one observes steep and large waves, especially during winter.

Waves arriving from the open sea deep-water regions are transformed in shallow waters in a complex way. This process is affected by local weather conditions and a seabed's relief. Three-dimensional waves are converted almost into

the two-dimensional ones. Big waves are reduced while the small ones increase in length and height [1].

Multi-tonnage vessels stay in shallow-water open harbors for partial loading, unloading, or bunkering. These vessels are exposed to the action of waves incident from the sea. At the same time, vessels prevent the propagation of waves. Secondary transformation zone of sea waves is formed around a vessel at outer anchorage. Such a wave field is a superposition of the incident and the diffracted sea waves.

Determining the characteristics of sea waves around vessels at outer anchorage is necessary for planning the operation of auxiliary ships, specifically: tugboats, bunkering barges, pilot and inshore boats, oil garbage collectors and boom crafts. The operation of these ships is linked to safe navigation (transfer of people and cargo from one ship to another) and environmental safety (elimination of oil spills).

Of special importance is the operational specificity of rescuer tugboats, oil garbage collectors and boom crafts under conditions of outer anchorage. It is necessary to be able to predict the safe motion trajectories and reasonable motions of auxiliary vessels at complex three-dimensional sea waves. It is also required to take into consideration that the height of waves is one of those parameters based on which the movement of an oil spot is predicted at oil spill [2].

Operation of auxiliary vessels significantly depends on the height and length of waves. Defining the characteristics of waves at outer anchorage is necessary for safe operation of auxiliary vessels, therefore, it is a relevant task.

2. Literature review and problem statement

Paper [3] reports a solution to the linear diffraction problem for an actual elongated vessel in shallow waters. The sea waves are incident to the vessel under an arbitrary angle. The work's results are the established wave fields around a vessel at different values for the depth of a water area and the length of waves. These results show the way the sea waves are transformed around a multi-tonnage vessel at outer anchorage. However, when applying a linear theory, it is impossible to take into consideration the relationship between the height of a wave and its length. Thus, one needs to find a solution to the problem on diffraction using the theory of waves with finite amplitude and to compare the results.

Most existing solutions to the nonlinear diffraction problem over recent years have addressed assessing the impact of waves on fixed or floating offshore structures.

Study [4] considers the basic equations that make it possible to define the transformation of waves in a coastal zone. The range under consideration varies from the linear periodic waves to the nonlinear random waves. The authors provided the systematic review of theories related to steady periodic waves (Stokes and cnoidal). They specified the scope of application of these theories. For random waves, a method is given for estimating the rectified spectra. They gave wave equations for the estimation of combined refraction and diffraction of linear periodic waves. They provided equations for calculating the nonlinear wave transformations in shallow waters. They showed the wave profiles defined by different theories. The results are given for several values of water depths, periods, and heights of waves. The authors explored sea waves in a harbor with a gently sloping bottom and near an underwater breakwater.

Diffraction of the cnoidal waves at vertical cylinders under conditions of shallow water was dealt with in [5]. The wave forces and moments were determined using the Boussinesq and Green-Naghdi equations for a single cylinder and a group of cylinders arranged in a row.

Solving the diffraction problem is important for devices and equipment, which utilize the energy of ocean waves to generate electricity, Oscillating Water Column (OWC). Paper [6] reports an analytical solution of first order to the diffraction problem of ocean waves against a hollow vertical cylinder in the ocean of a finite depth. For the same object, the authors in [7] determined a wave field created by a swinging cylinder. In addition, this work solved the combined problem on diffraction-radiation.

Research [8] addressed the interaction between nonlinear waves and a vertical cylinder and a group of four cylinders. The authors used the method of finite differences

(FDM), the finite element methods (Weakly Nonlinear and Weakly Dispersive FEM, Fully Nonlinear and Weakly Dispersive FEM). At propagation and collisions of single waves, they defined a rise of the free surface and the coefficients of hydrodynamic forces. They gave results from calculations and comparison with experimental data.

Article [9] considers vertical movement of the vertical circular cylinders. To solve the potential problems of the first and second orders, the authors used the method of finite elements. For a single cylinder and for groups of two and four cylinders, they defined wave fields and wave loads.

A nonlinear diffraction problem was considered in work [10]. The authors gave a complete analytical solution of second order to this problem for two-dimensional stationary rectangular cylinders at the free surface of a liquid with a finite depth. They defined the magnitudes for the vertical and horizontal forces of the first and second orders. Comparison between experiments and calculations by other authors has confirmed the probability of the solution.

It is worth noting that papers [6] and [10] employ the same system of inherent functions as is the case in work [3].

Operation of tension leg platforms (TLPs) requires solving a problem on the impact of waves on the groups of vertical cylinders. Numerical modelling of the interaction between nonlinear waves and a system of two vertical circular cylinders is described in article [11]. One cylinder is rigidly fixed at the bottom while a second one floats. The authors identified characteristics for wave forces and moments, as well as for the displacement of a floating cylinder.

Diffraction and refraction of waves in a fluid with a finite depth are explored in [12]. The problem was solved by a finite element method using the discrete non-local (DNL) boundary conditions. The studied objects included a channel of rectangular cross section, a circular cylindrical island with a parabolic bottom around it. The characteristics for sea waves were given.

Interaction between waves and a vertical cylinder was addressed in article [13]. The authors explored numerical modelling (CFD) of this process. They solved the Reynolds equation for the averaged turbulent fluid flow (Reynolds Averaged Navier-Stokes, RANS). They simulated the regular and irregular sea waves with a small and finite amplitude (second-order Stokes) at a numerical experimental pool. The work's results are the characteristics for waves and a wave force.

Experimental research into the influence of waves on a floating cylinder was reported in paper [14]. Ocean waves are modeled at a wave tray (small-sized wave flume). The authors studied the interaction between waves and a floating cylinder moored to the shore. They gave the characteristics for sea waves and the motions of a floating body.

Work [15] investigated the interaction (reflection and transmission) between a floating dock and the nonlinear water waves under conditions of shallow water. The authors gave wave amplitudes for several combinations of sizes of dock and water area depth (the amplitudes of the nonlinear reflected and transmitted waves). Theoretical solutions (analytic and using the method of matched-eigen function expansions) were confirmed by experiments.

Solution to a problem on the diffraction of monochromatic and bichromatic waves against a stationary horizontal cylindrical body that crosses a free surface is given in paper [16]. The depth of a fluid is infinite; the incident sea waves are lateral or arbitrary. The authors used a diffraction potential to determine the forces acting on a floating oil storage

unit (a body with half-elliptic waterlines in the bow part, rectangular frames in the middle part and a prismatic stern).

It should be noted that the examined objects from papers [5–15], namely the vertical and horizontal cylinders with a circular or rectangular cross-section and the floating oil storage unit from work [16], are of simplified shapes. The characteristics of waves around objects, reported in these studies, are hard to compare to the sea waves around bodies of a vessel-like shape. First, the rectangular and circular cross-sections of cylinders differ from the bubble, U- and V-like ship frame cross sections. Second, the shape of the fore and aft edges of a vessel differ considerably from the cylindrical shape.

In problems on vessel hydrodynamics, a diffraction problem is considered relatively rarely. In a general case, the region that is occupied by a fluid is non-stationary; a boundary condition at the free surface is nonlinear. Therefore, it is a difficult task to estimate the interaction between a vessel and such sea waves. In addition, it is known from [17] that in order to determine the hydrodynamic forces acting on a motion vessel it would suffice to derive a solution to a simpler problem on radiation.

Assessment of the vessel performance under strong and extreme sea waves was given in [17]. The authors accounted for a wetted surface and the relationships between different kinds of oscillations. They used a model of the elongated vessel; the velocity is limited ($Fr < 0.3$). The total potential is composed of the potential of the incident free waves with a finite amplitude and the potential of the linear disturbed motion of a fluid, caused by the presence of the vessel in it. The authors linearized the boundary problem and the distribution of hydromechanical forces. Diffracted forces at an instantaneous wetted surface are determined by using the solution to a problem on radiation.

Solving the diffraction problem for specific cases needs clarification and the specialization of boundary conditions.

For vessels, the diffraction problem is solved when determining the wave loads in deep water or for determining the hydrodynamic forces during vessel's motions in shallow water.

The potential of radiation and the diffraction potential at longitudinal pitching of a vessel that moves in considerable shallow waters are defined in article [18]. The authors used the matched asymptotic expansions method (MAEM). For motion in a quiet sea, they determined the potentials of radiation for surging, heaving, and pitching. In addition, the authors gave an expression for the components of diffraction potential for pitching.

Paper [19] reports a solution, derived using the numerical method, for a three-dimensional potential problem on vessel motions in the limited depth liquid. The authors investigated the influence of change in the depth on the value for added masses and damping coefficients; and gave the results from calculating these magnitudes for different types of vessels.

Oscillation of a frame contour in the limited depth fluid was investigated in study [20]. The authors considered a two-dimensional non-linear potential problem. Boundary conditions for the frame contour and the free surface of the fluid are non-linear. The nonlinear forces are determined to the second order accuracy. Calculations were performed for different frame contours. The authors investigated the influence of change in relative depth on the value for nonlinear forces.

Papers [17–20] examined vessel behavior during vessel's motions, but did not consider the profiles of waves around a ship.

Worth noting is the characteristics of waves around a vessel moving in deep water, reported in article [21]. The solution was derived using the improved matching method. The region, which is occupied by the fluid, is divided into a near-field and a far-field. The far field employs a radiation condition. Boundary condition at the free surface is linear. A vessel is replaced with the system of singularities. The characteristics for these singularities and the potential of velocities for the near field are determined using a double technique. A first one implies the use of conditions at the free surface and on a body in the near field. A second one employs the continuity of velocity potential and its normal derivative when traversing the matching surface. Such a technique is close to the one used in work [3].

The nonlinear models of wind waves during storm and the nonlinear hydromechanical loads on a vessel's hull were considered in [22]. The problem on interaction between a vessel and the moderately linear and irregular waves was solved by using a perturbation theory. The desired velocity potential of the disturbed fluid flow was expanded into a series based on powers of the small parameter. The expansion retained the assigned number of first terms in the series. Following the substitutions and transforms, the boundary value problem, nonlinear initially, was brought to a series of linear boundary value problems for the components of the potentials.

The scientific literature describes no efficient direct methods to solve problems on the interaction between vessels and the sea waves of finite amplitude in shallow waters. Approximated solutions are in one way or another related to the linearization of boundary conditions, specialization of a vessel hull's shape.

A wave field is transformed around multi-tonnage vessels at outer anchorage. Sea waves near a ship significantly differ from sea waves at a significant distance from the vessel.

3. The aim and objectives of the study

The aim of this study is to investigate the characteristics of a wave field around a vessel that floats without running at shallow outer anchorage under the influence of oblique waves with a finite amplitude. This would make it possible to determine the dimensions of the zone of sea waves transformation and the height of the waves in this zone. Such characteristics must be considered when estimating the safety of operation of auxiliary (relatively small) vessels.

To accomplish the aim, the following task has been set: to build, by using the method of matched asymptotic expansions, the wave profile equation at the assigned points around a vessel exposed to the incident oblique sea waves with a finite amplitude in considerable shallow waters.

4. Materials and methods to study characteristics of a wave field around a vessel in considerable shallow waters

4.1. Substantiation of the choice of a wave theory

When solving wave problems, the most often assigned are the depth of a water area and the period of a wave. According to statistical data [1], the average periods and distributions of wind sea waves during a transition from deep water to shallow water almost never change. The length, height, and other characteristics of waves are defined by the chosen wave theory type, namely small or finite amplitude.

In the problems on vessel dynamics at sea waves, the essential characteristics are the length and height of a wave at a given depth of a water area. The period and other characteristics are to be defined depending on the chosen theory of waves. Thus, to solve the diffraction problem, it is important to conveniently assign the equation of the profile of a wave with finite amplitude.

A significant portion of the Ukraine's sector of the Black Sea has a depth of less than 100 m; and the average depth of the sea exceeds 1,200 m. In the Black Sea, the period of wind waves typically does not exceed 9 s. The length of such a wave under conditions of deep water is $\lambda=126.5$ m. The repeatability of 6 m waves and larger is less than 1 %, of 5 m ones and larger is less than 2 %. The steepness of such waves is $h/\lambda=1/21$ and $h/\lambda=1/25$, respectively (h is the height of a wave).

Breaking of waves starts at critical depth $H_{cr}=2h$. The smaller initial slope of the deep water waves causes the greater waves' height and the smaller relative depth when waves' destruction begins.

At water depth of $0.5\lambda>H>H_{cr}$ (H is the water depth), calculations employ the theory of nonlinear waves of finite amplitude. For example, the fifth order Stokes theory is used in [23]. The characteristics of waves in that article were determined from formulae reported in [24]. For a very small depth ($H<0.1\lambda$), it is necessary to apply the theory of cnoidal waves.

The concept of "shallow water" in relation to a vessel is associated with the distance between the bottom of a vessel and the bottom of a water reservoir. It was shown in [23] that even under conditions of moderate (relative to a ship) shallow water, a wave length, when compared with the length of the ship, is five or more times larger the depth of the water area.

Let us accept that at a given depth the incoming sea waves do not break. We obtain then quite a range of lengths and heights of waves [23], for which the profile of an incident wave is quite accurately determined from the fifth order Stokes theory. We shall solve a diffraction problem for such values of lengths and heights of waves.

4. 2. Statement of the diffraction problem

Consider the interaction between a stationary vessel that floats in shallow water of depth H with sea waves of a finite amplitude that is incident at an arbitrary angle. We believe that the waves do not reach the breakage stage. Denote the length of a wave λ , the height of a wave h , the wave propagation velocity c .

Introduce two rectangular coordinate systems: motionless $O\xi_0\eta_0\zeta_0$ that characterizes the motion of a fluid, and a vessel-associated one $Oxyz$ (Fig. 1). The direction of the $O\xi_0$ axis coincides with a velocity vector of the incident sea waves. The course angle of the incident sea waves β (between the Ox and $O\xi_0$ axes) varies from 0 to 360°, positive direction is counterclockwise from $O\xi_0$ to Ox .

A wave profile equation, according to [24], is given in the form:

$$\zeta_w = \sum_{j=1}^5 \zeta_{(j)} = \frac{1}{k} \sum_{j=1}^5 a^j b_j \cos(j\theta), \tag{1}$$

where $k=2\pi/\lambda$ is the wave number (shape frequency); a is the parameter for a wave height, determined from ratio:

$$kh = 2 \left[a + a^3 B_{33} + a^5 (B_{35} + B_{55}) \right], \tag{2}$$

where b_j are the dimensionless coefficients that depend on the depth of a water area and the length of a wave; B_{33}, B_{35}, B_{55} are the wave profile parameters, determined according to [24];

$$\theta = k(\xi_0 - ct) = k\xi_0 - \sigma t.$$

Note that paper [17] stated a non-linear problem of vessel's motions. The authors assigned the potential of incident sea waves' velocities as the sum of a large number of harmonics of irregular sea waves. In equation (1) directions for the propagation of all components are the same.

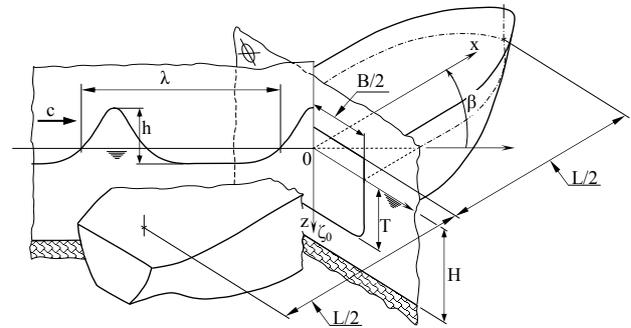


Fig. 1. Coordinate systems and characteristics of sea waves

A liquid is considered to be ideal, heavy and incompressible; its motion is potential. In a coordinate system associated with the vessel where $\xi_0 = x \cos\beta + y \sin\beta$, $\zeta_0 = z$, the disturbed fluid movement is described by the potential of velocities $\Phi^E(x, y, z, t)$. The domain for determining it E is limited by the water area bottom D , the vessel's wetted surface S , and the free surface of a liquid Σ . Represent the potential Φ^E as the sum

$$\Phi^E(x, y, z, t) = \Phi^*(x, y, z, t) + \Phi^d(x, y, z, t), \tag{3}$$

where $\Phi^*(x, y, z, t)$ is the potential of the incident sea waves velocities; $\Phi^d(x, y, z, t)$ is the potential of velocities of a diffracted wave motion.

The potential of the incident sea waves' velocity, according to the chosen notation of a wave profile, is recorded in the $Oxyz$ coordinate system in the form [24]

$$\begin{aligned} \Phi^*(x, y, z, t) &= \sum_{j=1}^5 \Phi^{*(j)} = \\ &= \frac{\sigma}{k^2} \sum_{j=1}^5 a^j A_j \operatorname{ch}(jk(z-H)) \sin[j(kx \cos\beta + ky \sin\beta - \sigma t)], \end{aligned} \tag{4}$$

where A_j are the parameters for a wave potential (dimensionless functions that depend on kH).

The relationship between a wave number and the sea wave frequency $\sigma=kc$ is determined from formula

$$\sigma^2 = kg \cdot \operatorname{th}(kH) \cdot (1 + a^2 C_1 + a^4 C_2) = kg \cdot \operatorname{th}(kH) \cdot C^*, \tag{5}$$

where C_1, C_2 are the parameters for a wave frequency determined from [24].

We shall assume that the diffraction of all $\zeta_{(j)}$ against a vessel's hull occurs mutually independently. Such an assumption can be accepted given the nature of the sea waves components in formula (1), as well as the expressions of potentials in the form (3) and (4).

The potential of first order is determined from formula

$$\Phi^{*(1)} = \frac{\sigma}{k^2} a \frac{A_1^*}{\text{sh}(kH)} \times \text{ch}[(k(z-H))] \cdot \sin[(kx \cos \beta + ky \sin \beta - \sigma t)], \quad (6)$$

where A_1^* is a parameter for a wave potential, determined from [24].

The solution to the diffraction problem for oblique waves of small amplitude $r=h/2$ that are incident to the actual ship under conditions of shallow waters, is given in work [3].

The corresponding incident sea waves potential takes the form:

$$\Phi^{*(1)} = \frac{rg}{\sigma} \frac{1}{\text{ch}(kH)} \times \text{ch}[k(z-H)] \cdot \sin(kx \cos \beta + ky \sin \beta - \sigma t). \quad (7)$$

The structure and dimensionality of expressions (6) and (7) are identical. Only the form of recording a dimensional multiplier is different. Formula (6) can be rewritten in the form:

$$\Phi^{*(1)} = K \frac{r_{(1)}^* g}{\sigma} \frac{\text{ch}[k(z-H)]}{\text{ch}(kH)} \times \sin(kx \cos \beta + ky \sin \beta - \sigma t), \quad (8)$$

where $r_{(1)}^* = aA_1^*/k$ is the amplitude coefficient of wave of first order;

$$K = \frac{\sigma^2}{kg \text{th}(kH)}$$

is the auxiliary factor.

Coefficient K is the same for all components of the potential in formula (4). We obtain for the waves of second order and higher

$$r_{(j)}^* = \frac{a^j A_j^*}{k} = \frac{a^j A_j^*}{k} \frac{\text{ch}(jkH)}{\text{ch}(jkH)}, \quad (9)$$

where A_j^* are the parameters for a wave potential, determined from [24].

Then

$$\begin{aligned} \Phi^*(x, y, z, t) &= \frac{g}{\sigma} \frac{K}{\text{ch}(kH)} \times \\ &\times \sum_{j=1}^5 r_{(j)}^* \text{ch}[jk(z-H)] \times \\ &\times \sin[j(kx \cos \beta + ky \sin \beta - \sigma t)] = \\ &= \sum_{j=1}^5 \frac{r_{(j)}^* g K_{(j)}}{\sigma} \frac{\text{ch}[jk(z-H)]}{\text{ch}(jkH)} \times \\ &\times \sin[j(kx \cos \beta + ky \sin \beta - \sigma t)]. \end{aligned} \quad (10)$$

The form of recording of all the components in the incident sea waves' velocity potential coincides with that used in [3]. In order to solve a diffraction problem, it is possible to apply a procedure given in [3]. Let us represent potential $\Phi^d(x, y, z, t)$ in the form

$$\Phi^*(x, y, z, t) = \sum_{j=1}^5 [\Phi_c^{*(j)} \cos(j\sigma t) + \Phi_s^{*(j)} \sin(j\sigma t)], \quad (11)$$

where the amplitude functions are equal to

$$\begin{aligned} \left\{ \begin{array}{l} \Phi_c^{*(j)} \\ \Phi_s^{*(j)} \end{array} \right\} &= \\ &= \frac{r_{(j)}^* g K_{(j)}}{\sigma} \frac{\text{ch}[jk(z-H)]}{\text{ch}(jkH)} \left\{ \begin{array}{l} \sin[j(kx \cos \beta + ky \sin \beta)] \\ -\cos[j(kx \cos \beta + ky \sin \beta)] \end{array} \right\}. \end{aligned} \quad (12)$$

For each $\Phi^{*(j)}$, $j=1, \dots, 5$, we determine the corresponding potential of velocities of the diffracted wave movement $\Phi^{d(j)}$.

Boundary problems for all components of the diffraction potential include harmony condition and the following boundary conditions:

- at the free surface of a liquid Σ ;
- at the wetted surface of a vessel S ;
- at the bottom of a water reservoir D .

There is also a condition for the attenuation of diffracted waves at an infinite distance from the ship. In addition, each potential $\Phi^{d(j)}$ must satisfy the principle of radiation. Represent $\Phi^{d(j)}$ in the form of a sum

$$\Phi^{d(j)}(x, y, z, t) = \Phi_c^{d(j)} \cos(j\sigma t) + \Phi_s^{d(j)} \sin(j\sigma t). \quad (13)$$

Then the amplitude functions $\Phi_c^{d(j)}$ and $\Phi_s^{d(j)}$ must satisfy the following differential systems:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Phi_{c,s}^{d(j)}(x, y, z) = 0, \quad (x, y, z) \in E; \quad (14)$$

$$\left(\frac{\partial}{\partial z} + \frac{(j\sigma)^2}{g} \right) \Phi_{c,s}^{d(j)}(x, y, 0) = 0, \quad (x, y, 0) \in \Sigma, \quad (15)$$

$$\frac{\partial \Phi_{c,s}^{d(j)}(x, y, H)}{\partial z} = 0, \quad (x, y) \in]-\infty; \infty[; \quad (16)$$

$$\lim_{r \rightarrow \infty} (\overline{\text{grad}} \Phi_{c,s}^{d(j)}(x, y, z)) = 0,$$

$$(x, y, z) \in E, \quad r = \sqrt{x^2 + y^2}; \quad (17)$$

$$\frac{\partial \Phi_{c,s}^{d(j)}(x, y, z)}{\partial N} = -\frac{\partial \Phi_{c,s}^{*(j)}(x, y, z)}{\partial N}, \quad (x, y, z) \in S. \quad (18)$$

Hereafter, index "d" in the descriptions of diffraction potentials is omitted.

We assume the ship to be an elongated body. Refine the form of a normal derivative from the potential at wetted surface S . Similar to the way it was performed in [3], we obtain:

$$\begin{aligned} \frac{\partial \Phi_c^{(j)}(x, y, z)}{\partial N} &= B_c^{EV(j)} + B_c^{OD(j)}, \\ \frac{\partial \Phi_s^{(j)}(x, y, z)}{\partial N} &= B_s^{EV(j)} + B_s^{OD(j)}. \end{aligned} \quad (19)$$

$$\begin{cases} B_c^{EV(j)} \\ B_s^{EV(j)} \end{cases} = \frac{r_{(j)} g K_{(j)}(jk)}{\sigma \operatorname{ch}(jkH)} \operatorname{sh}[jk(z-H)] \times \\ \times \cos(\bar{N}, z) \begin{cases} -\sin[j(kx \cos\beta + ky \sin\beta)] \\ \cos[j(kx \cos\beta + ky \sin\beta)] \end{cases}, \quad (20)$$

$$\begin{cases} B_c^{OD(j)} \\ B_s^{OD(j)} \end{cases} = -\frac{r_{(j)} g K_{(j)}(jk \sin\beta)}{\sigma \operatorname{ch}(jkH)} \operatorname{ch}[jk(z-H)] \times \\ \times \cos(\bar{N}, y) \begin{cases} \cos[j(kx \cos\beta + ky \sin\beta)] \\ \sin[j(kx \cos\beta + ky \sin\beta)] \end{cases}. \quad (21)$$

Thus, each diffraction potential $\Phi^{(j)}$ is equal to the sum of four components:

$$\Phi^{(j)} = \sum_{i=1}^4 \Phi_i^{(j)}. \quad (22)$$

Diffraction potentials $\Phi^{(j)}$ (each separately) are determined by the matched asymptotic expansions method (MAEM) as shown below.

4. 3. Determining the potentials of a diffracted wave motion

According to the procedure for applying MAEM, we shall conditionally divide the region occupied by a fluid into zones: external, where $(y/L) = O(1)$, and internal, where $(y/L) = O(\epsilon)$, $\epsilon \ll 1$.

A boundary transition $\epsilon \rightarrow 0$ at y and z fixed in the external zone converts a vessel hull into a segment $\Delta = \{-L/2 \leq x \leq L/2, y = z = 0\}$. Region E converts into region \bar{E}_0 (a layer of liquid $0 \leq z \leq H$ with a cut-out segment Δ). Free surface Σ converts into plane $\bar{\Sigma}_0$ (plane $z = 0$ with a cut-out segment Δ). In the external zone, edge problems do not include the boundary conditions at the wetted surface of a vessel.

In the internal zone, we introduce elongated coordinates $Y = y/\epsilon, Z = z/\epsilon, \epsilon \ll 1$. The movement of a liquid at accuracy of up to small $O(\epsilon)$ is considered to be two-dimensional. The region occupied by the liquid is a band with the omitted frame contour $L(x)$. This problem has no boundary condition at the infinite distance from the ship.

In each zone, individual edge problems are stated. Their solutions are asymptotically matched at the zones' interface. The derived approximated solution is uniformly suitable across the entire region occupied by a liquid.

The final expressions for the components of the diffraction potential are recorded as follows:

$$\Phi_{c,s(1)}^{(j)}(x, y, z) = \frac{jk}{2} \operatorname{ch}[jk(z-H)] V(jkH) \int_{-\frac{L}{2}}^{\frac{L}{2}} Q_c(\xi, jk) \begin{cases} N_0(jkR) \\ -J_0(jkR) \end{cases} d\xi; \quad (23)$$

$$\Phi_{c,s(3)}^{(j)}(x, y, z) = \frac{jk}{2} \operatorname{ch}[jk(z-H)] V(jkH) \int_{-\frac{L}{2}}^{\frac{L}{2}} Q_s(\xi, jk) \begin{cases} J_0(jkR) \\ N_0(jkR) \end{cases} d\xi; \quad (24)$$

$$\Phi_{c,s(2)}^{(j)}(x, y, z) = \frac{jk y}{2} \operatorname{ch}[jk(z-H)] V(jkH) \times \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{P_c(\xi, jk)}{R} \begin{cases} N_1(jkR) \\ -J_1(jkR) \end{cases} d\xi; \quad (25)$$

$$\Phi_{c,s(4)}^{(j)}(x, y, z) = \frac{jk y}{2} \operatorname{ch}[jk(z-H)] V(jkH) \times \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{P_s(\xi, jk)}{R}(\xi, jk) \begin{cases} J_1(jkR) \\ N_1(jkR) \end{cases} d\xi, \quad (26)$$

where ξ is the variable of integration lengthwise a vessel;

$$R = \sqrt{(x - \xi)^2 + y^2};$$

$$V(jkH) = \frac{\operatorname{ch}(jkH)}{2jkH + \operatorname{sh}(2jkH)};$$

J_0, N_0 are the Bessel and Neumann functions of zero order for a real argument;

J_1, N_1 are the Bessel and Neumann functions of first order for a real argument, respectively.

Functions $P_{c,s}$ and $Q_{c,s}$ are determined when solving problems in the internal zone.

Calculation formulae using the external variables are given below. When computing functions $P_{c,s}$ and $Q_{c,s}$, we used, respectively, $B_c^{EV(j)}$ for Q_c ; $B_s^{EV(j)}$ for Q_s ; $B_c^{OD(j)}$ for P_c ; $B_s^{OD(j)}$ for P_s .

The detailed determination of the potential of velocities of the diffracted wave motion of a fluid at a random course angle of sea waves is given in work [25].

4. 4. Determining the characteristics of sea waves near a vessel

The above theoretical solution to a diffraction problem was used to determine the profiles of waves at the assigned points around the hull of a floating immovable vessel in shallow water.

The equation of a wave profile is recorded as follows:

$$\zeta_w = -\frac{1}{g} \frac{d}{dt} \Phi^E(x, y, 0, t) = -\frac{1}{g} \frac{d}{dt} \left\{ \sum_{j=1}^5 \left[\Phi^{*(j)}(x, y, 0, t) + \sum_{i=1}^4 \Phi_{(i)}^{(j)}(x, y, 0, t) \right] \right\} = \sum_{j=1}^5 \left[r_c^{(j)} \cos(j\sigma t) + r_s^{(j)} \sin(j\sigma t) \right], \quad (27)$$

where the components of a wave profile are assigned by formulae:

$$r_c^{(j)} = r_{(j)} K_{(j)} \cos[j(kx \cos\beta + ky \sin\beta)] + \sum_{i=1}^4 \zeta_{c(i)}^{(j)}; \\ r_s^{(j)} = r_{(j)} K_{(j)} \sin[j(kx \cos\beta + ky \sin\beta)] - \sum_{i=1}^4 \zeta_{s(i)}^{(j)}. \quad (28)$$

The components of diffracted sea waves $\zeta_{c(i)}^{(j)}, \zeta_{s(i)}^{(j)}, i = 1 \div 4, j = 1 \div 5$ taking into consideration substitution $z = 0$ and

the parity of functions $\text{ch}(jkH)$, are determined from formulae:

$$\zeta_{c(1)}^{(j)} = \frac{\sigma jk}{2g} E(jkH) \int_{-\frac{L}{2}}^{\frac{L}{2}} Q_c(\xi, jk) N_0(jkR) d\xi; \quad (29)$$

$$\zeta_{c(2)}^{(j)} = y \frac{\sigma jk}{2g} E(jkH) \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{P_c(\xi, jk)}{R} N_1(jkR) d\xi; \quad (30)$$

$$\zeta_{c(3)}^{(j)} = \frac{\sigma jk}{2g} E(jkH) \int_{-\frac{L}{2}}^{\frac{L}{2}} Q_s(\xi, jk) J_0(jkR) d\xi; \quad (31)$$

$$\zeta_{c(4)}^{(j)} = y \frac{\sigma jk}{2g} E(jkH) \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{P_s(\xi, jk)}{R} J_1(jkR) d\xi; \quad (32)$$

$$\zeta_{s(1)}^{(j)} = -\frac{\sigma jk}{2g} E(jkH) \int_{-\frac{L}{2}}^{\frac{L}{2}} Q_c(\xi, jk) J_0(jkR) d\xi; \quad (33)$$

$$\zeta_{s(2)}^{(j)} = -y \frac{\sigma jk}{2g} E(jkH) \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{P_c(\xi, jk)}{R} J_1(jkR) d\xi; \quad (34)$$

$$\zeta_{s(3)}^{(j)} = \frac{\sigma jk}{2g} E(jkH) \int_{-\frac{L}{2}}^{\frac{L}{2}} Q_s(\xi, jk) N_0(jkR) d\xi; \quad (35)$$

$$\zeta_{s(4)}^{(j)} = y \frac{\sigma jk}{2g} E(jkH) \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{P_s(\xi, jk)}{R} N_1(jkR) d\xi, \quad (36)$$

where

$$E(jkH) = \frac{1}{\frac{2jkH}{\text{ch}^2(jkH)} + 2\text{th}(jkH)}.$$

When performing calculations, each frame contour $L(x)$ is assigned in the form of sets of points. The points divide the contour into rather small elements, each of which is considered as a straight segment. Calculation formulae are converted considering a transition from integration to summation along points.

Functions $P_{c,s}$ and $Q_{c,s}$ are computed for each frame contour $L(x)$, that is for fixed x :

$$\left. \begin{matrix} P_{c,s}^{(j)} \\ Q_{c,s}^{(j)} \end{matrix} \right\} = 2 \int_{\frac{b(x)}{2}}^{\frac{b(x)}{2}} e^{-jk\eta_0(u)} \left\{ \begin{matrix} T_1^{(j)}(u) \sin(jku) + T_2^{(j)}(u) \cos(jku) \\ T_1^{(j)}(u) \cos(jku) - T_2^{(j)}(u) \sin(jku) \end{matrix} \right\} du, \quad (37)$$

where $b(x)$ is the width of a frame contour $L(x)$; u is the variable of integration along width; $\eta_0(u)$ is the equation of contour $L(x)$;

$$T_1^{(j)} = jkA(u) + B(u) \sqrt{1 + \left[\frac{d\eta(u)}{du} \right]^2}; \quad (38)$$

$$T_2^{(j)} = jkA(u) \frac{d\eta_0(u)}{du}. \quad (39)$$

Formulae (38) and (39) include the value for a velocity potential $A(u)$ and the normal derivative from potential $B(u)$ along contour $L(x)$ for each component of the diffraction potential from formula (13). Indexes are omitted to simplify the notation. Values for the normal derivatives are defined by boundary conditions (19) to (21). Potentials along a contour are unknown, since these potentials are a solution to the problem. In accordance with the practice of applying the Cochin functions for wave problems, $A(u)$ in formulae (38) and (39) is substituted with a value for the potential at infinite frequency.

It should be noted that the applied estimation procedures are not oriented towards any special shape of frame contours.

5. Results of research into the wave field near a vessel

When solving a linear diffraction problem in work [3], the most essential changes in the characteristics of waves were observed from the incident side. It should be borne in mind that oil garbage collectors and boom crafts must operate at any place near a ship in emergency. Therefore, while solving a nonlinear diffraction problem, we must first analyze the waves from the side of incidence.

Compared with work [3], the number of variable parameters was reduced. We have chosen the two smallest values for the depth of a water area, because considerable shallow water is considered. Single smallest wavelengths of those reported in work [3] was chosen because one observed the largest height of standing waves for it. For the selected values of H/T and λ/L , the magnitude for the Ursella number [1] $N_{URS} = H\lambda^2/h^3$ is less than 26; it helps assess the suitability of theories of wave formation. Therefore, calculations may employ the fifth order Stokes theory.

In accordance with the practice for determining the heights of waves with a finite amplitude, we accepted: $h/\lambda = 1/20, 1/30, 1/40$, that is the respective heights $h = 5.04 \text{ m}, h = 3.36 \text{ m}, h = 2.52 \text{ m}$. According to [26], in the Ukraine's sector of the Black Sea the wave height of $h_{3\%} = 5 \text{ m}$ is seldom exceeded. This value for $h_{3\%}$ is matched with the height of "significant" waves $h_{3\%} = h_s = 3.76 \text{ m}$ or average height $\bar{h} = 2.37$. For such heights, at $\lambda = 100.8 \text{ m}$, the slope would be $h/\lambda = 1/27$ and $h/\lambda = 1/42$, accordingly. Thus, the values fit the assigned range. It should also be noted that for auxiliary vessels the assigned wavelength is rather large.

Based on the above formulae, we calculated wave profiles at the assigned points of observation near a vessel. The

selected points of observation are the same that were used when applying a linear theory in paper [3].

These points form a grid. The coordinates of the points are within $|x| \leq L, |y| \leq (L+B/2)$. Points inside the contour of the waterline of the vessel are excluded. A step along the grid's abscissas and ordinates is $0.0125L$. Such an arrangement of points was selected according to the results from numerical experiments. This provided satisfactory accuracy of subsequent calculations while the amount of source data is the smallest.

We determine the cosine and sinus components $r_c^{(j)}$ and $r_s^{(j)}$ (28) in the grid, which are further used for the calculation of wave profiles (27).

Similarly to work [3], the selected research object was a wave field around a bulk carrier, the type of "Zoya Kosmodemyanskya". Basic dimensions of the vessel are: length 215.4 m, length between perpendiculars $L=201.6$ m, width $B=31.8$ m, draught $T=11.73$ m.

Taking into consideration results from work [3], we calculated wave profiles for the relative wavelength $\lambda/L=0.5$. Other variable parameters are as follows:

- the relative depth of a water area $H/T=1.1, 1.3$;
- the course angles of the incident sea waves $\beta=90^\circ, 120^\circ, 135^\circ, 150^\circ$;
- time moments $t/\tau=0.25, 0.5, 1.0$.

Based on the results of calculations, we have derived the relative wave profiles for all combinations of source data

$$\bar{\zeta}_w(x, y, t) = 2\zeta_w(x, y, t) / h.$$

As an example, below are the wave profiles for $x=0$ (a plane of the middle cross section of a vessel) at different time moments, from the incidence side. Fig. 2 shows the influence of wave steepness on the wave profile at a constant depth. Fig. 3 shows the impact of change in depth on the wave profile at transverse incidence of waves of the same steepness. Fig. 4 demonstrates the impact of change in depth on the wave profile at oblique incidence of waves with the same steepness. In all figures, $y=0$ coincides with the ship's side.

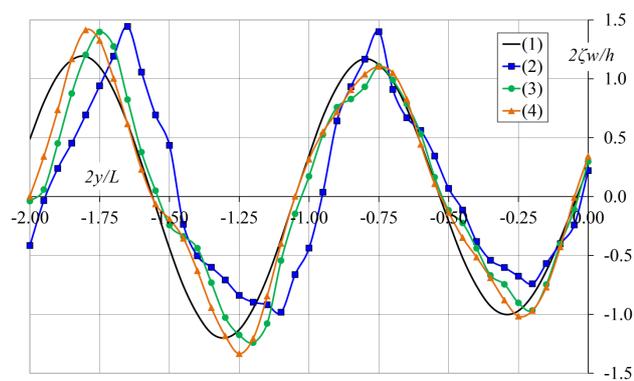


Fig. 2. Wave profiles around a vessel's hull (the incidence side) at different values of wave steepness $H/T=1.1, t=\tau, \beta=90^\circ$:
 1 – linear theory; 2 – $h/\lambda=1/20, N_{URS}=23.8$; 3 – $h/\lambda=1/30, N_{URS}=15.9$; 4 – $h/\lambda=1/40, N_{URS}=11.9$

For comparison, all figures show the results from calculating the corresponding wave profiles using a linear theory.

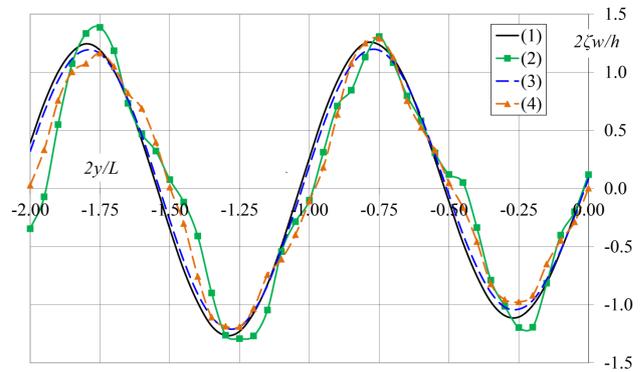


Fig. 3. Wave profiles around a vessel's hull (the incidence side) at different values of the water area depth $t=\tau/4, \beta=90^\circ$:
 1 – $H/T=1.1$, linear theory; 2 – $H/T=1.1, h/\lambda=1/30, N_{URS}=15.9$; 3 – $H/T=1.3$, linear theory; 4 – $H/T=1.3, h/\lambda=1/30, N_{URS}=9.6$

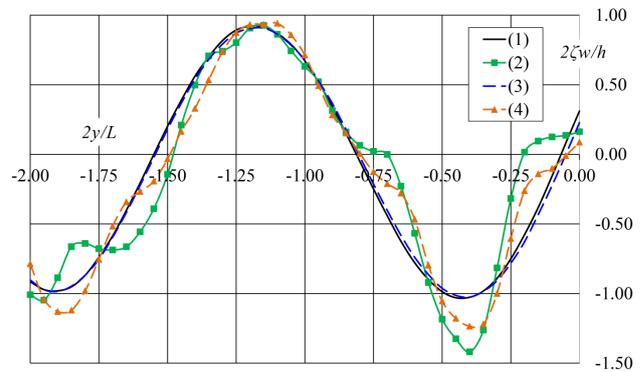


Fig. 4. Wave profiles around a vessel's hull (the incidence side) at different values of the water area depth $t=\tau/2, \beta=135^\circ$:
 1 – $H/T=1.1$, linear theory; 2 – $H/T=1.1, h/\lambda=1/30, N_{URS}=15.9$; 3 – $H/T=1.3$, linear theory; 4 – $H/T=1.3, h/\lambda=1/30, N_{URS}=9.6$

6. Discussion of results from determining the wave field near a vessel

When comparing the wave profiles determined by using the linear and nonlinear theories, the following common and different features have been established.

Common features:

- standing waves occur near a vessel from the side of incidence (Fig. 2–4);
- at antinodes, the heights of waves increase in comparison with the height of the waves far from the vessel (Fig. 2–4);
- the antinodes of waves are at a distance from each other that roughly equals a half the wavelength (Fig. 2–4);
- the arrangement of antinodes depends on the course angle of sea waves (Fig. 3, 4);
- the arrangement of wave antinodes changes little with a water area depth (Fig. 3);
- zones of the decreased oscillations are between the antinodes (Fig. 2–4).

Different features:

- an increase in the relative heights of crests of the nonlinear standing waves is more pronounced (from 10 to 20 %) than that for the linear waves (Fig. 2);

– the absolute heights of crests of the nonlinear standing waves are larger than those for the linear waves (Fig. 2), as expected;

– the antinodes of nonlinear waves occur closer to a vessel's hull in proportion to the steepness of waves (Fig. 2);

– the antinodes of nonlinear waves are closer to a vessel's hull than the antinodes of linear waves (Fig. 2).

The above common features of the linear and nonlinear wave profiles are observed at all frame cross sections. The different features are fully dependent on the arrangement of a frame cross section lengthwise a vessel.

Thus, the use of a nonlinear theory makes it possible to more accurately take into consideration those changes that a vessel introduces when it stays in considerable shallow waters to the wave field of outer anchorage. The current research allows the determination of the size and configuration of the zone of sea waves transformation around a vessel, as well as the height of waves in this zone.

It should be noted that the characteristics for a wave transformation zone depend mostly on the sea waves course angle. Therefore, at oblique incidence, it is necessary to increase the size of the grid, formed by the points of observation; the grid's limits should equal $|x| \leq 2L$, $|y| \leq (2L+B)/2$.

The limitations in the current research are associated with the use of the Stokes theory of waves of fifth order for the selected values of a water area depth. In particular, for depth $H=12.90$ m the wave length $\lambda=100.8$ m is the biggest permissible. For longer waves and other depths, one should

apply a cnoidal theory of waves or the Stokes theory of higher orders waves. Choosing a calculation procedure and the verification of results for long waves are implied at the next stage of our work.

7. Conclusions

We have given the equations of a wave profile at the assigned points around a vessel, derived by using the method of matched asymptotic expansions. The oblique sea waves of finite amplitude are incident on an elongated stationary vessel in considerable shallow waters. A technique for applying an existing solution to the linear diffraction problem for waves with finite amplitude has been devised. We have performed calculations of wave profiles at fixed points of time at the assigned points of observation in line with the nonlinear and linear theories. The varied parameters are the depth of water, the slope of wave, and the course angle of sea waves. The examples of relative wave profiles have been given. It has been shown that the linear and nonlinear wave fields near a vessel are qualitatively similar. The relative heights of crests of the nonlinear standing waves increase by 10–20 % in comparison with those for the linear waves. The relative heights of crests of the resulting standing wave do not exceed the relative heights of crests away from a vessel. Quantitative differences are more likely related to the influence of wave steepness, and to a lesser extent to the depth of water.

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