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Проводиться дослідження нелінійних гідродинамічних характеристик рушійно-стернового комплексу (РСК), які впливають на точність плоского траєкторного руху автономного ненаселеного підводного апарата (АНПА). При криволінійному русі підводного апарата його РСК працює у косому потоці води, що набігає. Це призводить до зниження сили упору РСК і негативно впливає на керований траєкторний рух підводного апарату. Дослідження було проведено для конкретного типу АНПА для режиму плоского криволінійного руху.

У якості методу дослідження було обрано метод математичного моделювання. З цією метою відому математичну модель руху АНПА доповнено системою керування, що імітує траєкторний рух АНПА. Розроблена модель складається з чотирьох основних блоків: удосконаленої моделі АНПА; блоку завдання швидкості руху апарату; блоку керування кутом повороту насадки; блоку, який містить заздалегідь підготовлені траєкторії руху АНПА.

Представлено результати дослідження гідродинамічних параметрів АНПА для декількох типів траєкторій його руху. До досліджуваних параметрів належать наступні: необхідний кут повороту насадки; дійсна траєкторія руху апарату; швидкість руху апарату; момент на валу гребного електродвигуна; упор гребного гвинта.

В результаті проведених досліджень побудовано діаграму залежності упору гребного гвинта від кута повороту насадки АНПА в діапазоні швидкості від 0,2 м/с до 1 м/с та при повороті насадки в діапазоні до 35°. Сформовано трохвимірну матрицю, яка описує залежність упору гребного гвинта від кута потоку води, що набігає, та швидкості руху апарату. Отримана залежність може бути використана при синтезі регуляторів систем автоматичного керування плоским маневровим рухом АНПА підвищеної точності

**Ключові слова:** автономний ненаселений підводний апарат, рушійно-стерновий комплекс, математичне моделювання, поворотна насадка

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# MATHEMATICAL MODELING OF AUTONOMOUS UNDERWATER VEHICLE PROPULSION AND STEERING COMPLEX OPERATION IN OBLIQUE (BEVELED) WATER FLOW

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## 1. Introduction

Today, in the world leading maritime countries, autonomous underwater vehicles (AUVs), which differ significantly by architectural and design type, mass-dimensional parameters and depths of application, are being created. However, all varieties of AUVs combine a common property – the ability of controlled trajectory (plane or spatial) motion.

The forces acting on the underwater vehicle during such motion determine its dynamics and essentially influence the vehicle manoeuvrability. Only by having the complete information about all the forces affecting the AUV, as well as about their control means, the conditions under which it is possible to construct vehicle effective automatic control systems, can be determined.

That is why, in recent years, more and more attention is being paid to the research and improvement of the PSC auto-

matic control systems (ACS) of such vehicles. Such a task is especially relevant for AUVs with PSC of “propeller – rotary nozzle” type, which provide improved controllability of the underwater vehicle.

To successfully solve this problem, it is expedient to apply the methods of mathematical modeling of the vehicle trajectory motion, taking into account the hydrodynamic characteristics of the propeller in the rotary nozzle.

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## 2. Literature review and problem statement

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In [1], attention is drawn to the wide application of AUVs in survey and search operations, which require the organization of precise motions along the given trajectories, and are determined by the search algorithm. Accordingly, when designing AUVs, their structure should reflect the nature of the tasks to be solved and the requirements for their development (adjustment), take into account the state of the external environment and the influence of external disturbances.

In [2], a simulator specially designed for AUVs is described. The simulator allows for experimental research and does not require the use of marine robotics functional prototypes, which significantly reduce the experiments cost. However, to achieve the versatility of this simulator, a rather complex mathematical model is used, which is complicated by the addition of new simulator functionalities.

Samples of the developed models of AUVs are also presented in [3, 4]. However, the question of simplifying the mathematical models of the presented samples and the researches of their operation in the oblique water flow remains unresolved.

In [5], a mathematical model of an underwater vehicle with undulatory motion is presented. This model is proposed to use for pre-setting the control system of new underwater vehicles. To conduct experiments, the use of CFD software packages for modeling is proposed, it increases the time spent on research and significantly complicates its implementation technology.

The mathematical model of the underwater robot, “Eco Mapper”, based on calculations using CFD packages is developed by the authors in [6]. The question of the need to control the vehicle important hydrodynamic parameters and the use of appropriate modeling environment is elaborated in the paper. However, the issue of clarifying the energy characteristics of the underwater vehicle PSC during maneuvering is not considered.

The model framework presented in [7] contains three main units: the underwater vehicle model unit, the sensor model unit, and the environment model unit. The environment model includes the hydrostatic and hydrodynamic components that affect the AUV. Based on this platform, the user is offered specially designed scenarios of the underwater vehicle behavior. However, the issue of changing the PSC thrust forces during the AUV maneuvering motion is not considered in the paper.

In [8, 9], the issue of improving the accuracy of modeling, taking into account the hydrodynamic characteristics of autonomous underwater vehicles, is elaborated. To achieve the required maneuverability of the underwater vehicle, it is necessary to apply an accurate hydrodynamic model and a complex control system. In the presented works, CFD softwares are used to account for the dynamic flow. However, the question of determining the AUV nonlinear hydrodynamic characteristics during curvilinear motion remains

unresolved. To account for the important vehicle motion dynamics parameters, complex mathematical models are used.

In [10], the authors present the kinematic and dynamic equations of an underwater robot model with two rotating thrusters. The proposed equations are implemented using the virtual environment realized in MATLAB and LabVIEW. The advantage of such approach is the ability to use fairly simple and well-tested engineering packages, and the disadvantage is the lack of modules in them for the PSC operation research in the oblique incident water flow.

The work [11] is aimed at reflecting the AUV dynamic model calculation based on a set of experiments. The model is designed for the synthesis of control algorithms and navigation systems. The authors indicate that in order to develop a high-precision control system, it is necessary to take into account all the parameters that affect the vehicle during the assigned mission execution. Particular attention should be paid to those parameters that have nonlinear characteristics, for example, the propeller thrust force, when operating in the oblique water flow. The technique proposed by the authors allows the development of accurate control systems. However, it is quite complicated to implement and requires considerable computational resources.

The review performed shows that despite the well-studied issues of AUVs mathematical modeling, the issue of such vehicle PSC operation in the oblique incident water flow remains unexplained in the scientific literature. This PSC operation mode occurs during the AUV maneuvering and results in a change in the PSC characteristics in comparison with the rectilinear motion mode.

Studying the PSC operation peculiarities in the oblique incident water flow will allow synthesizing the spatial motion control systems of high accuracy. An effective tool for such a study is the method of mathematical modeling of the AUV motion. Thus, an important condition is the use of simplified models, which can be researched using widely available software packages, in particular, the Simulink package, MathWorks product, USA.

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## 3. The aim and objectives of the study

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The aim of the study is to determine the dependence of the AUV PSC “propeller – rotary nozzle” thrust force on the speed and angle of the incident water flow. The dependence established should serve as a theoretical basis for the synthesis of high-precision AUV control systems.

To achieve this goal, the following objectives were set:

- to develop a mathematical model of an AUV plane curvilinear motion with PSC of “propeller in a rotary nozzle” type;
- to investigate the AUV plane curvilinear motion with the selected type of PSC by the mathematical modeling method;
- to synthesize the dependences of the PSC thrust force on the AUV traverse speed and the angle of the incident water flow.

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## 4. Investigation of AUV propulsion and steering complex hydrodynamic parameters during plane curvilinear motion

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### 4.1. Simulation of AUV plane curvilinear motion in Simulink system

Application of mathematical modeling to investigate the force characteristics of the AUV PSC during curvilinear

motion will allow obtaining preliminary results before the full-scale test.

The following equations system was based on the AUV plane curvilinear motion model [12]:

$$L \frac{di}{dt} = K_u U_x - r_a i - \frac{c\Phi}{k_p} \omega, \quad (1)$$

$$J_{PS} \frac{d\omega}{dt} = M_{EM} - Q = \frac{c\Phi}{k_p} i - \frac{K_Q \rho D^5}{4\pi^2} \omega^2, \quad (2)$$

$$\begin{aligned} (m_{AUV} + \lambda_x) \frac{dV_x}{dt} &= T - F_x - F_{Dx} = \\ &= \frac{K_T \rho D^4}{4\pi^2} \omega^2 - \frac{\rho C_x \Omega}{2} V_x^2 - F_{Dx}, \end{aligned} \quad (3)$$

where:

–  $i$  is the instantaneous value of the propulsion motor armature current;

–  $u_x$ ,  $K_u$  are the voltage regulator control signal and its gain factor, respectively;

–  $L$ ,  $r_a$ ,  $c$ ,  $\Phi$  are the electromagnetic parameters of the propulsion motor;

–  $\omega$  represents the propeller angular speed;

–  $k_p$  is the gear ratio;

–  $J_{PS}$  is the moment of inertia of the “propeller electric motor – gearbox – shafting – propeller screw” system applied to the screw;

–  $M_{EM} = c\Phi i / k_p$  is the driving moment of the propulsion motor;

–  $Q$  represents the braking torque generated by the AUV PSC propeller;

–  $K_Q$  is the non-dimensional nonlinear coefficient characterizing the propeller torque;

–  $\rho$  is the water specific density;

–  $D$  represents the propeller diameter;

–  $m_{AUV}$ ,  $\lambda_x$  are the AUV mass and the water added mass;

–  $V_x$  is the AUV current motion velocity along the  $x$  axis;

–  $P_{PS}$  is the propeller driving force (thrust);

–  $K_T$  is the dimensionless nonlinear coefficient, propeller characterizing thrust;

–  $F_x = \rho C_x \Omega V_x^2 / 2$  is the water resistance force to AUV movement;

–  $C_x$  is the AUV hull hydrodynamic coefficient along the  $x$  axis;

–  $\Omega$  is the area of the wetted surface of the AUV outer hull;

–  $F_{Dx}$  is the external disturbance force acting on the AUV hull as it moves along the  $x$  axis.

Typically, the hydrodynamic AUV hulls configurations are chosen so that their flow around the water flow is slightly laminar in nature. Besides, the PSCs are carried out behind the AUV hull so that the propeller operates in a practically unturbulent flow.

An example of such AUV design is the underwater vehicle, “Skarus-1” project produced by the Admiral Makarov National University of Shipbuilding, Ukraine (Fig. 1). Such AUV has a compressed “fluid-drop”-shaped form, in the aft part of which there are two PSCs of the “propeller – rotary nozzle” type. The mentioned thrusters are maximally distant from the vehicle main hull and operate at speeds of up to 1 m/s in a laminar flow.



Fig. 1. Appearance of AUV “Skarus-1” project

The Simulink-implementation of the equations system (1)–(3) has the form:

$$\left( K_u * u(1) - r_a * u(3) - \left( c\Phi / k_p \right) u(2) \right) * \Delta t + u(3), \quad (4)$$

$$\left( \left( c\Phi / k_p \right) * u(1) - u(2) \right) / J_{PS} * \Delta t + u(3), \quad (5)$$

$$\left( u(1) - 0,5 * \rho * C_x * \Omega * u(2) * \text{abs}(u(2) + u(3)) \right) / (m_{AUV} + \lambda_x) * \Delta t + u(2), \quad (6)$$

where  $u(1)$ ,  $u(2)$ ,  $u(3)$  are the signals received at the inputs of the corresponding blocks [14]. The implementation of this model in Simulink is discussed in more detail in [12].

The presented mathematical model takes into account the nonlinearity of the propeller action curves and approximates these dependences using a multilayer neural network with direct signal propagation [13].

## 4. 2. Supplementation of AUV model by PSC equations for curvilinear motion modeling

### 4. 2. 1. Determination of the propeller parameters in the rotary nozzle

The propeller screw in the rotary nozzle was selected as the PSC, the general view of which is shown in Fig. 2 [17].

The thrust force  $P_K$ , created by the propeller operation in the rotary nozzle can be represented as the sum of the propeller thrust forces  $P_{PS}$  and the nozzle itself  $P_H$ :

$$P_K = P_{PS} + P_H = P_{PS} \cdot (1 + t_H), \quad (7)$$

where  $t_H = \frac{P_H}{P_{PS}}$  – suction coefficient (ratio) of the rotary nozzle.

The nozzle suction coefficient depends on the degree of load of the whole complex by the thrust, which is characterized by the coefficient  $\sigma_c$ . The indicative value  $t_H$  can be determined from Table 1 [16].

Table 1

The coefficient value  $t_H$

$\sigma_c$	1	2	3	4	5	6	7	8	10	$\infty$
$t_H$	0.23	0.32	0.39	0.44	0.48	0.52	0.55	0.58	0.62	0.9

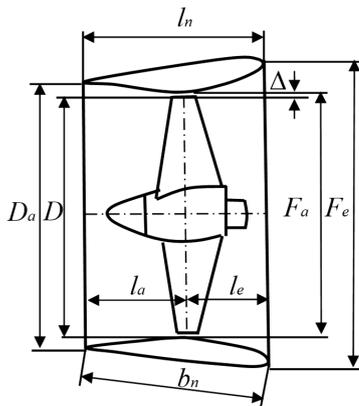


Fig. 2. Geometric parameters of the rotary nozzle:  $D$  is the propeller diameter;  $D_n$  is the area of the nozzle working section;  $D_o$  is the area of the nozzle inlet opening;  $D_e$  is the area of the nozzle outlet opening;  $l_n$  is the nozzle length;  $l_e$  is the length of the inlet part of the nozzle;  $l_o$  is the length of the tail part of the nozzle;  $b_n$  is the nozzle profile chord,  $\Delta$  is the gap between the propeller and the nozzle

Supplementing the propeller thrust calculation equation (8) with the nozzle thrust equations (7), we obtain the design model of PSC “propeller – rotary nozzle” operation.

**4. 2. 2. Development of a mathematical model of PSC “propeller – rotary nozzle”, which operates in the oblique incident water flow**

Since the PSC operates in the laminar flow, when the nozzle rotation angle changes, the complex begins to operate in the oblique but not perturbed (turbulent) water flow. As a rule, the calculation of the propeller thrust is performed under the PSC operation conditions in the coaxial water flow. However, due to the change of the flow angle approaching the propeller, the PSC hydrodynamic characteristics change significantly. When shifting the nozzle to an angle  $\delta_n$ , the flow symmetry of the propulsion flow is broken, which leads to the appearance of the flow velocity component,  $\vec{V}_n$ , the average direction of which can be taken perpendicular to the nozzle axis. The complex flow axis, according to the equality:

$$\vec{V}_\delta = \vec{V}_x - \vec{V}_n, \tag{8}$$

deviates from the propeller shaft axis in the same direction as the nozzle at an angle  $\delta_c$ . The complex thrust  $\vec{P}_K$ . in this case is decomposed into two components: the deflected reaction to the flow angle  $\delta_c$  flow  $\vec{P}_\delta$ , and the additional reaction  $\vec{R}_\delta$ , that occurs when part of the flow approaches the rotary nozzle deflected profile.

Applying the vector form of the law of conservation of momentum to equality (8) allows obtaining the following result [14]:

$$\vec{P}_\delta = \vec{P}_K \vec{R}_\delta. \tag{9}$$

The design of vectors (8) and (9) on the diametrical and horizontal planes (Fig. 3) gives the following result:

$$P_x = P_\delta \cos \delta_c = P_K (1 - \sin^2(\theta_n \delta_n)), \tag{10}$$

$$P_y = P_\delta \sin \delta_c = P_K \sin(\theta_n \delta_n) \cos(\theta_n \delta_n), \tag{11}$$

where  $\theta_n$  – coefficient for the nozzle without stabilizer, depending on the nozzle relative length  $l_n = \frac{L_n}{F_a}$ , (Fig. 2).

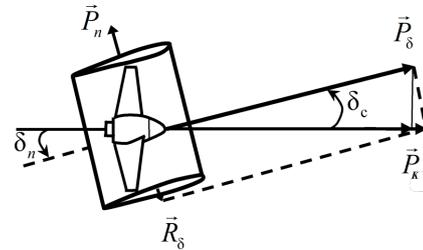


Fig. 3. Scheme of forces generated by the propeller operating in the oblique water flow

Then

$$P_K = \sqrt{(P_x)^2 + (P_y)^2}. \tag{12}$$

Due to the increase in thrust force, when the flow angle changes, the force on the propeller shaft also increases.

Besides, the torque on the propeller shaft can be determined as follows [14]:

$$M'_{EM} = M'_c (1 + \sin^2(\theta_n \delta_n)), \tag{13}$$

where  $M'_c$  – the propeller torque in the coaxial flow.

The parameters under consideration include the PSC hydrodynamic characteristics operating in the oblique water flow: the propeller thrust and torque on the propulsion motor shaft.

The mathematical equations (10)–(13) obtained in Simulink have the following form:

$$T_x = u(1) * (1 - \sin(u(2) * 0,6))^2, \tag{14}$$

$$T_y = u(1) * \sin(u(2) * 0,6) * \cos(u(2) * 0,6), \tag{15}$$

$$T = \text{sqrt}((u(1))^2 + (u(2))^2), \tag{16}$$

$$Q = u(2) * (1 + (\sin(u(1) * 0,6))^2), \tag{17}$$

where  $u(1)$ ,  $u(2)$  – the incoming signals received at the inputs of the corresponding blocks;  $T$  is the propeller thrust;  $Q$  – moment on the propulsion (propeller) motor shaft.

The implementation of this model in Simulink is discussed in more detail in [14].

**4. 2. 3. Analysis of typical types of autonomous underwater vehicle trajectory motion**

Consider the most common types of trajectories.

From the conducted analysis [12, 18], the following typical modes of AUVs spatial (trajectory) motion, which determine the requirements for their automatic control systems (Fig. 4) can be noted:

- in a straight line – with stabilization on course, depth, height above the ground;
- tacks – triangular, rectangular;
- along a flat curve – in a spiral, along a piecewise-broken trajectory;
- motion along a curve with distance control;
- height stabilization between the underwater object and the trajectory;

- along vertical structures;
- movement along the supports of maritime stationary platforms;
- trajectory multiple motion between the sea surface and a given seabed point; complex spatial motion in cramped navigational conditions.

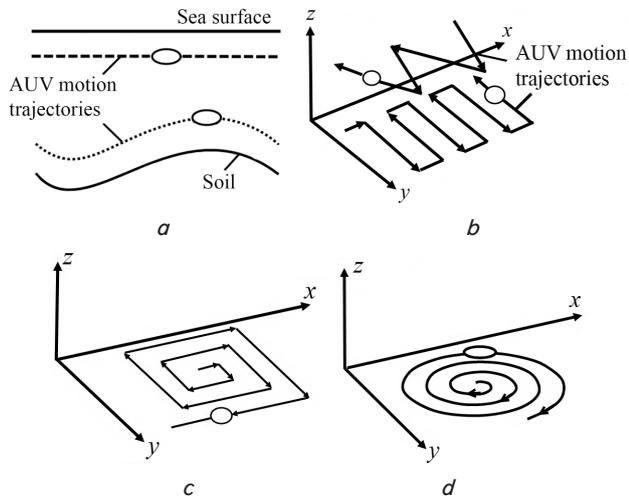


Fig. 4. Typical trajectories of stabilized AUV motion: *a* – linear motion with stabilization of the depth of immersion and trajectory motion with height stabilization above the ground; *b, c, d* – correspond to plane curvilinear motion

Therefore, the trajectories type 4, *b – d* correspond to plane curvilinear motion, so they can be used in modeling the AUV trajectory motion.

### 4.3. Development of an additional model structure for simulating autonomous underwater vehicle plane trajectory motion

Based on AUV mathematical models ((1)–(3), [12]) and PSC of “propeller – rotary nozzle” type ((9)–(13), [14]) a Simulink model was created to simulate AUV motion on a given trajectory. The general appearance of the model is presented in Fig. 5.

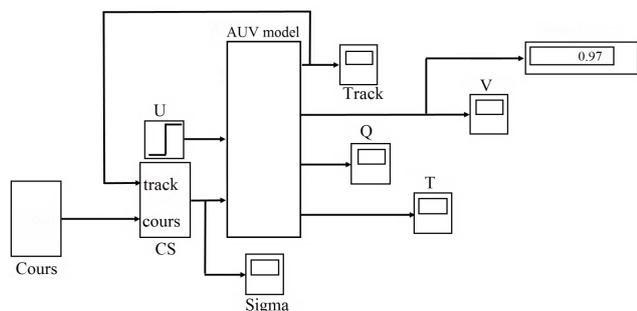


Fig. 5. Mathematical model of AUV trajectory motion

The developed model consists of four main blocks (units): “U”, “Cours”, “CS” and “AUV model”.

The AUV model unit is the AUV model according to (4)–(6), which has been improved by taking into account the PSC operation in the oblique water flow according to (13)–(16). In this case, the AUV moves under the action of the propeller thrust in the rotary nozzle, which is driven

through a gear box of the DC motor with independent excitation.

Unit “U” is intended to set the required AUV speed “V” by applying the appropriate supply voltage to the propulsion motor.

Unit “CS” is the structure added to the AUV model, to simulate the underwater vehicle trajectory motion. This unit controls the rotation angle of the rotary nozzle “sigma”. The “Sigma” unit input receives the signal “track”, which corresponds to the value of the vehicle current (real) motion trajectory  $y(t)$ . The course value, which corresponds to the value of a given motion trajectory enters the “cours” input. This unit calculates the error and, depending on its value and sign, determines the nozzle rotation angle.

The “Cours” unit includes the AUV trajectories prepared in advance in the form of functions  $y(t)$ , which are selected and connected alternately by the operator.

The parameters that are observed include: “Sigma” – nozzle rotation angle, deg.; “Track” – real vehicle motion trajectory  $y(t)$ ; “V” – speed, m/s; “Q” – moment on the propulsion motor shaft, Nm; “T” – propeller thrust, N.

### 4.4. Modeling of autonomous underwater vehicle trajectory motion

The research considers only trajectory motion in the horizontal plane (stabilized in depth) (Fig. 6, *b–d*) [18].

Let us simulate the trajectory motion of the AUV “Skarus-1” project functioning layout, with the following characteristics:

- mass – 600 kg;
- speed – 1 m/s;
- propeller diameter – 160 mm;
- diameter of the rotary nozzle – 166.4 mm;
- relative length of the rotary nozzle – 1.0.

In the first experiment, we choose a piecewise-broken trajectory (Fig. 4, *c*). The simulation results are presented in Fig. 6. Speed is 1 m/s, controlled steering angle.

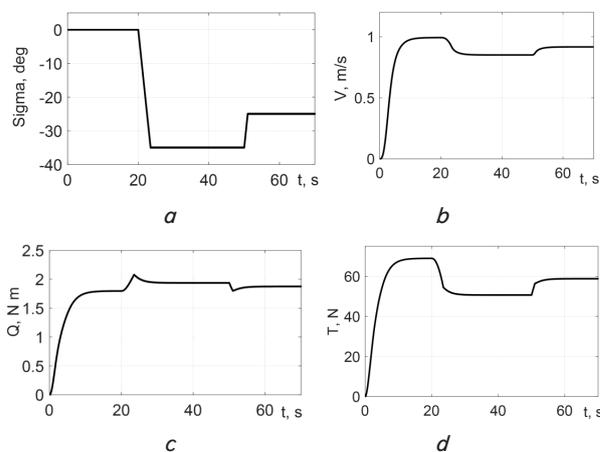


Fig. 6. AUV motion along a piecewise-broken trajectory: *a* – required nozzle rotation angle; *b* – vehicle speed; *c* – moment on the shaft; *d* – propeller thrust

In the second experiment, we choose a trajectory with obstacles avoidance. The simulation results are presented in Fig. 7.

In the next experiment, we will model the AUV trajectory motion with rectangular tacks (Fig. 4, *c*). The simulation result is presented in Fig. 8.

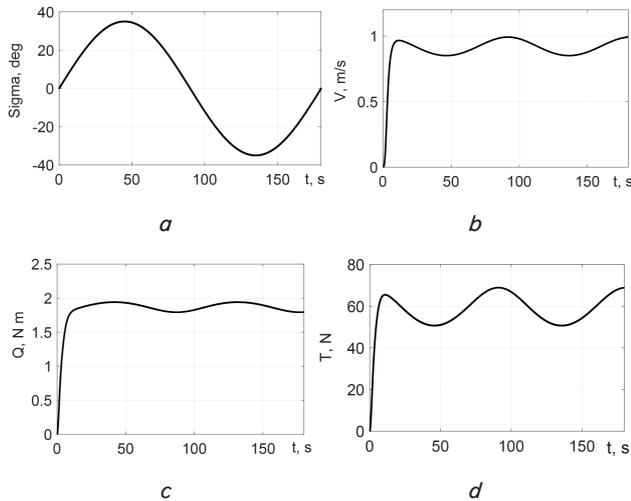


Fig. 7. AUV motion along a trajectory with obstacle avoidance: *a* – required nozzle rotation angle; *b* – vehicle speed; *c* – moment on the shaft; *d* – propeller thrust

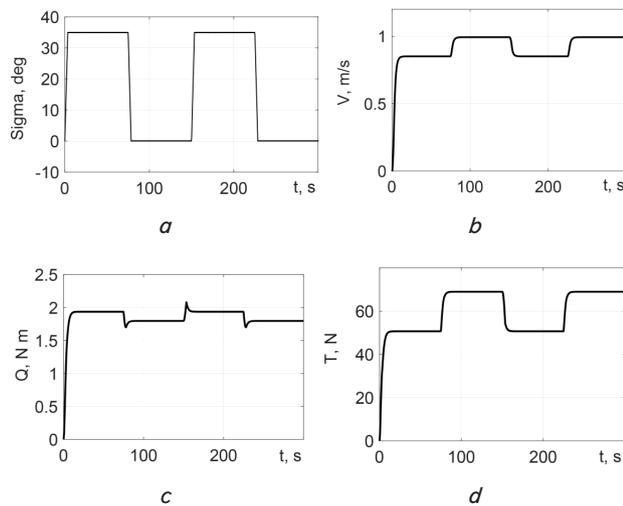


Fig. 8. AUV trajectory motion by rectangular tacks: *a* – required nozzle rotation angle; *b* – vehicle speed; *c* – moment of the shaft; *d* – thrust

As can be seen from the presented Fig. 8, the developed mathematical model allows to investigate a number of important parameters: speed, thrust, moment – during the AUV curvilinear motion.

### 5. Synthesis of the dependence of the propulsion and steering complex thrust on the autonomous vehicle speed, and the incident water flow angle

The developed mathematical model was used to investigate characteristics of nonlinear dependence of the PSC thrust on the incident water flow angle (changes in the nozzle rotation angle to 35°) and the AUV speed in the range of 0.2–1.0 m/s (Fig. 9).

The analysis of the obtained results shows that at low AUV speeds, the PSC thrust is practically independent of the nozzle rotation angle. However, with the increase in the AUV speed (flow rate), the PSC thrust decreases by essentially nonlinear law.

Thus, it is deduced that the greatest deviation is observed at a speed of 1 m/s and with a change in the nozzle rotation angle by 35°. In comparison with the zero angle of the nozzle deflection, the thrust decreases by 7.3 % (decreases from 68.99 N to 50.7 N).

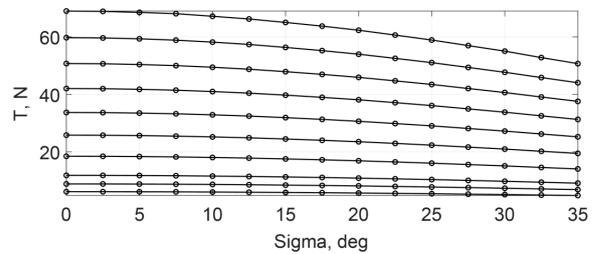


Fig. 9. Diagram of the thrust dependence on the nozzle rotation angle in the speed range [0.2...1] m/s

Using the designed model, a series of computer experiments were carried out and a database, in the form of a three-dimensional matrix, is formed. The resulting matrix describes the dependence of the propeller thrust on the incident flow angle, and the vehicle speed.

According to the obtained data, a surface was constructed, which clearly shows in which zones of velocities and angles of the incident water flow, the most significant occurrence of nonlinear dependences between these variables is observed (Fig. 10).

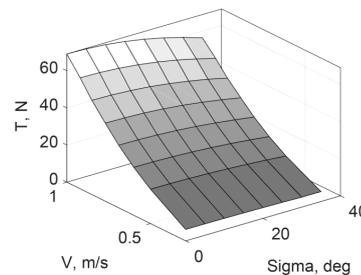


Fig. 10. Dependence of the thrust on the incident flow angle and AUV speed

The surface presented above is a graphical form of the “T – V – Sigma” three-dimensional data matrix obtained in the Simulink system, which can be used for neural network training of the AUV automatic control system. This matrix can be used for the PSC thrust deflection compensation and, as a result, improvement of the underwater vehicle plane curvilinear motion control accuracy.

### 6. Discussion of the results of mathematical modeling of propulsion and steering complex operation of an autonomous underwater vehicle in the oblique water flow

Based on the research results, it was found that the developed mathematical model allows investigating the PSC hydrodynamic parameters, which operates in a beveled flow with the AUV plane curvilinear motion. The presented graphs in Fig. 9 show that a change in the nozzle rotation angle significantly affects the PSC thrust, and with an increase in the vehicle motion, the PSC thrust decreases according to substantially nonlinear law. This can be explained by the fact that the maximum value of the propeller thrust is

reached when the PSC operates in the coaxial flow. When the nozzle is rotated at some angle, the propeller starts to operate in the oblique water flow, which leads to a decrease in the thrust force.

In this case, the AUV motion simulation on plane circulation was carried out in the range of water flow velocity relative to the vehicle hull from 0.2 m/s to 1 m/s. When the vehicle speed is less than 0.2 m/s, the deviation of the thrust value is rather small and is almost linear, therefore this influence can be neglected. The mathematical model implemented in Simulink describes an underwater vehicle with specific mass and size parameters (subsection 4.4.), with the maximum vehicle speed of 1 m/s. The determined speed corresponds to the technical specifications for AUV creation.

It should also be noted that the simulation was carried out for curvilinear motion with small accelerations, when the water added masses can be neglected. Also, due to the AUV design features, the laminar nature of the incident water flow onto the PSC is adopted in the modeling.

In more detail, the effect of the nozzle rotation angle on the PSC thrust can be analyzed from the graphs obtained during the AUV trajectory motion modeling (Fig. 6–8).

As can be seen from the graph (Fig. 6, *a*), the AUV moves rectilinearly for 20 s, since the equipment is not deviated from the diametrical plane of the vehicle. After 20 s, the nozzle begins to rotate by 35°. At this time, the AUV speed starts to decrease, and the moment on the propulsion motor shaft increases due to the decrease in the nozzle efficiency in the oblique water flow. Upon the nozzle reaching a rotation angle of 35°, the vehicle performs circulation with reduced thrust and speed, and the nozzle operates in a steady, oblique water flow.

Similar changes in the PSC thrust are also observed in other considered AUV flat trajectory motion modes – in obstacle avoidance and movement by rectangular tacks (Fig. 7, 8).

This indicates the necessity to take into account the PSC identified functioning properties in the high-precision control systems synthesis for AUV plane motion. The advantage of the resulting model is the possibility to take into

account the dependence nonlinearity of the propeller thrust on the nozzle rotation angle, which describes the nature of the linearized model dependence much more accurately. The introduction of the “T – V – Sigma” matrix (Fig. 10) into the structure of the AUV motion automatic control system allows to compensate for the deviations of the thrust force during the AUV curvilinear motion. This will improve the accuracy of the controlled AUV motion.

It should be noted that the obtained “T – V – Sigma” dependence is calculated for a specific type of AUV (the characteristics of which are presented in subsection 4.4.), using simplified mathematical models of the vehicle and PSC hydrodynamics and reflects nonlinear dependencies with some assumptions.

Further obtained dependence refinement is planned to be carried out using computational fluid dynamics packages (for example, CFD-package “Flow Vision”) and experiments with PSCs in the research basin.

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## 7. Conclusions

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1. The AUV mathematical model is implemented in the Simulink system for plane curvilinear motion research with small accelerations, when the water added masses can be neglected. The AUV model includes a PSC mathematical model of “propeller – rotary nozzle” type, which operates in the oblique water flow.

2. A research of AUV functioning was carried out for three plane trajectory motion modes – piecewise-broken trajectory, obstacle avoidance trajectory, and trajectory on rectangular tacks. The PSC thrust dependence on AUV motion and rotary nozzle deviation angle is shown. It was shown that the thrust maximum deviation at a speed of 1 m/s and a nozzle deviation angle of 35° reaches 7.3 %.

3. Based on the simulation results, a three-dimensional surface, “PSC thrust – water flow velocity – nozzle rotation angle”, is constructed. The resulting three-dimensional surface can be included in the AUV automatic control system for the PSC thrust precise control during plane trajectory motion.

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