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DEVELOPMENT OF A METHOD FOR GEOMETRICAL MODELING OF THE AIRFOIL PROFILE OF AN AXIAL TURBOMACHINE BLADE

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Запропоновано метод геометричного моделювання обводів спинки та коритця профілів лопаток осьових турбомашин, які описуються складеними кривими і формуються двома ділянками. Кожна ділянка обводу профілю моделюється кривою, що подається у натуральній параметризації. Для вхідної частини профілю застосовано кубічний закон розподілу кривини, для вихідної частини профілю – квадратичний закон. Стикування вхідних і вихідних частин профілів спинки і коритця відбувається із забезпеченням третього порядку гладкості, який передбачає рівність значень функцій, похідних від функцій, кривини та похідних від неї в точці стикування. При моделюванні профілю лопатки застосовуються тринадцять кінематичних і геометричних параметрів. Невідомі коефіцієнти квадратичних і кубічних законів розподілу, а також довжини дуг ділянок спинки і коритця профілю, визначаються в процесі моделювання решітки профілів на задані параметри. Задача розв'язується шляхом мінімізації відхилень побудованих кривих від базових точок модельованого профілю, розташованих у горлі міжлопаткового каналу та на колі, яке визначає максимальну товщину профілю.

На підставі запропонованого методу розроблено програмний код, який, окрім цифрової інформації по модельованій середній лінії профілю лопатки турбомашини, також видає отримані результати в графічному вигляді на екран монітора комп'ютера. Проведені розрахункові дослідження підтвердили працездатність запропонованого удосконаленого методу моделювання обводів спинки і коритця профілів лопаток осьових турбін. Розроблений метод може бути корисним організаціям, які займаються проектуванням та виготовленням лопаткових апаратів осьових газових турбін газотурбінних двигунів

Ключові слова: осьова турбіна, профіль лопатки, геометричне моделювання, натуральна параметризація, закони розподілу кривини

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1. Introduction

The problem of saving fuel and energy resources is inextricably linked with creation of highly efficient axial gas turbomachines. These machines have achieved widespread use in structural schemes of avionics, ships, locomotives, and stationary gas-turbine engines (GTE). Modern turbomachines are characterized by a reduction in the number of stages with a simultaneous increase in heat transfer and efficiency. The increase in heat transfer causes subsonic and even supersonic modes of gas flow. When creating axial turbomachines GTE, one of the main tasks facing designers is the design of flowing parts, which are formed by stator and rotor blades.

It is known that efficiency of the axial turbomachine significantly depends on the degree of aerodynamic perfection of blade apparatus. In turn, aerodynamic excellence is largely determined by geometry of blade airfoil profiles. Designing of blade apparatus of axial turbomachines is a labor-intensive process requiring high design proficiency and relevant skills.

Complexity and laboriousness of designing axial turbomachine blade apparatus is explained by versatile and mutually independent requirements of gas dynamics, strength and manufacturability of a structure. Working part (blade airfoil) of blades that interacts with gas must satisfy these requirements. Besides, in order that the designed turbomachine provided specified parameters and had high efficiency, blade apparatus should provide estimated turn and flow acceleration with as little energy loss as possible.

Regarding manufacturability of the blade airfoil, it can be mentioned that introduction of modern high-speed machining centers with programmed numerical control somewhat reduced requirements to geometry of suction and pressure sides of turbine blades. At present, it is possible to manufacture turbomachine blades of very complex geometric shapes, but in turn, this requires development of perfect software products. Therefore, latest manufacturing equipment has made it possible to apply more advanced methods of representing airfoil profiles of turbine blades, develop new ones and improve existing methods of geometric modeling.

The design of axial turbomachines blade apparatus is a complex multi-stage process whose key component consists in construction of blade airfoil profiles and turbine cascades based on them.

In order to improve efficiency of axial turbomachines, measures are taken to change blade geometry such as offset [1] and leading edge configuration [2], giving the blades a bow-shaped form [3] and meridional profile [4].

Studies in this direction are carried out by introducing the mentioned changes of geometric shape of the blade airfoil into the methods used in designing these important components of axial turbomachine flow parts. For example, study [5] proposes a method for improving axial turbomachine blade airfoil profile by solving an optimization problem and constructing cascade of profiles with optimal profiling of blade crown.

All of this determines relevance of carrying out studies directed to development of methods for mathematical description of contours of the suction and pressure sides of the airfoil profile in axial turbomachines which will improve the GTE efficiency.

2. Literature review and problem statement

Various analytical methods of modeling of the suction and pressure sides' of the airfoil profiles are widely used in blade apparatus designing of axial turbomachines. They differ mainly in their underlying mathematical dependences.

Methods of modeling blade airfoil profile cascades of axial turbomachines can be divided into three groups:

1. Selection of airfoil profiles from special atlases containing profile cascades well-developed by experimental and theoretical methods, for example that in [6]. Profile atlases are used in many organizations designing turbomachine flow parts. Such atlases contain aerodynamic characteristics of profiles as a function of certain geometric and operation mode parameters. However, in this case, choice of airfoil profile cascades is limited and level of their efficiency is determined by the design time.

2. Designing profile cascade by flexing a special aerodynamic profile along some camber. Thickness of the sought aerodynamic profile is distributed along the camber. This approach to profile modeling is based on the theory of wing streamlining and is mainly used in compressor engineering although blade profiles in earlier turbines were designed by constructing a camber and distributing along it the thickness variation between the leading to trailing edges of the profile according to certain laws. Despite their historical antiquity, methods of this group are still being used in turbine engineering. For example, convex and concave parabolic lines are used in [7] to construct the camber. At their endpoints, these lines have tangent inclination angles corresponding to the angles of inlet and outlet of the gas flow from the profile cascade. Thicknesses of the modeled profile are distributed along the constructed line. These thicknesses correlate with the radii of leading and trailing edges and should provide necessary angles of sharpening of these edges. It is also proposed to describe the profile camber by a parabolic curve however the parabola degree is not specified [8].

3. Modeling profiles by direct construction of the blade suction and pressure sides, leading and trailing edges. This approach is widely used in profiling turbine cascades whose theory is based on calculation of the working fluid flows in channels. As it is usual in turbine engineering, the blade channel is modeled as a whole but not separate airfoil profiles as in the above approach.

It should be noted that the methods referred to the third group are now widely used in designing blade apparatus both for gas and steam turbines for various purposes. This was largely facilitated by implementation of computer-aided turbine design which in addition to direct calculations has made it possible to display numerical results in a graphical form.

Various curves are widely used for analytical representation of the curves describing the airfoil profile suction and pressure sides: parabolas, hyperbolas, Lame ellipses [9], lemniscate [9, 10], polynomials of various degrees [11, 12], hyperbolic spirals [13], circle arcs, splines [14]. The airfoil profile suction side is modeled independently of the pressure side. Leading and trailing edges of the airfoil profile are described by circle arcs. In general, this is a generally accepted approach to building turbine blade airfoil profiles.

Two methods were proposed in [9] for modeling the suction and the pressure side of a turbine blade profile using Lame ellipses and modified Bernoulli lemniscates. The Lame ellipses are constructed in oblique coordinates with axes drawn through points C_1 and C_2 where tangents touch circles of the leading and trailing edges, respectively. Here, the $\bar{x}$ axis is parallel to the tangent in point C_2 and the $\bar{y}$ axis is parallel to tangent in point C_1. The oblique coordinate center is located at point $\bar{O}_c$ at intersection of straight lines drawn parallel to the corresponding tangents. In the oblique coordinate system $(\bar{x}\bar{O}_c\bar{y})$, segments $\bar{O}_cC_1$ and $\bar{O}_cC_2$ determine magnitudes of the major and minor semi-axes (a_c and b_c) of the Lame ellipse which describes the airfoil profile suction side and has the following equation:

$$\left(\frac{\bar{x}}{a_c}\right)^m+\left(\frac{\bar{y}}{b_c}\right)^n=1.$$

A similar approach is also used when modeling the airfoil profile pressure side.

The second method involves modeling of airfoil profiles of axial turbomachine blades using modified Bernoulli lemniscates:

$$\rho=e\sqrt[m]{\cos(n\varphi)-\cot an\left(\pi-\varphi_{C_1}\right)\sin(n\varphi)},$$

where e is the lemniscate parameter; φ is the polar angle.

Both in the case of the Lame ellipse and the modified lemniscate, exponents m and n are determined in the process of modeling the suction and the pressure side of the profile.

According to [9], geometric angles of flow inlet and outlet, wedging angles of the leading and trailing edges, radii of the edge rounding, stagger angle, profile chord or axial length of the cascade, pitch of profile step length in the cascade, maximum thickness of the airfoil profile and its location, as well as the channel throat and the unguided turning angle are used as initial data in airfoil profile modeling. Therefore, 13 parameters are involved in airfoil profile modeling.

It is proposed in [10] to represent suction and pressure sides of the turbine blade profiles using Bernoulli lemniscates. However, unlike [9], to obtain desired combination of angles at inlet and outlet of the airfoil profile cascade, lemniscate is deformed along the axis of ordinates. Function of the angle at the outlet of the airfoil profile cascade serves as coefficient of deformation. A point at which the angle of tangent inclination corresponds to the geometric angle of the stream inlet was sought in the obtained lemniscate.

Modeling of the suction and the pressure sides of the turbine blade profiles by high-order polynomials is provided in [11]. It should be noted that high-order polynomials are characterized by some problems associated with the so-called oscillation which leads to curve arching which is an undesirable phenomenon. A similar phenomenon occurs in [12] where polynomials of high orders are also used to represent the profile suction and pressure sides.

Modeling of the suction and the pressure sides of profiles of the turbomachine blades by means of hyperbolic helix sections which can be joined in certain circumstances with circle arcs is considered in [13]. Even though the method is algorithmized and a calculation program has been developed according to it, the following can be noted. There are breaks of curvature in the junction points of sections of the applied curves which adversely affects flow of the working medium in blade channels of the profile cascades.

It should be noted that when modeling profiles of axial turbine blades, reference is very often made to study [14] in which 11 geometric parameters are used for analytical representation of contours of the suction and the pressure side of the profile. The suction and the pressure side are analytically described by polynomial curves of the third order and, if necessary, by circle arcs. In these cases, discontinuity in curves of the suction and the pressure side occurs. This is not critical for streamlining of the profile pressure side but discontinuity in the profile suction curves can lead to flow detachment and therefore increase in loss of the working medium power.

In recent years, Bezier curves have become widely used in modeling contours of the turbine blade profiles [15–19]. Bernstein polynomials form mathematical basis for these curves.

The suction and the pressure side of the profile were modeled in [15] with the help of Bezier curves of fourth or fifth order. This required very careful positioning of vertices of the characteristic Bezier polyline which outlined the modeled curve in first approximation. Failure to place vertices of characteristic polyline can result in non-structural profiles.

The study [16] which uses composite Bezier curves with order not exceeding three is consistent with the previous study. It is clear that there is also a problem of location of the characteristic polyline vertices. Construction of curves of the suction and the pressure side by means of compound curves relates to curvature discontinuity at the points of junction of individual sections.

It is proposed in study [17] to delineate profile of the turbine blade by three Bezier curves: one parabola (second-order Bezier curve) for the profile side and two parabolas (second-order Bezier curves) for the pressure side taking into account maximum profile thickness. This study also has disadvantages identified in analysis of studies [15, 16].

Authors of study [18] used the Bezier polylines of the third order in modeling profiles of axial turbomachine blades. Similar curves can be constructed in presence of four vertices of a characteristic polyline. Two vertices coincide with points of junction of the profile suction or the pressure side with circles of leading and trailing edges. In these points, they have angles of tangents inclination determined by corresponding angles of inlet and outlet flows taking into account angles of edge sharpening. Coordinates of these points are easily determined. Two intermediate points are taken in tangents to the leading and trailing edges and they are a fraction of the segments whose lengths are measured from the point of intersection of the tangents to the points of contact with edges of the profiles. This is a significant drawback since it is practically impossible to obtain profiles with a large angle of flow turn.

Profiles of the turbine blades are modeled in [19] using nine vertices of a characteristic polyline separately for the suction and the pressure side resulting in Bezier curves of eighth-order. Algorithm of determining coordinates of intermediate points of the characteristic polyline is not discussed. Analyzing the figure showing the profile with vertices of the characteristic polyline it can be concluded that these vertices are taken spontaneously, especially for the pressure side.

Beside the Bezier curves, rational parametric curves [20] and NURBS curves [21] are used to represent contours of the turbine blade profiles.

It should be noted that the rational parametric curves used in [20] are in fact a separate case of NURBS curves. The approach proposed in [21] for modeling the suction and the pressure side of the blade profile of an axial turbomachine based on NURBS curves also assumes presence of control vertices. In addition, NURBS curves apply weight coefficients that also influence the modeled curve in some way. An optimization problem was solved to minimize the number of control vertices and determine values of the weight coefficients.

It should be noted that Bezier curves, B-splines and NURBS curves have a certain analytical basis, so there is no need to save coordinates of each point in the modeled curve. This allows one to create rather complex curvilinear objects with few control vertices. However, the most appropriate positioning of the controlling vertices is a time-consuming, ambiguous task. Solution of this problem depends on the designer's skill not only in geometry of curves but also in his subject field.

Authors of [22] demonstrated modeling of an axial turbine blade in CATIA V5 software environment. Geometric model of the blade profile was constructed using splines and then extruded to obtain a solid model.

To complete analysis of the published data, it should be noted that all methods of analytical profiling blades of axial turbomachines have inherent advantages and disadvantages limiting their widespread use. Within some of these methods, it is fundamentally impossible to maintain some important parameters of profiles and their cascade or some of the parameters may be obtained with poor accuracy. In those methods involving construction of the profile suction or the pressure side using a compound curve, the issue of ensuring junction of sections of curves of the second order of smoothness and especially the third order of smoothness in the junction point has not even been considered. Most often, developers of the methods for modeling profile contours by means of composite curves in the point of section junction confined themselves to ensuring equality of values of functions and their derivatives corresponding to the first degree of smoothness.

3. The aim and objectives of the study

The study objective was to develop an advanced method of geometric modeling profiles of the suction and the pressure side of axial turbomachine blades by means of a compound curve. Sections of this curve should be presented in a natural parameterization with the use of square and cubic laws of curvature distribution depending on the proper arc length. Junction of sections should occur with smoothness of the third order.

To achieve this objective, the following tasks were set:

– to determine position of reference points and angles of tangent inclination in these points for subsequent drawing of a compound curve through them;

– to construct leading section of the suction of turbomachine blade profile using natural parameterization and the square law of curvature distribution;

– to construct leading section of the side of turbomachine blade profile using the cubic law of curvature distribution and provide smoothness of the third order at the junction with the trailing section of the profile side;

– to construct a pressure side of the blade profile by means of a compound curve using the quadratic and cubic laws of curvature distribution for the trailing and leading sections of the pressure side, respectively, and ensure smoothness of the third order for the jointed sections;

– to implement the proposed method of modeling contours of the axial turbine blade profile in a form of a computer code and ensure visualization of results obtained on a computer screen.

4. Modeling contours of profile of an axial turbomachine blade with the use of curves described in natural parameterization

As in [23], description of curves in natural parameterization will be used in modeling contours of an axial turbomachine blade profile. Curvature k of the curve depending on the proper arc length s is determined by the following dependence:

$$k(s) = \mathrm{d}\varphi/\mathrm{d}s,$$

where φ is the angle of tangent inclination.

Applying provisions of differential geometry, write the expressions for finding angle of tangent inclination and coordinates of points of the modeled curve depending on the length of its arc:

$$\varphi(s) = \varphi(0) + \int_0^s k(s)\mathrm{d}s; \quad (1)$$

$$x(s) = x(0) + \int_0^s \cos\varphi(s)\mathrm{d}s;$$

$$y(s) = y(0) + \int_0^s \sin\varphi(s)\mathrm{d}s, \quad (2)$$

where $\varphi(0)$, $x(0)$, $y(0)$ is the angle of the tangent inclination and coordinates of the starting point of the modeled curve.

Integrals in expressions (2) are not analytically taken but are calculated by numerical integration method.

4. 1. Location of reference points of the axial turbine blade profile

When modeling cascade of turbine profiles, geometric parameters are used. Their content is clear and does not require special explanation (Fig. 1). Note that 13 geometric parameters (linear and angular) are used to model the axial turbomachine blade.

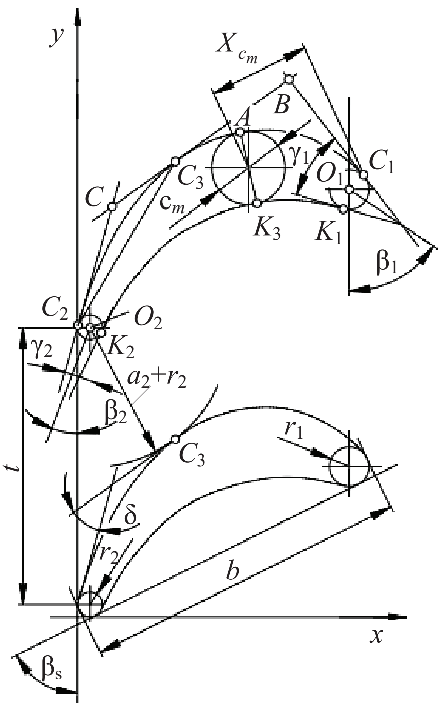

Fig. 1. Profiles of turbine blades and their geometric parameters

Modeling of contours of the axial turbomachine blade profile starts from placement of so-called reference points according to certain conditions. The reference points include centers O_1 and O_2 of circles of leading and trailing edges, points C_1 and K_1 where tangents touch the circle of the leading edge, points C_2 and K_2 where tangents touch the circle of the trailing edge and point C_3 in the channel throat.

Coordinates O_2 and O_1 of centers of circles of the trailing and the input edges are determined based on their location in the chosen coordinate system:

$$x_{O_2} = r_2; \;\; y_{O_2} = r_2;$$

$$x_{O_1} = B - r_1;$$

$$y_{O_1} = y_{O_2} + (r_1 - r_2)/\sin\beta_s + (B - r_1 - r_2)/\tan\beta_s.$$

In these expressions, B means axial length of the profile cascade and β_s means the profile angle measured from the vertical axis.

Coordinates of the reference points C_2, K_2 and C_1, K_1 are expressed by:

$$x_{C_2} = x_{O_2} - r_2\cos\beta_{C_2}; \;\; y_{C_2} = y_{O_2} + r_2\sin\beta_{C_2};$$

$$x_{K_2} = x_{O_2} + r_2\cos\beta_{K_2}; \;\; y_{K_2} = y_{O_2} - r_2\sin\beta_{K_2};$$

$$x_{C_1} = x_{O_1} + r_1\cos\beta_{C_1}; \;\; y_{C_1} = y_{O_1} + r_1\sin\beta_{C_1};$$

$$x_{K_1} = x_{O_1} - r_1\cos\beta_{K_1}; \;\; y_{K_1} = y_{O_1} - r_1\sin\beta_{K_1},$$

where

$$\beta_{C_2} = \beta_2 - \gamma_2/2;$$

$$\beta_{K_2} = \beta_2 + \gamma_2/2;$$

$$\beta_{C_1} = \beta_1 - \gamma_1/2;$$

$$\beta_{K_1} = \beta_1 + \gamma_1/2.$$

Angles β_1 and β_2 are also measured from the vertical axis y.

Coordinates of the reference point C_3 located in the channel throat are determined as follows:

$$x_{C_3} = x_{O_2} + (a_2 + r_2)\cos(\beta_{C_2} + \delta);$$

$$y_{C_3} = y_{O_2} + t - (a_2 + r_2)\sin(\beta_{C_2} + \delta).$$

Locations of reference points of the test example of modeling the nozzle blade profile are shown in Fig. 2.

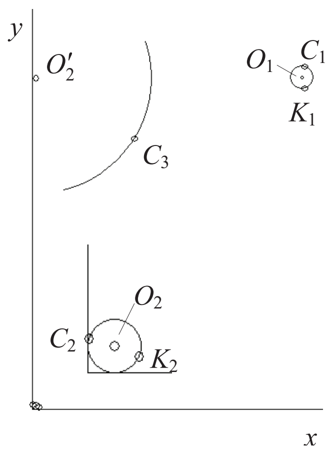

Fig. 2. Reference points of the profile

Because of very small size of the trailing edge of the profile, it is impossible to see the reference points on it, so the trailing edge in Fig. 2 is enlarged. This has made it possible to show locations of the reference points on this edge.

4. 2. Modeling the trailing section of the profile side

Take the following initial data for modeling the first (trailing) section of the side section located between points C_2 and C_3 (Fig. 1): coordinates of points C_2 and C_3 and the angles of tangent inclination in them. These data are sufficient to model a curve by means of the linear law of curvature distribution.

Apply the square law of curvature distribution to model the trailing section of the profile of the turbine blade side taken in the form:

$$k(s) = a_{1c}s^2 + b_{1c}s + c_{1c}, \tag{3}$$

where a_{1c}, b_{1c} and c_{1c} are unknown coefficients to be determined in the process of curve modeling.

Under these circumstances, number of unknowns in equation (3) is exaggerated. Of course, one could take dependence of the curvature without a free term c_{1c}, but this would indicate that the curve of the contour degenerates into a straight line at the point C_2. This is an undesirable phenomenon. So, let us define this coefficient as some inverse portion of radius of the trailing edge circle.

Substitution of equation of the square law of curvature distribution in the form (3) to expression (1) results in a dependence of the angle of tangent inclination to the abscissa axis on the arc length:

$$\varphi(s) = \varphi(0) + \frac{a_{1c}s^3}{3} + \frac{b_{1c}s^2}{2} + c_{1c}s. \tag{4}$$

Since angles of tangents at points C_2 and C_3 are known, the following expression for the coefficient a_{1c} is obtained from dependence (4):

$$a_{1c} = \frac{3}{S_{23}}\left(\frac{\varphi_{C_3} - \varphi_{C_2}}{S_{23}^2} - \frac{b_{1c}}{2} - \frac{c_{1c}}{S_{23}}\right),$$

where S_{23} is length of the arc of the curve constructed between points C_2 and C_3.

Note that angles of tangent inclination in the initial C_2 and end C_3 points of the modeled curve are equal to:

$$\varphi_{C_2} = \pi/2 - \beta_{C_2}; \quad \varphi_{C_3} = \beta_{C_3}.$$

Taking into account the coefficient a_{1c}, expression for distribution of the angle of tangent inclination to the curve depending on the arc length will look like the following:

$$\varphi = \varphi_{C_2} + \frac{s^3}{S_{23}}\left(\frac{\varphi_{C_3} - \varphi_{C_2}}{S_{23}^2} - \frac{b_{1c}}{2} - \frac{c_{1c}}{S_{23}}\right) + \frac{b_{1c}s^2}{2} + c_{1c}s. \tag{5}$$

Substitution of the point C_3 coordinates into expressions (2) and taking into account dependence (5) results in two equations:

$$x_{C_3} = x_{C_2} + \int_0^S \cos\left(\varphi_{C_2} + \frac{s^3}{S_{23}}\left(\frac{\varphi_{C_3} - \varphi_{C_2}}{S_{23}^2} - \frac{b_{1c}}{2} - \frac{c_{1c}}{S_{23}}\right) + \frac{b_{1c}s^2}{2} + c_{1c}s\right)ds;$$

$$y_{C_3} = y_{C_2} + \int_0^S \sin\left(\varphi_{C_2} + \frac{s^3}{S_{23}}\left(\frac{\varphi_{C_3} - \varphi_{C_2}}{S_{23}^2} - \frac{b_{1c}}{2} - \frac{c_{1c}}{S_{23}}\right) + \frac{b_{1c}s^2}{2} + c_{1c}s\right)ds.$$

Their numerical solution determines two unknown values: arc length S_{23} and coefficient b_{1c}.

By arbitrarily specifying values of the arc length S_{23} and the coefficient b_{1c}, coordinates $x(s)$ and $y(s)$ of some point distant from point C_3 by the distance d are obtained as defined by the expression:

$$d = \pm\sqrt{\left(x(s) - x_{C_3}\right)^2 + \left(y(s) - y_{C_3}\right)^2}.$$

Sign in this expression is taken depending on location of the point with coordinates $x(s)$ and $y(s)$ relative to the straight line connecting points C_2 and C_3.

By varying length of the arc S_{23} and the coefficient b_{1c}, their values can be found that will make it possible to determine deviation d with a specified accuracy.

Purposeful character can be imparted to the process of selection of S_{23} and b_{1c} quantities if the expression for determining deviation d is taken as the objective function and one of the methods of minimizing function of several variables is applied to it.

To find a minimum of the function, the algorithm proposed in [24] was applied which makes it possible to minimize function of several variables by a direct search method and does not permit calculation of derivatives of the minimized function. Calculation practice has shown high efficiency of this algorithm in solving the problem of function minimization.

When values of the coefficients of distribution of the curve curvature depending on the length of its own arc are determined, it is possible to calculate coordinates of any point of the modeled curve, curvature of the curve in that point and the angle of tangent inclination.

4. 3. Modeling the leading section of the profile side

Upon completion of modeling the first section of the suction contour, construction of its second part located between points C_3 and C_1 is started. A distinctive feature of modeling this part of the suction contour consists in that distribution of curvature k along the contour length s is represented by parabolic dependence of the third degree:

$$k(s)=a_{2c}s^3+b_{2c}s^2+c_{2c}s+d_{2c}, \qquad (6)$$

where a_{2c}, b_{2c}, c_{2c} and d_{2c} are unknown coefficients to be determined in the process of curve modeling.

In modeling the leading section of the side, it is assumed that the curvilinear coordinate s is zero in point C_3.

Growth of the degree of representation of curvature dependence on the arc length is determined by desire to provide third order of smoothness of the curve junction in point C_3.

It is known that the third order of smoothness of curve junction implies equality of values of functions, function derivatives, curvature and curvature derivative in a common point.

Since curvature of the curve in point C_3 becomes known as a result of construction of the trailing section of the side, a value equal to curvature of the trailing section in the endpoint is given to coefficient d_{2c}, that is $d_{2c}=k_{C_3}$, where:

$$k_{C_3}=a_{1c}S_{23}^2+b_{1c}S_{23}+c_{1c}.$$

Curvature derivative is also known in point C_3, so value of this derivative is given to the coefficient c_{2c}:

$$c_{2c}=2a_{1c}S_{23}+b_{1c}.$$

Substitution of equation of the cubic law of curvature distribution in the form (6) to expression (1) gives dependence of the angle of tangent inclination to the abscissa axis on the arc length:

$$\varphi(s)=\varphi(0)+\frac{a_{2c}s^4}{4}+\frac{b_{2c}s^3}{3}+\frac{c_{2c}s^2}{2}+d_{2c}s. \qquad (7)$$

Since angles of tangent inclination in points C_1 and C_3 are known, then, just as was done above, it is possible to write:

$$\varphi_{C_1}=\varphi_{C_3}+\frac{a_{2c}S_{31}^4}{4}+\frac{b_{2c}S_{31}^3}{3}+\frac{c_{2c}S_{31}^2}{2}+d_{2c}S_{31}$$

and find expression for the coefficient a_{2c}:

$$a_{2c}=\frac{4}{S_{31}^2}\left(\frac{\varphi_{C_1}-\varphi_{C_3}}{S_{31}^2}-\frac{b_{2c}S_{31}}{3}-\frac{c_{2c}}{2}-\frac{d_{2c}}{S_{31}}\right).$$

In these expressions, S_{31} means length of the curve arc between points C_3 and C_1.

Taking into account the a_{2c} coefficient, the dependence that determines distribution of the tangent inclination angle in the leading section of the suction profile will take the form:

$$\varphi=\varphi_{C_3}+\frac{s^4}{S_{31}^2}\left(\frac{\varphi_{C_1}-\varphi_{C_3}}{S_{31}^2}-\frac{b_{2c}S_{31}}{3}-\frac{c_{2c}}{2}-\frac{d_{2c}}{S_{31}}\right)+$$
$$+\frac{b_{2c}s^3}{3}+\frac{c_{2c}s}{2}+d_{2c}s.$$

Substitution of this expression to equations (2) written with respect to the C_1 point, the following dependences are obtained for determining the arc length and the coefficient b_{2c} by a numerical method:

$$x_{C_1}=x_{C_3}+$$
$$+\int_0^S\cos\left\{\begin{aligned}&\varphi_{C_3}+\frac{s^4}{S_{31}^2}\left(\frac{\varphi_{C_1}-\varphi_{C_3}}{S_{31}^2}-\frac{b_{2c}S_{31}}{3}-\frac{c_{2c}}{2}-\frac{d_{2c}}{S_{31}}\right)+\\&+\frac{b_{2c}s^3}{3}+\frac{c_{2c}s}{2}+d_{2c}s\end{aligned}\right\}ds;$$

$$y_{C_1}=y_{C_3}+$$
$$+\int_0^S\sin\left\{\begin{aligned}&\varphi_{C_3}+\frac{s^4}{S_{31}^2}\left(\frac{\varphi_{C_1}-\varphi_{C_3}}{S_{31}^2}-\frac{b_{2c}S_{31}}{3}-\frac{c_{2c}}{2}-\frac{d_{2c}}{S_{31}}\right)+\\&+\frac{b_{2c}s^3}{3}+\frac{c_{2c}s}{2}+d_{2c}s\end{aligned}\right\}ds.$$

The arc length S_{31} between points C_3 and C_1 as well as the coefficient b_{2c}, are determined in the process of solving the problem of minimizing deviation d of the obtained intermediate endpoint of the contour from point C_1.

A test example of geometric modeling of the contour of turbine profile side characteristic of a nozzle blade is shown in Fig. 3. It can be seen in this figure that each of the modeled sections of the profile side is located within a respective triangle. Sides of each triangle are tangents to the side contour at the so-called reference points C_2, C_3 and C_1. The sides of the triangles that are connected to the bases are segments of straight lines connecting the mentioned reference points.

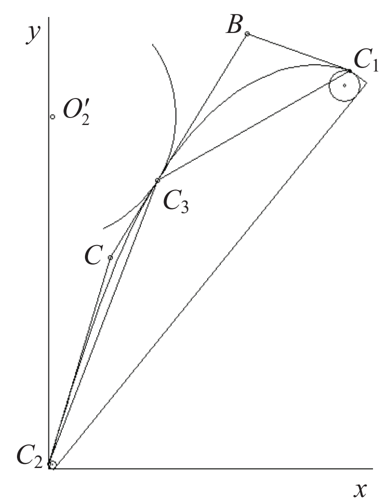

Fig. 3. Result of modeling the suction side of profile

A test example of a nozzle blade profile is shown in Fig. 4 with graphs of distribution of curvature, its derivative and angle β formed between the tangent to the side of the modeled profile and the axis of ordinates. These graphs were plotted depending on the relative length of the curve arc. Circles of small radius show positions of the points in which sections of the profile side are joined. The curves of distribution of the angle β and curvature k are monotone. The curve of distribution of the curvature derivative depending on the arc length is also continuous. There is a break of the curve in the point of junction but there is no curve discontinuity. All this indicates that there is junction of the compound curve sections in a common point with assured smoothness of the third order. This shows the fact that the problem of modeling the profile side was solved.

The final step in modeling the suction side of the profile involves finding by numerical method the point of contact of the circle of radius $c_m/2$ with leading section of the side (point A in Fig. 1) This makes it possible to determine position of the point K_3 (Fig. 5) which is diametrically opposite to the point A and has the same angle of tangent inclination in it.

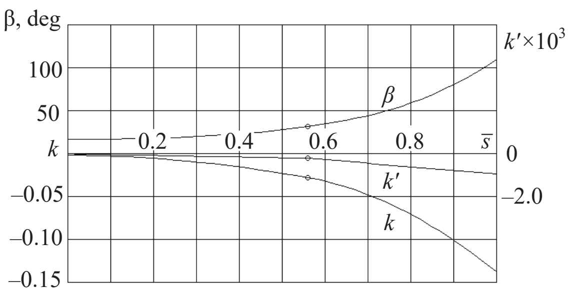

Fig. 4. Distribution of curvature k, its derivative k' and angle β depending on relative curve length for the suction side of the nozzle blade profile

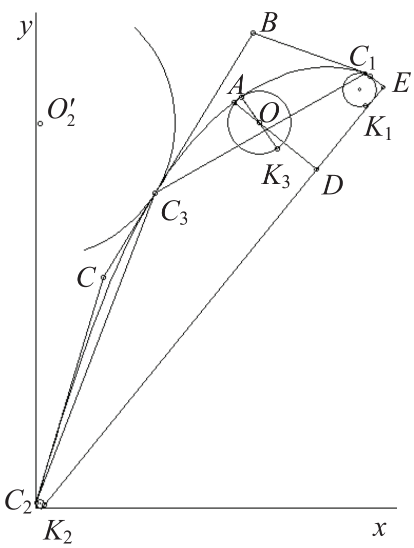

Fig. 5. Reference points of the profile pressure side

Equality of angles of tangent inclination in points A and K_3 ensures achievement of a specified thickness c_m at a distance X_{c_m} from the leading edge of the profile.

4. 4. Modeling the pressure side of the profile

The pressure side of the profile is modeled similarly to the profile suction side. At first, the trailing section K_2K_3 and then the leading section K_3K_1 is built. Angles of the tangent inclination in all reference points of the profile pressure side are known except coordinates. Junction of the sections occurs in point K_3 with assurance of the third order of smoothness.

The trailing section is modeled according to the square law of curvature distribution similar to (3) and the leading section is modeled according to the cubic law of curvature distribution similar to (6). The unknown coefficients of these laws are determined by numerical method. All of the above dependences are true for the trailing and leading sections of the profile pressure side.

Curve of the trailing section is located in the triangle K_2FK_3 (Fig. 6) and that of the leading section in the triangle K_3GK_1. To see a complete picture of modeling the profile of the turbine blade, Fig. 6 also contains the profile suction side saved from the previous construction.

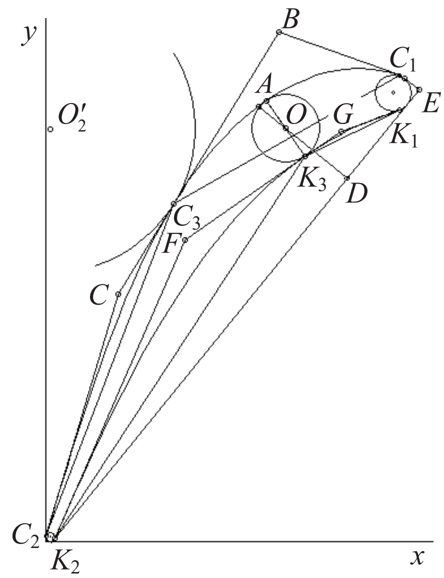

Fig. 6. Result of modeling the profile pressure side

Because of small scale of Fig. 6, the curve of the leading section of the profile side almost coincides with sides of the triangle K_3GK_1. To confirm the fact that there is a curved line in the leading section of the profile between points K_3 and K_1, Fig. 7 shows the leading section of the modeled profile substantially enlarged. The figure clearly shows the polyline between points K_3 and K_1.

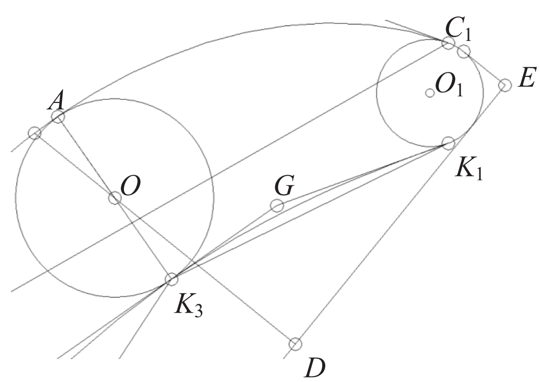

Fig. 7. Leading section of the profile

Just as it was done for the profile side, Fig. 8 shows graphs of distribution of curvature k, curvature derivative k' and angle β depending on relative length of the curve for the profile pressure side.

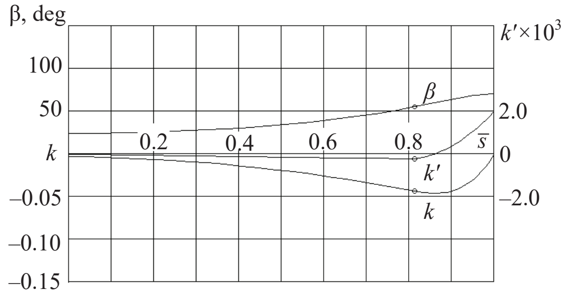

Fig. 8. Distribution of curvature *k*, curvature derivative *k′* and angle β depending on the relative length of curve for the pressure side of the nozzle blade profile

It follows from Fig. 8 that nature of distribution of curvature and its derivative essentially differs from similar curves plotted for the profile side. Both curves show growth after the juncture point. As to the angle β, it can be concluded that it grows with movement to the leading edge but the growth rate decreases somewhat after the junction point.

The results are purely demonstrative. Their purpose was to confirm feasibility of the proposed method of modeling profiles of the axial turbomachine blades.

5. The results of modeling the profile of the axial turbine blade

Based on the proposed method of building axial turbine blade profiles, a software code was developed in the Fortran PowerStation environment that enables calculation and visualization of obtained results on a computer screen.

To further confirm efficiency of the proposed method, a test example of a profile characteristic of the rotor blades was modeled (Fig. 9).

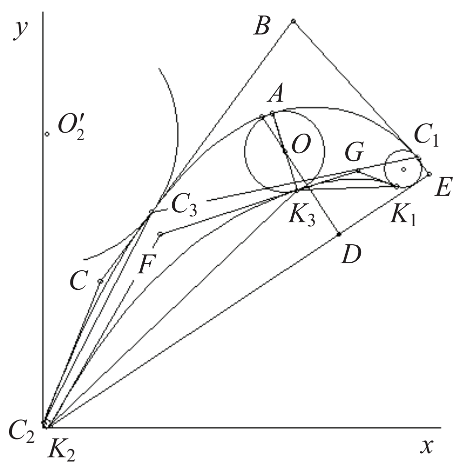

Fig. 9. Results of modeling the rotor blade profile

Fig. 10, 11 show graphs of distribution of curvature *k*, curvature derivative *k′* and angle β respectively for the suction and the pressure side of the profile plotted depending on the relative length of the curve. These data show that conjugation of sections of compound curves occurs in the junction points (indicated by small circles) with smoothness of the third order. Somewhat unusual nature of distribution of the curvature and its derivative is observed on the profile pressure side. To ensure smoothness of the curvature derivative, level of the law of curvature distribution should be raised. However, this is not stipulated by the conditions imposed on the curve junction smoothness of the third order.

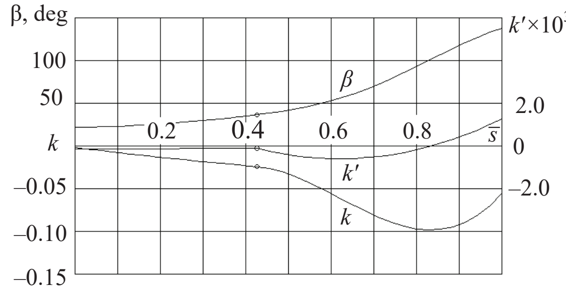

Fig. 10. Distribution of curvature *k*, curvature derivative *k′* and angle β depending on the relative length of the curve for the suction side of the rotor blade profile

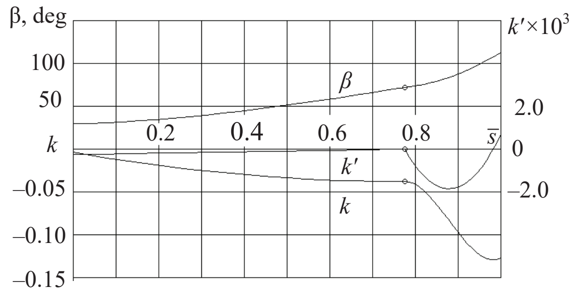

Fig. 11. Distribution of curvature *k*, curvature derivative *k′* and angle β depending on the relative length of the curve for the pressure side of the rotor blade profile

The two channels formed by profiles of the nozzle and rotor blades are shown in Fig. 12.

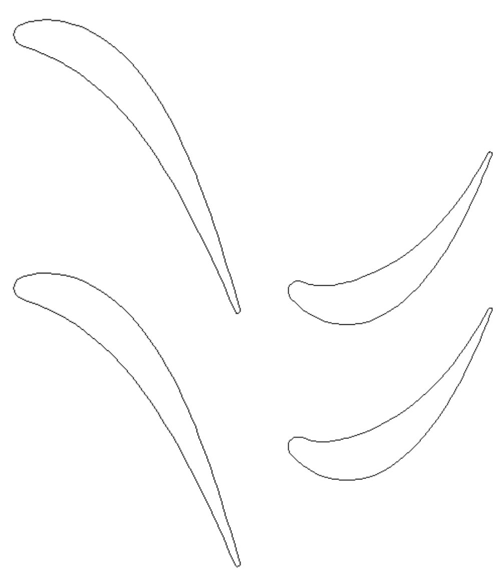

Fig. 12. Profiles of the nozzle and working blades of the turbine stage

The profiles shown in Fig. 12 indicate that all of the above geometric parameters completely satisfy conditions of modeling profiles of the axial turbomachine blades. However, it should be noted that choice of values of geometric parameters must be approached carefully. First of all, this refers to placement of the circle of the profile of maximum thickness and its radius.

It should be noted that Fig. 2–12 are screenshots obtained in running the software developed for modeling profiles of the axial turbomachine blades.

6. Discussion of the method for modeling profiles of axial turbine blades

A new method of geometric modeling contours of the axial turbomachine blade profiles was proposed in this paper. Peculiarity of the method consists in that the modeled contours of the suction and the pressure side are represented by compound curves that pass through corresponding reference points with ensuring specified angles of tangent inclination. Sections of the compound curves of the suction (Fig. 3) and the pressure side (Fig. 6) of the nozzle blade profile are joined with smoothness of the third order which is confirmed by the graphs of distribution of curvature and its derivative (Fig. 4, 8). Similar conclusions can be drawn from consideration of Fig. 9–11 for the rotor blade profile. This is the fundamental difference between the proposed method and the existing solutions for this issue.

Advantage of the proposed method consists in that it enables observing of the graphs of distribution of curvature, its derivative and angle β depending on the length of the proper arc in the process of profile modeling. If an unsatisfactory nature of distribution of these differential characteristics along the arc of the curve contour is obtained, an acceptable result can be achieved by introducing a slight change in the input data except geometric angles of the modeled profiles.

Initial data must be chosen carefully. Practical calculations have shown that even with unsuccessful choice of initial data, no program stops were observed. It is clear that the profile may be nonconstructive in this case. However, this is not a limitation of the proposed method of geometric modeling profiles of axial turbine blades. When determining initial data for their modeling, developers of the blade profile projects use statistical profile parameter dependences built on the basis of well-developed turbine stages.

Further development of the proposed method of modeling profiles of axial turbine blades may be associated with elaboration of measures concerning analysis of geometric perfection of blade channels formed by the modeled profiles of the nozzle and rotor blades of axial turbomachines.

7. Conclusions

1. A method of geometric modeling contours of the suction and the pressure side profiles of flat sections of the axial turbomachine blades in a form of a curve formed of two sections was developed. Each section is described in a natural parameterization applying laws of curvature distribution depending on the length of the proper arc. The modeled sections of curves are joined with assurance of smoothness of the third order.

2. An algorithm for determining positions of reference points through which the modeled curves of the profile suction and the pressure side should pass was developed. Coordinates of the K_3 point were determined by numerical method, provided that position of a circle of maximum radius inscribed in the profile is ensured at a specified distance from the leading edge.

3. The trailing sections of the suction and the pressure side were modeled using a square law of distribution of curvature depending on the length of the curve arc. Since coordinates of two points and the tangent inclination angles are not sufficient to determine unknown coefficients of the linear law of curvature distribution, an additional condition was adopted when applying the square law of curvature distribution. In particular, the free member of this law is taken as some fraction of the inverse value of radius of the trailing edge rounding.

4. Leading sections of the suction and the pressure side were modeled using a cubic law of curvature distribution. The free member of this law was equal to the curvature in the endpoint of the leading section, the penultimate term was equal to the derivative value, also in the endpoint of the leading section. These conditions are sufficient to ensure junction of the suction or the pressure side sections of the profile with smoothness of the third order. Like in the case of the trailing sections, unknown coefficients of the cubic laws of distribution of arc curvature and length were determined by solution of the problem of minimizing deviation of the intermediate endpoints of the sections from the specified reference points of the profile.

5. The method of modeling contours of the suction and the pressure side profile of axial turbomachine blades was implemented as a program code with visualization of the results obtained on a computer screen. The calculations has confirmed operability of the proposed method of geometric modeling the profiles of axial turbomachine blades which was proved by visualization of profiles of the nozzle and rotor blades. Numerical methods used in the program code have enabled obtaining of results with an error less than 10^{-6}. It should be noted that modern manufacturing equipment makes it possible to process turbomachine blades with a minimum tolerance of 0.03–0.05 mm.

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