Аналітично досліджені режими застрягання вантажів (куль, роликів, маятників) в рамках плоскої моделі врівноваженого ротора на ізотропних пружно-в'язких опорах, що несе автобалансир з багатьма однаковими вантажами.

Описана фізико-математична модель системи ротор-автобалансир. Записані диференціальні рівняння руху системи щодо системи координат, що обертається з постійною швидкістю обертання. Знайдено всі усталені режими руху, в яких вантажі застряють на постійній швидкості обертання. В системі координат, що синхронно обертається з вантажами, ці рухи стаціонарні.

Проведені теоретичні дослідження показують, що режими застрягання вантажів в системі ротор-автобалансир є однопараметричними сім'ями усталених рухів.

Кожен режим застрягання характеризується певною конфігурацією вантажів і відповідною частотою застрягання.

В системі координат що синхронно обертається з вантажами:

– переміщення ротора є сталим;

– параметром є кут, що визначає напрямок вектора переміщення ротора;

– вантажі займають певні фіксовані положення відносно вектора переміщення ротора і ці положення залежать від швидкості обертання ротора.

У автобалансира з n_b однаковими вантажами різних конфігурацій вантажів n_b+1. Загальна кількість різних режимів застрягання вантажів:

– $2(n_b+1)$, якщо nb непарне;

– $2n_b+1$, якщо nb парне.

Загальна кількість різних частот застрягання:

– $3(n_b+1)/2$, якщо nb непарне;

– $3n_b/2+1$, якщо nb парне.

Загальна кількість різних характерних швидкостей – n_b+2. Характерні швидкості є точками біфуркацій рухів, бо при їх переході зароджуються чи зникають однопараметричні сім'ї рухів, що відповідають певному режиму застрягання. В цих точках режими застрягання можуть набувати, або втрачати стійкість

Ключові слова: пасивний автобалансир, ефект Зоммерфельда, інерційний віброзбудник, резонансна вібромашина, біфуркація рухів

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STUDYING THE LOAD JAM MODES WITHIN THE FRAMEWORK OF A FLAT MODEL OF THE ROTOR WITH AN AUTO-BALANCER

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1. Introduction

Passive auto-balancers, ball (roller), pendulum, etc. [1–13], are used for balancing high-rotational rotors at operation. The same devices with a single or more loads can be used in vibration machines in order to excite vibrations [14–18].

The use of auto-balancers for different purposes is possible due to that a rotor machine with loads in the form of balls (rollers), pendulums can execute various steady modes of motion that correspond to:

– auto-balancing or synchronous rotation of loads together with the rotor (stationary movements) [1–4];

– load jams (caused by the Sommerfeld effect) [5–16];

– parametric and other oscillations of loads [17, 18].

Theoretically, the rotor machine–auto-balancer system can execute various steady movements. Among all possible steady movements, over time, the system would execute only steady movements. Therefore, when constructing an analytical theory for such machines, one searches for all possible steady movements of the system and investigates their stability.

The most complete information on the origin, disappearance, conditions of existence and stability of different motion modes of the system is provided by the bifurcation theory. Typically, the speed of rotor rotation is accepted as a bifurcation parameter. In this case, at certain speeds of rotor rotation an auto-balancer can balance the rotor, and at certain – to excite vibrations. To build a bifurcation theory, it is necessary to find and investigate all possible steady movements of the system; this is, however, a complicated mathematical problem.

Today, most analytical results were obtained in the framework of a flat model of the rotor on isotropic elastic-viscous supports carrying an auto-balancer with identical loads. However, there remain the insufficiently studied modes of load jams caused by the Sommerfeld effect. Only those jam modes have been found and investigated at which loads are combined.

It is a relevant task to analytically find, within the specified model of the rotor and auto-balancer, all possible modes of load jams. This is important for building analytical theories both for passive auto-balancing and resonance vibration machines in which an auto-balancer is used as a vibration exciter.

2. Literature review and problem statement

Initially, passive auto-balancers were intended for balancing high-rotational rotors [1]. The history of emergence and development of passive autobalancing, methods for investigating the autobalancing process, etc., were described in [2].

For a long time, it was believed that the system a rotor on isotropic supports – auto-balancer can only execute stationary steady movements. During stationary movements, loads synchronously rotate with the rotor and take one of the possible positions of relative equilibrium. A procedure for studying stationary movements of the rotor on isotropic supports with auto-balancers is described in [3]. The procedure makes it possible to find all possible stationary motion modes of the system and to assess their stability. Paper [3] examined, also within the framework of a flat model of the rotor on isotropic elastic-viscous supports, which carries an auto-balancer with many loads (balls, rollers, pendulums), the stationary movements; the authors derived the rotor rotation speeds, which give rise to the emergence or disappearance of various stationary movements. In terms of the bifurcation theory, in the transition of these speeds, stationary movements can acquire or lose stability. It was found that among all stationary movements, at the pre-resonance speeds of rotor rotation, only such a motion can be steady during which loads unbalance the rotor to the fullest, and at the over-resonance speeds – the multiparametric family of basic movements (during which a rotor is balanced by an auto-balancer). However, the construction of a complete bifurcation theory necessitates identifying and investigating all other possible steady motion modes of the system.

Study [4] provides an analytical statement of the problem on building a nonlinear bifurcation theory for the considered system. However, the bifurcation analysis was carried out by numerical methods, for the case of two loads. It was found that there are boundary cycles and chaotic movements in the system along with the stationary movements.

To estimate the degree of completeness of the analytical bifurcation theory built for the system considered, we shall review some studies that have reported the new steady motion modes of rotor machines with auto-balancers.

Paper [5] established, experimentally and from a computing experiment, the modes of pendulum jams in the rotor pendulum auto-balancer system. Under these modes, the pendulums are combined, they cannot accelerate and get stuck at one of the resonance speeds of rotor rotation. It was found that jam modes occur at low resistance forces in the system. The emergence of modes was explained by the manifestations of the Sommerfeld effect [6]. Next, the pendulum jam modes were explored in the following papers:

– [7] – theoretically for the rotor on isotropic supports, which executes spatial motion and is balanced by a single or two two-pendulum auto-balancers;

– [8] – experimentally for the pendulum freely mounted onto the shaft of an electric motor.

It was established that when a rotor rotates at a constant speed the pendulums get stuck at one of the resonance speeds of rotor rotation (natural frequency of the system oscillations).

Paper [2] experimentally identified and analytically examined the modes of load jams in a multi-ball auto-balancer. Theoretical study was carried out for a flat model of the rotor on isotropic supports and an auto-balancer. The authors considered the case when loads were assembled together. It was theoretically found that, depending on the speed of rotor rotation, there are one or three possible speeds when loads get stuck. For the case of three jam speeds, two speeds are close to the resonance speed of rotor rotation, and one is somewhat less the speed of rotor rotation. Among all possible regimes of load jams, only such a mode would be stable under which the speed of load rotation is the lowest, and pre-resonance at the same time. It should be noted that within the framework of the considered model the authors derived exact analytical solutions for jam regimens for the case of a balanced rotor.

The modes when balls get stuck were theoretically and from a computing experiment detected and investigated within the following models:

– a rotor on isotropic supports that executes spatial motion and is statically balanced with a double-ball auto-balancer [9];

– a rotor on isotropic supports that executes a flat motion and is balanced with a double-ball auto-balancer, by using the modified incremental harmonic balance method [10];

– a rotor on isotropic supports that executes a flat motion and is balanced with a double-ball auto-balancer, by applying the Limit-Cycle Analysis [11];

– a rotor on anisotropic supports that executes flat motion and is statically balanced with a two-ball auto-balancer [12];

– a rotor mounted on isotropic supports atop a platform that moves in a straight-line direction, with the rotor balanced with a two-ball auto-balancer.

The effect of jamming was analytically investigated in vibration machines, in which:

– one or two inertial vibration exciters are mounted on a bearing body exercising flat motion [14];

– a wind wheel with an unbalanced mass, mounted on a vibration platform [15];

– auto-balancer [13].

Papers [9–15] established that loads get stuck at one of the resonance speeds of rotor rotation; for the case of several loads, such jam modes were investigated at which the loads are assembled together.

Authors of [16] analytically derived and examined load jam modes within the framework of a single-mass vibration machine with straight forward motion of the platform and a vibration exciter in the form of an auto-balancer with many loads. They explored only those jam modes under which the loads are assembled together. It was determined that depending on the speed of rotor rotation there are one or three possible frequencies of load jams in the system. For the case of three frequencies, two frequencies are close to the resonance frequency of platform oscillations, and one – to the frequency of rotor rotation. It should be noted that it is impossible, within the framework of the considered model, to derive precise analytical solutions for jam modes. Therefore, the frequencies of jams were determined approximately, by the method of decomposition based on powers for a small parameter.

Let us analyze the results obtained. Detecting, in papers [2, 16], the three, rather than one, possible frequencies of load jams is due to that these speeds were determined from exact formulae [2] or in higher approximation [16]. In studies [5, 7–15] the frequencies of load jams were determined approximately, by different approximation methods. Therefore, in the lowest approximations, two smaller frequencies of load jams coincided with the resonance speed of rotor rotation, and the highest – with the speed of rotor rotation.

Studying only such a jam mode under which loads are assembled together can be explained as follows. Computational or field experiments [2, 5, 8, 14] can only detect stable steady motion modes of the system. This is how the jam modes were found under which the loads are assembled together. Therefore, the analytical theory was built exactly for such a regime; the issue of the existence of other jam modes was not addressed [7, 9–13, 16].

It should be noted that a rotor system with an auto-balancer can execute the parametric (steady) oscillations. Such oscillations, for example, were detected and investigated for the following cases:

– a single-mass [17] vibration machine with a vibration exciter in the form of a ball auto-balancer mounted on the rotor with eccentricities;

– a flat model of the rotor on isotropic supports with a two-ball auto-balancer [18].

The above results show that building an analytical bifurcation theory for the considered system is impossible yet, since not all possible steady motion modes of the system have been analytically identified and investigated. The necessary stage in the construction of an analytical bifurcation theory of the system under consideration is to investigate all possible modes of load jams.

3. The aim and objectives of the study

The aim of this study is to find all possible modes of load jams in a passive auto-balancer with many identical loads in the framework of a flat model of the balanced rotor on isotropic elastic-viscous supports. This would make it possible both to avoid such modes when balancing rotors automatically and to enable such modes in vibration machines.

To accomplish the aim, the following tasks have been set:

– to describe a mechanical-mathematical model of the rotor–auto-balancer system, to record the differential equations of motion and the equations of stationary movements in the movable coordinate system;

– under condition for load jams in the auto-balancer, to find all possible solutions to the differential equations of the system motion and to test the established jam modes in a computational experiment.

4. Methods for finding all possible modes of load jams in an auto-balancer

To construct a mechanical-mathematical model of the rotor – auto-balancer system, we shall use elements from the theories of rotor machines with passive auto-balancers [1, 2], classical mechanics [19], disturbances [20], bifurcation of movements [21].

Differential equations of the system motion are recorded with respect to the coordinate system that rotated at a constant angular velocity.

In such a coordinate system:

– the mechanical system motion is described by a system of regular nonlinear autonomous differential equations;

– all load jam modes are stationary movements, provided that the rotation speed of the moving coordinate system coincides with the angular velocity of load jams.

Finding all possible modes of load jams comes down to solving a nonlinear system of algebraic equations. At the same time, one searches for all possible frequencies of load jams (angular velocities of rotation of the moving coordinate system), positions of loads relative to the rotating coordinate system, corresponding deviations of the rotor.

To solve the system of nonlinear algebraic equations, we shall use the method of expanding the roots of equations in powers of the small parameter [20]. In this case, different ratios of smallness between the parameters of the system will be considered.

A bifurcation parameter to be applied is the angular velocity of rotor rotation. The load jams modes will be searched for depending on the angular velocity of rotor rotation. The occurrence and disappearance of different jam modes will be studied from the point of view of the theory of movements bifurcation [21].

The results to be obtained from a theoretical study will be tested using specific numerical calculations.

5. Load jam modes in an auto-balancer – a flat model of rotor on isotropic supports

5. 1. Mechanical-mathematical model of the system

5. 1. 1. Description of the mechanical-mathematical model of the system

To study the system dynamics, we have adopted the so-called flat model of the rotor and auto-balancer. Under it, a rotor is a symmetrical flat disk of mass M, mounted on a completely rigid shaft perpendicular to its plane (Fig. 1). The rotor is located vertically, moves in a flat parallel direction in the horizontal plane, and rotates at a constant angular velocity ω. For the case of a pendulum auto-balancer (Fig. 1, *a*), the rotor shaft hosts n_b identical pendulums. The mass of the pendulum is m, physical length l and the main central axial moment of inertia I_C. For the case of a ball (roller) auto-balance (Fig. 1, *b*), n_b balls (rollers) roll without sliding along a circular track. The ball (roller) mass is m, radius R, distance from the shaft axis to the center of the ball (roller) is l. As it is accepted in the analytical theory of passive auto-balancers, it is assumed that the loads do not interfere with each other's motion.

At a stationary rotor, the shaft is aligned with the axis of rotation. In the process of motion, the shaft, point O, deviates from the axis of rotation, point K, and it is exposed to the action of a renewing force, and the force of the medium viscous resistance. Coefficients of rigidity and damping in the shaft supports are c, b.

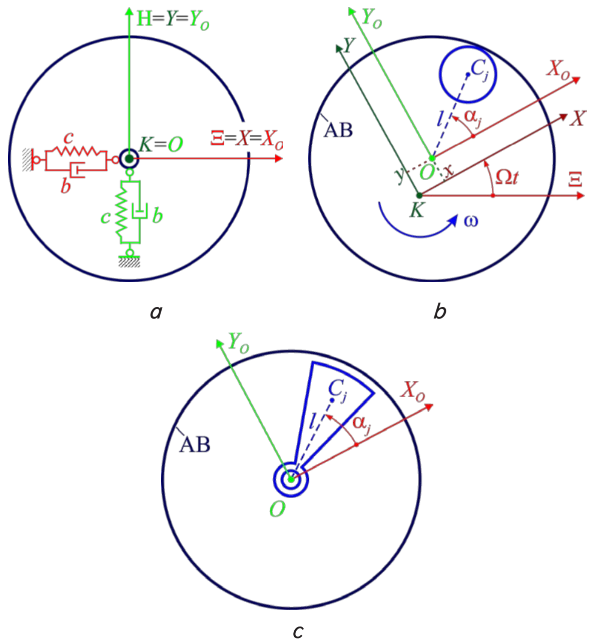

Fig. 1. A flat model of rotor and auto-balancer:
a – rotor on isotropic elastic-viscous supports;
b – kinematics of the rotor motion and a ball or a roller;
c – kinematics of the pendulum motion

To describe the system motion, the following systems of axes are used:

– $O\Xi H$ – right-hand system of stationary rectangular axes;

– OXY – right-hand system of moving rectangular axes that rotates around the rotation axis (point K) at a constant angular velocity;

– OX_OY_O – right-hand system of moving rectangular axes, starting at the center of the disc and parallel to the system of OXY axes.

The rotation angle of the OXY axes around point K is equal to Ωt, where t is the time. The rotation angle of the rotor is ωt. Position of the j-th load is determined, relative to the system of the OX_OY_O axes, by angle α_j. When the j-th pendulum rotates around the shaft, it is exposed to the momentum of force of viscous resistance $\beta l^2(\omega-\Omega-\dot{\alpha}_j)$, where β is the coefficient of forces viscous resistance, $(\omega-\Omega-\dot{\alpha}_j)$ is the pendulum rotation speed around the shaft (rotor): a dot over the magnitude defines a derivative for time. When the j-th ball (roller) moves along a track, it is exposed to a force of viscous resistance $\beta l(\omega-\Omega-\dot{\alpha}_j)$, where β is the coefficient of viscous resistance forces, and $l(\omega-\Omega-\dot{\alpha}_j)$ is the motion speed of the center of a ball (roller) along a track relative to the rotor.

The mass of the system, the total imbalance due to loads in the projections onto the moving X and Y axes, the resonance rotation speed of the rotor, are, respectively:

$$M_\Sigma = M + n_b m,$$

$$s_x = ml\sum_{j=1}^{n_b}\cos\alpha_j,$$

$$s_y = ml\sum_{j=1}^{n_b}\sin\alpha_j,$$

$$\omega_0 = \sqrt{c/M_\Sigma}. \tag{1}$$

Projections of speed of the rotor mass center onto the X and Y axes are, respectively

$$v_{Ox} = \dot{x} - \Omega y,\ v_{Oy} = \dot{y} + \Omega x. \tag{2}$$

5. 1. 2. Differential equations of the system motion

The differential equations of the system motion are conditionally divided into two groups [2]:

– the differential equations of load motion

$$L_j = \kappa m l^2\ddot{\alpha}_j + \beta l^2(\dot{\alpha}_j + \Omega - \omega) - -ml\begin{bmatrix}(\ddot{x} - 2\Omega\dot{y} - \Omega^2 x)\sin\alpha_j - \\ -(\ddot{y} + 2\Omega\dot{x} - \Omega^2 y)\cos\alpha_j\end{bmatrix} = 0, \quad /j = \overline{1, n_b}/, \tag{3}$$

where, for a ball, a roller, or a pendulum, respectively,

$$\kappa = \frac{7}{5},\quad \kappa = \frac{3}{2},$$

$$\kappa = 1 + I_C/(ml^2); \tag{4}$$

– the differential equations of rotor motion

$$L_x = M_\Sigma(\ddot{x} - 2\Omega\dot{y} - \Omega^2 x) + b(\dot{x} - \Omega y) + cx - -ml\sum_{j=1}^{n_b}\left[\ddot{\alpha}_j\sin\alpha_j + (\dot{\alpha}_j + \Omega)^2\cos\alpha_j\right] = 0;$$

$$L_y = M_\Sigma(\ddot{y} + 2\Omega\dot{x} - \Omega^2 y) + b(\dot{y} + \Omega x) + cy - -ml\sum_{j=1}^{n_b}\left[-\ddot{\alpha}_j\cos\alpha_j + (\dot{\alpha}_j + \Omega)^2\sin\alpha_j\right] = 0. \tag{5}$$

Note that for a mathematical pendulum $I_C = 0,\ \kappa = 1$.

5. 1. 3. The generalized potential and dissipation function of the system

Potential energy of the system

$$V = \frac{1}{2}c(x^2 + y^2). \tag{6}$$

At steady motion, the kinetic energy of the rotor is the sum of the kinetic energies of the progressive motion together with the center of masses (point O) and the rotation around the center of masses [19]:

$$T_r = M(x^2 + y^2)\Omega^2/2 + I_O^{(r)}\omega^2/2, \tag{7}$$

where $I_O^{(r)}$ is the axial momentum of rotor inertia relative to the longitudinal axis.

At steady motion, loads stop moving relative to the moving axes. Loads behave as a completely rigid body rotating at a constant angular velocity Ω around the axis of rotation. Their kinetic energy can be represented in the form

$$T_b = I_K^{(b)}\Omega^2/2, \tag{8}$$

where $I_K^{(b)}$ is the axial momentum of inertia of loads relative to the axis of rotation.

In its turn

$$I_K^{(b)} = \sum_{j=1}^{n_b} \left\{ I_C + m\left[\left(x + l\cos\alpha_j \right)^2 + \left(y + l\sin\alpha_j \right)^2 \right] \right\} =$$
$$= n_b \left[I_O + m\left(x^2 + y^2 \right) \right] + 2xs_x + 2ys_y, \tag{9}$$

where $I_O = I_C + ml^2$ is the axial momentum of inertia of the load relative to the O axis.

Thus, the kinetic energy of the system at steady motion

$$T_0 = T_r + T_b = M_\Sigma \left(x^2 + y^2 \right) \Omega^2 / 2 +$$
$$+ I_K^{(r)} \omega^2 / 2 + \left(n_b I_O + 2xs_x + 2ys_y \right) \Omega^2 / 2. \tag{10}$$

The generalized potential:

$$\Pi = V - T_0 = \begin{bmatrix} \left(c - M_\Sigma \Omega^2 \right) \left(x^2 + y^2 \right) - I_K^{(r)} \omega^2 - \\ - \left(n_b I_O + 2xs_x + 2ys_y \right) \Omega^2 \end{bmatrix} / 2. \tag{11}$$

This function generalizes the analog of the potential energy derived in paper [3] and extends it for the case of movements at which loads get stuck.

A dissipation function

$$D = \frac{1}{2} b v_O^2 + \frac{1}{2} \sum_{j=1}^{n_b} \beta u_j^2 =$$
$$= \frac{1}{2} b \left[\left(\dot{x} - \Omega y \right)^2 + \left(\dot{y} + \Omega x \right)^2 \right] + \frac{1}{2} \beta l^2 \sum_{j=1}^{n_b} \left(\omega - \Omega - \dot{\alpha}_j \right)^2. \tag{12}$$

The component, linear relative to the generalized velocities, is

$$D_1 = b\Omega \left(-\dot{x} y + \dot{y} x \right) - \beta l^2 \left(\omega - \Omega \right) \sum_{j=1}^{n_b} \dot{\alpha}_j. \tag{13}$$

This function is an analog of the dissipative function derived in paper [3] and extends it for case of movements at which loads get stuck.

5. 1. 4. Equations of the system stationary movements

The equations of stationary motions for the considered system will take the form

$$\tilde{L}_j = \frac{\partial \Pi}{\partial \alpha_j} + \frac{\partial D_1}{\partial \dot{\alpha}_j} = \beta l^2 \left(\Omega - \omega \right) + ml\Omega^2 \left(\tilde{x} \sin\tilde{\alpha}_j - \tilde{y} \cos\tilde{\alpha}_j \right) = 0,$$

$$/ j = \overline{1, n_b} /, \tag{14}$$

$$\tilde{L}_x = \frac{\partial \Pi}{\partial x} + \frac{\partial D_1}{\partial \dot{x}} = \left(c - M_\Sigma \Omega^2 \right) \tilde{x} - \Omega^2 \tilde{s}_x - b\Omega \tilde{y} = 0,$$

$$\tilde{L}_y = \frac{\partial \Pi}{\partial y} + \frac{\partial D_1}{\partial \dot{y}} = \left(c - M_\Sigma \Omega^2 \right) \tilde{y} - \Omega^2 \tilde{s}_y + b\Omega \tilde{x} = 0. \tag{15}$$

When loads get stuck at angular velocity Ω, the generalized coordinates are constant

$$\alpha_j = \tilde{\alpha}_j, \ \dot{\alpha}_j = \ddot{\alpha}_j = 0, \ \ / j = \overline{1, N} /; \ \ x = \tilde{x}, \ \ y = \tilde{y}. \tag{16}$$

Therefore, the system of equations (14), (15) can be derived from the system of differential equations of motion (3), (5).

This system of (n_b+2) nonlinear algebraic equations relative to (n_b+3) unknowns $\tilde{x}, \ \tilde{y}, \ \tilde{\alpha}_j, \ / j = \overline{1, n_b} /, \ \Omega$.

Stationary movements are the solutions to a system of algebraic equations (14), (15).

The system may execute the following stationary movements:

– basic and side at which loads rotate synchronously with the rotor Ω≠ω [3];

– side movements, at which loads rotate non-synchronously with the rotor Ω≠ω.

The stationary movements at which loads rotate synchronously with rotor were studied in paper [3]. Next, we shall examine the stationary movements at which loads rotate non-synchronously with the rotor Ω≠ω.

Note that we are considering movements that are stationary relative to different coordinate systems that rotate at certain stable angular velocities. These movements are not stationary relative to a single, particular, rotating coordinate system.

5. 2. Finding load jam modes

5. 2. 1. Problem statement

Introduce an angle ϑ between vector $\overline{KO}$ (the rotor displacement vector) and the X axis.

Then

$$\cos\vartheta = \tilde{x} / \tilde{r}, \ \sin\vartheta = \tilde{y} / \tilde{r}, \ \tilde{r} = \sqrt{\tilde{x}^2 + \tilde{y}^2}, \tag{17}$$

and the equations of steady movements (14), (15) are transformed to the form:

$$\tilde{L}_j = \beta l^2 \left(\Omega - \omega \right) - ml\Omega^2 \tilde{r} \sin\varphi_j = 0,$$

$$\left(\sum_{j=1}^{n_b} \tilde{L}_j = n_b \beta l^2 \left(\Omega - \omega \right) - ml\Omega^2 \tilde{r} \sum_{j=1}^{n_b} \sin\varphi_j = 0 \right); \tag{18}$$

$$\tilde{x}\tilde{L}_x + \tilde{y}\tilde{L}_y = \left(c - M_\Sigma \Omega^2 \right) \tilde{r}^2 - ml\Omega^2 \tilde{r} \sum_{j=1}^{n_b} \cos\varphi_j = 0,$$

$$\tilde{x}\tilde{L}_y - \tilde{y}\tilde{L}_x = b\Omega \tilde{r}^2 + ml\Omega^2 \tilde{r} \sum_{j=1}^{n_b} \sin\varphi_j = 0, \tag{19}$$

where

$$\varphi_j = \vartheta - \tilde{\alpha}_j, \ / j = \overline{1, n_b} /. \tag{20}$$

We have derived a system of (n_b+2) nonlinear algebraic equations relative to (n_b+2) unknowns $\tilde{r}$, Ω, φ_j, $/ j = \overline{1, n_b} /$.

A first group of equations – (18) would hold if

$$\sin\varphi_j = \sin\chi, \ \cos\varphi_j = \left(-1 \right)^{k_j} \cos\chi, \ \ / j = \overline{1, n_b} /, \tag{21}$$

where $k_j \in \{0, 1\}, \ / j = \overline{1, n_b} /;$ χ – some angle to be found.

Equations (21) would hold if

$$\varphi_j = \begin{cases} \chi, \ k_j = 0; \\ -\pi - \chi, \ k_j = 1. \end{cases} \tag{22}$$

Using (21), we obtain from (18) and (19) the following system of equations to determine $\chi, \ \tilde{r}, \ \Omega$:

$$\tilde{L}_j = \beta l^2 \left(\Omega - \omega \right) - ml\Omega^2 \tilde{r} \sin\chi = 0, \tag{23}$$

$$\tilde{x}\tilde{L}_x + \tilde{y}\tilde{L}_y = \left(c - M_\Sigma \Omega^2 \right) \tilde{r}^2 - ml\Omega^2 \tilde{r} n_{AB}(k) \cos\chi = 0,$$

$$\tilde{x}\tilde{L}_y - \tilde{y}\tilde{L}_x = b\Omega \tilde{r}^2 + n_b ml\Omega^2 \tilde{r} \sin\chi = 0, \tag{24}$$

where

$$n_{AB}(k)=\sum_{j=1}^{n_b}(-1)^{k_j},\quad k=k_nk_{n-1},\ldots,k_1. \qquad (25)$$

A binary number $k_n, k_{n-1}, \ldots, k_1$, (or its decimal equivalent k) characterizes a certain configuration of loads in the moving coordinate system OX_OY_O under a jam mode. The introduced angle θ is the parameter. Therefore, each jam mode is a single-parametric family of movements at which loads get stuck in a specific configuration at a single particular rotation speed.

In the decimal calculation system $0\le k\le 2^{n_b}-1$. Therefore, different configurations of loads may not exceed 2^{n_b}.

As the loads are the same, the movements at which for the same quantity of loads $\varphi_j=\chi$ are not fundamentally different. We shall enumerate the fundamentally different configurations of loads with index $i=0,\ldots, n_b$. Under configuration 0, all loads are inclined at angles $\varphi_j=\chi$, at motion 1 – one load is inclined at $-(\pi+\chi)$, and the rest – at χ, etc.

The total quantity of different configurations of loads is:

$$N=n_b+1. \qquad (26)$$

In this case

$$n_{AB}(i)=n_b-2i,\quad /\, i=\overline{0,n_b}\,/ \qquad (27)$$

and parameter $n_{AB}(i)$ accepts the following values:

– $n_b,\ldots, 3, 1, -1, -3,\ldots, -n_b$, if n_b is even;

– $n_b,\ldots, 4, 2, 0, -2, -4,\ldots, -n_b$, if n_b is odd.

5. 2. 2. Jam modes for the case when $n_{AB}(i)\neq 0$

The general sequence of problem solving. Note that at the odd quantity of loads, it is always $n_{AB}(k)\neq 0$.

We shall solve the system of equations (23), (24). Represent it in the form

$$\tilde r^2=-\frac{n_b ml\Omega}{b}\tilde r\sin\chi,\quad \tilde r\cos\chi=-\frac{M_\Sigma\Omega^2-c}{ml\Omega^2 n_{AB}(k)}\tilde r^2,$$

$$\tilde r\sin\chi=\frac{\beta l}{m\Omega^2}(\Omega-\omega). \qquad (28)$$

We derive from the first and third equations

$$\tilde r^2=-\frac{n_b ml\Omega}{b}\frac{\beta l}{m\Omega^2}(\Omega-\omega)=\frac{n_b\beta l^2}{b\Omega}(\omega-\Omega). \qquad (29)$$

It follows from (29) that loads can lag behind the rotor only ($\Omega<\omega$). Then it follows from the first equation in (28) that $\sin\chi<0$, therefore $\chi\in(-\pi,0)$.

By using the first and second equation in (28), introduce the angle

$$\gamma(k)=\arctan\left(\frac{\sin\chi(k)}{\cos\chi(k)}\right)=$$

$$=\arctan\left(\frac{n_{AB}(k)}{n_b}\frac{b\Omega(k)}{M_\Sigma\Omega^2(k)-c}\right),$$

$$\gamma(k)\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right). \qquad (30)$$

Then, we find $\chi(k)\in(-\pi, 0)$:

$$\chi(k)=\begin{cases}\gamma(k),\ \gamma(k)\le 0;\\ \gamma(k)-\pi,\ \gamma(k)>0.\end{cases} \qquad (31)$$

Use the identity

$$\tilde r^2=(\tilde r\sin\chi)^2+(\tilde r\cos\chi)^2=$$

$$=\frac{\beta^2 l^2}{m^2\Omega^4}(\Omega-\omega)^2+\frac{\left(M_\Sigma\Omega^2-c\right)^2}{m^2l^2\Omega^4 n_{AB}^2(k)}\tilde r^4.$$

Substitute $\tilde r^2$ from (29) in this equation, we obtain

$$\frac{n_b\beta l^2}{b\Omega}(\omega-\Omega)=\frac{\beta^2l^2}{m^2\Omega^4}(\Omega-\omega)^2+$$

$$+\frac{\left(M_\Sigma\Omega^2-c\right)^2}{m^2l^2\Omega^4n_{AB}^2(k)}\frac{n_b^2\beta^2l^4}{b^2\Omega^2}(\omega-\Omega)^2. \qquad (32)$$

This equation can be satisfied in the following two cases:

1) $\omega-\Omega=0$ – loads rotate synchronously with the rotor;

2) $\frac{n_b}{b}=\frac{\beta}{m^2\Omega^3}(\Omega-\omega)+\frac{\left(M_\Sigma\Omega^2-c\right)^2}{m^2\Omega^5n_{AB}^2(k)}\frac{n_b^2\beta}{b^2}(\omega-\Omega)$ – loads lag behind the rotor.

Paper [3] examined the motion modes at which loads rotate synchronously with the rotor. Below, we study the load jam modes.

The second case produces the following equation

$$n_b bm^2\Omega^5 n_{AB}^2(k)-$$

$$-\left[\left(M_\Sigma\Omega^2-c\right)^2n_b^2+b^2\Omega^2n_{AB}^2(k)\right]\beta(\omega-\Omega)=0. \qquad (33)$$

We find, from (33), frequencies $\Omega_i(k)$ at which loads can get stuck.

Then, from equation (29), we find

$$\tilde r(k)=l\sqrt{n_b\beta\left[\omega-\Omega(k)\right]/\left[b\Omega(k)\right]}. \qquad (34)$$

We find from (20) and (22):

$$\tilde\alpha_j(k,\vartheta)=\vartheta-\varphi_j(k)=\begin{cases}\vartheta-\chi(k),\ k_j=0;\\ \vartheta+\pi+\chi(k),\ k_j=1,\end{cases}$$

$$/\, j=\overline{1,n_b}\,/. \qquad (35)$$

We find from (17)

$$\tilde x(k,\vartheta)=\tilde r(k)\cos\vartheta,\ \tilde y(k)=\tilde r(k)\sin\vartheta. \qquad (36)$$

Magnitudes (34) to (36) are calculated for the specific frequency of jamming.

Frequencies of load jams. Convert equation (33) to the form

$$P(\nu,k)=\chi(k)\nu^5-(n-\nu)\left[\left(1-\nu^2\right)^2+4h^2(k)\nu^2\right]=$$

$$=a_0(k)\nu^5+a_1(k)\nu^4+a_2(k)\nu^3+a_3(k)\nu^2+$$

$$+a_4(k)\nu+a_5(k)=0, \qquad (37)$$

where

$$\nu=\frac{\Omega}{\omega_0},\quad n=\frac{\omega}{\omega_0},\quad \delta(k)=\frac{n_{AB}(k)}{n_b},$$

$$\chi(k) = n_b \frac{m^2 b}{M_\Sigma^2 \beta} \delta^2(k),$$

$$h(k) = \frac{b\delta(k)}{2M_\Sigma \omega_0};$$

$$a_0(k) = 1 + \chi(k),$$

$$a_1 = -n,$$

$$a_2 = -2\left[1 - 2h^2(k)\right],$$

$$a_3 = 2n\left[1 - 2h^2(k)\right],$$

$$a_4 = 1, \quad a_5 = -n. \qquad (38)$$

The roots of the equation that exactly coincides with equation (37) were investigated in [2, 16]. The effect of jamming occurs at small viscous resistance forces (in the supports and forces that prevent the motion of loads relative to the body of an auto-balancer). According to the results from papers [2, 16], in this case, for the configuration of loads k, there are three characteristic rotor rotation speeds

$$n_1(k) \approx 1 + \frac{3}{4}\sqrt[3]{4\chi(k)} + \sqrt[3]{2\chi^2(k)} + \frac{53}{48}\chi(k) - \frac{h^2(k)}{\sqrt[3]{4\chi(k)}},$$

$$n_2 = \frac{\chi(k)}{4h^2(k)} + 1,$$

$$n_3(k) \approx \frac{\chi(k)}{4h^2(k)} + 1 + \frac{9}{16}\chi(k) + \frac{3}{2}\left(1 + \frac{27}{32}\chi(k)\right)h^2(k). \qquad (39)$$

At their transition, the quantity or properties of possible frequencies of load jams change. In this case,

$$1 < n_1(k) << n_2 < n_3(k) << n$$

and at rotor rotation speed:

– lower the $n_1(k)$ $\left(0 < n < n_1(k)\right)$, there is the single frequency of load jams $\nu_1(k)$, and $0<\nu_1(k)<1$;

– exceeding $n_1(k)$, but lower than n_2 $\left(n_1(k) < n < n_2\right)$, there are three frequencies of load jams $\nu_{1,2,3}(k)$, such that $0<\nu_1(k)<1<\nu_2(k)<\nu_3(k)<n$;

– exceeding n_2, but lower than $n_3(k)$ $\left(n_2 < n < n_3(k)\right)$, there are three frequencies when loads jam $\nu_{1,2,3}(k)$, such that $1<\nu_1(k)<\nu_2(k)<<\nu_3(k)<n$;

– exceeding $n_3(k)$ $\left(n > n_3(k)\right)$, there is a single frequency of load jams $\nu_3(k)$, such that $1<<\nu_3(k)<n$.

Note that n_2 does not depend on the configuration of loads relative to the rotor, but $\tilde{n}_1(k)$, $\tilde{n}_3(k)$ do depend. We find from (38) and (39)

$$n_2 = \frac{n_b m^2 \omega_0^2}{b\beta} + 1. \qquad (40)$$

Table 1 gives the formulae intended for the approximate calculation of frequencies of load jams at different ratios of smallness between the system parameters.

In Table 1, ε is a dimensionless positive magnitude that is much less than 1 (0<ε<<1). It is introduced to determine the ratios of smallness between the system parameters.

Table 1

Dependence of dimensionless frequencies of load jams $\nu_j(k)$ on rotor rotation speed (n)

No. of entry	Ratio of smallness between parameters	Frequencies of load jams – expanding the roots of a polynomial (39)
1	n~ε	$\nu_1(k) \approx n\left[1 - \chi(k)n^4\right]$
2	n~1/ε	$\nu_3(k) \approx \frac{n}{1+\chi(k)} - \frac{2\chi(k)\left[1-2h^2(k)\right]}{n}$
3	$\|n-1\|$~1, χ~ε, h~ε	$\nu_{1/2} \approx 1 \mp \frac{1}{2}\sqrt{\frac{\chi(k)}{n-1}} + \frac{\chi(k)(4n-3)}{8(n-1)^2}$, $\nu_3 \approx n - \frac{\chi(k)n^5}{(n^2-1)^2}$
4	$n \approx n_1$: $(n-1) \sim \sqrt[3]{\varepsilon}$, $\chi, h \sim \varepsilon$	$\nu_1(k) \approx 1 - \frac{1}{4}\sqrt[3]{4\chi(k)}\left[1 - \frac{w(k)}{3}\right]$, $\nu_{2/3}(k) \approx 1 + \frac{1}{2}\sqrt[3]{4\chi(k)}\left[1 \mp \sqrt{w(k)} + 2\frac{w(k)}{3}\right]$, $w(k) = \frac{4}{3}\frac{(n-1)}{\sqrt[3]{4\chi(k)}} - 1$
5	$n \approx n_2$: n~1/ε[2], h~ε, σ~1 – parameter	$n \approx \frac{\chi(k)}{4h^2(k)} + 1 + \frac{9}{16}\chi(k) + \sigma h^2(k)$, $\nu_{1/2} \approx 1 + \frac{3}{2}h^2(k) \mp h^3(k)\sqrt{\frac{96 + 81\chi(k) - 64\sigma}{16\chi^2(k)}}$, $\nu_3 \approx \frac{n}{1+\chi(k)} - \frac{2\chi(k)}{n}$

Number of characteristic speeds, angular velocities and load jam modes

It is clear from (33) that the characteristic speeds and angular velocities of load jams are the same for configurations of loads for which $n_{AB}^2(i)$. are the same. It is clear from (27) that $n_{AB}^2(i) = n_{AB}^2(n_b - i)$. The case $n_{AB}(i) = 0$ is possible only at the even number of loads. Thus, there are the fundamentally different options for which $n_{AB}^2(i) \neq 0$:

– $(n_b+1)/2$, if n_b is odd;

– $n_b/2$, if n_b is even.

Note that in the same number of configurations $n_{AB}(i) > 0$ or $n_{AB}(i) < 0$.

Each option is matched with its own three speeds of load jams. Therefore, for the case $n_{AB}(i) \neq 0$, the total number of different speeds of load jams:

– $3(n_b+1)/2$, if n_b is odd;

– $3n_b/2$, if n_b is even.

Since the characteristic speed n_2 does not depend on load configuration, for the case $n_{AB}(i) \neq 0$, there will be the following number of different characteristic speeds:

– n_b+2, if n_b is odd;

– n_b+1, if n_b is even.

One can see from (30), (31) and (35) that the configurations in which:

– $n_{AB}(i)>0$ are implemented for a load jam frequency of $0<\nu_1(k)<1$;

– $n_{AB}(i)<0$ are implemented for load jams frequencies $1<\nu_1(k)<\nu_2(k)<<\nu_3(k)$.

Therefore, the total number of jam modes at which $n_{AB}(i)\neq 0$:

$$n_{\Sigma 1}=\begin{cases}(1+3)(n_b+1)/2=2(n_b+1), \text{ if } n_b \text{ is even;}\\ (1+3)n_b/2=2n_b, \text{ if } n_b \text{ is odd.}\end{cases} \qquad (41)$$

The case when $n_{AB}(i)=0$ is considered below.

5. 2. 3. Jam modes for the case when $n_{AB}(k)=0$

Note that the case $n_{AB}(k)=0$ is possible only at an even number of loads, when half of the loads are inclined at angle $\varphi_j=\chi$. In this case, from the first equation in (24), we obtain the following frequency of load jams

$$\Omega=\omega_0. \qquad (42)$$

The remaining equations in (23), (24) take the form

$$\tilde{L}_j=\beta l^2(\omega_0-\omega)-ml\omega_0^2\tilde{r}\sin\chi=0, \qquad (43)$$

$$\tilde{x}\tilde{L}_y-\tilde{y}\tilde{L}_x=b\omega_0\tilde{r}^2+n_b ml\omega_0^2\tilde{r}\sin\chi=0. \qquad (44)$$

Hence

$$\tilde{x}\tilde{L}_y-\tilde{y}\tilde{L}_x+n_b\tilde{L}_j=b\omega_0\tilde{r}^2+n_b\beta l^2(\omega_0-\omega)=0.$$

We find from this equation

$$\tilde{r}=l\sqrt{n_b\beta(\omega-\omega_0)/(b\omega_0)}. \qquad (45)$$

Consequently, this mode can exist only at the over-resonance speeds of rotor rotation. We find from (43)

$$\sin\chi=-\frac{\beta l(\omega-\omega_0)}{m\omega_0^2\tilde{r}}=$$

$$=-\frac{\beta l(\omega-\omega_0)}{m\omega_0^2 l\sqrt{n_b\beta(\omega-\omega_0)/(b\omega_0)}}=-\frac{1}{m\omega_0}\sqrt{\frac{\beta b(\omega-\omega_0)}{n_b\omega_0}}.$$

Hence

$$\chi(\omega)=-\arcsin\left[\frac{1}{m\omega_0}\sqrt{\frac{\beta b(\omega-\omega_0)}{n_b\omega_0}}\right]. \qquad (46)$$

Motion disappears when

$$\frac{1}{m\omega_0}\sqrt{\frac{\beta b(\omega-\omega_0)}{n_b\omega_0}}=1.$$

Hence, we find the characteristic (dimensional) speed

$$\tilde{\omega}_2=\omega_0+\frac{n_b m^2\omega_0^3}{\beta b}=n_2\omega_0. \qquad (47)$$

Consequently, this jam mode exists in the range of dimensionless speeds of rotor rotation $n\in[1,n_2]$. At point $n=1$ $(\omega=\omega_0)$, this mode occurs and at point $n=n_2$ $(\omega=\tilde{\omega}_2)$, it disappears.

We find from (46) $\chi(1)=0,\ \ \chi(n_2)=-\pi/2$.

We find from (35):

– when $n=1$

$$\tilde{\alpha}_j=\begin{cases}\vartheta,\ k_j=0,\\ \vartheta+\pi,\ \ k_j=1;\end{cases} \qquad (48)$$

– when $n=n_2$

$$\tilde{\alpha}_j=\vartheta+\pi/2. \qquad (49)$$

Thus, for the case $n_{AB}(k)=0$, there is only one jam mode. When this regime emerges at the resonance speed, a half of the loads is inclined towards the rotor inclination and another half – in the opposite direction. When these modes disappear, all the loads are equally inclined at 90° relative to rotor inclination.

Considering (41), the total number of jam modes:

$$n_\Sigma=\begin{cases}n_{\Sigma 1}=2(n_b+1), \text{ if } n_b \text{ is even;}\\ n_{\Sigma 1}+1=2n_b+1, \text{ if } n_b \text{ is odd.}\end{cases} \qquad (50)$$

In this case, each jam mode is characterized by the i-th configuration of loads and the corresponding jamming frequency ν_j. We designate the fundamentally different modes of load jams as follows:

$$i\nu_j,\ \ /i=\overline{0,n_b},\ \ \ j=\overline{1,3}\ /. \qquad (51)$$

Then, for example, mode $0\nu_1$ implies that all loads are inclined in one direction so that $n_{AB}(0)=n_b>0$ and the loads get stuck at pre-resonance speed ν_1 ($\nu_1<1$).

5. 2. 4. Computational experiment

Estimated data: $n_b=3$, $\kappa=1$, $m=0.33$ kg, $l=0.1$ m, $M_\Sigma=4$ kg, $c=10{,}000$ N/m, $b=4$ N·s/m, $\beta=0.15$ N·s/m.

Then

$$M=M_\Sigma-n_b m=3.88 \text{ kg},$$

$$\omega_0=\sqrt{c/M_\Sigma}=50 \text{ rad/s=15.9154943 Hz}.$$

Calculations show that the system (Table 2) has:

– $n_b+2=3+2=5$ different characteristic speeds;

– $n_{\Sigma 1}=3(n_b+1)/2=3(3+1)/2=6$ different speeds of load jams;

– $2(n_b+1)=2(3+1)=8$ different modes of load jams.

Table 2

Load jam modes

Magnitudes	Numeric values of magnitudes for different load configurations							
k	0 (000)	7 (111)	1 (001)	2 (010)	4 (100)	3 (011)	5 (101)	6 (110)
i	0	3	1			2		
$n_{AB}(i)$	3	–3	1			–1		
$n_{AB}^2(i)$	9		1					
$n_1(i)$	1.23595		1.10291					
n_2	12.25							
$n_3(i)$	12.25268		12.25030					
Modes	$\nu_1<1$	$\nu_1>1$, ν_2, ν_3	$\nu_1<1$			$\nu_1>1$, ν_2, ν_3		

Fig. 2 shows the built graphs of dependences of angular velocities of load jams on rotor rotation speed for load configurations k=1÷6 $\left(n_{AB}^2(k)=1\right)$.

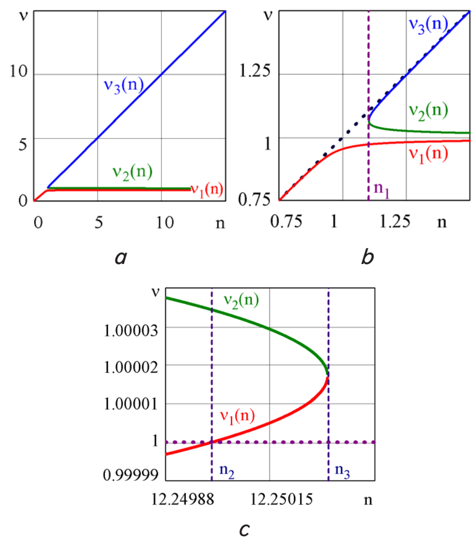

Fig. 2. Graphs of dependences of angular velocities of load jams on rotor rotation speed for load configurations k=1÷6 $\left(n_{AB}^2(k)=1\right)$: *a* – general view; *b* – in the vicinity of characteristic angular velocity $n_1(k)$; *c* – in the vicinity of characteristic angular velocities n_2, $n_3(k)$

Almost the same form is demonstrated by graphs of dependences of angular velocities of load jams for loads configurations k=0.7 $\left(n_{AB}^2(k)=9\right)$.

Fig. 3, *a* shows the emergence of motion $0\nu_1$ (n~0) and its merging with motions $1\nu_1$ and $3\nu_1$ (n=n_2).

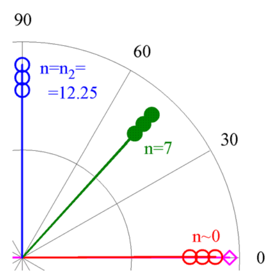

Fig. 3. Emergence of motion $0\nu_1$ (n~0) and its merging with motions $1\nu_1$ and $3\nu_1$ (n=n_2)

Fig. 4 shows the emergence of motions $3\nu_2$, $3\nu_3$ (n=n_1), the merging of motion $3\nu_2$ with motion $3\nu_1$ (n=n_3), motion $3\nu_3$ at n=n_3.

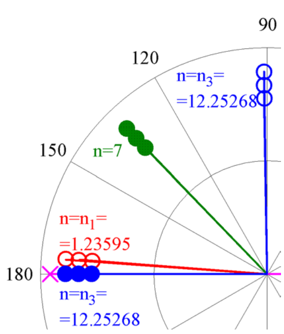

Fig. 4. Emergence of motions $3\nu_2$, $3\nu_3$ (n=n_1), the merging of motion $3\nu_2$ with motion $3\nu_1$ (n=n_3), motion $3\nu_3$ at n=n_3

Fig. 5 shows the emergence of motion $1\nu_1$ (n~0) and its merging with motions $0\nu_1$, $3\nu_1$ (n→n_2).

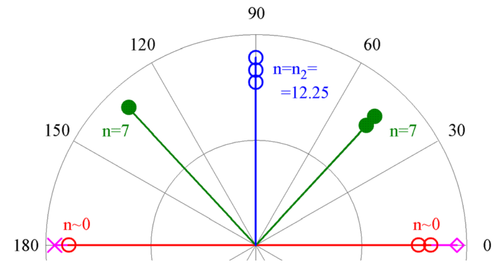

Fig. 5. Emergence of motion $1\nu_1$ (n~0) and its merging with motions $0\nu_1$, $3\nu_1$ (n→n_2)

Fig. 6 shows the emergence of motions $2\nu_2$, $2\nu_3$ (n=n_1), the merging of motion $2\nu_2$ with motion $2\nu_1$ (n=n_3), motion $2\nu_3$ at n=n_3.

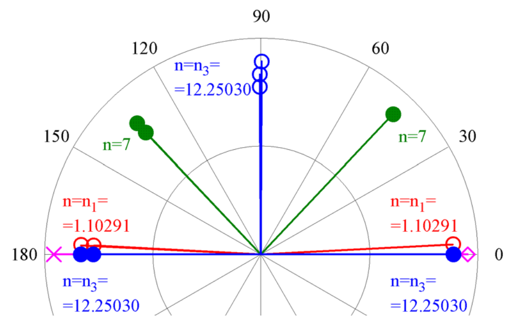

Fig. 6. Emergence of motions $2\nu_2$, $2\nu_3$ (n=n_1), the merging of motion $2\nu_2$ with motion $2\nu_1$ (n=n_3), motion $2\nu_3$ at n=n_3

Fig. 3–6 are drawn, for convenience, for angle ϑ=0 (the rotor, due to the total load imbalance, has shifted along the x axis). In addition, red color shows the positions of loads at the lowest rotor rotation speed, green – middle, blue – the largest.

6. Discussion of results obtained from studying load jam modes

Our theoretical study shows that the load jam modes in the rotor–auto-balancer system are the single-parametric families of steady movements.

Each jam mode is characterized by a certain load configuration and the appropriate frequency of jamming. In the coordinate system that rotates synchronously with loads:

– the rotor displacement is constant;

– the parameter is the angle defining the direction of the rotor displacement vector;

– loads take certain fixed positions relative to the rotor displacement vector and these positions depend on rotation speed of the rotor.

The auto-balancer with n_b+2 of the same loads haven_b+1 different load configurations. The total number of different modes of load jams is:

– 2(n_b+1), if n_b is odd;

– 2n_b+1, if n_b is even.

The total number of different jamming frequencies is:

– 3(n_b+1)/2, if n_b is odd;

– 3n_b/2+1, if n_b is even.

The total number of different characteristic speeds is n_b+2. Characteristic speeds are the points of movement bifurcations, because their transitions give rise to the emergence or disappearance of single-parametric families of movements that correspond to a certain jam mode. At these points, the jam modes may acquire or lose stability.

It should be noted that the results were obtained for the cases of small forces of viscous resistance in the system or at low mass of loads in comparison with the system mass. However, this assumption is relevant for practice. In addition, the assumption was accepted at the stage of finding the expansion of characteristic speeds and jam velocities into series based on the small parameter power.

The solved problem can act as a model problem, particularly in order to estimate:

– efficiency of approximated methods for studying the dynamics and stability of movements of mechanical systems;

– the completeness of solving the problems on studying jam modes within other models of rotor machines with auto-balancers, the unbalanced vibration exciters, etc.

The results obtained make it possible both to reduce the regions of the existence of jam modes and to increase them. This could be used in the design of auto-balancers for balancing rotors or vibration exciters in the form of auto-balancers.

Among all theoretically possible jam modes, only stable movements would be executed in practice. Therefore, in the future it is planned to investigate the stability of the established jam modes and to conduct computational experiments. Note that the study can be carried out using the fixed-motion stability theory for nonlinear autonomous systems. At the same time, it is possible to analytically find the "exact" boundaries of movement stability regions in the parameter space.

7. Conclusions

1. Within the framework of a flat model, the dynamics of a balanced rotor on isotropic supports and a ball (roller) or a pendular auto-balancer are described by the autonomous system of differential equations.

With respect to the coordinate system that rotated synchronously with loads, both the load jam modes and the synchronous rotation modes of loads with the rotor are the stationary solutions to the system of differential equations. Thus, any such regime is a state of relative equilibrium of the mechanical system relative to the moving coordinate system.

2. The load jams modes in the rotor-auto-balancer system are the single-parametric families of steady movements.

Each jam mode is characterized by a certain load configuration and the appropriate frequency of jamming.

In the coordinate system that rotates synchronously with loads:

– the rotor displacement is constant;

– the parameter is the angle defining the direction of the rotor displacement vector;

– loads take certain fixed positions relative to the rotor displacement vector and these positions depend on rotation speed of the rotor.

The auto-balancer with n_b of the same loads has n_b+1 different load configurations. The total number of different modes of load jams is:

– $2(n_b+1)$, if n_b is odd;

– $2n_b+1$, if n_b is even.

The total number of different jamming frequencies is:

– $3(n_b+1)/2$, if n_b is odd;

– $3n_b/2+1$, if n_b is even.

The total number of different characteristic speeds is n_b+2. Characteristic speeds are the points of movement bifurcations, because their transition gives rise to the emergence or disappearance of single-parametric families of movements that correspond to a certain jam mode. At these points, the jam modes can acquire or lose stability.

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