На базі методу аргумент функцій та методу функцій комплексного змінного отримані узагальнюючі рішення плоскої задачі теорії пружності з використанням інваріантних диференціальних співвідношень, здатних замкнути результат для поставленої системи рівнянь. Наведено підходи, за допомогою яких визначають не самі дозволяючи функції, а умови їх існування. Це дозволяє розширити коло гармонійних функцій різної складності, що задовольняють всіляким крайовим умовам прикладних задач, що постійно оновлюються. До розгляду взято дві базові функції: тригонометрична та фундаментальна, аргументи яких є невідомими координатними залежностями. Введення до розгляду аргумент функцій змінює підходи визначення дозволяючих залежностей, тому що задача істотно спрощується при виявленні диференціального зв'язку поміж ними у вигляді співвідношень Коші-Рімана та Лапласа. Показано кілька аналітичних рішень різної складності, яким відповідають різні граничні умови. Зіставлення з результатами досліджень інших авторів, при однакових вихідних даних, призводить до однакового результату, а при розгляді тестової задачі взаємодії металу з пружнім напівпростором – до збігу визначальних схем силового впливу на пружне середовище.

Таким чином, запропоновано новий підхід рішення плоскої задачі теорії пружності, пов'язаний з використанням аргумент функцій, що дозволяє замкнути задачу через диференціальні співвідношення Коші-Рімана та Лапласа. Ці узагальнення розширюють коло гармонійних функцій, що відповідають різним граничним умовам прикладних задач

Ключові слова: теорія пружності, аргумент функції, співвідношення Коші-Рімана, рівняння Лапласа, граничні умови

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STUDYING THE STRESSED STATE OF ELASTIC MEDIUM USING THE ARGUMENT FUNCTIONS OF A COMPLEX VARIABLE

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1. Introduction

Papers [1–4] propose a new approach to solving problems on continuum mechanics by using argument functions for some basic dependences. This makes it possible to expand the range of examined applied problems for various purposes.

Using the argument functions simplifies solving the systems of equations from the theory of plasticity, elasticity, and dynamic problems. Common provisions are employed to solve differential equations in private derivatives of different types, such as hyperbolic and elliptical. A certain invariance is defined that is associated with finding the conditions for the existence of permitting functions.

The theoretical approaches presented are in line with the fast-paced industrial production, which constantly puts forward new requirements to materials, technologies, equipment, as well as their theoretical and experimental justification. There are a lot of fields in mechanics that are explained by the variety of applied problems. In this regard, new methods and procedures to solve them are being developed, including current trends in their advancement. The level of examined problems is both fundamental and applied in character. These include: contact problems with a different geometry of the tool and varying friction conditions; studies of heterogeneous fields of deformations and stresses, including additive deformation of bodies with different shapes; the interaction between bodies with a different loading.

Separately considered is the classical and non-classical theories of elasticity, which in the latter case are defined by different directions, including: asymmetrical, microstructural, micromolar, multimolar, gradient, as well as other theories. This necessitates consideration of a new set of problems on the theory of elasticity, related to the subsequent transition to more complex and technologically-substantiated applied production issues. The new processes represent problems with a wide variety of boundary conditions that must be matched with solutions to problems from the theory of elasticity. At this stage, it is more effective to find not the solution itself, but the conditions for its existence through the defining differential and integrated ratios.

It is a relevant task to devise generalized approaches to solving the problems on the theory of elasticity, using argument functions under conditions of complex interaction between deformable bodies.

2. Literature review and problem statement

Paper [5] stated the basic generalizing approaches to problems on the theory of elasticity, supported by analytical solutions. Using a complex variable function [6] makes it possible to expand the range of approaches when solving problems on the theory of elasticity. Work [7] outlines solutions to contact problems in a semi-infinite space. These also include modern structural solutions to the problem [8], integrated ratios [9] for assessing kinematic perturbations, which close parameters [10] that define the overall form of a gradient solution. It should be emphasized that although the papers include elements of generalization, their authors failed to define differential ratios for variables capable of closing the result of permitting differential equations.

An option to overcome the related difficulties might be to use the argument functions introduced for consideration for the basic variables. Such approaches were used in papers [11–13]. Study [11] reports the generalized Cauchy-Riemann conditions, but the argument functions that close the solution to the problem were not introduced for consideration.

The conditions for the existence of solutions can be determined through the differential ratios shown in work [12]; in this case, the approaches from the cited work do not make it possible to consider the generalization using argument functions.

The generalizations reported in article [13] are primarily related to the theory of plasticity, which limits obtaining a specific result for the theory of elasticity.

Publication [14] examines the contact interaction between a sample and a punch. Common approaches to the problem were complemented by the relationship conditions. The lack of conditions for the existence of solutions to the problem complicates calculation and obtaining a reliable result.

The structure of stating a practical problem is determined within the general problem and the periodically changing obvious and boundary conditions [15]. From the argument functions point of view, this makes it possible to predict one of the basic dependences that should include a trigonometry variable.

Paper [16] studies cyclical loading for the case of a simple shift, which finds a cyclical response from internal stresses. Periodic exposure can also be determined from the basic trigonometrical function in the proposed method.

An analysis of change in the load pattern for the thickness of a sample exposed to the compact tension is given in [17]. The maximum zone is closer to the surface, which indicates the uneven stressed state of the material. Accounting for the heterogeneity of the stressed state of an alloy is characterized in theory by introducing coordinate functions for consideration or, in this case, the argument functions.

The local problem on loading at the discontinuity base was considered in [18] by using a general approach defined by the state of the medium. The repeated heterogeneity of the stressed state or a change in obvious conditions show the need to use coordinate functions in a solution in combination with periodic dependences. For the case of an argument function method, this represents a combination of basic functions, including a trigonometrical one and the corresponding argument function.

Paper [19] shows that changing the characteristics of external loading leads to a change in the characteristics of internal response under the exponential law. In the method of argument functions, for the case of solving a linear equation in partial derivatives, it is advisable to use a fundamental substitute for a second basic function, which includes the same dependence.

Varying stresses and deformations during loading are the main reasons for a decrease in the strength and durability of articles [20]. That renders relevance to solving applied problems that characterize the stressed state of articles by applying classical equations from the theory of continuum mechanics.

The scientific literature cited above [8, 10, 11] allows one to construct mathematical dependences for the basic variables characterized by argument functions. However, there are the unresolved issues related to determining general results, which could show not the solutions themselves but the conditions for their existence, identifying the invariant ratios between the argument functions themselves, connected to the conditions for the existence of closing solutions.

3. The aim and objectives of the study

The aim of this study is to devise new approaches to solving problems on the theory of elasticity, which are distinguished by a significant variety of boundary conditions, by using argument functions.

To accomplish the aim, the following tasks have been set:

– to confirm the argument function method by solving the problems on the theory of elasticity as an example;

– to solve in an analytical form, using the argument functions, the theoretical and applied problem on the theory of elasticity;

– to identify generalizing dependences that would make it possible to derive conditions for the existence of closing solutions to the problems on the theory of elasticity;

– to test the obtained result using an example of applied problems, and to compare with studies by other authors.

4. Substantiating the argument function method in the theory of elasticity

Known statement of the flat problem from the classical theory of elasticity is used. It includes: two differential equilibrium equations, a deformations continuity equation, boundary conditions for stresses in a trigonometrical form. We introduce unknown argument functions, for the exponential and trigonometrical basic dependences that close the solution to the flat problem. At the same time, the trigonometric and fundamental substitutions must be confirmed in the course of obtaining the ultimate result. Differential ratios between the argument functions demonstrated by the solution to the problem are the Cauchy-Riemann conditions, which occur in the analytical functions of complex variables, that is

$$\theta_x = -A\Phi_y,\ \theta_y = A\Phi_x,$$

where θ_x, θ_y, $A\Phi_y$, $A\Phi_x$ are the particular derivatives from argument functions θ and $A\Phi$ by coordinates.

By using the derivatives, we obtain the elliptical Laplace equations

$$\theta_{xx} + \theta_{yy} = 0,\quad A\Phi_{xx} + A\Phi_{yy} = 0.$$

A kind of invariant connection is established between the physical functions, which underlies mathematical transformations when using the method of complex variables.

A feature of this approach is that the ratios resulting from inference make it possible to close the solution in accordance with the boundary conditions.

The proposed solutions typically begin with the statement of the problem and the formulation of the boundary conditions. For a flat problem, we obtain:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0,\ \ \frac{\partial\tau_{yx}}{\partial x}+\frac{\partial\sigma_y}{\partial y}=0,$$

$$\nabla^2\left(\sigma_x+\sigma_y\right)=\nabla^2\left(2\cdot\sigma_0\right)=0. \tag{1}$$

Boundary conditions for stresses can be brought to form [3]

$$\tau_n=-\frac{\sigma_x-\sigma_y}{2}\cdot\sin 2\varphi+\tau_{xy}\cdot\cos 2\varphi, \tag{2}$$

where σ_0 is the mean normal stress or hydrostatic pressure.

It should be emphasized that the above statement of the elastic problem is acceptable for both the flat-stressed and the flat-deformed state of a material. Analysis of expression (2) shows that, in order to simplify the solution and boundary conditions, a trigonometrical substitution should be used, in the following form:

$$\tau_{xy}=T_i\cdot\sin\left(\text{A}\Phi\right),\ \ \sigma_x-\sigma_y=2\cdot T_i\cdot\cos\left(\text{A}\Phi\right), \tag{3}$$

which must be subsequently confirmed by the solution to the problem and by meeting the boundary conditions. Substituting (3) in (2), we obtain:

$$\tau_n=-T_i\cdot\sin\left(\text{A}\Phi-2\varphi\right), \tag{4}$$

where $\text{T}_i=\text{T}_i(x, y)$ is a function of the coordinates for a deformation site, coinciding in functional terms with the intensity of tangential stresses; A is a constant factor that determines the elastic state of a deformable environment; Φ is a function of coordinates, one of the argument functions introduced for consideration that characterizes contact tangential stresses; φ is the angle of pad's incidence.

Given that the differential equation system is linear, it is possible to use a fundamental substitution in determining the intensity of tangential stresses T_i [21]. It should be noted that paper [21] accepted a linear dependence on coordinates in the exponent. It is proposed to introduce a second argument function θ, that is the exponent, in the form of an arbitrary continuous function of coordinates, whose value at this stage of solution is unknown. Typically, elastic deformations define the stressed state in such a way that the intensity of stresses is a variable quantity. In this regard, it is advisable to introduce into consideration a spatial influence factor associated with the coordinates for a deformation site, that is:

$$T_i=C_\sigma\cdot\exp\left(\pm\theta\right)=C_\sigma\cdot\left[\text{ch}\left(\theta\right)\pm\text{sh}\left(\theta\right)\right]. \tag{5}$$

It should be added that the θ exponent is an unknown dependence and is represented by a second argument function. Taking into consideration comments (3) to (5), we have:

$$\tau_{xy}=C_\sigma\cdot\exp\left(\theta\right)\cdot\sin\left(\text{A}\Phi\right),$$

$$\sigma_x-\sigma_y=2\cdot C_\sigma\cdot\exp\left(\theta\right)\cdot\cos\left(\text{A}\Phi\right). \tag{6}$$

In expressions (6), two basic functions (trigonometric and exponential) and two unknown argument functions (θ and AΦ) are considered, which are largely the defining ones. If there is a mathematical relation between them, for example, a Cauchy-Riemann condition, then there is sufficient certainty to derive an analytical solution and the possibility of establishing the conditions for its existence for the system of equations (1).

The boundary conditions show that the difference between normal stresses (6) should have a specific mathematical notation, confirmed by the solution to the flat problem from the theory of elasticity. According to (1) and (6), taking into consideration the deviator component for normal stresses [22], it is possible to record

$$\sigma_x=-\int\frac{\partial\tau_{xy}}{\partial y}\text{d}x+\sigma_0^{'}+C,\ \ \sigma_y=-\int\frac{\partial\tau_{xy}}{\partial x}\text{d}y+\sigma_0^{'}+C,$$

then

$$\sigma_x-\sigma_y=-\int\frac{\partial\tau_{xy}}{\partial y}\text{d}x+\int\frac{\partial\tau_{xy}}{\partial x}\text{d}y=2\cdot C_\sigma\cdot\exp\left(\theta\right)\cdot\cos\left(\text{A}\Phi\right).$$

This is possible if

$$\int\frac{\partial\tau_{xy}}{\partial y}\text{d}x=-C_\sigma\cdot\exp\left(\theta\right)\cdot\cos\left(\text{A}\Phi\right),$$

$$\int\frac{\partial\tau_{xy}}{\partial x}\text{d}y=C_\sigma\cdot\exp\left(\theta\right)\cdot\cos\left(\text{A}\Phi\right),$$

or

$$\sigma_x=C_\sigma\cdot\exp\left(\theta\right)\cdot\cos\left(\text{A}\Phi\right)+\sigma_0^{'}+f\left(y\right)+C,$$

$$\sigma_y=-C_\sigma\cdot\exp\left(\theta\right)\cdot\cos\left(\text{A}\Phi\right)+\sigma_0^{'}+f\left(y\right)+C. \tag{7}$$

Requirements (3) to (7) are set to the solution to the problem from boundary conditions (2).

Let us consider the mean normal stress that is employed in the deformation continuity equation. This parameter should be given special attention, given its presence in the statement and solution to the problem. Taking into consideration (7), we write

$$\sigma_x^{'}+\sigma_y^{'}=C_\sigma\cdot\exp\left(\theta\right)\cdot\cos\left(\text{A}\Phi\right)+\sigma_0^{'}+C+$$
$$+\left[-C_\sigma\cdot\exp\left(\theta\right)\cdot\cos\left(\text{A}\Phi\right)+\sigma_0^{'}+C\right]=2\sigma_0^{'}+2C.$$

If $2\sigma_0^{'}+2C=2\sigma_0^{'}=0$ or equals a constant, $2\sigma_0$=const, then the deformation continuity equation is identically satisfied. However, these are not the only solutions to the continuity equation. The integrated ratios shown above (7) are of interest.

Similar to Mora's circles, there may be shifts along the abscissa axis of the stressed state towards negative or positive values, due to the mean stress σ_0. For the shift process to be obvious, hydrostatic pressure must be represented in a comparable form to the "core" of the solution, in expressions (6), (7). Establish under what limitations for the argument functions the deformation continuity equation holds.

There is a need to determine conditions for the existence of a solution to the Laplace equation of the following form

$$\nabla^2(\sigma_0)=\nabla^2(C_\sigma\cdot\exp(\theta)\cdot\cos(A\Phi))=0. \quad (8)$$

We shall analyze dependences (6), (7) for the mean normal stress in terms of the problem in statement (1). We obtain σ_0 in the form

$$\sigma_0=C_\sigma\cdot\exp(\theta)\cdot\cos(A\Phi).$$

The last expression makes it possible, in formulae (7), to eliminate uncertainty when finding stresses σ_x and σ_y.

5. Solving the problem from the theory of elasticity using the argument function of a complex variable

Let us return to the differential equilibrium conditions. Given that expressions (6), (7) include exponential dependences, including a complex one, we shall write down a tangential stress via an exponential function in the form:

$$\tau_{xy}=C_\sigma\cdot\frac{\exp(\theta+iA\Phi)-\exp(\theta-iA\Phi)}{2i}. \quad (9)$$

To determine normal stresses, one needs to substitute the expression for tangential stresses (9) in the equilibrium equation. Partial derivatives take the following form

$$\frac{\partial\tau_{xy}}{\partial y}=C_\sigma\cdot\frac{(\theta_y+iA\Phi_y)\cdot\exp(\theta+iA\Phi)-(\theta_y-iA\Phi_y)\cdot\exp(\theta-iA\Phi)}{2i},$$

$$\frac{\partial\tau_{xy}}{\partial x}=C_\sigma\cdot\frac{(\theta_x+iA\Phi_x)\cdot\exp(\theta+iA\Phi)-(\theta_x-iA\Phi_x)\cdot\exp(\theta-iA\Phi)}{2i}.$$

After substituting the derivatives into differential equilibrium equations and separating the variables, we obtain

$$d\sigma_x=-C_\sigma\cdot\frac{(\theta_y+iA\Phi_y)\cdot\exp(\theta+iA\Phi)-(\theta_y-iA\Phi_y)\cdot\exp(\theta-iA\Phi)}{2i}\cdot dx,$$

$$d\sigma_y=-C_\sigma\cdot\frac{(\theta_x+iA\Phi_x)\cdot\exp(\theta+iA\Phi)-(\theta_x-iA\Phi_x)\cdot\exp(\theta-iA\Phi)}{2i}dy.$$

By applying the analyticity condition for bracketed functions($\theta_x=-A\Phi_y$), ($\theta_y=-A\Phi_x$), we obtain the opportunity to move from one integration variable to another. We obtain

$$d\sigma_x=-C_\sigma\cdot\frac{(A\Phi_x-i\theta_x)\cdot\exp(\theta+iA\Phi)-(A\Phi_x+i\theta_x)\cdot\exp(\theta-iA\Phi)}{2i}\cdot dx,$$

$$d\sigma_y=-C_\sigma\cdot\frac{(-A\Phi_y+i\theta_y)\cdot\exp(\theta+iA\Phi)-(-A\Phi_y-i\theta_y)\cdot\exp(\theta-iA\Phi)}{2i}dy.$$

Under such a statement, the integrands are recorded with a single variable. One can show that

$$A\Phi_x-i\theta_x=\frac{\theta_x+iA\Phi}{i},\quad A\Phi_x+i\theta_x=-\frac{\theta_x-iA\Phi_x}{i},$$

$$-A\Phi_y+i\theta_y=-\frac{\theta_y+iA\Phi_y}{i},\quad -A\Phi_y-i\theta_y=\frac{\theta_y-iA\Phi_y}{i}.$$

After substituting the ratios obtained above, integration

$$\sigma_x=C_\sigma\cdot\frac{\exp(\theta+iA\Phi)+\exp(\theta-iA\Phi)}{2}+C,$$

$$\sigma_y=-C_\sigma\cdot\frac{\exp(\theta+iA\Phi)+\exp(\theta-iA\Phi)}{2}+C.$$

Moving on to physical functions, we obtain

$$\sigma_x=C_\sigma\cdot\exp\theta\cdot\cos A\Phi+C,$$

$$\sigma_y=-C_\sigma\cdot\exp\theta\cdot\cos A\Phi+C. \quad (10)$$

If the integration determined not the stresses but the stress deviators $s_x=\sigma_x-\sigma_0$, $s_y=\sigma_y-\sigma_0$, according to [22], then:

$$\sigma_x=C_\sigma\cdot\exp\theta\cdot\cos A\Phi+\sigma_0,$$

$$\sigma_y=-C_\sigma\cdot\exp\theta\cdot\cos A\Phi+\sigma_0, \quad (11)$$

at $\theta_x=-A\Phi_y$, $\theta_y=A\Phi_x$, $\theta_{xx}+\theta_{yy}=0$, $A\Phi_{xx}+A\Phi_{yy}=0$.

It should be emphasized that expressions (11) correspond to expressions (6), which was required when stating the problem. By representing the deviator component in the form, we obtain:

$$s_x=\sigma_x-\sigma_0-f(y),\quad s_y=\sigma_y-\sigma_0-f(x),$$

then

$$\sigma_x=C_\sigma\cdot\exp\theta\cdot\cos A\Phi+\sigma_0+f(y),$$

$$\sigma_y=-C_\sigma\cdot\exp\theta\cdot\cos A\Phi+\sigma_0+f(x). \quad (12)$$

The above Cauchy Riemann conditions for the argument functions completely close the solution to the problem both in terms of boundary conditions (6) and equilibrium equations (1). The unknown functions θ and AΦ, introduced in consideration, are determined from the Laplace equations according to (11), (12), which provides sufficient certainty for their finding. Differential relations

$$\theta_x=-A\Phi_y,\quad \theta_y=A\Phi_x,$$

$$\theta_{xx}+\theta_{yy}=0,\quad A\Phi_{xx}+A\Phi_{yy}=0, \quad (13)$$

are the invariants of argument functions that limit the solution to the problem. By using (13), there is a tool to obtain additional capabilities for analytical and numerical solution. An entire class of argument functions emerges, that is of new dependences that meet the boundary conditions and equations of the system equilibrium (1).

However, the problem is not finalized, as the mean normal stresses that are included in (7), (11), (12) through the deformation continuity condition (8) have not been determined. In this case, the problem is set to determine at which values for argument functions the continuity equation (8) holds. Let us write (8) via a complex variable function

$$\nabla^2\left(C_\sigma\cdot\frac{\exp(\theta+i\mathrm{A}\Phi)+\exp(\theta-i\mathrm{A}\Phi)}{2}\right)=0.$$

We describe the derivatives with respect to coordinates

$$\frac{\partial^2\left[C_\sigma\cdot\frac{\exp(\theta+i\mathrm{A}\Phi)+\exp(\theta-i\mathrm{A}\Phi)}{2}\right]}{\partial x^2}=$$
$$=C_\sigma\frac{\left[(\theta_{xx}+i\mathrm{A}\Phi_{xx})+(\theta_x+i\mathrm{A}\Phi_x)^2\right]\exp(\theta+i\mathrm{A}\Phi)}{2}+$$
$$+\frac{\left[(\theta_{xx}-i\mathrm{A}\Phi_{xx})+(\theta_x-i\mathrm{A}\Phi_x)^2\right]\exp(\theta-i\mathrm{A}\Phi)}{2},$$
$$\frac{\partial^2\left[C_\sigma\cdot\frac{\exp(\theta+i\mathrm{A}\Phi)+\exp(\theta-i\mathrm{A}\Phi)}{2}\right]}{\partial y^2}=$$
$$=C_\sigma\frac{\left[(\theta_{yy}+i\mathrm{A}\Phi_{yy})+(\theta_y+i\mathrm{A}\Phi_y)^2\right]\exp(\theta+i\mathrm{A}\Phi)}{2}+$$
$$+\frac{\left[(\theta_{yy}-i\mathrm{A}\Phi_{yy})+(\theta_y-i\mathrm{A}\Phi_y)^2\right]\exp(\theta-i\mathrm{A}\Phi)}{2}.$$

After substituting the derivatives in the deformation continuity equation and upon contractions, we obtain

$$\begin{aligned}&\exp(\theta+i\mathrm{A}\Phi)\times\\&\times\begin{bmatrix}(\theta_{xx}+\theta_{yy})+(\mathrm{A}\Phi_{xx}+\mathrm{A}\Phi_{yy})\cdot i+\\+(\theta_x+i\mathrm{A}\Phi_x)^2+(\theta_y+i\mathrm{A}\Phi_y)^2\end{bmatrix}+\\&+\exp(\theta-i\mathrm{A}\Phi)\times\\&\times\begin{bmatrix}(\theta_{xx}+\theta_{yy})-(\mathrm{A}\Phi_{xx}+\mathrm{A}\Phi_{yy})\cdot i+\\+(\theta_x-i\mathrm{A}\Phi_x)^2+(\theta_y-i\mathrm{A}\Phi_y)^2\end{bmatrix}=0.\end{aligned}\tag{14}$$

Operators in (14) that are adjacent to the exponents contain the same second derivatives for coordinates and non-linearity. If for some reason the operators are zero, then there is an identity. Let us show it. We shall describe the non-linearities in the operators and regroup them.

$$\begin{aligned}&(\theta_x+i\mathrm{A}\Phi_x)^2+(\theta_y+i\mathrm{A}\Phi_y)^2=\\&=(\theta_x+\mathrm{A}\Phi_y)\cdot(\theta_x-\mathrm{A}\Phi_y)+2i(\theta_x\cdot\mathrm{A}\Phi_x+\theta_y\cdot\mathrm{A}\Phi_y)+\\&+(\theta_y+\mathrm{A}\Phi_x)\cdot(\theta_y-\mathrm{A}\Phi_x),\end{aligned}$$

$$\begin{aligned}&(\theta_x-i\mathrm{A}\Phi_x)^2+(\theta_y-i\mathrm{A}\Phi_y)^2=\\&=(\theta_x+\mathrm{A}\Phi_y)\cdot(\theta_x-\mathrm{A}\Phi_y)-2i(\theta_x\cdot\mathrm{A}\Phi_x+\theta_y\cdot\mathrm{A}\Phi_y)+\\&+(\theta_y+\mathrm{A}\Phi_x)\cdot(\theta_y-\mathrm{A}\Phi_x).\end{aligned}$$

By taking in the products of brackets one to be equal to zero, we move away from non-linearity, then $\theta_x=-\mathrm{A}\Phi_y$, $\theta_y=\mathrm{A}\Phi_x$, which was observed when solving differential equations of equilibrium. The expression for both operators automatically turns into zero

$$\theta_x\cdot\mathrm{A}\Phi_x+\theta_y\cdot\mathrm{A}\Phi_y=-\mathrm{A}\Phi_y\cdot\mathrm{A}\Phi_x+\mathrm{A}\Phi_x\cdot\mathrm{A}\Phi_y=0.$$

The continuity equation (14) is significantly simplified and takes the form

$$\begin{aligned}&\exp(\theta+i\mathrm{A}\Phi)\cdot\left[(\theta_{xx}+\theta_{yy})+(\mathrm{A}\Phi_{xx}+\mathrm{A}\Phi_{yy})\cdot i\right]+\\&+\exp(\theta-i\mathrm{A}\Phi)\cdot\left[(\theta_{xx}+\theta_{yy})-(\mathrm{A}\Phi_{xx}+\mathrm{A}\Phi_{yy})\cdot i\right]=0.\end{aligned}$$

We determine second derivatives from the Cauchy-Riemann conditions, which show that:

$$\theta_{xx}+\theta_{yy}=0,$$

$$\mathrm{A}\Phi_{xx}+\mathrm{A}\Phi_{yy}=0,$$

that is the deformation continuity equation is identically satisfied. Therefore, the solution to the deformation continuity equation at

$$\theta_x=-\mathrm{A}\Phi_y,\ \theta_y=\mathrm{A}\Phi_x,\ \theta_{xx}+\theta_{yy}=0,\ \mathrm{A}\Phi_{xx}+\mathrm{A}\Phi_{yy}=0$$

is

$$\sigma_0=n\cdot C_\sigma\cdot\exp\theta\cdot\cos\mathrm{A}\Phi,\tag{15}$$

where n is any number.

The solution (15) is subject to the same constraints as (11), (12) at the same parameters. It should be emphasized that the solution to the deformation continuity equation allows the presence in the expression of the mean normal stress of two exponents simultaneously with the opposite signs of argument function θ. We show it:

$$\begin{aligned}&\sigma_0=n\cdot C_\sigma\cdot\exp(-\theta)\cdot\cos\mathrm{A}\Phi=\\&=n\cdot C_\sigma\cdot\frac{\exp(-\theta+i\mathrm{A}\Phi)+\exp(-\theta-i\mathrm{A}\Phi)}{2}.\end{aligned}$$

By substituting in the deformation continuity equation, we obtain

$$\begin{aligned}&\exp(-\theta+i\mathrm{A}\Phi)\times\\&\times\begin{bmatrix}-(\theta_{xx}+\theta_{yy})+(\mathrm{A}\Phi_{xx}+\mathrm{A}\Phi_{yy})\cdot i+\\+(\theta_x-i\mathrm{A}\Phi_x)^2+(\theta_y-i\mathrm{A}\Phi_y)^2\end{bmatrix}+\\&+\exp(-\theta-i\mathrm{A}\Phi)\times\\&\times\begin{bmatrix}-(\theta_{xx}+\theta_{yy})-(\mathrm{A}\Phi_{xx}+\mathrm{A}\Phi_{yy})i+\\+(\theta_x+i\mathrm{A}\Phi_x)^2+(\theta_y+i\mathrm{A}\Phi_y)^2\end{bmatrix}=0.\end{aligned}\tag{16}$$

By comparing (14) and (16), we make sure that the operators before the exponents are of the opposite sign in comparison to function θ, but, in terms of solution, they almost have not changed. We group

$$\begin{aligned}&\exp(-\theta+i\mathrm{A}\Phi)\times\\&\times\left[-(\theta_{xx}+\theta_{yy})+(\mathrm{A}\Phi_{xx}+\mathrm{A}\Phi_{yy})\cdot i+(\theta_x+\mathrm{A}\Phi_y)\cdot(\theta_x-\mathrm{A}\Phi_y)-\right.\\&\left.-2i\cdot(\theta_x\mathrm{A}\Phi_x+\theta_y\mathrm{A}\Phi_y)+(\theta_y+\mathrm{A}\Phi_x)\cdot(\theta_y-\mathrm{A}\Phi_x)\right]+\\&+\exp(-\theta-i\mathrm{A}\Phi)\times\\&\times\left[-(\theta_{xx}+\theta_{yy})-(\mathrm{A}\Phi_{xx}+\mathrm{A}\Phi_{yy})\cdot i+(\theta_x+\mathrm{A}\Phi_y)\cdot(\theta_x-\mathrm{A}\Phi_y)+\right.\\&\left.+2i\cdot(\theta_x\mathrm{A}\Phi_x+\theta_x\mathrm{A}\Phi_x)+(\theta_y+\mathrm{A}\Phi_x)\cdot(\theta_y-\mathrm{A}\Phi_x)\right]=0.\end{aligned}$$

By using the Cauchy-Riemann conditions, $\theta_x=\text{A}\Phi_y$, $\theta_y=-\text{A}\Phi_x$, the continuity equation is significantly transformed and takes the form

$$\exp(-\theta+i\text{A}\Phi)\cdot\left[-(\theta_{xx}+\theta_{yy})+(\text{A}\Phi_{xx}+\text{A}\Phi_{yy})\cdot i\right]+$$
$$+\exp(-\theta-i\text{A}\Phi)\cdot\left[-(\theta_{xx}+\theta_{yy})-(\text{A}\Phi_{xx}+\text{A}\Phi_{yy})\cdot i\right]=0.$$

The established differential relation between the argument functions makes it possible to determine second derivatives

$\theta_{xx}=\text{A}\Phi_{yx}$, $\theta_{yy}=-\text{A}\Phi_{xy}$, $\text{A}\Phi_{xx}=\theta_{yx}$, $\text{A}\Phi_{yy}=-\theta_{yy}$.

By substituting derivatives in the continuity equation, we make sure that (14) turns into an identity.

We have stated and solved from a unified position the flat problem on the theory of elasticity, and identified the generalizing ratios (13) determining the conditions for the existence of the assigned class of solutions through the invariants of differential ratios of argument functions.

The result is the notation

$$\sigma_x=\pm C_\sigma\cdot\exp(\pm\theta)\cdot\cos\text{A}\Phi+\sigma_0+f(y)+C,$$
$$\sigma_y=\mp C_\sigma\cdot\exp(\pm\theta)\cdot\cos\text{A}\Phi+\sigma_0+f(x)+C,$$
$$\tau_{xy}=C_\sigma\cdot\exp(\pm\theta)\cdot\sin(\text{A}\Phi),$$
$$\sigma_0=\pm n\cdot C_\sigma\cdot\exp(\pm\theta)\cdot\cos\text{A}\Phi, \qquad (17)$$

at $\theta_x=\mp\text{A}\Phi_y$, $\theta_y=\pm\text{A}\Phi_x$, $\theta_{xx}+\theta_{yy}=0$, $\text{A}\Phi_{xx}+\text{A}\Phi_{yy}=0$.

Analysis reveals that solution (17) can be further strengthened and represented in the form

$$\sigma_x=\pm\exp(\pm\theta)(C_1\cos\text{A}\Phi-C_2\sin\text{A}\Phi)+\sigma_0+f(y)+C=$$
$$=\pm\left[\text{ch}(\pm\theta)\pm\text{sh}(\pm\theta)\right](C_1\cos\text{A}\Phi-C_2\sin\text{A}\Phi)+$$
$$+\sigma_0+f(y)+C,$$

$$\sigma_y=\mp\exp(\pm\theta)(C_1\cos\text{A}\Phi-C_2\sin\text{A}\Phi)+\sigma_0+f(x)+C=$$
$$=\mp\left[\text{ch}(\pm\theta)\pm\text{sh}(\pm\theta)\right](C_1\cos\text{A}\Phi-C_2\sin\text{A}\Phi)+$$
$$+\sigma_0+f(x)+C,$$

$$\tau_{xy}=\exp(\pm\theta)\cdot(C_1\sin\text{A}\Phi+C_2\cos\text{A}\Phi)=$$
$$=\left[\text{ch}(\pm\theta)\pm\text{sh}(\pm\theta)\right](C_1\sin\text{A}\Phi+C_2\cos\text{A}\Phi),$$

$$\sigma_0=\pm n\cdot\exp(\pm\theta)\cdot(C_1\cos\text{A}\Phi\mp C_2\sin\text{A}\Phi)=$$
$$=\pm n\cdot\left[\text{ch}(\pm\theta)\pm\text{sh}(\pm\theta)\right](C_1\cos\text{A}\Phi\mp C_2\sin\text{A}\Phi), \qquad (18)$$

at $\theta_x=\mp\text{A}\Phi_y$, $\theta_y=\pm\text{A}\Phi_x$, $\theta_{xx}+\theta_{yy}=0$, $\text{A}\Phi_{xx}+\text{A}\Phi_{yy}=0$.

As a particular case, expression (18) can be considered as a function of stresses to employ for a comparative analysis. Indeed, a biharmonic equation for a flat problem can be represented

$$\nabla^4\phi=\nabla^2(\nabla^2\phi)=0. \qquad (19)$$

Since $\nabla^2(\sigma_0)=0$, hence

$$\nabla^2\left[\nabla^2(\sigma_0)\right]=\nabla^2[0]=0. \qquad (20)$$

Paper [23] gave the solutions to a flat problem using series. The stress function φ takes the form

$$\phi=\sin(\alpha x)\cdot\begin{bmatrix}C_3\cdot\text{ch}(\alpha y)+C_4\cdot\text{sh}(\alpha y)+\\+C_5\cdot y\cdot\text{ch}(\alpha y)+C_6\cdot y\cdot\text{sh}(\alpha y)\end{bmatrix}. \qquad (21)$$

We shall reduce expressions (18) and (21) to a comparable form, that is $C_5=C_6=0$, $\text{A}\Phi=\alpha x$, $\theta=\alpha y$, $n=1$, $C_1=0$, $C_2=-1$. In (18), a plus sign is chosen before the expression. In this case, expressions (18) and (21) coincide, therefore, for both of them the Cauchy-Riemann conditions and the Laplace equations must hold, those that were obtained from the current solution

$$\theta_x=-\text{A}\Phi_y,\ \ \theta_y=\text{A}\Phi_x,\ \ \theta_{xx}+\theta_{yy}=0,\ \ \text{A}\Phi_{xx}+\text{A}\Phi_{yy}=0.$$

Indeed, $\theta_x=0$, $\text{A}\Phi_y=0$, $\theta_y=\alpha$, $\text{A}\Phi_x=\alpha$. The Cauchy-Riemann conditions for a known solution then also hold, $0=-0$, $\alpha=\alpha$, that is the functions presented in work [23] are tested with respect to (18). This follows from the fact that functions αx and αy are the simplest solution to the Laplace equation, which allows a whole class of harmonic functions in different combinations. It is assumed that the functions are not necessarily linear and may depend on multiple coordinates at the same time. For example, the more complex function of AΦ is the function of second order. Let us consider several options. Trigonometric argument functions are:

$$\text{A}\Phi_1=\text{AA}_6x;\ \text{A}\Phi_2=\text{AA}_6xy;\ \text{A}\Phi_3=\mp\text{AA}_{13}(x^2-y^2). \qquad (22)$$

Second argument functions θ are determined from the Cauchy-Riemann condition, in the form

$$\theta_1=\text{AA}_6y;\ \theta_2=-\frac{1}{2}\text{AA}_6(x^2-y^2);\ \theta_3=\pm\text{AA}_{13}(xy). \qquad (23)$$

Let us check all the functions for harmony, substituting alternately in the Laplace equation:

$$\frac{\partial^2\text{A}\Phi_1}{\partial x^2}+\frac{\partial^2\text{A}\Phi_1}{\partial y^2}=0+0=0;\ \ \frac{\partial^2\text{A}\Phi_2}{\partial x^2}+\frac{\partial^2\text{A}\Phi_2}{\partial y^2}=0+0=0;$$
$$\frac{\partial^2\text{A}\Phi_3}{\partial x^2}+\frac{\partial^2\text{A}\Phi_3}{\partial y^2}=(\text{AA}_{13}-\text{AA}_{13})=0. \qquad (24)$$

Next

$$\frac{\partial^2\theta_1}{\partial x^2}+\frac{\partial^2\theta_1}{\partial y^2}=0+0=0,\ \ \frac{\partial^2\theta_2}{\partial x^2}+\frac{\partial^2\theta_2}{\partial y^2}=-\text{AA}_6+\text{AA}_6=0,$$
$$\frac{\partial^2\theta_3}{\partial x^2}+\frac{\partial^2\theta_3}{\partial y^2}=0+0=0. \qquad (25)$$

All three options satisfy the Cauchy-Riemann conditions and the Laplace equations (22) to (25), as was defined by the solution to the problem. It follows from the last analysis that there can be as many solutions as the defined harmonic functions. Then, in a general form, one can write down:

$$\sigma_x=\pm\sum_{i=1}^{n}\left\{\exp(\pm\theta_i)(C_{i1}\cos\text{A}_i\Phi_i-C_{i2}\sin\text{A}_i\Phi_i)+\sigma_{i0}\right\}=$$
$$=\pm\sum_{i=1}^{n}\left\{\left[\text{ch}(\pm\theta_i)\pm\text{sh}(\pm\theta_i)\right](C_{i1}\cos\text{A}_i\Phi_i-C_{i2}\sin\text{A}_i\Phi_i)+\sigma_{i0}\right\},$$

$$\sigma_y = \mp\sum_{i=1}^{n}\left\{\exp(\pm\theta_i)\left(C_{i1}\cos A_i\Phi_i - C_{i2}\sin A_i\Phi_i\right)+\sigma_{i0}\right\}=$$

$$=\mp\sum_{i=1}^{n}\left\{\left[\operatorname{ch}(\pm\theta_i)\pm\operatorname{sh}(\pm\theta_i)\right]\left(C_{i1}\cos A_i\Phi_i - C_{i2}\sin A_i\Phi_i\right)+\sigma_{i0}\right\},$$

$$\tau_{xy} = \sum_{i=1}^{n}\left\{\exp(\pm\theta_i)\cdot\left(C_{i1}\sin A_i\Phi_i + C_{i2}\cos A_i\Phi_i\right)\right\}=$$

$$=\sum_{i=1}^{n}\left\{\left[\operatorname{ch}(\pm\theta_i)\pm\operatorname{sh}(\pm\theta_i)\right]\left(C_{i1}\sin A_i\Phi_i + C_{i2}\cos A_i\Phi_{ii}\right)\right\}, \quad (26)$$

$$\sigma_{i0} = \pm\sum_{i=1}^{n} n_i\cdot\exp(\pm\theta_i)\cdot\left(C_{i1}\cos A_i\Phi_i \mp C_{i2}\sin A_i\Phi_i\right)=$$

$$=\pm\sum_{i=1}^{n} n_i\cdot\left[\operatorname{ch}(\pm\theta_i)\pm\operatorname{sh}(\pm\theta_i)\right]\left(C_{i1}\cos A_i\Phi_i \mp C_{i2}\sin A_i\Phi_i\right),$$

at

$$\theta_{ix} = \mp A_i\Phi_{iy},\ \theta_{iy} = \pm A_i\Phi_{ix},\ \theta_{ixx}+\theta_{iyy}=0,\ \ A\Phi_{ixx}+A\Phi_{iyy}=0.$$

Different harmonic functions exert different influences on the ultimate result; they can be characterized by different boundary conditions for problems. In contrast to the expressions obtained by the method of separating variables, the argument functions may be non-linear and, at the same time, depend on two coordinate variables.

For analysis, we shall use the simplest variants of argument functions (22), (23). Let us show the impact of their construction on the distribution of contact stresses in the elastic zone. We examine the stressed state of an elastic semi-space under the influence of force P from a massive die of width $2b$ (Fig. 1). It is assumed that there is a continuous distribution of stresses throughout the volume of zone of elastic deformation, which will make it possible, in a contact with a die, to find the law of distribution of tangents and normal stresses.

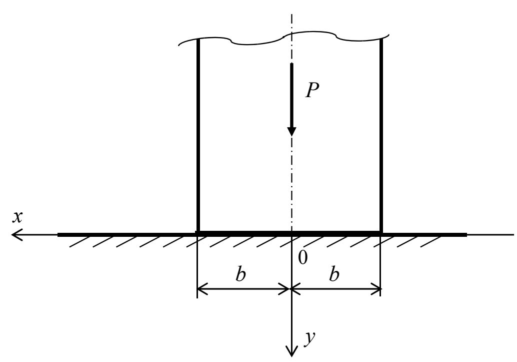

Fig. 1. Effect of a flat die on elastic semi-space

Choose the simplest first variant of expressions (22). By accepting expression (18), considering $C_1=C_\sigma$, $C_2=0$, we obtain differential ratios in the form:

$\theta_x=A\Phi_y$, $\theta_y=-A\Phi_x$, $\theta_{xx}+\theta_{yy}=0$, $A\Phi_{xx}+A\Phi_{yy}=0$.

Stresses take the form:

$$\sigma_x = -\exp(-\theta)C_\sigma\cos A\Phi + \sigma_0,$$

$$\sigma_y = \exp(-\theta)C_\sigma\cos A\Phi + \sigma_0,$$

$$\tau_{xy} = \exp(-\theta)C_\sigma\sin A\Phi,\ \ \sigma_0 = \pm n\cdot\exp(-\theta)\cdot C_\sigma\cos A\Phi.$$

If, in the process of loading, there forms the stressed state of one sign, for instance, compression, then:

$$\sigma_x = -\exp(-\theta)C_\sigma\cos A\Phi + n\cdot\exp(-\theta)\cdot C_\sigma\cos A\Phi,$$

$$\sigma_y = \exp(-\theta)C_\sigma\cos A\Phi + n\cdot\exp(-\theta)\cdot C_\sigma\cos A\Phi,$$

$$\tau_{xy} = \exp(-\theta)C_\sigma\sin A\Phi.$$

In this case, the minimum integer n=2, then:

$$\sigma_x = \exp(-\theta)C_\sigma\cos A\Phi,\ \sigma_y = 3\exp(-\theta)C_\sigma\cos A\Phi,$$

$$\tau_{xy} = \exp(-\theta)C_\sigma\sin A\Phi.$$

If, given the assumptions accepted, it is necessary to obtain stress σ_x=0, then n=1, hence:

$$\sigma_y = 2\exp(-\theta)C_\sigma\cos A\Phi,\ \ \tau_{xy} = \exp(-\theta)C_\sigma\sin A\Phi.$$

Functions AΦ and θ are determined considering the Cauchy-Riemann conditions and the Laplace equations (22) to (25):

$$A\Phi = AA_6x,\ \theta = -AA_6y.$$

Substituting argument functions in the last expressions for stresses, we obtain:

$$\sigma_y = 2\exp(-AA_6y)C_\sigma\cos(AA_6x),$$

$$\tau_{xy} = \exp(-AA_6y)C_\sigma\sin(AA_6x). \quad (27)$$

Analysis of expressions (27) shows that the solution implies the presence of friction in the contact, which changes according to the sinusoid law. If one accepts that in dependences (27) the constant AA_6 equals zero, the heterogeneity of the stressed state disappears. Then

$$\sigma_y = 2\cdot C_\sigma,\ \ \tau_{xy}=0.$$

It follows from the above formulae (27) that the constant AA_6 characterizes the heterogeneity of the stressed state, which in the deformation zone is determined by contact friction. The stress σ_y does not change. The equilibrium equation can yield a connection between constant C_σ and power P. Returning to expressions (27), it is clear that function θ depends only on a single variable y; as it increases the exponent decreases. This characterizes the damping of the impact from the die deep into the semi-space, both for the normal stress and tangential one. Contact normal stress changes according to the cosinusoidal law. It should be noted that the stresses in contact for (27) accept maximum values.

Consider more complex dependences for argument functions in (22):

$$A\Phi_2 = AA_6xy,\ \ \theta_2 = \mp\frac{1}{2}AA_6\left(x^2-y^2\right).$$

By adopting the Cauchy-Riemann conditions in form $\theta_x=A\Phi_y$, $\theta_y=-A\Phi_x$, we shall write an expression for stresses in the following form

$$\sigma_y = 2\exp\left[\frac{1}{2}\cdot AA_6\left(x^2-y^2\right)\right]C_\sigma\cos(AA_6xy),$$

$$\tau_{xy}=\exp\left[\frac{1}{2}\cdot AA_6\left(x^2-y^2\right)\right]C_\sigma\sin\left(AA_6xy\right). \quad (28)$$

In Fig. 1, the coordinates origin is at the contact surface; the tangent stresses in this case are zero, which excludes them from consideration. We consider the problem on the impact exerted by a flat die on an elastic semi-space without taking into consideration contact friction. It should be emphasized that the change in coordinate functions changes the ability to meet boundary conditions in contact, that is a change in the problem. As in the previous case, the constant AA_6 being zero leads to an even distribution of stresses across the elastic deformation zone. However, the value for AA_6 is no longer determined by contact friction, but by other indicators. It is evident that the extreme values of trigonometrical functions are derived not only in contact but also in the depth of a half-space. In this case, expressions (28) are recorded:

$$\sigma_y=2\exp\left[\frac{1}{2}\cdot\frac{\pi}{2by_0}\left(x^2-y^2\right)\right]C_\sigma\cos\left(\frac{\pi}{2by_0}xy\right),$$

$$\tau_{xy}=\exp\left[\frac{1}{2}\cdot\frac{\pi}{2by_0}\left(x^2-y^2\right)\right]C_\sigma\sin\left(\frac{\pi}{2by_0}xy\right), \quad (29)$$

where y_0 is the position of a point deep in a half-plane for the case of an extreme value for the trigonometrical function.

Hence, it follows that the uneven distribution of stresses in the deformation zone is primarily associated with the fading effect of the die on the elastic half-space. If one assigns position in the contact, that is, $y_0=0$, we shall also obtain extreme values for trigonometrical functions. In this case, (29) take the form:

$$\sigma_y=2\exp\left[\frac{1}{2}\cdot\frac{\pi}{2by_0}\left(x^2\right)\right]C_\sigma,\quad \tau_{xy}=0. \quad (30)$$

The lowest stress in contact is obtained at $x=0$. An increase in y_0 decreases the exponent, which indicates a significant impact exerted by this parameter on the heterogeneity of the stressed state.

Fig. 2 show the distribution of normal stresses in contact and in the depth of a half-space. The diagrams are constructed using relative magnitudes $x/2b$ and σ_y/σ_{y0}, where σ_{y0} is the minimum value for normal stresses in the deformation zone.

In contact, the distribution of stresses is determined from expressions (30), the tangent stresses are absent, which is set by the condition to the problem. The dependence of normal stress is defined by a change in the exponent. The stress diagram is concave in shape. As the y coordinate increases, the stresses decrease; they would ultimately equal zero, which corresponds to the damping effect of the die on the elastic half-space. Ultimately, such a damping effect over long distances is reminiscent of the Saint-Venan principle.

It is interesting to compare result (29), (30) to the already known theoretical and experimental data. The proposed variant qualitatively correctly reflects the distribution of stresses, which is confirmed in studies by many authors [6–8]. Indeed, we have obtained the following law of stress distribution under a flat hard die without taking friction into consideration:

$$q(x)=\frac{P}{\pi\sqrt{b^2-x^2}}.$$

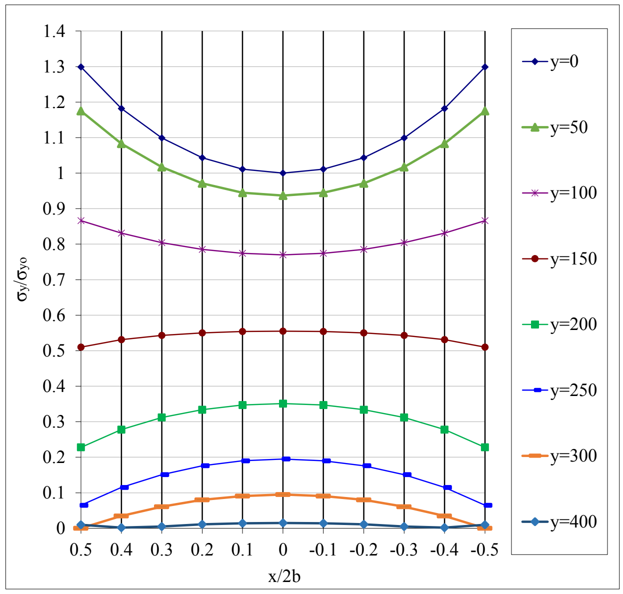

Fig. 2. Distribution of normal stresses in contact and in the depth of a half-space exposed to the action of a flat die without taking friction into consideration

The contact stress diagram is a concave curve, with a minimum, similarly to case (30), at point $x=0$. Hence, it follows that all qualitative characteristics of the stress distribution under a flat smooth die correspond to data from the scientific literature, except for point $x=b$. In this case, $q(x)$ turns into infinity. However, there are papers [26] that show that the stress remains limited at point b. In the current study, such a limitation is the parameter y_0.

6. Discussion of results of studying the stressed-state using the argument functions of a complex variable

The use of the same generalizing approaches in the form of differential ratios and equations in partial derivatives (13) in the theory of plasticity, elasticity, and dynamic problems, makes it possible to extend the method of argument functions to the continuum mechanics.

The solutions considered (18) are a test confirming the validity of the new result.

It is possible to use the result from study (22), (23) to solve problems:

– with different patterns of external power distribution;
– with different schemes of loading an elastic semi-space;
– with different boundary conditions and the stressed state schemes;
– with a complicated geometry of the tool;
– considering friction in contact.

It should be emphasized that the represented solution does not fully exploit the biharmonic equation (19), (20): it is satisfied at an earlier stage.

At the same time, there is a possibility of losing some solutions.

The current study can be advanced if the biharmonic equation is to be completely closed using argument functions.

By summing up, one can note that the introduction of the argument functions into consideration, which make it possible to close the problem and to identify their generalizing characteristics in the form of Cauchy-Riemann conditions and the Laplace equations (13), brings sufficient certainty to obtaining the ultimate result (15).

7. Conclusions

1. We have advanced the method of argument functions when solving various applied problems in the mining industry, metal treatment under pressure, instrumental and manufacturing engineering.

2. The argument functions method has been applied to demonstrate generalized conditions for the existence of solutions to the problems from the theory of elasticity. This expands practical possibilities in the operation of facilities, machinery and assemblies, in the design and construction of equipment for various purposes in metallurgy, machine engineering.

3. We have defined the differential dependences between argument functions in the form of the Cauchy-Riemann conditions, the Laplace equations, which make it possible to close the problem in a general form. There is a possibility to use them to solve different types of differential equations in partial derivatives in mathematics, continuum mechanics. To solve applied problems on the interaction between elastic bodies in the regions of intensive loading for transport engineering, aerospace engineering.

4. The result that is comparable to studies by other authors has been shown, used in geomechanics when loading massive bodies on the ground of a semi-infinite space.

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