

Розглянуто результати виявлення періодичних сигналів з використанням теорії хаосу, на основі дискретної обробки атрактора Дуффінга у перерізі Пуанкаре.

Для проведення досліджень обрано хаотичну систему Дуффінга, яка характеризується високою чутливістю до сигналів періодичної форми, і може бути реалізована відносно нескладною схемою.

Проведено аналіз реакції системи Дуффінга на періодичний вплив. Показано, що при збільшенні амплітуди періодичних складових вхідного сигналу на частоті задаючих коливань відбувається зсув фазової траєкторії вздовж перерізу Пуанкаре, який характеризується фрактальною геометрією. Визначено типи змін атрактора Дуффінга, які виникають внаслідок дії періодичного сигналу на вході. У фазовій площині, утвореній вихідним сигналом і його похідною, виділено контрольні області для фіксації типів динаміки фазової траєкторії. Відповідно до характеристик отриманих фазових траєкторій побудовано таблицю істинності, яка дозволяє виконувати оцінку впливу періодичної складової із достатньо великим кроком дискретизації за часом, що важливо для забезпечення швидкодії пристроїв обробки сигналів. Отримано функціонали, які описують процес виявлення періодичних сигналів шляхом дискретної обробки атрактора системи Дуффінга у перерізі Пуанкаре.

На основі сформульованих функціоналів та таблиці істинності запропоновано структурну схему пристрою для виявлення періодичного сигналу в шумі. У запропонованому пристрої в якості вхідного блоку може використовуватися реалізація системи Дуффінга на основі аналогового електричного кола.

Отримано значення дискретних оцінок амплітуди періодичної складової вхідного сигналу за зміщенням фазової траєкторії системи Дуффінга відносно атрактора у перерізі Пуанкаре. Згідно з результатами проведеного моделювання, запропонована схема дозволяє виявляти періодичні сигнали при низьких значеннях відношення сигнал/шум

Ключові слова: виявлення сигналів, хаотична система, гармонічний сигнал, аттрактор, цифрова обробка сигналів

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CHAOS-BASED SIGNAL DETECTION WITH DISCRETE-TIME PROCESSING OF THE DUFFING ATTRACTOR

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1. Introduction

Development of advanced information technologies, communication and control systems [1] induces experts to focus more and more attention to the use of chaotic signals and information processing systems [2]. Signal detection methods based on chaos theory have been known for the last 20 years. Such methods can be characterized by a significant level of information capacity, improved correlation properties and super-broad bands (super-broad band systems, SBS) under various conditions [3, 4]. Currently, signal detection methods using chaotic systems are being investigated with various types of modulation (BPSK, QPSK and others) [1, 5–7]. Software and hardware implementations of the signal detection methods based on chaos theory are known. However, practical application of such methods is complicated because of errors in control and estimation of chaos [8–10]. Present-day studies related to detection of signals using the theory of chaos are focused on the problems of identification and

control of chaos [11–13]. Their solution makes it possible to improve sensitivity and noise immunity.

The main advantage of using chaotic systems for detecting signals consists in their high sensitivity to weak signals against the background of noise, in particular when the signal-to-noise ratio is less than 0 [4]. However, the known methods of processing attractors [9, 10] do not enable taking advantage of chaotic systems to a full extent because of insufficient development of algorithms of estimation of the input signal parameters by analyzing the output chaotic oscillations [12]. In particular, because of complexity of dynamics of the chaotic systems in detection of periodic signals, the process of transition of the system from a chaotic state to a periodic state [14] which is characterized by low sensitivity [2] is used as an indicator of signal presence. Coupled with sensitivity of the critical state parameters to the form of non-periodic input signals, this disadvantage substantially limits the possibility of practical application of chaotic systems to detection of periodic signals.

Thus, the task of analyzing and developing new methods and means of detecting periodic signals by means of processing attractors of chaotic systems is relevant.

2. Literature review and problem statement

The results obtained in studies of signal detection capabilities using a chaotic system were presented in [15]. It was shown that a chaotic system is characterized by high sensitivity to weak periodic signals. The methods are based on sensitivity of the Lyapunov exponents of the chaotic system to weak input signals [12]. The results presented in [12] suggest that chaotic systems have significant prospects for application in signal processing. However, it has been shown that the process of identifying a chaotic state of oscillations can introduce a significant statistical error and brings about substantial signal processing complications. Thus, current methods of analysis of chaotic signals were developed insufficiently and require further improvement.

A theory of nonlinear systems with chaotic dynamics was developed in [14] for the problems of signal detection. It was shown that the Duffing oscillator described by the differential equation of the second order is characterized by noise immunity. It was proved that the Duffing system response to periodic signals is stronger than the response to non-periodic noise with uniform distribution. Let us consider the Duffing oscillator equation [14] given by expression (1):

$$x''(t) + k \cdot x'(t) - x(t) + (x(t))^3 = s(t), \quad (1)$$

where $s(t)$ is the input signal; $x(t)$ is the output signal; k is the attenuation factor. It is known that such Duffing system is extremely sensitive to small variations of periodic components of the input signal [14].

As amplitude of the periodic component of the input signal grows, chaotic state of the Duffing system oscillation (Fig. 1, *a, b*) changes for the periodic state of oscillations (Fig. 1, *c, d*). The graphs shown in Fig. 1 were obtained at signal-to-noise ratios smaller than 0. Thus, transition of the Duffing system from chaotic state to periodic in the known signal detection means serves as a function of the indicator of presence of a periodic signal.

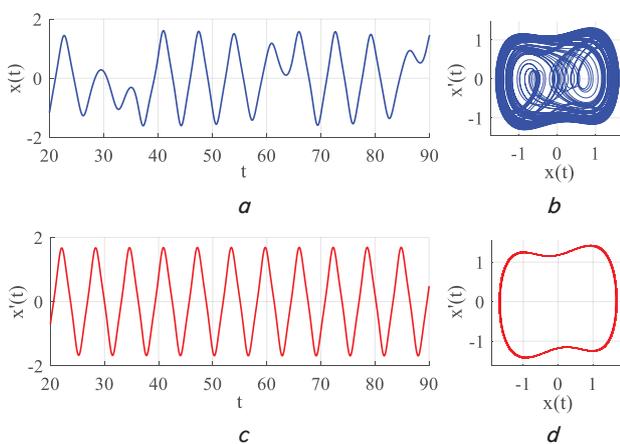


Fig. 1. States of the Duffing system oscillations: *a* – chaotic state; *b* – phase portrait of chaotic oscillations; *c* – periodic state; *d* – phase portrait of periodic oscillations

Relative simplicity of implementation is advantage of the presented detection method consisting, practically, in distinguishing between forms of the phase portrait in Fig. 1, *b*, and Fig. 1, *d*. However, information contained in the form of chaotic oscillations (Fig. 1, *a, b*) is practically not used to detect signals. Thus, benefits of the chaotic system are not utilized completely.

Ways of realization of oscillating systems with bi-stable potential functions were described. Methods are known for detecting signals with analog implementation of the Duffing system as an input circuit of the receiver [16]. Digital implementation of the Duffing system can be based on the use of a programmable logic matrix [17]. The methods presented in [16] and [17] are characterized by drawbacks inherent in the method [14] (Fig. 1), which relate to incomplete use of information about the input signal which can be derived from the form of oscillations of the output signal in a chaotic state.

One of the most important trends in development of chaos-based signal detection methods is communication where chaotic systems are used as highly sensitive input circuits of receivers [6]. Besides, high sensitivity of chaotic systems is used to diagnose mechanical faults [18] for the purpose of non-destructive testing. Application of a chaotic system to seismic signal processing was described in [19]. There is also an example of use of chaotic mapping in analysis of Internet traffic in order to detect LDoS attacks [20]. Studies [6, 18–20] also used a detection method using transition to the periodic state which is a significant drawback. Insufficient practical implementation of these methods is associated with the difficulty of maintaining a stable threshold of signal detection and the need to solve the problem of chaos identification. Methods of estimation of parameters of an input signal of a chaotic system proceeding from characteristics of an output signal need further advancement for an ample use of the information contained in chaotic oscillations.

In addition, issues related to insufficient noise immunity of the described signal detection methods remain unresolved. This may be caused by objective difficulties associated with the use of transition from chaotic state to periodic as an indicator of presence of signals which manifest themselves in essentially impossible detection of weak signals in the periodic state and in the costs associated with maintaining critical oscillation state [11] which makes relevant studies inappropriate. A variant of overcoming the above difficulties can consist in advancement of methods of chaos identification according to the characteristics of phase portraits. Namely this approach was used in [13], however, the problem of maintaining a stable value of the detection threshold in the critical state remains unresolved because of high sensitivity of the chaotic system to weak influences which is described in [21]. According to the results reported in [21], critical state of a chaotic system depends on weak influences and therefore value of the signal detection threshold may differ for different forms of noise. Variation of the threshold value impairs signal detection characteristics and increases value of the required minimum signal-to-noise ratio.

Thus, the present study addresses the problem of detecting periodic signals by processing the Duffing attractor in a chaotic state without using transfers to the periodic state which enables additional increase in sensitivity due to processing of chaotic oscillations.

3. The aim and objectives of the study

The study objective was to develop a method for detecting signals using a discrete processing of the Duffing attractor without transitions to the periodic state of oscillations.

To achieve this objective, the following tasks were set:

- study response of the Duffing system to periodic and non-periodic signals;
- determine relationship between the amplitude of the periodic component of the input signal and dynamics of the Duffing system attractor;
- construct a block diagram of a device for detecting periodic signals by discrete estimation of the phase trajectory shift of the Duffing system.

4. The task on detecting periodic signals by discrete processing of the Duffing attractor

Let us consider equation of the Duffing oscillator [2, 12] given by expression (1) in a form of transform $F_D[u(t)]$:

$$\begin{aligned} x''(t) + k \cdot x'(t) - x(t) + (x(t))^3 = \\ = s_0(t) + u(t) \Leftrightarrow x(t) = F_D[u(t)], \end{aligned} \quad (2)$$

where $s_0(t)$ is the driving signal that determines state of oscillations of the Duffing system; $x(t)$ is the output signal; k is the attenuation factor; $u(t)$ is the input signal that is an external influence on the Duffing system.

Input of the Duffing system receives signal $s(t)$ which is a sum of the driving signal $s_0(t)$ and the external influence $u(t)$:

$$s(t) = s_0(t) + u(t). \quad (3)$$

The driving signal has a harmonic form:

$$s_0(t) = A_0 \sin(\omega_0 t + \varphi_0). \quad (4)$$

The external input signal contains a useful periodic signal $s_{inf}(t)$ and aperiodic noise $\xi(t)$:

$$u(t) = s_{inf}(t) + \xi(t). \quad (5)$$

Useful periodic signal is described by expression:

$$s_{inf}(t) = A_{inf} \sin(\omega_{inf} t + \varphi_{inf}). \quad (6)$$

Amplitude A_{inf} is the informative parameter of the useful signal $s_{inf}(t)$.

The problem of detecting the periodic signal $s_{inf}(t)$ in composition of the input signal $u(t)$ can be expressed as a problem of finding transformation $G[x(t)]$ resulting in a binary value Q , which is 0 in the absence of signal and 1 when signal is present:

$$Q = G[x(t)] = G[F_D[u(t)]] = Y[u(t)], \quad (7)$$

$$Q = \begin{cases} 0, & A_{inf} < A_{thr}, \text{ decision: signal } s_{inf}(t) \text{ is absent,} \\ 1, & A_{inf} \geq A_{thr}, \text{ decision: signal } s_{inf}(t) \text{ is present.} \end{cases} \quad (8)$$

Transformation $G[x(t)]$ can be expressed as transformation $Y[u(t)]$ performed on the input signal.

5. Study of the Duffing system response to periodic and non-periodic signals

To solve the problem of periodic signal detection, it is necessary to study response of the Duffing system to periodic and non-periodic signals at the following parameters:

$$k = \frac{1}{2}; \quad \omega_0 = 1; \quad \varphi_0 = 0. \quad (9)$$

However, it should be noted that a same form of oscillation of the Duffing system can be obtained for different values of frequency ω_0 and amplitude A_0 when scaling coefficients are introduced into equation (2) as shown in [23].

Input signal of the Duffing system meets the conditions [14, 24]:

$$A_{inf} < 0.02A_0; \quad \omega_{inf} = \omega_0 \pm 0.05\omega_0; \quad \varphi_{inf} \approx \varphi_0. \quad (10)$$

Input noise $\xi(t)$ is a random variable whose distribution is uniform in the passband of the Duffing system (2).

According to solution of equation (2), the Duffing system is in a chaotic state if $0.35 < A_0 < 0.83$. It was established that chaotic state is stable in the amplitude range $0.4 < A_0 < 0.7$ and does not change to periodic state as a result of weak external influences [22]. In this state, chaotic oscillator has the highest sensitivity to changes in the input harmonic component $s_{inf}(t)$ at frequencies close to ω_0 [2, 24].

Oscillograms of the Duffing system output signal in a chaotic state ($A_0=0.4$) and in a periodic state ($A_0=0.8$) are shown in Fig. 2.

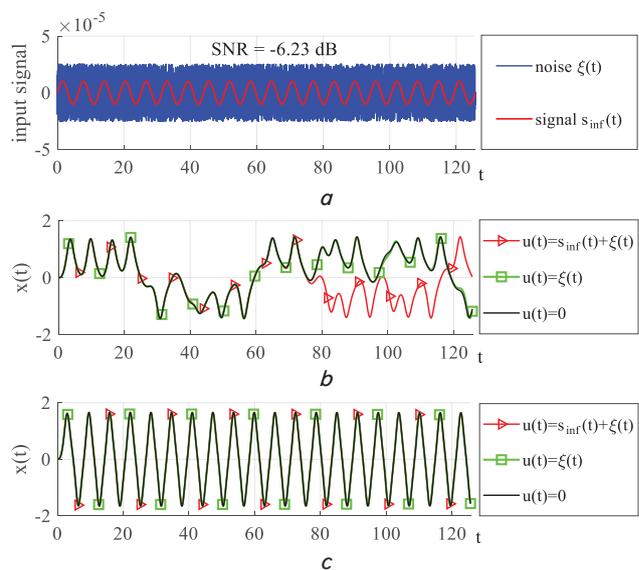


Fig. 2. Duffing system response to periodic and non-periodic signals: *a* – input signals; *b* – chaotic state; *c* – periodic state

A signal $u(t)$ described by expression (5) enters the Duffing system input. Oscillograms of chaotic oscillations almost coincide (Fig. 2, *b*) in a complete absence of signal ($u(t)=0$) and in presence of non-periodic noise ($u(t)=\xi(t)$). When adding a weak periodic signal ($u(t)=s_{inf}(t)+\xi(t)$, $SNR=-6.23$ dB), a significant change of the form of chaotic oscillations is observed.

Thus, in the chaotic oscillation state, the Duffing system is characterized by high sensitivity to the influence of

periodic signals and resistance to the influence of non-periodic noise. This enables detection of weak signals against the background of noise by distinguishing changes in the shape of oscillations of the Duffing system which arise because of action of different types of input signals.

Known methods of detecting periodic signals using the Duffing system are based on the use of the process of the system transition from chaotic state to periodic as a result of increase in amplitude of the periodic component of the input signal above a certain threshold level (Fig. 1) as an indicator of signal presence. This approach was used, in particular, in [5, 7, 10–13].

A substantial drawback of this approach consists in a significant reduction of the Duffing system sensitivity in a periodic state. This is confirmed by the graphs in Fig. 2, *c* where it is shown that oscillations of the Duffing system in the periodic state practically do not differ under the action of the same input signals ($u(t)=0$, $u(t)=\xi(t)$), $u(t)=s_{inj}(t)+\xi(t)$, SNR=−6.23 dB).

6. Detection of periodic signals using discrete processing of the Duffing attractor

6.1. Establishment of a relationship between amplitude of the periodic component of the input signal and dynamics of the Duffing system attractor

Let us consider the Duffing attractor in the Poincaré section defined as a set of points $(x(mT+\varphi); x'(mT+\varphi))$ for which $m=[t/T]=0, 1, 2, 3, \dots$; T is the period of the driving signal; $T=2\pi/\omega_0$; φ is the phase of the driving oscillation; $\varphi=t-mT$. Fig. 3 shows geometric location of the Poincaré section points of the Duffing attractor for three different values of the driving signal phase (0, $\pi/2$, π).

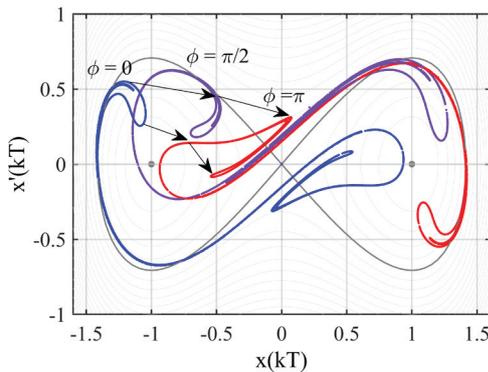


Fig. 3. Dynamics of the Poincaré section of the Duffing system: $\varphi=0$ (blue); $\varphi=\pi/2$ (purple); $\varphi=\pi$ (red)

The Poincaré section of the Duffing system changes over time according to the phase of the driving signal. Structure of the Duffing attractor has properties of fractal geometry that manifest themselves in a connection with the homoclinic forms of phase trajectories. A more detailed analysis of the fractal geometry of the Duffing attractor is presented in [2, 21, 23] with schematic images.

Based on the results of numerical experiments, a diagram of motion of points of the Poincaré section of the Duffing attractor was constructed.

Possible types of movement of the Poincaré points are shown in Fig. 4 by green arrows. The points can rotate

around one of the centers $(-1, 0)$, $(1, 0)$ or move from one center orbit to the orbit of the other center. Accordingly, during each period T of the driving signal $s_0(t)$, phase trajectories of the Duffing system diverge which leads to an increase in influence of single-phase periodic signals at frequencies close to ω_0 . The continuous process of divergence of phase trajectories can be seen in Fig. 1, *b*.

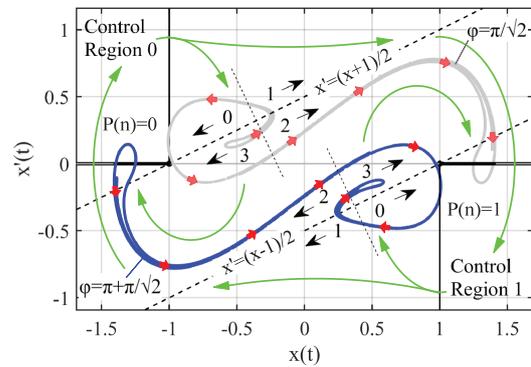


Fig. 4. A diagram of movement of the points of the Poincaré section of the Duffing attractor

Red arrows indicate the direction of shift of the Poincaré cross-section points which results from an increase in the periodic signal amplitude at the input (at a frequency close to ω_0).

Thus, points of phase trajectories of the Duffing system are characterized by four types of dynamics with respect to the centers $(-1, 0)$ and $(1, 0)$:

- 1) rotation around one of the centers without changing the shift direction with an increase in amplitude of the periodic component of the input signal;
- 2) rotation around one of the centers with a change of the shift direction as the amplitude of the periodic component of the input signal grows;
- 3) transition from one center of rotation to another without changing the shift direction as the amplitude of the periodic component of the input signal grows;
- 4) transition from one center of rotation to another with a change in the shift direction when amplitude of the periodic component of the input signal grows.

Accordingly, using sequences of types of dynamics 1–4 in time, it is possible to estimate and compare magnitudes of the shift of phase trajectories caused by influence of different forms of the input signal $u(t)$ which is also confirmed by the results obtained in [23, 24].

In order to determine the shift direction corresponding to growth of the useful signal amplitude, two control regions (Control Region 0, Control Region 1) were selected on the phase plane and two auxiliary lines $x'=(x+1)/2$, $x'=(x-1)/2$ were drawn which make it possible to divide the Poincaré section of the Duffing attractor into segments 0, 1, 2, 3 as shown in Fig. 4 for the phases of the driving signal $\varphi=\pi/\sqrt{2}$ and $\varphi=\pi+\pi/\sqrt{2}$.

If amplitude of the periodic component of the input signal grows, then the phase trajectory of the Duffing system shifts sequentially from segment 0 to segment 3 regardless of the driving signal phase.

The sequence number of entry of the phase trajectory point into control region 0 or 1 is indicated by variable n . Position of the phase trajectory point with respect to the

structure of the Duffing attractor is determined each time the point enters one of the control regions (Fig. 4) at time moments t_n where $n=0, 1, 2, \dots, M$. Then, position of the point of the phase trajectory with respect to the Duffing attractor can be described for each n in the binary numeration system.

Value of $P(n)$ is an indicator of entry of the phase trajectory point into the control regions (Fig. 4).

$$P(n) = \begin{cases} 1, & x(t) < 1 \cap x'(t) < 0 \quad (\text{control region 0}), \\ 0, & x(t) < -1 \cap x'(t) > 0 \quad (\text{control region 1}). \end{cases} \quad (11)$$

The value of $V(n)$ is an indicator of direction of shift of the phase trajectory as amplitude of the periodic component of the input signal grows (Fig. 4) for the point of time t_n when the phase trajectory point enters one of the control regions.

$$V(n) = \begin{cases} 0, & \varphi(t_v) = \frac{\pi}{2} + \pi P(n) \cap x'(t_v) > (-1)^{P(n)} \cdot (x(t_v) + (-1)^{P(n-1)}) / 2, \\ 1, & \varphi(t_v) = \frac{\pi}{2} + \pi P(n) \cap x'(t_v) \leq (-1)^{P(n)} \cdot (x(t_v) + (-1)^{P(n-1)}) / 2, \end{cases} \quad (12)$$

$$L = \sum_{n=0}^M E(n) \cdot 4^{-n} = W_L[E] = Y_L[u(t)], \quad (16)$$

where $t_{n-1} < t_v < t_n$, quantity $P(n)$ takes value 0 or 1 according to (11). Value $V(n)=0$ corresponds to clockwise direction and the value $V(n)=1$ corresponds to counterclockwise direction.

Thus, quantities P and V are arrays of binary quantities of $1 \times M$ size which are defined as transforms from the input signal:

$$P = G_p[x(t)] = Y_p[u(t)], \quad (13)$$

$$V = G_v[x(t)] = Y_v[u(t)]. \quad (14)$$

From the binary quantities $P(n)$, $V(n)$ and $P(n-1)$, $V(n-1)$ determined at the moments of the n -th and $n-1$ -th entry of the phase trajectory point into the control regions, the number $E(n)$ of the Poincare section segment which characterizes the type of point movement can be determined using the truth table (Table 1).

Table 1

Truth table for determining number of a segment of the Poincare section

$P(n-1)$	$V(n-1)$	$P(n)$	$V(n)$	$E(n)$		
				Binary system		Quaternary system
				$E_1(n)$	$E_0(n)$	
0	0	0	0	1	1	3
0	0	0	1	0	0	0
0	0	1	0	0	1	1
0	0	1	1	1	0	2
0	1	0	0	1	1	3
0	1	0	1	0	0	0
0	1	1	0	0	1	1
0	1	1	1	1	0	2
1	0	0	0	0	1	1
1	0	0	1	1	0	2
1	0	1	0	1	1	3
1	0	1	1	0	0	0
1	1	0	0	0	1	1
1	1	0	1	1	0	2
1	1	1	0	1	1	3
1	1	1	1	0	0	0

The two-element output vector $E(n)$ in Table 1 is the number of the segment in the Poincare section (Fig. 4) which contains the point of phase trajectory $(x(t); x'(t))$. Values of $E(n)$ are given in binary and quaternary numeration systems. $E_1(n)$ and $E_0(n)$ are the higher- and lower-order bits of the binary number $E(n)$, respectively. An array of indexes of $1 \times M$ size which can be expressed as a transform W_E for all $n=0, 1, 2, \dots, M$ is the result of successive application of the truth table (Table 1):

$$E = W_E[G_p[x(t)], G_v[x(t)]]. \quad (15)$$

Estimation of the phase trajectory shift resulting from influence of the periodic component of the input signal is given by this expression:

where L is the value of the shift estimate; $E(n)$ are the weight coefficients that can take values 0, 1, 2, 3 according to the truth table (Table 1).

Fig. 5 shows dependence of the value of quantity L on amplitude A_{inf} of the useful signal $s_{inf}(t)$.

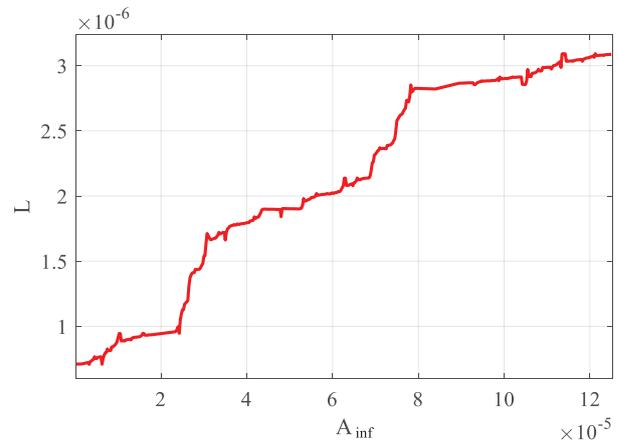


Fig. 5. Dependence of estimate of the magnitude of shift of the phase trajectory on the amplitude of the periodic component of the input signal

Thus, dependence of dynamics of the attractor of the Duffing system on amplitude of the periodic component of the input signal is reflected by the estimate of shift L of the phase trajectory described by expression (16), Fig. 5. The growing form of the dependence shown in Fig. 5 is remaining at signal-to-noise ratios up to -10 dB. Therefore, a weak periodic influence of $s_{inf}(t)$ causes a larger shift of the phase trajectory of the Duffing system than the much stronger non-periodic influence of $\xi(t)$.

6. 2. The process of detecting periodic signals by discrete processing of the Duffing attractor in the Poincare section

As shown in Section 6. 1, dependence of shift of the phase trajectory of the Duffing system on amplitude of the periodic component of the input signal is described by expression (16)

for the Poincare section. Since application of the transform $Y_L[u(t)]$ results in a growing dependence of the quantity L on amplitude of the useful signal A_{inf} (Fig. 5), it is advisable to use two threshold values L_{thr0}, L_{thr1} which will allow one to uniquely determine position of the A_{inf} value relative to the threshold A_{thr} . Then, the decision-making procedure is described by the expression:

$$\begin{cases} L < L_{thr0} \Rightarrow A_{inf} < A_{thr}, \text{ decision: signal } s_{inf}(t) \text{ is absent,} \\ L \geq L_{thr1} \Rightarrow A_{inf} \geq A_{thr}, \text{ decision: signal } s_{inf}(t) \text{ is present.} \end{cases} \quad (17)$$

Thus, the desired transformation (7) which describes the signal detection process has the form of expression (18):

$$Q = G[x(t)] = Y[u(t)] = \begin{cases} 0, Y_L[u(t)] < L_{thr0}, \text{ decision: signal } s_{inf}(t) \text{ is absent,} \\ 1, Y_L[u(t)] \geq L_{thr1}, \text{ decision: signal } s_{inf}(t) \text{ is present.} \end{cases} \quad (18)$$

Expressions (11) to (18) can be used to further develop devices and algorithms for detecting periodic signals by analyzing the Duffing attractor.

7. Block diagram of a device for detecting periodic signals by discrete estimation of the phase trajectory shift of the Duffing system

Implementation of the Duffing system [16] is shown in Fig. 6. The electric circuit in Fig. 6 is implementation of a chaotic system characterized by a typical Duffing attractor in the phase space.

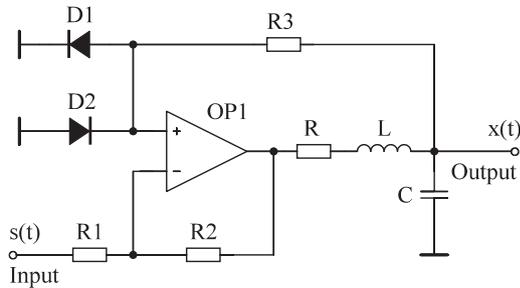


Fig. 6. Implementation of the Duffing system in a form of an electric circuit

Voltage of the input signal $s(t)$ is fed to the inverting input of the operational amplifier OP1. Voltage of the output signal $x(t)$ is taken from the capacitor C . The linear part of the Duffing system is realized by a resistor R , an inductor L and a capacitor C . The nonlinear part of the Duffing system is implemented by a resistor $R3$ and diodes $D1, D2$. The addition operation is implemented by the OP1 operational amplifier circuit [16].

The amplitude estimation unit of the periodic component of the input signal can be digitally implemented. A logic circuit that determines discrete value L of the amplitude estimation of a periodic component of the input signal is the main part of the evaluation unit.

The logic circuit model is described by the truth table (Table 1).

The signal detection device can be implemented in a form of the structure shown in Fig. 7.

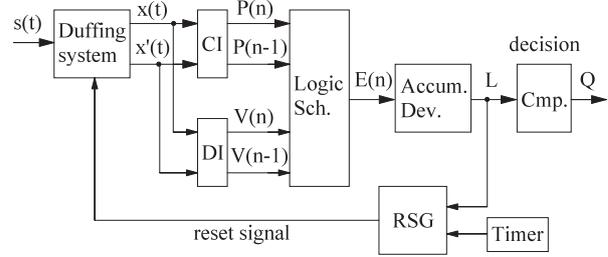


Fig. 7. Block diagram of a device for detecting periodic signals

The analog part of the signal detection device consists of a Duffing system (Fig. 6) and a driving signal generator. The digital part consists of a unit of the control region indicator (CI), a unit of the indicator of the phase trajectory shift with growing amplitude A_{inf} of the periodic component of the input signal (DI), a logic circuit that implements the truth table (Table 1), a device for accumulation of counts, a comparator unit (Cmp.), a restart signal generator (RSG) and a timer unit.

A binary quantity $V(n)$ is the output signal of the unit of the shift direction indicator. The unit of the control region indicator consists of comparison circuits that determine values of $P(n)$.

The CI and DI units produce new values of the output signals when phase trajectories intersect the line $x'=0$ when entering the control regions according to Fig. 4.

Counts of the logic output signal $E(n)$ are fed to the accumulation unit which generates value of the L quantity in accordance with expression (16). The comparator unit (Cmp.) compares the quantity L with the threshold value L_{thr} and issues a decision Q on presence or absence at the input of a periodic signal $s_{inf}(t)$ with amplitude A_{inf} not less than the threshold value A_{thr} .

If the value of the estimate of L is much greater than the threshold value L_{thr} (typical condition is $L > 2L_{thr}$), then the RSG generates a restart signal that sets zero values at the input and output of the chaotic system. The timer block restarts the Duffing system when the L value remains below the threshold value L_{thr} during a relatively long time (typically 20–30 periods of the driving signal).

Thus, the presented structure of the signal detection device implements the transform (18) by discrete processing of the output signal of the analog Duffing system. The result of operation of this device is a binary quantity Q which is 0 in the absence and 1 in the presence of a useful periodic signal with an amplitude not less than the specified threshold value.

The device model (Fig. 7) was studied in the MATLAB software environment. The obtained model parameters are given in Table 2.

Table 2
Basic parameters of the model of the device for detection of periodic signals

Parameter	Value
Amplitude of the driving signal, A_0	0.4
Amplitude of the useful signal, A_{inf}	$10^{-5} \dots 10^{-4}$
Signal-to-noise ratio (SNR)	-10 dB
Threshold value of the useful signal amplitude, A_{thr}	$6 \cdot 10^{-5}$
Lower threshold of estimate of the phase trajectory shift, L_{thr0}	$1.5 \cdot 10^{-6}$
Upper threshold of estimate of the phase trajectory shift, L_{thr1}	$2.5 \cdot 10^{-6}$
Number of elements of the array of coefficients E, M	24

8. Discussion of results obtained in signal detection using discrete processing of the Duffing attractor in the Poincare section

The obtained results of detection of periodic signals using discrete processing of the Duffing attractor are explained by the fact that periodic influences cause a much greater shift of the phase trajectory of the Duffing system than non-periodic influences (Fig. 2, 5). The proposed approach features an implementation of detecting periodic signals using the Duffing system in a chaotic state, without transitions to the periodic state. This makes it possible to avoid the errors associated with instability of the critical state and low sensitivity in the periodic state. The minimum signal-to-noise ratio at which signals can be detected is limited by own noise of the Duffing system circuit and the digit capacity of the digital estimation device.

Dependence of the quantity L on amplitude of the useful signal A_{inf} (Fig. 5) is similar in form to the dependence obtained in [23] by integration according to the A_{inf} amplitude which confirms correctness of the performed calculations.

The described approach provides detection of periodic signals at low signal-to-noise ratios in the bandwidth of the Duffing system. Additional linear filters can be used to obtain more complex characteristics of frequency selectivity at the input. On the other hand, the linear part of the Duffing system can also be modified to obtain the necessary pass band and delay band according to the theory of linear filtering.

It is known that the Duffing attractor is characterized by the properties of fractal geometry [3]. For further studies, development of mathematical models based on fractal geometry to analyze the Duffing system response to various forms of the input signal is an interesting task. Such models can greatly increase efficiency of the above signal detection method.

9. Conclusions

1. Response of the Duffing system to periodic signals was investigated. The ranges of parameters necessary to maintain the chaotic oscillation state were determined. It was found that the Duffing system is characterized by the highest sensitivity to weak periodic influences in the amplitude range $0.4 < A_0(t) < 0.7$ where the chaotic state does not change to the periodic state due to weak external influences.

2. Dependence of dynamics of the Duffing system attractor on amplitude of the periodic component of the input signal was determined using a discrete estimation of the phase trajectory shift in the Poincare section. The phase trajectory shift estimate was presented in a quaternary numeration system according to the four main types of phase trajectory dynamics of the Duffing system trajectory. The obtained estimation makes it possible to make decisions on presence or absence of a periodic signal with a specified amplitude range at the input.

3. A block diagram of a device for detecting periodic signals based on a logical truth table which implements discrete estimation of the phase trajectory shift in the Duffing system under influence of the periodic component of the input signal was constructed. The proposed block diagram enables detection of periodic signals at a signal-to-noise ratio of -10 dB in the bandwidth of the Duffing system.

Thus, a method for detecting signals by means of discrete processing of the Duffing attractor was developed which makes it possible to increase sensitivity due to the absence of the periodic state of low sensitivity. The results obtained can be used when developing high-sensitivity digital devices for receiving signals.

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