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FORMING A METODOLOGY FOR TRANSFORMING A MODEL AS THE BASIS FOR EXPANDING ITS INFORMATIVENESS

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Поставлена та розв'язана задача побудови методології перетворення неявної форми моделі, що підвищує ефективність заміни складних нелінійних форм математичних моделей зведенням їх до рекурентної послідовності у вигляді аналітичних виразів, які допускають швидкі експрес обчислення.

Запропоновано нові явні форми моделей, що дозволяють застосовувати рекурентні послідовності для представлення розв'язку задачі і формування виразу оцінки похибки та іншої додаткової інформації. У зв'язку з тим, що утворюючими для багатьох ознак є роз'язок та оцінка похибки, аналітичність виразів відкриває нові властивості і можливості. Грунтуючись на таких факторах як достовірність, точність, глибина, суттєвість та повнота, адекватність моделі представлено єдиним аналітичним виразом, що дозволить у подальшому спрощувати процес порівняння, за рахунок застосування кількісних методів. Представлення перетворень, відповідно до яких встановлено зв'язок між похибкою двох послідовних наближень та залежністю від номеру наближення, зумовлено необхідністю аналізу динаміки збіжності за номером наближення. Другим не менш важливим варіантом, що може характеризувати динаміку збіжності, є зв'язок похибки першого наближення та довільного. На підставі загального розвинення неявної форми моделі та теореми про середнє встановлено зв'язок між двома послідоними похибками або нормами. Продемонстровано, що якщо похибка або норма похибки задана, то оцінки першої та другої похідних дозволять визначати граничний номер ітерації, починаючи з якої похибка менша за задану.

Наведено приклад виводу оцінки для загальної моделі величини максимально можливої похибки, граничного номеру ітерації, починаючи з якої похибка набуває значення менше за задане. Виведено також комплексну аналітичну оцінку адекватності за єдиним виразом.

Представлення інформаційних ознак у кількісній формі обумовлено новими можливостями, що буде утворено за рахунок отриманих інструментів для кількісного аналізу.

Проведено чисельне моделювання та досліджено характер динаміки нових інформаційних показників та ознак. Представлені дані для дев'яти ітерацій демонструють ефективність та повноту інформації. для швидкого аналізу і висновку. На підставі динаміки кількісних ознак відносної похибки та нових, що пропонуються за результатами імплементації методології перетворення моделі, показано, що відкриваються можливості швидкого аналізу і висновку. Продемонстровано, що введені ознаки розширюють інформаційність реалізації методології для подальшого представлення нелінійної моделі у вигляді рекурентної послідовності

Ключові слова: нелінійна вектор-функція, рекурентна модель, інформаційні ознаки, аналітичні вирази, оцінка похибки, гранична ітерація, оцінка адекватності

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1. Introduction

It is generally known that studying an object, as well as structures and processes, implies forming their simplified representation in the form of a model [1]. Various forms of model representation in automated systems are reduced to elementary binary numerical display in code and the defined set of mathematical operations [2]. Despite the formal simplicity of such sets, their totality in most models of real-life processes [3] takes complex linear or non-linear mathematical forms [4]. In the process of working with models, there is a need to quickly calculate them based on the results of parameter measurements in real time [5]. Prediction of characteristics, error estimation, the estimation of error effect and the assessment of risk from an incorrect forecast – this is far from a complete list of tasks [6] that accompany the construction and operation projects of modern industries [7]. This is especially relevant for automated systems [8]. Under these conditions, the choice of a model type based on the information about its properties is one of the effective directions in the development of decision support systems [6–9]. Reducing the time due to the simplicity and analyticity of models, which allows the implementation of rapid computations [6], as well as successful introduction of algorithms that operate with long numbers [10], open the way for the implementation of long numbers. Implementation of the latter would make it possible to represent the model's informational image in the form of a single number with an ordered structure [11]. It is obvious that such an informational description of modeling and coding results [12] must satisfy the condition of completeness. Thus, solving the specified tasks opens us a possibility and a necessity to improve the completeness of a description, to form an informational support based on the totality of attributes [13]. The current success of database and knowledge base creation demonstrates new opportunities and capabilities to meet the proposed requirements to the implementation of the new paradigm of prescribing [14] and to expanding the informativeness of data on a model for effective simulation [15].

The above is a justification for the need to create preconditions for reviewing and searching for methods that could represent complex implicit and nonlinear forms by analytical expressions suitable for rapid computation. A series of informational attributes as additions [15] and the integration of data from a multiservice corporate network with classes of the post-relational DBMS Caché [16], together with the application and development of predicates algebra [3], reveals new possibilities to formalization and coordination control [5]. However, successful execution of coordination control requires the existence of explicit mathematical models and their informational characteristics for structural elements of the system [15]. In this regard, it is a relevant area of research to explore the process of transforming an implicit form of a model to explicit one and to formalize attributes that could quantitatively describe the model and represent it in the forms that allow quantification.

2. Literature review and problem statement

Analysis of the current state of the issue on transforming a model, as well as theoretical substantiation, is impossible without a reference to earlier works that underlie functional analysis. Analysis of metric spaces as the basis for representing the implicit forms of a model and operations within them are the main results of the study that created the basis for a new direction [17]. However, the subsequent search for the relationship among linear operators of varying dimensionalities, as well as reducing them to one-dimensional ones, including within the Banach spaces, was one of the fields of research reported in work [18]. However, the limited practical application stimulates the development of linear approximation methods [19]. It should be noted that despite the successes in examining the operators for applying the Freshet, Gato derivatives, in the development of linear and nonlinear operators, scientists established the inapplicability of a fixed-point method to solving practical model transformation problems [20]. At the same time, success of the Kuhn-Tucker duality theory together with the theory of positive and monotonous operators and the theory of generalized convexity and differential irregularities had substantiated the new goals [21]. Work [22] analyzes and demonstrates the development of indirect methods of mathematical physics and elucidates their feasibility and constraints.

Studies [23, 24] demonstrated a practical appeal to solve nonlinear problems. Given the representation, in [23], of the solution in the form of numerical sequence under conditions of limitations imposed on second derivatives, the convergence of such sequences was proven. Work [24] reported a new approach to piecewise-linear approximations, which was theoretically justified for solving nonlinear algebraic and differential forms. However, the type of the considered operator is limited. The introduction of existing methods by Newton-Kantorovich [19] or quasilinearization by R. Bellman-R. Kalaba [24] for the generalized forms of operators implements the idea of introducing a unified approach to the analysis of nonlinear models [25]. Using a piecewise-linear approximation of the nonlinear operator in the functional space [26] represents a solution to nonlinear equations through a sequence of linear solutions. The effectiveness of this approach was repeatedly demonstrated for problems on finding a root, boundary-value problems, as well as variational problems [24]. However, the applicability of the Newton-Kantorovich method [19] and, consequently, the quasilinearization, was limited by the condition of monotony and strict convexity [26]. In this case, the linear approximation condition comes in contradiction with the requirement for accuracy at each step of the recurrent approximation [27]. In addition, the zero first derivative from the image formed by the operator's effect on the proto-image, makes their application impossible for some types of the operator, and especially for oscillatory ones [28]. The models that are considered in work [29] and which are associated with the analysis of oscillatory temporal ECG series are especially relevant. Simultaneous application of techniques to build and transform a model and form attributes based on the information technology has made it possible to conduct the analysis and diagnose deviations in patients' health at the initial stages of disease development [29]. At the same time, the development of methods of the Laplace integral transformation, as well as others [30], and the cosine and sine transformations by Fourier, Hankel, allowed the representation of integrated differential models as systems of algebraic equations. These transformations substantially simplify models [30]. In recent years, scientists actively explore other types of transformations based on the generalized ateb sinus and cosine functions [31]. Their applicability to the simplification of models, namely the creation of Gabor filters, is effective for the models of information security systems [31]. The result of the synthesis of low noise signals [32] is not less important. In addition, the formation of modulated screening technology results in the simplification of images in models of graphic representation, which at present provides for another application – high-quality printing [33]. However, despite the specified advantages, application of integrated transformations is limited only to cases when the variables in models, the relation among which is represented in the form of differential equations with partial derivatives, could be divided [18]. Of course, this property depends both on the type of an equation and the choice of the type of transformation [30], and is an obstacle, which complicates the use of integrated transformations.

The approach based on piecewise-nonlinear approximation has been proposed and investigated in paper [28]. The authors also demonstrated the advantages of using an apparatus of finite-integrated transformations to solving nonlinear boundary-value problems and representing them in an analytical form, which allows express computation.

In addition, paper [28] investigated convergence for different schemes of linear and quadratic approximation and established a relation between the error assessment of the first and subsequent approximations. The authors examined the influence exerted by an error of natural numbers in the problem. They proposed a recurrent sequence to represent the error as a solution to linear boundary problems. Its convergence was investigated. Such a recurrent approximation linked two contradictory requirements for linearity and convergence and a nonlinear operator over the entire region of their determining. In this case, it was demonstrated that the maximum simplicity and the commonality of the algorithm for simulation problem solving are kept [15]. However, as further analysis shows, the piecewise-linear approximation is unsuitable for problems with a complicated root. This approach was improved by author of work [15] who applied it for the case when the region of values for a first derivative in the determining interval contains zero values, that is the image formed as a result of the operator action has not simple roots or local extrema.

Thus, the preconditions for modeling technological complexes and robotic technical systems at the design stage could be created [5]. There are possibilities for the development of algorithms of functioning based on analytical approximate solutions of the system of partial differential equations at special choice of kernels by methods of finite-integral transformations [28]. At the same time, there is a need for effective assessment of the error in such models and in forming information about the convergence speed of approximations and reducing the complexity of the simulation [6]. An example of a successful integration of models at the stage of design of technical means is work [34]. The authors implemented, in conjunction with the use of transformed models, data compression methods, which made it possible to significantly simplify the algorithm of control over adaptive technological complexes for ecopyrolysis. Equally important is the task on simplifying flock movement algorithms, implemented for unmanned devices based on graphic-analytic models [35]. Important for the design and configuration of such decision-making systems for effective management is to have additional informational features [36] that are developed based on the coordination management paradigm [5].

In terms of the need for transformed models, also significant are the automated geoinformation systems of high-level security for especially important objects [37]. Thus, work [38] successfully employs fast transformations of visual models in the course of dynamic representation of scenes by means of accelerated rotation of complex character images.

The search for methods for transforming an implicit form of models into explicit is accompanied by the simultaneous search for methods of approximate constructing of expressions for the norm of an error of arbitrary kernel in a finite-integrated transformation or for the sine and cosine finite-integral transformation. The task on representing an error in the form of a recurrent sequence becomes relevant due to the need to select a model reasonably. Such a task is especially important for hybrid decision-making systems, since the application of error norms as criteria for deciding on the type of a selected model is a priority when it comes to introducing it to a knowledge base [6].

The works analyzed show that modern methods for exact or approximate solving of nonlinear problems are intended to represent a model as a function of problem parameters and in best cases represent the estimation of an error [18–28]. that is a significant drawback, which complicates the further use of such models. The development of model building theory [1–6] demonstrates the need to supplement a model also with other informational attributes that are necessary for its analysis, selection, and operation [9]. The literature demonstrates [24] that existing types of models are mainly reduced to a nonlinear algebraic equation, a system of nonlinear algebraic equations, nonlinear boundary-value problems, and a system of nonlinear differential equations [24, 28].

The oscillations observed in the operation of objects with ACS and the models that describe them make it impossible to apply the methods by Newton-Kantorovich and the quasilinearization by R. Bellman and R. Kalabi. As shown by author of work [39], the simultaneous application of several schemes of approximations with the subsequent checking of conformity of values obtained at the point of each approximation to the root rules out an error of determining a root. Numerical experiments for the example, given in [24, 39, 40], as well as other cases, in which images formed by an operator contain several local extrema, suggest the impossibility of direct application of linear and quadratic approximation schemes. Application of the Newton-Rafson-Kantorovich method, the quasilinearization [23, 24], Halley [27], is impossible. In this regard, the task on modification and finding new approaches [28, 39, 41] is relevant and requires further research into the ways of transforming non-monotonous models. By using an example of solving a problem on finding the root of oscillating functions [39], through the intelligent process, a promising area of research has been demonstrated, which implements a comparative idea [3]. Thus, works [6, 9] proposed introducing a rule for triggering a three-level comparator, by analogy to the two-level comparative idea [3]. In addition, paper [39] shows that the introduction of three-level comparators and a simultaneous implementation of the ternary calculus system expands the possibilities for descriptions, as well as the functions and operational modes of neural networks and automated control systems [40].

It should be noted, as shown in [40], that the main advantage provided by the analyticity of transformed model representations, regardless of the physical type of an object, is the expansion of a tasks list. Even for such specific processes as the activation of hemostasis [41], the analyticity of expressions for the transformed models analytically determines the conditions for the detachment of photo-acceptor electrons. In work [42], the explicit form and analyticity as a property of the model, obtained as a result of the recurrent representation, makes it possible to predict the conditions under which the probability of detachment would be greatest. Another example that demonstrates the relevance of application of the transformed models to the analytical training of a recurrent network and calibration of feedback sensors is automated systems and interactive simulators or imitators [43].

Thus, the transformation of models that are recorded in the quantitative analytical form, is performed by the methods of piecewise-linear, piecewise-quadratic approximation [24–28] or by methods that use a preliminary decomposition into a Taylor series. The lack of methodology for transforming a model to an analytical expression and constructing such informative analytical expressions–attributes as error, adequacy, iteration number, its creation, is the main unresolved issue. Solving a task on constructing an analytical explicit representation of the model and attributes would enable the selection and application of the model in different processes of analysis and operation. In this case, the main obstacle to such a transformation and subsequent use is the requirement of differentiation. Additional awareness about the object or model avoids, in certain cases, domains with uncertain differentiation. It should be noted that awareness of the behavior of the model, its adequacy in the form of a single analytical expression [44] prior to and after the transformation, has remained insufficiently studied up to now. Thus, additional detailed studying and reconsideration of the processes of transforming the implicit, nonlinear models is an integral precondition and part of the design of a unified model transformation methodology. The formation of explicit forms of the model, represented in the analytical form, together with the representation of expressions of attributes in the analytical form, is the main unresolved task of our time.

3. The aim and objectives of the study

The aim of this study is to improve the efficiency of transforming complex nonlinear forms of mathematical models by reducing them to a recurrent sequence in the form of analytical expressions that make it possible to perform rapid computations. It is expected that using such sequences would help derive new forms for representing additional information about a source model's properties based on analytical expressions.

To accomplish the aim, the following tasks have been set:

– to investigate the applicability of expansion into a recurrent series, and to define the conditions for the development of the model, represented in the implicit form of a nonlinear vector-function, into a recurrent sequence;

– to solve a problem on developing a model, represented in the implicit form, for a nonlinear vector-function into a recurrent sequence, which allows both the numerical and analytical representation;

– to derive estimates of the error norm in a form suitable for rapid computation;

– to give an expression for the express computation of a value for model adequacy;

– to derive an expression for the express computation of the number of a boundary iteration, starting with which an error would be less than the chosen one or the assigned one.

4. Stating and solving a problem on forming a transformation methodology that would ensure an increase in the informativeness of the model

4. 1. Statement of the problem on developing a model, represented in the implicit form of a nonlinear vector-function, into a recurrent sequence

Let is consider a domain in the n-dimensional metric space into which a n-dimensional strategy vector $\bar{X}(t)$ is mapped and for which the following expression for determining a distance is assigned

$$d=\left[\sum_{i=1}^{n}\left(x_{i2}^{2}-x_{i1}^{2}\right)\right]^{1/2} \tag{1}$$

and norms

$$\|N\|=\left[\int_{0}^{1}x_i^2\mathrm{d}x_i\right]^{1/2}. \tag{2}$$

Into this space, the vector-functions of constraints are mapped, which separate in it the regions of possible solutions and split it into half-spaces. Assume that an m- dimensional space is additionally assigned, which maps the m-dimensional vector of operating influences $\bar{Y}(t)$ and in which the identical expressions for determining distance (1) and norm (2) operate. All the components of vectors of strategies and controlling influences are normalized and dimensionless, and their modules vary from zero to unity. Thus, the space that maps the vector of controlling influences is also metric and confined. Assume that the generalized model is nonlinear and is represented in one of the implicit forms:

$$F\left(\bar{X}(t),\bar{Y}\left(\bar{X}(t),t\right),A,\Omega(t)\right)=0, \tag{3}$$

where A is the parameter matrix, $\Omega(t)$ is the function-matrix of disturbances. Assume that the vector function in the left-hand side of equation (3) allows the expansion into a Taylor series. Thus, in the region of defining a task, the conditions for the existence, continuity, and differentiation must be satisfied.

4. 2. Reducing the model, represented in the implicit form of a nonlinear vector function, to a recurrent sequence

Assume the model is represented in the generalized implicit form (3). According to the special properties of form (3), it is possible to choose one of the methods by Newton-Kantorovich, the quasilinearization by Bellman-Kalabi, Halley, which makes it possible to represent the approximated solution. However, as set out in works [23, 24, 27], the presence of not simple roots restricts the use of such approaches. In this regard, let us assume the possibility of existence of not simple roots and the oscillation of the vector function. Under these initial conditions, we expand the vector function, according to the recurrent approximation method, in the form [28]:

$$\begin{aligned}&\bar{F}\left(\bar{X}(t),\bar{Y}\left(\bar{X}(t),t\right),A,\Omega(t)\right)=\\&=\bar{F}\left(\bar{X}(t),\bar{Y}\left(\bar{X}(t),t\right),A,\Omega(t)\right)\Big|_{\Delta x=0}+\\&+\|B\|\Delta\bar{X}(t)+\frac{1}{2}\|C\|\Delta\bar{X}(t),\end{aligned} \tag{4}$$

where $\|B\|$ and $\|C\|$ denote the value for square matrices of nxn dimensionality at expansion point $\bar{X}(t)$:

$$\|B\|=\left\|\nabla_x^T F_k\right\|_{\Delta\bar{x}=0};$$

$$\|C\|=\left\|\sum_{j=1}^{n}\Delta x_j\frac{\partial}{\partial x_j}\nabla_x^T F_k\Big|_{x_i}\right\|_{\Delta\bar{x}=0}. \tag{5}$$

We should note that the overall expression of the line in matrix $\|B\|$ is the value of the transposed gradient at point $\bar{X}_n(t)$ for the n-th approximation. The common element of matrix $\|C\|$ is the value at point $\bar{X}_n(t)$ of the n-th approximation of the sum of products of second mixed derivatives for the component of vector x_j and x_i from each component of the gradient for the k-th component of vector function $\bar{F}_k$ by Δx_j, computed for the n–1-th approximation. Thus, a solution to the modeling problem is formally represented as the solution to problem (4) in the form of a recurrent sequence [28]:

$$\begin{aligned}&\bar{X}_{n+1}=\\&=\bar{X}_n-\left[\|B\|+\frac{1}{2}\|C\|\right]^{-1}\bar{F}\left(\bar{X}(t),\bar{Y}\left(\bar{X}(t),t\right),A,\Omega(t)\right)\Big|_{\Delta x=0}.\end{aligned} \tag{6}$$

Such a notation allows both the analytical and numerical representation, which is inherently limited by the ability to represent the initial approximation of strategy-vector in the form of time functions $\bar{X}_n(t)$. Practical implementation of the solution in form (6) faces the task of constructing matrices $\|B\|$ and $\|C\|$, due to the differentiation in the implicit form of the source nonlinear model through their inversion and multiplication.

4. 3. Construction of an estimate for an error norm in the form suitable for rapid computation

The convergence of sequence (6), which explicitly represents the original implicit model, is evaluated based on the expression of difference between two consecutive approximations derived in development (4):

$$\begin{aligned}&2\bar{\delta}\left(\bar{X}(t),\bar{Y}\left(\bar{X}(t),t\right),A,\Omega(t)\right)=\\&=\left[2\|B\|+\|C\|\right]\left[\Delta X_{n+1}(t)-\Delta X_n(t)\right],\end{aligned}$$

where designations were introduced

$$\begin{aligned}&\bar{\delta}\left(\bar{X}_n(t),\bar{Y}\left(\bar{X}_n(t),t\right),A,\Omega(t)\right)=\\&=\bar{F}\left(\bar{X}_{n+1}(t),\bar{Y}\left(\bar{X}_{n+1}(t),t\right),A,\Omega(t)\right)-\\&-\tilde{F}\left(\bar{X}_n(t),\bar{Y}\left(\bar{X}_n(t),t\right),A,\Omega(t)\right).\end{aligned}$$

Application of the norm determined from expression (2) would make it possible to assess the magnitude of the upper limit and the convergence dynamics of vector of strategies according to changes in the original form (3) and its differential properties. Thus, the estimation of an error norm in the general form, suitable for express computation, will be represented as:

$$\left\|\left[\Delta\bar{X}_{n+1}(t)-\Delta\bar{X}_{n}(t)\right]\right\|\leq\left\|2\left|\bar{\delta}\left(\begin{matrix}\bar{X}(t),\\ \bar{Y}\left(\bar{X}(t),t\right),\\ A,\Omega(t)\end{matrix}\right)\right|\right\|\times$$

$$\times\left[2\|B\|+\|C\|^{-1}\right]_{\min}. \quad (7)$$

Along with the properties of the model itself, the characteristic features defining the influence of magnitude exerted by the standard error of the explicit model are the components of vectors of first and second order gradients. Matrices (5), formed based on the values for their components at points of the n–1-th, n-th, n+1-th approximation, would determine changes in the norm of an error in the interval, which expands the overall awareness of model properties based on the new form of a recurrent sequence.

4. 4. Adequacy of recurrent sequence to the original generalized form of the model

It is known that the process of creating a model is considered to be one of the types of technological processes [44]. Its effectiveness – adequacy – is estimated with respect to the object based on the properties of the created model. At present, it is generally accepted to assess adequacy using basic indicators from the group: authenticity, accuracy and completeness, depth and materiality, simplicity and applicability to a convenient solution to the task on a phenomenon study [45]. The substantiation of the unified method for constructing a complex criterion in the form of a unified expression for a given technology has already been discussed in paper [44].

Thus, if one uses different types of measures to assess the authenticity and to evaluate the norm of deviation due to change in several derivatives, then, under these conditions, the adequacy would be assessed as one that takes into consideration several norms simultaneously:

$$E=\sum_{j=1}^{1+k_{ji\min}}P_{mj}\left\{\left(\frac{\sigma_{mj}}{X^{(mj)}\left(\bar{Y},\ldots\right)_{j\max}}\right)^{-2}\right\}^{0.5}\times$$

$$\times\sum_{m=1}^{N_{m\max}}\left(1+k_{mj\max}\right)\frac{N_{mj\max}}{N}\frac{\left|\dfrac{\partial X^{(j)}\left(\bar{Y},\ldots\right)}{\partial x_i}\Delta x_i\right|}{N_{m\max}\left|X^{(j)}\left(\bar{Y},\ldots\right)\right|_{j\max}}, \quad (8)$$

where m denotes the current number of the norm, which determines accuracy, M indicates the number of norms that are used simultaneously in the construction of a model of the j-th derivative and the i-th factor. Additionally, the notation is introduced:

– $N_{m\max}$ is the total number of factors that affect the physical quantity;

– $N_{mj\max}$ is the number of factors that would be taken into consideration by a mathematical model;

– $X^{(mj)}\left(\bar{Y},\ldots\right)$ is the formalized notation of the model constructed, which is determined after the model is transformed in line with (6) $X^{(mj)}\left(\bar{Y},\ldots\right)_{\max}$;

– $X^{(mj)}\left(\bar{Y},\ldots\right)_{j\max}$ are the largest values for a physical quantity and its derivative, respectively, described by the model;

– σ_j is the standard deviation in the values for the j order derivative of a physical quantity;

– P_j is the confidence probability of the fact that the confidence interval covers the values for an j order derivative of a physical quantity;

– $k_{ji\max}$ is the maximum order of the derivative chosen from the total number of its values for each factor in the list of considered $N_{mji\max}$ as the largest magnitude of

$$k_{ji\max}=\sup\left\{k_{ji},i=\overline{1,N_{\max}}\right\}.$$

4. 5. Constructing an expression for the express computation of the number of the boundary iteration starting from which an error would be less than the chosen one

It should be noted that if the norm of an error is used as a parameter of unconditional execution, its maximum possible error would be determined by a doubtful category. Under these conditions, overall efficiency is significantly increased, as the actual error is always less than the maximum possible. In addition, it should be noted that in this case – when a condition of equality is satisfied regardless of the form of a norm – the confidence probability acquires its greatest value – unity. Given the set task on assessing the effectiveness of transforming the original generalized form of model (3), we shall use representation (4) for the difference between two approximations, taking into consideration the mean value theorem [20]:

$$\bar{F}\left(\bar{X}_{n+1}(t),\bar{Y}\left(\bar{X}_{n+1}(t),t\right),A,\Omega(t)\right)-$$
$$-\bar{F}\left(\bar{X}_{n}(t),\bar{Y}\left(\bar{X}_{n}(t),t\right),A,\Omega(t)\right)=$$
$$=\left[\bar{X}_n(t)-\bar{X}_{n-1}(t)\right]^T\left\{\nabla\left[\|B\|+\frac{1}{2}\|C\|\right]\right\}\Bigg|_{\bar{X}_{n-1}}\times$$
$$\times\left[\bar{X}_n(t)-\bar{X}_{n-1}(t)\right]+\left[\|B\|+\frac{1}{2}\|C\|\right]\Bigg|_{\bar{X}_{n}}\left[\bar{X}_{n+1}(t)-\bar{X}_n(t)\right]. \quad (9)$$

Thus, the standard deviation could be estimated based on values for direct and inverse matrices $\|B\|$ and $\|C\|$ at points of the n–1-th, n-th and n+1-th approximations. If one denotes

$$\sigma_{n+1}=\left[X_{n+1}(t)-X_n(t)\right],$$

it is recorded then in the general form:

$$\sigma_{n+1}=\left\{\begin{matrix}\delta\left(\bar{X}_{n+1}(t),\bar{Y}\left(\bar{X}_{n+1}(t),t\right),A,\Omega(t)\right)-\\ -\left[\bar{X}_n(t)-\bar{X}_{n-1}(t)\right]^T\left\{\nabla\left[\|B\|+\frac{1}{2}\|C\|\right]\right\}\Big|_{\bar{X}_{n-1}}\times\\ \times\left[\bar{X}_n(t)-\bar{X}_{n-1}(t)\right]\end{matrix}\right\}\times$$
$$\times\left\{\left[\|B\|+\frac{1}{2}\|C\|\right]\Bigg|_{\bar{X}_{n}}\right\}^{-1}. \quad (10)$$

It should be noted that for the assigned value of a model error magnitude, which is represented in implicit form (3), following its transformation to explicit form (6), the error takes the form of a vector and is calculated from (10), including using norm (2). Simultaneous application of operations of squaring and the specified integration in the assigned interval and extracting the square root leads to the formation of a magnitude of the mean integral square error:

$$\sigma_{n+1}=\left\{\begin{array}{l}\bar{\delta}\left(\bar{X}_{n+1}(t),\bar{Y}\left(\bar{X}_{n+1}(t),t\right),A,\Omega(t)\right)-\\-\left[\bar{X}_{n}(t)-\bar{X}_{n-1}(t)\right]^{T}\times\times\left\{\nabla\left[\|B\|+\frac{1}{2}\|C\|\right]\right\}\Big|_{\bar{X}_{n-1}}\times\\\times\left[\bar{X}_{n}(t)-\bar{X}_{n-1}(t)\right]\end{array}\right\}\times$$
$$\times\left[\begin{array}{l}\bar{F}\left(\bar{X}_{n}(t),\bar{Y}\left(\bar{X}_{n}(t),t\right),A,\Omega(t)\right)-\\-\bar{F}\left(\bar{X}_{n-1}(t),\bar{Y}\left(\bar{X}_{n-1}(t),t\right),A,\Omega(t)\right)\end{array}\right]^{-1}\sigma_{n}. \qquad (11)$$

Such a representation of the error of two consecutive approximations should be recorded via an error of first approximation and the number of approximation:

$$\sigma_{n+1}=\left\{\begin{array}{l}\bar{\delta}\left(\bar{X}_{n+1}(t),\bar{Y}\left(\bar{X}_{n+1}(t),t\right),A,\Omega(t)\right)-\\-\left[\bar{X}_{n}(t)-\bar{X}_{n-1}(t)\right]^{T}\left\{\nabla\left[\|B\|+\frac{1}{2}\|C\|\right]\right\}\Big|_{\bar{X}_{n-1}}\times\\\times\left[\bar{X}_{n}(t)-\bar{X}_{n-1}(t)\right]\end{array}\right\}\times$$
$$\times\left[\begin{array}{l}\bar{F}\left(\bar{X}_{n}(t),\bar{Y}\left(\bar{X}_{n}(t),t\right),A,\Omega(t)\right)-\\-\bar{F}\left(\bar{X}_{n-1}(t),\bar{Y}\left(\bar{X}_{n-1}(t),t\right),A,\Omega(t)\right)\end{array}\right]^{-1}\sigma_{1}^{n}, \qquad (12)$$

which makes it possible to calculate the number of approximation, starting with which the error is less than the assigned one. Thus, the iteration number is computed from expression:

$$n=\frac{\ln\sigma_{n+1}-\ln\bar{\delta}_{n}^{-1}-\ln\left\{\begin{array}{l}\bar{\delta}\left(\bar{X}_{n+1}(t),\bar{Y}\left(\bar{X}_{n+1}(t),t\right),A,\Omega(t)\right)-\\-\left[\bar{X}_{n}(t)-\bar{X}_{n-1}(t)\right]^{T}\left\{\nabla\left[\|B\|+\frac{1}{2}\|C\|\right]\right\}\Big|_{\bar{X}_{n-1}}\times\\\times\left[\bar{X}_{n}(t)-\bar{X}_{n-1}(t)\right]\end{array}\right\}}{\ln\sigma_{1}}. \qquad (13)$$

Thus, one more of the components of attributes has been additionally determined, which describes properties of the transformed model. The specified totality of properties gives a fuller idea of behavior and represents it in the form of a sequence that, if desired, takes the analytical form. In addition, it gives, by the unified expressions, information about its properties, such as the estimation of error, adequacy, the number of iteration, starting with which the error would be less than the assigned one.

5. Convergence modelling and analysis of results for individual cases of electrical process models

In recent years, modeling the dynamics of direct-current contactless electric motors has become critically important for such technological sectors as manipulators and drives for various purposes in robotic-technical systems, drones, quadro-copters [45]. Given this, let us choose for a numerical experiment a model of electrical processes in the coils of direct current contactless electric motors [46]. Particular relevance of this study is due to the need to build force physiotherapy devices [40] and robots that design mobile physiotherapy complexes of early diagnosis, prevention and treatment of spine diseases [41]. Such a model that considers the dynamic component of inductance takes the form:

$$u-L\frac{\mathrm{d}i}{\mathrm{d}t}-\left(R+\mu_{0}n^{2}Sl\frac{\mathrm{d}\mu}{\mathrm{d}i}\right)i-K_{w}\frac{\mathrm{d}\alpha}{\mathrm{d}t}=0, \qquad (14)$$

the following designations are introduced: u – voltage drop on electric motor's winding; i – amperage; R, L, n, S, l – resistance, inductance, number of turns per unit length, cross-sectional area, winding length, respectively. In addition, μ, K_w, $\frac{\mathrm{d}\alpha}{\mathrm{d}t}$ – relative magnetic permeability of the substance of the core, the coefficient of all-inductance and angular velocity of the motor shaft, respectively. Table 1 gives simulation results for a brushless DC motor.

Table 1

Dependences of approximations of amperage and additional informational parameters of nonlinear electrical processes

No.	i_3, A	i_4, A	i_5, A	i_6, A	i_7, A	ε_6	E_6	E_7
1	0.25051	0.25142	0.25120	0.25125	0.25124	0.00088	2,069859	263,5081
2	0.25205	0.25353	0.25317	0.25326	0.25324	0.00142	1,278819	157,8831
3	0.25434	0.25549	0.25521	0.25528	0.25526	0.00112	1,620353	194,9365
4	0.25811	0.25709	0.25736	0.25729	0.25731	0.00104	1,752651	207,4221
5	0.26442	0.25783	0.25972	0.25926	0.25939	0.00731	248.8591	29,48255
6	0.27468	0.25650	0.26249	0.26113	0.26150	0.02282	79.66422	9,781877
7	0.29056	0.25004	0.26571	0.26284	0.26368	0.05899	30.82263	4,309477
8	0.31368	0.23051	0.26798	0.26475	0.26583	0.13982	13.00380	3,368113
9	0.34477	0.17556	0.25856	0.27211	0.26638	0.32101	5.663956	633.8321
10	0.38232	0.00582	0.17167	0.33048	0.21359	0.96608	1.882019	31.11004
11	12.5016	22.2599	22.0522	23.1925	32.6321	0.00942	193.0591	0.385225
12	7.93713	7.16059	8.14159	12.1624	11.4224	0.12049	15.08957	4.913611
13	7.86429	7.09902	8.08080	12.0782	11.3323	0.12150	14.96508	4.874903
14	54.4919	60.4871	57.8263	61.7019	57.1048	0.04601	39.51367	0.791000
15	37.5	37.5	37.5	37.5	37.5	2.6E-12	6.82E+11	3.6E+10
16	37.5	37.5	37.5	37.5	37.5	2.6E-12	6.81E+11	3.6E+10

The modelling is made for the set of parameters:

$$K_w = 0.04\ \text{Vs/rad};\ \ \frac{d\alpha}{dt} = 300\ \text{rad/s};\ \ R = 0.4\ \Omega;$$

$$L = 0.004\ \text{H};\ \ u = 27\ \text{V};\ \ n = 5\cdot 10^4\ \text{turns/m};\ \ Sl = 6\cdot 10^6\ \text{m}^3$$

and the law of change in the relative magnetic permeability of the core's substance:

$$\mu = \mu_{sat} - \left(\mu_{sat} - \mu_{min}\right)e^{-f(ni)},$$

whose properties could be obtained by means of data approximation using methods that are outlined, for example, in works [47, 48]. In these conditions, the magnitude of amperage is represented as a recurrent sequence in the analytical form:

$$i_{n+1} = i_n + \frac{\left[u - L\frac{di}{dt} - \left(R + \mu_0 n^2 Sl\frac{d\mu}{di}\right)i - K_w\frac{d\alpha}{dt}\right]\Bigg|_{i_n} - L\Delta\frac{di}{dt}}{\left(R + \mu_0 n^2 Sl\frac{d\mu}{di}\right) + \mu_0 n^2 Sl\frac{d^2\mu}{di^2}i_n + \left(2\mu_0 n^2 Sl\frac{d^2\mu}{di^2} + \mu_0 n^2 Sl\frac{d^3\mu}{2di^3}i_n\right)\Delta i_n}. \quad (15)$$

Data on the simulation results, in the form of dependences of amperage on relative error ε and adequacy E_n are given in Table 1. For four approximations of the amperage magnitude, which were calculated from recurrent expression (15) as time functions, are given in columns 2–6. Columns 7 and 8 give values of relative error and adequacy for the sixth approximation. Column 9 gives values of adequacy for the seventh approximation. Analysis of the approximation character shows that despite some fluctuations in values between approximations, their amplitude decreases rapidly. Typically, the necessary accuracy is achieved over five to six approximations. However, for individual values, this method requires an increase in the number of approximations. Thus, experiments, for example, lines 8–9 and 12– 3, require an increase in the number of approximations to eight, or the use of a specialized means to intelligently expand into a Taylor series, as proposed in works [9, 39]. The presence of such an informational attribute as an iteration number, starting with which an error value would be less than the assigned one, calculated from expression (13), substantially complements the informational component of the devised methodology for model transformation. Additional data to Table 1 about the dynamics of informational parameters are given in Table 2.

Data from a numerical experiment, given in Table 2, demonstrate how a growth in the approximation number n affects relative error ε_n and the adequacy of transformed model E_n for cases of worse convergence. The experiment numbers are kept similar to Table 1. An analysis of data shows that over three approximations (from 6 to 9) a relative error reduces to the magnitude less than a per cent, while adequacy increases by two to three orders. The latter, in its sensitivity, render these values the attributes of the indicators, and their totality for their trends of changes only improves the accuracy of the predicted attributes during simulation. In addition, adequacy, despite its comprehensiveness in terms of its definition, is a more sensitive indicator than the usual relative error from two iterations that are sequentially determined. Iteration number n, which is indicated in Tables 1, 2 by the lower index of magnitudes, in its essence defines the labor cost of volume of computation. Given this, the value of the number of iterations (13), determined in advance and connected uniquely to the amount of error without computing, is also an informational attribute. Especially so when it indicates a boundary starting with which the error is less than the assigned one.

6. Discussion of results of studying the formation of the model transformation methodology as the basis for expanding its informativeness

Simultaneous presence of the totality of four magnitudes (analytical expression of the sequence itself (6), expression of error estimation (7), expression of the number of boundary iteration (13), expression of adequacy (8)) increases awareness about model properties. This distinguishes the proposed methodology for representing nonlinear models in the form of a recurrent sequence (6) and additional informational parameters (7) to (8) and (12), (13) for choosing a model from a knowledge base. In addition to the obtained opportunities to represent information on the dependences of an objective function or a strategy vector, not less important is the information space that quantitatively represents a model in the form of an additional set of parameters. Especially important are the possibilities that are opened for implementation of, for example, the method of self-organization of complex systems models [49]. It should be expected that reasoning on the basis of comparisons, as a process of harmonizing the results from deduction analysis based on the proposed set of quantitative information indicators (7), (8) and (12) to (13) in the systems for choosing options and evaluating results [50], would be greatly simplified.

Table 2

Dependence of informational parameters of the transformed model of nonlinear electrical processes

No.	ε_6	E_6	ε_7	E_7	ε_8	E_8	ε_9	E_9
6	0.0228	79.6642	0.001422	1,278.973	0.000369	4,925.5	9.70E-05	18,743.
7	0.0590	30.8226	0.003200	568.1609	0.000831	2,189.0	0.00022	8,221.5
8	0.1398	13.0038	0.004061	447.6739	0.001062	1,711.5	0.00029	6,354.4
9	0.3210	5.66396	0.021538	84.41901	0.006323	287.57	0.00164	1,107.3
10	0.9661	1.88202	0.547243	3.322437	0.208484	8.7210	4.89E-05	37,139
11	0.0094	193.059	0.289273	6.285344	0.006405	283.86	0.03339	54.444
12	0.1205	15.0896	0.064790	28.06251	0.073529	24.727	0.23766	7.650
13	0.1215	14.9651	0.065824	27.62189	0.073736	24.658	0.236753	7.6797
14	0.0460	39.5136	0.080504	22.58493	0.040611	44.771	0.157621	11.536

Together with the development of inductive, training algorithms for complex modelling systems [51], given the quantitative possibilities, obtained in the work, for additional two- [52] or multilevel monitoring [53], the process of modeling based on the presented indicators (7), (8) and (12), (13) would acquire additional informational content.

At last, even for the problems based on the latest achievements in fuzzy logic [54], there emerge additional opportunities for model transformation. In this regard, the success of applying such transformed models, together with the analytic membership functions and transformed operations [55], would acquire new applications in comparison with those that have been already widely used [56]. The increased sensitivity of the integrated criterion of adequacy (8) and the established connection between the properties of an implicit source model and the explicit one with the predefined accuracy (12), as well the presence of an expression about the number of the boundary iteration (13), would create reasonable requirements that could ensure noise resistance in modelling processes [57]. Of course, the main obstacle to further implementation is the presence of regions and points that are not differentiated.

Practical implementation of solution in form (6) is associated with the task on constructing matrices $\|B\|$ and $\|C\|$, due to the differentiation of the implicit form of the original nonlinear model, their inversion and multiplication in the construction of expressions of the transformed one and new attributes (7), (8), (10), and (12), (13). Of course, when forming a methodology of transformation and practical realization of results (15), the differentiation of the nonlinear forms of operators or images formed based on them remained the main obstacle. It is obvious that the search for other approaches whose implementation does not use partial derivatives or the Freshet, Gato derivatives, would open up new opportunities, especially in the optimization problems. The search for ways that could expand the application of the transformation methodology to the analysis non-deterministic, for example, stochastic, models, is also one of the promising directions for further development. The latter is likely to evolve based on the general theory of errors, including random ones.

7. Conclusions

1. The proposed methodology for transforming a nonlinear model reduces the initial implicit nonlinear form to the explicit one in the form of a recurrent sequence by the method of recurrent approximation and allows both the numerical and analytical representation.

2. The analytical expression for a sequence, derived from the model transformation methodology, in the form of a recurrent solution to the problem on the development of the model, makes it possible, owing to analyticity, to form expressions of informational attributes. The presented expressions for error estimation, the number of a boundary iteration, adequacy, improve the informational completeness of model description based on the totality of four comprehensive attributes.

3. Estimations of an error norm, which are obtained for the generalized, nonlinear, implicit model, and applied for contactless direct current electric motors, clearly demonstrate the character of dependence of convergence based on the number of an iteration, as well as their informational effectiveness.

4. The integrated magnitude of adequacy, represented by a single expression, is more sensitive than the commonly applied error from two iterations that are consistently determined.

5. The expression for the number of boundary iteration (13) as the limit starting from which an error is less than the chosen one is the informational attribute that characterizes the rate of approximations.

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Reference

1. Glushkov, V. M. (1974). Vvedenie v ASU. Kyiv: Tehnika, 312.
2. Tolk, A. (2015). Learning Something Right from Models That Are Wrong: Epistemology of Simulation. Simulation Foundations, Methods and Applications, 87–106. doi: https://doi.org/10.1007/978-3-319-15096-3_5
3. Petrov, K. E., Kryuchkovskiy, V. V. (2009). Komparatornaya strukturno-parametricheskaya identifikatsiya modeley skalyarnogo mnogofaktornogo otsenivaniya. Herson: Oldi-plyus, 294.
4. Fisun, M. T. (1987). Avtomatizatsiya protsesov proektirovanya ASUP v sudostroenii. Leningrad: Sudostroenie, 78.
5. Hodakov, V. E., Sokolova, N. A., Kiriychuk, D. L. (2014). O razvitii osnov teorii koordinatsii slozhnyh sistem. Problemy informatsiynykh tekhnolohiy, 2, 12–22.
6. Trunov, A. (2016). Criteria for the evaluation of model's error for a hybrid architecture DSS in the underwater technology ACS. Eastern-European Journal of Enterprise Technologies, 6 (9 (84)), 55–62. doi: https://doi.org/10.15587/1729-4061.2016.85585
7. Kupin, A., Kuznetsov, D., Muzyka, I., Paraniuk, D., Serdiuk, O., Suvorov, O., Dvornikov, V. (2018). The concept of a modular cyberphysical system for the early diagnosis of energy equipment. Eastern-European Journal of Enterprise Technologies, 4 (2 (94)), 71–79. doi: https://doi.org/10.15587/1729-4061.2018.139644
8. Zhuravska, I. (2017). Automatized system of large-size cargo registrations based on devices with limited computing capabilities. Electrical and computer systems, 26, 60–67.doi: https://doi.org/10.15276/eltecs.26.102.2017.7

9. Trunov, A. (2017). Recurrent transformation of the dynamics model for autonomous underwater vehicle in the inertial coordinate system. Eastern-European Journal of Enterprise Technologies, 2 (4 (86)), 39–47. doi: https://doi.org/10.15587/1729-4061.2017.95783
10. Kudin, V., Onyshchenko, A., Onyshchenko, I. (2019). Algorithmizing the methods of basis matrices in the study of balace intersectoral ecological and economic models. Eastern-European Journal of Enterprise Technologies, 3 (4 (99)), 45–55. doi: https://doi.org/10.15587/1729-4061.2019.170516
11. Fisun, M., Smith, W., Trunov, A. (2017). The vector rotor as instrument of image segmentation for sensors of automated system of technological control. 2017 12th International Scientific and Technical Conference on Computer Sciences and Information Technologies (CSIT). doi: https://doi.org/10.1109/stc-csit.2017.8098828
12. Trunov, A., Fisun, M., Malcheniuk, A. (2018). The processing of hyperspectral images as matrix algebra operations. 2018 14th International Conference on Advanced Trends in Radioelecrtronics, Telecommunications and Computer Engineering (TCSET). doi: https://doi.org/10.1109/tcset.2018.8336305
13. Fisun, M. T., Kravets, I. O. Kazmirchuk, P. P., Nikolenko, S. H. (2016). Intelektualnyi analiz danykh. Lviv: Novyi svit-2000, 160.
14. Fradkov, A. L. (2005). Application of cybernetic methods in physics. Uspekhi Fizicheskih Nauk, 175 (2), 113–138. doi: https://doi.org/10.3367/ufnr.0175.200502a.0113
15. Trunov, A. (2016). Realization of the paradigm of prescribed control of a nonlinear object as the problem on maximization of adequacy. Eastern-European Journal of Enterprise Technologies, 4 (4 (82)), 50–58. doi: https://doi.org/10.15587/1729-4061.2016.75674
16. Fisun, M. T., Zhuravska, I. M., Horban, H. V. (2011). Intehratsiya danykh merezhevoho trafiku multiservisnoi korporatyvnoi merezhi z klasamy postreliatsiinoi SKBD Caché. Naukovi pratsi, 173 (161), 105–110.
17. Banakh, S. (1948). Kurs funktsionalnoho analizu (liniyni operatsiyi). Kyiv: Radianska shkola, 216.
18. Kantorovich, L. V., Krylov, V. I. (1962). Priblizhennye metody vysshego analiza. Moscow; Leningrad: Fizmatgiz, 708.
19. Akilov, G. P., Kantorovich, L. V. (1984). Funktsional'niy analiz. Moscow: Nauka, 752.
20. Kollatts, L. (1969). Funktsional'niy analiz i vychislitel'naya matematika. Moscow: Mir, 447.
21. Kolmohorov, A. M., Fomin, S. V. (1974). Elementy teoriyi funktsiy i funktsionalnoho analizu. Kyiv: Vyshcha shkola, 455.
22. Luchka, A. Yu., Luchka, T. F. (1985). Vozniknovenie i razvitie pryamyh metodov matematicheskoy fiziki. Kyiv: Naukova dumka, 239.
23. Bellman, R. (1962). Quasi-linearization and upper and lower bounds for variational problems. Quarterly of Applied Mathematics, 19 (4), 349–350. doi: https://doi.org/10.1090/qam/130585
24. Bellman, R. E., Kalaba, R. E. (1965). Quasilinearization and nonlinear boundary-value problems. Elsiver, 218.
25. Traub, Dzh. (1985). Iteratsionnye metody resheniya uravneniy. Moscow: Mir, 264.
26. Dzyadik, V. K. (1988). Approksimatsionnye metody resheniya differentsial'nyh i integral'nyh uravneniy. Kyiv: Naukova dumka, 304.
27. Halley, E. (1694). A new, exact, and easy method of finding the roots of any equations generally, and that without any previous reduction. Philos. Trans. Roy. Soc. London, 18, 136–145.
28. Trunov, O. M. (1999). Zastosuvannia metodu rekurentnoi aproksymatsiyi do rozviazku neliniynykh zadach. Naukovi pratsi, III, 135–142.
29. Shebanin, V., Atamanyuk, I., Kondratenko, Y., Volosyuk, Y. (2017). Canonical mathematical model and information technology for cardio-vascular diseases diagnostics. 2017 14th International Conference The Experience of Designing and Application of CAD Systems in Microelectronics (CADSM). doi: https://doi.org/10.1109/cadsm.2017.7916170
30. Tranter, K. D. (1956). Integral'nye preobrazovaniya v matematicheskoy fizike. Moscow: Gostehizdat, 204.
31. Dronyuk, I., Nazarkevych, M., Poplavska, Z. (2017). Gabor Filters Generalization Based on Ateb-Functions for Information Security. Man-Machine Interactions 5, 195–206. doi: https://doi.org/10.1007/978-3-319-67792-7_20
32. Dronyuk, I., Nazarkevych, M., Fedevych, O. (2016). Synthesis of Noise-Like Signal Based on Ateb-Functions. Distributed Computer and Communication Networks, 132–140. doi: https://doi.org/10.1007/978-3-319-30843-2_14
33. Dronjuk, I., Nazarkevych, M., Troyan, O. (2016). The Modified Amplitude-Modulated Screening Technology for the High Printing Quality. Computer and Information Sciences, 270–276. doi: https://doi.org/10.1007/978-3-319-47217-1_29
34. Kondratenko, Y. P., Kozlov, O. V. (2016). Mathematical Model of Ecopyrogenesis Reactor with Fuzzy Parametrical Identification. Studies in Fuzziness and Soft Computing, 439–451. doi: https://doi.org/10.1007/978-3-319-32229-2_30
35. Zhuravska, I., Kulakovska, I., Musiyenko, M. (2018). Development of a method for determining the area of operation of unmanned vehicles formation by using the graph theory. Eastern-European Journal of Enterprise Technologies, 2 (3 (92)), 4–12. doi: https://doi.org/10.15587/1729-4061.2018.128745
36. Petrov, E. G. (2014). Koordinatsionnoe upravlenie (menedzhment) protsessami realizatsii resheniy. Problemy informatsiynykh tekhnolohiy, 2, 6–11.
37. Vasiukhin, M. I., Vasyliev, I. V., Lobanchykova, N. M. (2007). Interaktyvna avtomatyzovana heoinformatsiyna systema vysokoho rivnia bezpeky osoblyvo vazhlyvykh obiektiv. Naukovi pratsi Donetskoho natsionalnoho tekhnichnoho universytetu. Seriya: Obchysliuvalna tekhnika ta avtomatyzatsiya, 56–60.
38. Vasyuhin, M. I., Kapshtyk, O. I., Kredentsar, S. M. (2008). Metod uskorennogo povorota slozhnogo simvola pri postroenii dinamicheskoy zritel'noy stseny v aeronavigatsionnyh geoinformatsionnyh sistemah real'nogo vremeni. Vestnik Hersonskogo natsional'nogo tehnicheskogo universiteta, 30, 281–287.
39. Trunov, A. (2016). Recurrent approximation as the tool for expansion of functions and modes of operation of neural network. Eastern-European Journal of Enterprise Technologies, 5 (4 (83)), 41–48. doi: https://doi.org/10.15587/1729-4061.2016.81298

40. Trunov, A., Belikov, A. (2015). Application of recurrent approximation to the synthesis of neural network for control of processes phototherapy. 2015 IEEE 8th International Conference on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications (IDAACS). doi: https://doi.org/10.1109/idaacs.2015.7341389
41. Trunov, A. (2016). Peculiarities of the interaction of electromagnetic waves with bio tissue and tool for early diagnosis, prevention and treatment. 2016 IEEE 36th International Conference on Electronics and Nanotechnology (ELNANO). doi: https://doi.org/10.1109/elnano.2016.7493041
42. Trunov, A. (2017). Theoretical predicting the probability of electron detachment for radical of cell photo acceptor. 2017 IEEE 37th International Conference on Electronics and Nanotechnology (ELNANO). doi: https://doi.org/10.1109/elnano.2017.7939776
43. Trunov, A., Malcheniuk, A. (2018). Recurrent network as a tool for calibration in automated systems and interactive simulators. Eastern-European Journal of Enterprise Technologies, 2 (9 (92)), 54–60. doi: https://doi.org/10.15587/1729-4061.2018.126498
44. Trunov, A. (2015). An adequacy criterion in evaluating the effectiveness of a model design process. Eastern-European Journal of Enterprise Technologies, 1 (4 (73)), 36–41. doi: https://doi.org/10.15587/1729-4061.2015.37204
45. Filaretov, V. F. (2000). Samonastraivayushchiesya sistemy upravleniya manipulyatorami. Vladivostok: izd-vo DVGTU, 304.
46. Trunov, A. (2018). Transformation of operations with fuzzy sets for solving the problems on optimal motion of crewless unmanned vehicles. Eastern-European Journal of Enterprise Technologies, 4 (4 (94)), 43–50. doi: https://doi.org/10.15587/1729-4061.2018.140641
47. Batuner, L. A., Pozin, M. E. (1971). Matematicheskie metody v himicheskoy tehnike. Leningrad: Himiya, 824.
48. Marchuk, G. I. (1977). Metody vychislitel'noy matematiki. Moscow: Nauka, 456.
49. Ivahnenko, A. G. (1981). Induktivnyy metod samoorganizatsii modeley slozhnyh sistem. Kyiv: Naukova dumka, 296.
50. Ivahnenko, A. G. (2005). Obraznoe myshlenie kak soglasovanie rezul'tatov deduktivnogo myshleniya i variantov induktivnogo myshleniya. Upravlyayushchie sistemy i mashiny, 2, 3–7.
51. Madala, H. R. (2019). Inductive learning algorithms for complex systems modeling. CRC Press, 380. doi: https://doi.org/10.1201/9781351073493
52. Ivahnenko, A. G., Savchenko, E. A., Ivahnenko, G. A., Sinyavskiy, V. L. (2007). Problemy induktivnogo dvuhurovnevogo monitoringa slozhnyh protsessov. Upravlyayushchie sistemy i mashiny, 3, 13–21.
53. Krutys, P., Gomolka, Z., Twarog, B., Zeslawska, E. (2019). Synchronization of the vector state estimation methods with unmeasurable coordinates for intelligent water quality monitoring systems in the river. Journal of Hydrology, 572, 352–363. doi: https://doi.org/10.1016/j.jhydrol.2019.02.038
54. Gil-Lafuente, A. M. (2005). Fuzzy Logic In Financial Analysis. Springer. doi: https://doi.org/10.1007/3-540-32368-6
55. Dykhta, L., Kozub, N., Malcheniuk, A., Novosadovskyi, O., Trunov, A., Khomchenko, A. (2018). Construction of the method for building analytical membership functions in order to apply operations of mathematical analysis in the theory of fuzzy sets. Eastern-European Journal of Enterprise Technologies, 5 (4 (95)), 22–29. doi: https://doi.org/10.15587/1729-4061.2018.144193
56. Solesvik, M., Kondratenko, Y., Kondratenko, G., Sidenko, I., Kharchenko, V., Boyarchuk, A. (2017). Fuzzy decision support systems in marine practice. 2017 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE). doi: https://doi.org/10.1109/fuzz-ieee.2017.8015471
57. Ivahnenko, A. G., Stepashko, V. S. (1985). Pomehoustoychivost' modelirovaniya. Kyiv: Naukova dumka, 216.