

*Інтенсифікація сільськогосподарського виробництва в сучасних умовах передбачає застосування надпотужних мобільних технічних засобів, що призводить до підвищення рівнів техногенного впливу на ґрунт і, як наслідок, погіршення його родючості. Тому, найактуальніший запит сучасного аграрного виробництва спрямований на вирішення проблеми покращення експлуатаційних показників роботи колісних ходових систем мобільних технічних засобів. Це спонукає до необхідності аналітичного дослідження процесів деформування ґрунту під еластичними рушійми мобільної колісної техніки сільськогосподарського призначення.*

*Запропонована методика графо-аналітичного поетапного моделювання процесу деформування ґрунту під пневматичними шинами рушіїв мобільної сільськогосподарської техніки з урахуванням змінюваної форми еластичної оболонки шини. Використання відповідних графічних моделей дозволяє поетапно досліджувати процес ущільнення ґрунту у профілі утворюваної ним колії. Встановлено, що в зоні контакту «деформований ґрунт – поверхня еластичного колісного рушія мобільного засобу» найвищий рівень ущільнення спостерігається в шарі ґрунту, який безпосередньо контактує з еластичним рушієм. Глибина переущільненого шару ґрунту на «дні колії» залежить від типорозміру шини колісного рушія і не перевищує значення 0,075 ширини шини. Найвищий рівень переущільнення ґрунту спостерігається в зоні, яка є безпосередньо прилеглою до осі колії. Визначено, що найнебезпечнішою конструкцією пневматичної шини, з точки зору переущільнення ґрунту в колії, є форма еластичної оболонки шини, яка описується кривою овалу Кассіні з чотирма точками перегину. Окреслено характерні особливості рекомендацій щодо визначення експлуатаційних значень робочого тиску у шинах залежно від конкретних фізико-механічних та агротехнологічних властивостей ґрунту і характеру виконуваних технологічних операцій*

*Ключові слова: ущільнення ґрунту, еластичний деформатор, поле сил тиску, нелінійність деформування ґрунту*

# GRAPH-ANALYTICAL OPTIMIZATION OF THE TRANSVERSE VERTICAL CROSS-SECTION OF A CONTACT ZONE BETWEEN SOIL AND AN ELASTIC WHEELED MOVER

**B. Sheludchenko**

PhD, Professor

Scientific-Innovative Institute of Engineering of Agro-Industrial Production and Energy Efficiency  
Staryi Blvd., 7, Zhytomyr, Ukraine, 10008

**E. Šarauskiš**

Full Member of the Lithuanian Academy of Sciences, Professor\*

**G. Golub**

Doctor of Technical Sciences, Professor, Head of Department  
Department of Tractors, Cars and Bioenergosistem\*\*

**S. Kukharets**

Doctor of Technical Sciences, Professor, Head of Department  
Department of Mechanics and Agroecosystems Engineering\*\*\*

E-mail: kikharets@gmail.com

**O. Medvedskyi**

PhD

Department of Processes, Machines and Equipment in Agroengineering\*\*\*

**V. Chuba**

PhD, Assistant Professor

Department of Transport Technologies and Means in AIC\*\*

**A. Zabrodskyi**

Doctoral Student\*

\*Institute of Agricultural Engineering and Safety  
Vytautas Magnus university, Agriculture Academy

Studentų str., 15a, Akademija, Kaunas r., Lithuania, LT – 53362

\*\*National University of Life and

Environmental Sciences of Ukraine

Heroiv Oborony str., 15, Kyiv, Ukraine, 03041

\*\*\*Zhytomyr National Agroecological University

Staryi Blvd., 7, Zhytomyr, Ukraine, 10008

Received date 25.09.2019

Accepted date 01.11.2019

Published date 10.12.2019

Copyright © 2019, B. Sheludchenko, E. Šarauskiš, G. Golub,

S. Kukharets, O. Medvedskyi, V. Chuba, A. Zabrodskyi

This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0>)

## 1. Introduction

The most characteristic feature of intensification of agricultural production under modern conditions is the stable

acceleration of energy saturation of technological processes in plant growing, which predetermines the use of super powerful mobile technical means, whose mass per unit will grow further. This leads to the intensification of anthropogenic

impact on the fertile layers of soil, which reduces their fertility: their natural structural organization is overcompacted, water-air soil regime is disrupted, the physical, chemical, technological properties deteriorate, etc. [1–3]. The above renders special significance to one of the most urgent problems in modern agrarian production, which relates to improving the technological indicators in the operation of running systems of wheeled mobile agricultural machinery.

It is possible to resolve this task on the condition of solving the problem related to reducing the levels of compacting action on agricultural soils by wheeled agricultural machinery to the agrotechnological permissible values [4–6]. To this end, in addition to applying results from experimental research and to employing practical experience of using mobile wheeled agricultural machinery, it is necessary to devise the scientifically grounded methods to analytically study the interaction between wheeled movers and soil. These methods should be based on the results from theoretical and experimental research into interaction between soil and the elastic equipment of agricultural machinery movers. And should form the basis for constructing algorithms aimed at designing modern wheeled movers.

Thus, solving the issue of improving the technological indicators in the operation of running systems of wheeled mobile farming machinery necessitates analytical study of processes of soil deformation under the elastic movers of mobile wheeled agricultural machinery.

**2. Literature review and problem statement**

It was established in work [7] that investigating the mechanical and mechanical-technological properties of soils implies their phenomenological idealization, which comes down to the linearization of these properties. This relates to the stability of rheological coefficients during their deformation, and, therefore, completely eliminates the variability of elasticity and plasticity modules, coefficient of transverse deformation, the structural strength limits, etc. [8, 9].

Phenomenological modelling of soil deformation processes with the idealized mechanical-technological properties employs different models of rheological bodies, most acceptable of which is the rheological elastic-plastic Prandtl's body [10, 11], whose model is shown in Fig. 1.

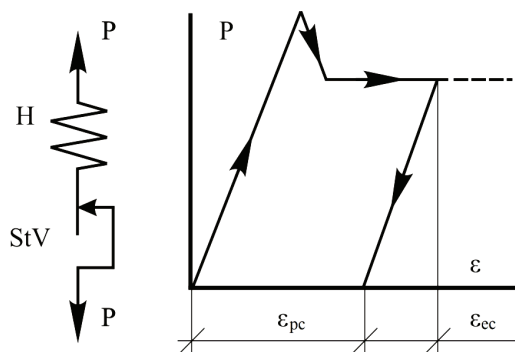


Fig. 1. Prandtl's rheological body and a corresponding diagram of its deformation:  $Pr$  – constructed Prandtl's body;  $H$  – simple rheological Hook's body;  $StV$  – simple rheological Saint-Venan body;  $P$  – external load;  $\epsilon_{pc}$  and  $\epsilon_{ec}$  – plastic and elastic components of deformation

The Prandtl's rheological body is composed of two simple rheological bodies, Hook's body ( $H$ ) and Saint-Venan body ( $StV$ ), and is described by structural formula:

$$Pr = H - StV. \tag{1}$$

A characteristic feature of the rheological Prandtl's body is that at stresses less than  $\sigma_i = \sigma_{pr}$  ( $\tau_i = \tau_{pr}$ ) one observes an elastic character of its deformation, and under larger stresses the body performs as a plastic medium (Fig. 2).

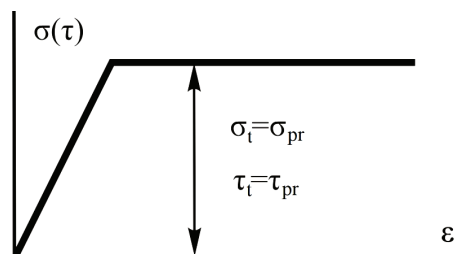


Fig. 2. Idealized advancement of Prandtl's diagram

It was established by a study [12] that actual soils undergo structural rearrangement in the process of deformation. Namely, there is an increase or decrease in density, disruption of internal bonds in varying degrees, and, consequently, there is a change in the rheological coefficients of a soil environment in general. At the same time, the change of rheological coefficients depends not only on the absolute magnitude of deformations, but on the deformation rate as well. Consequently, the processes of soil deformation are essentially physically nonlinear processes. Physical nonlinearity of the processes of soil deformation predetermines the need to apply the rheological quasi-linear equations with adjustable coefficients [10]. It was established in work [13] that the physical nonlinearity of soil deformation predetermines geometric nonlinearity, which implies the existence of deformations of second and higher orders of smallness, for cases of macro-deformation of soil. In turn, the physical nonlinearity of soil deformation is determined by the absence of linear connection between stresses and deformations and the rate of deformation. The presence of deformations of second and higher orders of smallness requires a transition to considering the states of final deformations of geometric parameters for the deformed environment, as described in papers [8, 11].

The geometric nonlinearity of the processes of soil deformation under the movers of mobile machinery predetermines the need for a graph-analytical study of the deformation dynamics not only of the deformed environment (soil) but the elastic [2, 9, 14] deformer (mover). In this case, the dynamics of a contour of contact surfaces of soil and the elastic mover sheath can be considered to be a certain generalization of the geometric characteristics of shape change in the soil microrelief under the mover of a mobile technological vehicle.

It was established in [14] that under the condition for an absolute elasticity of a mover's material, the most rational shape of its sheath is the equally stressed ovaloid of equal pressure, whose size ratio corresponds to the condition of being «fold-free» [15]. The condition of being «fold-free» is the basic limiting constraint in the geometric modelling of elastic sheaths [14]. The specified modeling condition is met by the family of Cassini ovals [16]. A feature of these flat curves is that the Cassini ovals (Fig. 3) outline the meridian

of an equally stressed surface of the potential field of pressure forces of compressed environment, which is located inside a deformed elastic sheath.

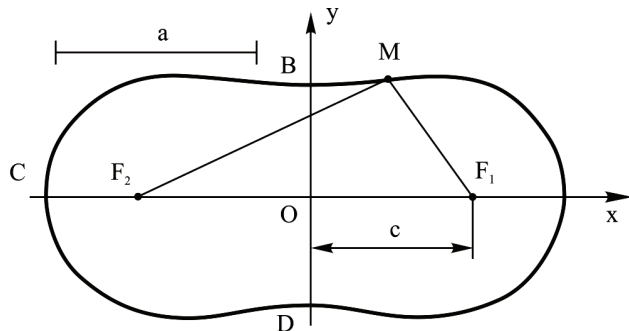


Fig. 3. Cassini oval

The Cassini ovals, at certain predefined values of constants, are separate cases of the Perseus’s curves, that is the algebraic lines of fourth order, for which the axes of coordinates are the axes of symmetry:

$$(x^2 + y^2)^2 - 2c^2(x^2 - y^2) - (a^4 - c^4) = 0, \tag{2}$$

or, in polar coordinates:

$$\rho^2 = c^2 \cos 2\varphi \pm \sqrt{c^4 \cos^2 2\varphi + (a^4 - c^4)}, \tag{3}$$

where  $c$  is the distance from a coordinate origin to the focus (Fig. 3);  $a$  is a parameter derived from:

$$a = \sqrt{|MF_1| \times |MF_2|}. \tag{4}$$

The geometric definition of a Cassini oval can be formulated as follows: point  $M$  of a plane rests on the curve if the product  $a$  of its distances to the fixed points  $F_1$  and  $F_2$  is constant while coordinates  $F_1$  and  $F_2$  are  $F_1(c, O)$  and  $F_2(-c, O)$ . Thus, the shape of the curve of a Cassini oval is dependent on ratio  $a/c$ , and the spatial equations based (2) and (3) are the mathematical models of elastic «fold-free» sheaths.

It follows from our analysis that the processes of soil deformation are nonlinear and are described by the rheological quasi-linear equations. The coefficients of these equations are not clearly defined, which complicates deriving adequate results from analytical modeling of soil deformation. Therefore, there is a need to conduct a graph-analytical study considering the states of final deformations of geometric parameters of the deformed environment. Subsequent modeling of a transverse vertical cross-section of the contact area «soil – the surface of an elastic wheeled mover of a mobile vehicle» is possible by using the contour of Cassini ovals at different ratios  $a/c$ . At the same time, it is possible to obtain the formalization of a profile of elastic «fold-free» sheaths, which could reduce pressure on soil from wheeled movers.

### 3. The aim and objectives of the study

The aim of this study is the structural and technological optimization of a transverse vertical cross-section of the contact area «soil – the surface of an elastic wheeled mover of the mobile vehicle». This would make it possible to reduce the

levels of technogenic stress on soil from the wheeled movers of mobile agricultural machinery.

To accomplish the aim, the following tasks have been set:

- to build a graph-analytical model and a corresponding modeling procedure of the process of a layer-wise shape change (deformation) in soil under the movers of mobile wheeled agricultural machinery with elastic peripherals;
- to define the basic geometric parameters that determine the character of soil environment deformation under a deformer outlined by the meridians of an equally stressed surface of the potential field of pressure forces of the compacted environment, located inside a deformed elastic sheath;
- to establish, based on the generalization of results from modeling the process of a layer-wise shape change (deformation) of soil under the movers of mobile wheeled agricultural machinery with elastic peripherals, the generalized requirements to the structural-technological parameters of the movers equipped with pneumatic tires.

### 4. Substantiation of the method for a graph-analytical study of transverse soil profile deformation

According to the geometric definition of the Cassini oval, the shape of the contact profile «soil – the surface of an elastic wheeled mover of a mobile vehicle» is determined by the  $a/c$  ratio. At the same time, the characteristic value of this ratio, which defines two types of the shape of a fourth-order curve is [16]:

$$\frac{a}{c} = \sqrt{2} \approx 1.41. \tag{5}$$

At the values of  $a/c$  exceeding the magnitude  $\sqrt{2}$ , the Cassini ovals are transformed into a regular convex oval, which at the further increase in the  $a/c$  ratio is approaching the shape of a circle with radius  $a$ . By decreasing the value of  $a/c$  to the magnitude 1.0, a Cassini oval degenerates to a Bernoulli lemniscate [16]. Thus, the boundaries of the geometrical model of the profile «soil – the surface of an elastic wheeled mover of the mobile vehicle» can be determined from:

$$\frac{a}{c} = \frac{1}{\sqrt[n]{2}}, \tag{6}$$

where  $1 < n < \infty$ .

Table 1 gives the value of parameter  $n$ , according to dependence (6), and the established corresponding values of the  $a/c$  ratio for Cassini ovals.

Table 1

Values for the  $a/c$  ratio chosen to construct graphic models of the vertical transverse cross-section of the contact’s profile «soil – the surface of an elastic wheeled mover of the mobile vehicle»

Value of parameter $n$	1.2	1.5	2.0	3.0	10.0
Value of ratio $a/c$	1.7	1.6	1.4	1.2	1.1

Based on the data given in Table 1, we propose a diagram of the layer-wise deformation of the transverse lateral soil profile by an elastic wheeled mover of the mobile vehicle, which is shown in Fig. 4.

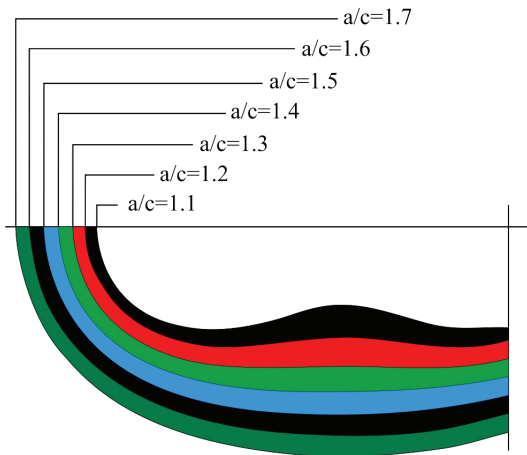


Fig. 4. Schematic of the layer-wise deformation of the transverse soil profile by an elastic wheeled mover of the mobile vehicle

We adjusted the model of the layer-wise deformation of soil profile in the contact area «soil – the surface of an elastic wheeled mover of the mobile vehicle» based on the distribution of stresses in line with the symmetrical channel model by I. I. Kandaurov [10], which is shown in Fig. 5.

If one assumes  $P=1$ , the values of efforts perceived by each subsequent layer that is at a single distance from a deformer would equal  $(1/2)^n$  (Fig. 5). In this case, the soil layer which is in contact with the deformer is considered to be zero.

Schematic of the implementation of a discrete layer-wise modeling of the deformation of a transverse profile of the soil layer under an elastic mover of the mobile vehicle is shown in Fig. 6.

The scheme implies the development of a set of graphic models at five key points, which were determined based on the value of the  $a/c$  ratios (Table 1) and  $(1/2)^n$  (Fig. 5).

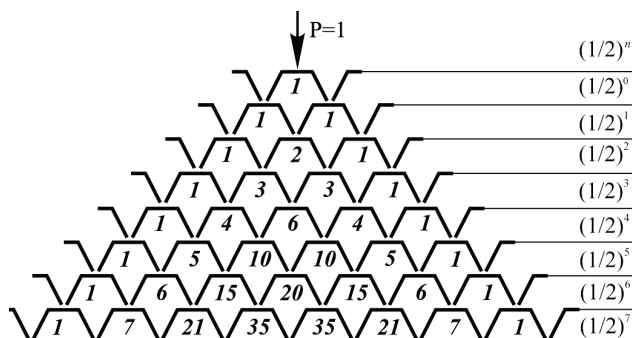


Fig. 5. Distribution of stresses in the symmetric channel model:  $P$  – force applied to a soil layer in contact with a deformer;  $n$  – a soil layer’s serial number

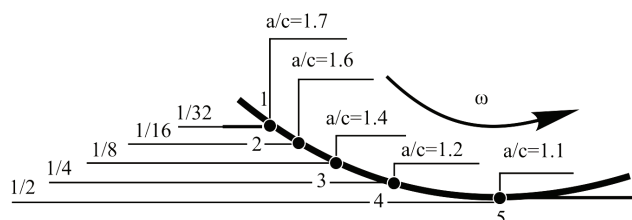


Fig. 6. Schematic of implementing a modelling experiment to study the deformation of a soil profile in the contact area «soil – the surface of an elastic wheeled mover of the mobile vehicle»;  $\omega$  – direction of mover rotation

- Specifically:
- at point 1 following the contact between a mover and a soil surface at  $a/c=1.7$  and  $(1/2)^n=1/32$ ;
  - at point 2 at  $a/c=1.6$  and  $(1/2)^n=1/16$ ;
  - at point 3 at  $a/c=1.4$  and  $(1/2)^n=1/8$ ;
  - at point 4 at  $a/c=1.2$  and  $(1/2)^n=1/4$ ;
  - at point 5 where the contact «mover – soil (track bottom)» is broken at  $a/c=1.1$  and  $(1/2)^n=1/2$ .

## 5. Results from simulating a layer-wise soil compaction inside a track under an elastic mover of the mobile vehicle

### 5.1. Modelling values of the levels of layer-wise soil compaction in a track under an elastic mover of the mobile vehicle

The results of graphic simulation to study the deformation of a soil profile in the contact area «soil – the surface of an elastic wheel mover of the mobile vehicle», performed in accordance with the scheme of model experiment implementation (Fig. 6), are shown in Fig. 7.

It was established, based on the results from graphic modeling of the soil profile deformation process in the contact zone «soil – the surface of an elastic wheel mover of the mobile vehicle», that the highest level of soil compaction is observed in the layer directly in contact with a deformer (an elastic mover). Thus, in particular, if we determine the level  $z_1$  of compaction of the soil layer in contact with the mover in the track as  $z_1=1$ , the level of compaction  $z_n$  in the  $n$ -th soil layer will be determined in accordance with the scheme of implementation of the model experiment shown in Fig. 6:

$$z_n = z_1 \cdot \left(\frac{a}{c}\right)^{-1} \cdot \left(\frac{1}{2}\right)^n, \tag{7}$$

where  $a/c$  is a geometric parameter that defines the side shape of a deformer (the mover) as a Cassini oval.

The values of levels  $z_n$  of the layer-wise soil compaction in a track under an elastic mover of the mobile vehicle, derived from (7), are given in Table 2 and marked in Fig. 7 as  $z \cdot k$ .

The geometric modeling scale  $\mu$  was determined in accordance with Fig. 7; it equals:

$$\mu = \frac{h_0}{b} \approx 0.025, \tag{8}$$

where  $h_0$  is the depth of the considered soil layer, mm;  $b$  is the width of an undeformed elastic tire of a mover, mm.

According to Fig. 8, the strength of the soil crust at the bottom of the track, formed after a single run of the mobile vehicle, does not exceed  $0.025b$ . At the same time, the total depth of the compacted soil layer does not exceed  $0.075b$ , which perceives more than 95 % of the total load on the soil.

It was established that the profile of the formed soil crust at the bottom of the track, which is formed by the mover of the mobile vehicle, is variable in width and can be sufficiently reliably described by an exponential function in the form (Fig. 9):

$$h = -b \cdot \exp\left\{-\left(\theta \cdot x\right)^2\right\}, \tag{9}$$

where  $b$  is the track’s width, mm;  $x$  is the horizontal coordinate along the mover’s axis;  $\theta \neq 0$  is a parameter that generalizes the rheological properties of a specific soil (module of elasticity, ductility module, transverse deformation coefficient, structural strength limit).

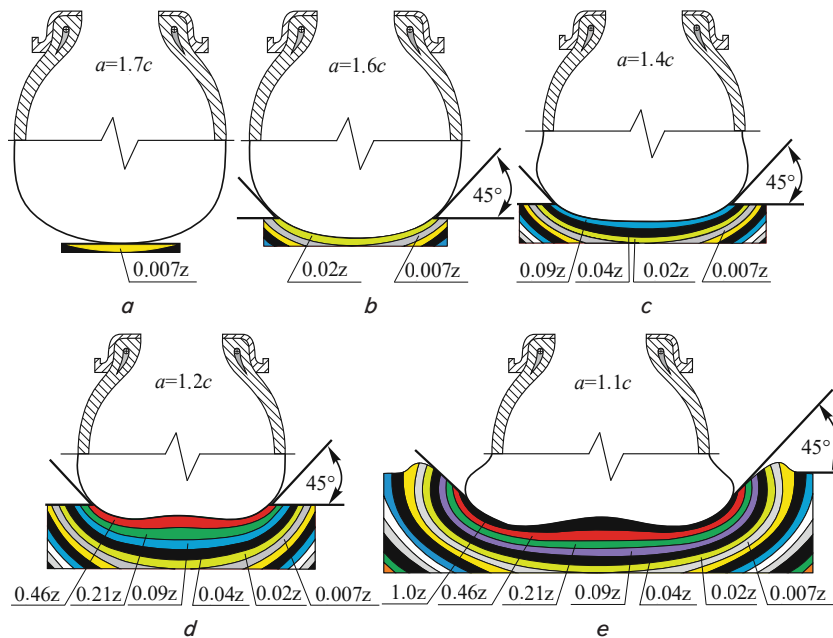


Fig. 7. Graphic models of soil deformation in the contact zone «soil – the surface of an elastic wheel mover of the mobile vehicle»: a –  $a/c=1.7$ ,  $(1/2)^n=1/32$ ; b –  $a/c=1.6$ ,  $(1/2)^n=1/16$ ; c –  $a/c=1.4$ ,  $(1/2)^n=1/8$ ; d –  $a/c=1.2$ ,  $(1/2)^n=1/4$ ; e –  $a/c=1.1$ ,  $(1/2)^n=1/2$

Table 2

Values of levels  $z_n$  of the layer-wise soil compaction in a track under an elastic mover of the mobile vehicle

Geometric parameter of the shape of a deformer (mover)		Value of layer-wise stress distribution $(1/2)^n$	Level $z_n$ of soil compaction $(a/c)^{-1} \cdot (1/2)^n$
$a/c$	$(a/c)^{-1}$		
1.0	1.0	1.0	1.0
1.1	0.909	0.500	0.455
1.2	0.833	0.250	0.208
1.4	0.714	0.125	0.089
1.6	0.625	0.063	0.039
1.7	0.588	0.031	0.018
1.9	0.526	0.016	0.008

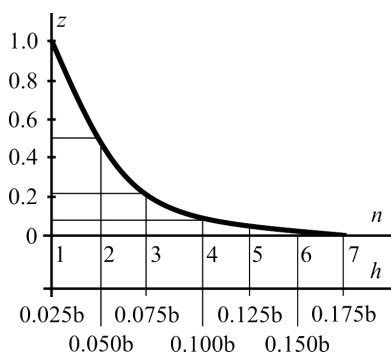


Fig. 8. Histogram of soil compaction distribution for depth  $h$  from an elastic wheeled mover of width  $b$

In this case, as a separate case, equation (9) takes the form of a Gaussian curve – the curve of normal distribution, as evidenced by the results of numerous number of field experiments [10]:

$$h = -\frac{b}{\vartheta\sqrt{2\pi}} \cdot \exp\left\{-\left(\vartheta\sqrt{2} \cdot x\right)^2\right\}. \quad (10)$$

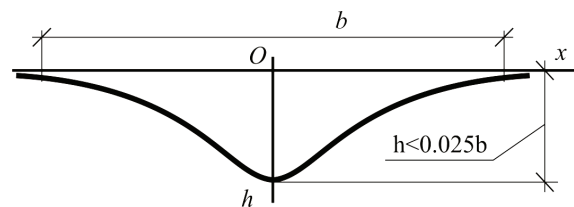


Fig. 9. Profile of soil crust at the bottom of the track formed by an elastic mover of the mobile vehicle

Changing a shape of the contact line «soil – the surface of an elastic wheeled mover of the mobile vehicle» in the vertical transverse plane of the soil profile, predetermined by a decrease in the  $a/c$  parameter from  $a/c=1.7$  to  $a/c=1.1$ , leads to an increase in the contact width  $\delta$  from 0 to  $1.32b$ .

### 5. 2. Defining basic geometric parameters of the character of soil environment deformation

Based on the results from graphic modelling of soil profile deformation in the contact zone «soil – the surface of an elastic wheeled mover of the mobile vehicle», we built a histogram in cyclic coordinates  $a/c-b-h-z$ , shown in Fig. 10.

A characteristic feature of the curve of contact «soil – the surface of an elastic wheeled mover of the mobile vehicle» with parameter  $(a/c) < \sqrt{2}$  is the existence of four inflection points, which predetermines a significant intensification of transverse displacements of the structural elements of the soil environment. If the parameter  $(a/c)$  value is less than the magnitude  $(a/c) < \sqrt{2}$ , there is a change in the characteristic features of this curve (Fig. 11).

The transverse deformation of soil under a mover of the mobile wheeled vehicle leads to the intensification of a track formation process and the formation of the overcompacted crust at the bottom part of the track with the profile shown in Fig. 9. The following condition is the start of the process of intensification of the formation of a track of depth  $h_{min}$  under a mover of the mobile vehicle (Fig. 12):

$$\varphi_{h_{\min}} > \theta_0, \tag{11}$$

where  $\theta_0$  is the angle of internal soil friction for specific physical and mechanical and agro-technological properties;  $\varphi_{h_{\min}}$  is the angle between the tangent to the contact line's curve «soil – the surface of an elastic wheeled mover of the mobile vehicle» at point  $M$  and a horizontal surface of the soil.

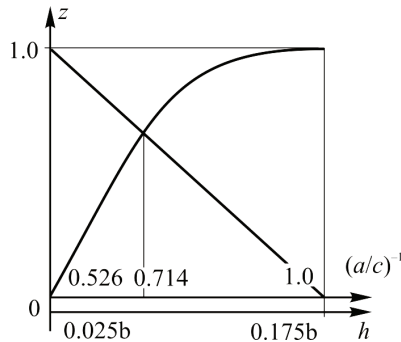


Fig. 10. Histogram of parameters' functional relation:  $(a/c)$  – shape of an elastic mover;  $b$  – width of a deformed elastic tire of the mover;  $h$  – depth of the considered soil layer;  $z$  – level of layer-wise soil compaction in a track under an elastic mover of the mobile vehicle

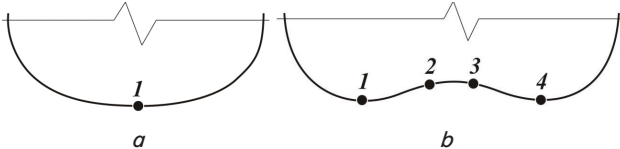


Fig. 11. The character of changes in the shape of contact line «soil – the surface of an elastic wheeled mover of the mobile vehicle» at different values of parameter  $(a/c)$ :  $a - (a/c) > \sqrt{2}$ ;  $b - (a/c) < \sqrt{2}$ ; 1, 2, 3, 4 – inflection point of the curve

In this case, the tangent to the curve of contact line «soil – the surface of an elastic wheeled mover of the mobile vehicle» at point  $M$  is determined as a perpendicular to symmedian  $MS$  of triangle  $\Delta F_1MF_2$ , formed by point  $M$  and the focal points of Cassini oval  $F_1$  and  $F_2$ .

We assessed the adequacy of the devised graphic models based on the comparison of the measure of the principal part of deformation using the experiment results and the measure of the principal part of deformation according to the criteria by Genkie and Swinger [10]:

– measure of the final (principal) part of deformation by Genkie:

$$\varepsilon^H = \ln \lambda = (\lambda - 1) - \frac{1}{2}(\lambda - 1)^2 + \frac{1}{3}(\lambda - 1)^3 - \dots \tag{12}$$

– measure of the (final) principal part of deformation by Swinger:

$$\varepsilon^S = 1 - \frac{1}{\lambda} = (\lambda - 1) - (\lambda - 1)^2 + (\lambda - 1)^3 - \dots \tag{13}$$

where  $\lambda$  is the principal part of deformation, which is determined from:

$$\lambda = \frac{l}{l_0}, \tag{14}$$

where  $l_0$  is the initial geometric size of the considered element (a soil layer) before deformation;  $l$  is the resulting geometric size of the considered element (a soil layer) after deformation.

As indicated in [10], the measures of the principal part of deformation by the criteria of Genkie and Swinger are the most acceptable criteria to estimate a shape change in viscous-plastic environments during their deformation. In this case, if we determine the measure of the principal part of deformation according to the Cauchy  $\varepsilon^C$  criterion as  $\varepsilon^C = 1$ , then the measure of the principal part of deformation by the criteria of Genkie  $\varepsilon^H$  and Swinger  $\varepsilon^S$  will equal:

$$\begin{cases} \varepsilon^C = 100 \%, \\ \varepsilon^H = 66 \%, \\ \varepsilon^S = 50 \%. \end{cases} \tag{15}$$

The  $\varepsilon^E$  measure of the principal part of deformation was determined, based on the results of model experiment, from:

$$\varepsilon^E = \frac{\left| \left[ \left( \frac{a}{c} \right) \cdot \left( \frac{1}{n} \right)^{-1} \right]_{k-1} \left[ \left( \frac{a}{c} \right) \cdot \left( \frac{1}{n} \right)^{-1} \right]_k \right|}{\left[ \left( \frac{a}{c} \right) \cdot \left( \frac{1}{n} \right)^{-1} \right]_{k-1}} \cdot 100 \%, \tag{16}$$

where

$$\left[ \left( \frac{a}{c} \right) \cdot \left( \frac{1}{n} \right)^{-1} \right]_k$$

is the value  $z_n$  for the magnitude of shape change (compaction) of the considered  $k$ -th layer of soil of depth  $h$ ;

$$\left[ \left( \frac{a}{c} \right) \cdot \left( \frac{1}{n} \right)^{-1} \right]_{k-1}$$

is the value  $z_n$  for the magnitude of shape change (compaction) in the above located  $(k-1)$ -th layer of soil of depth  $h$ .

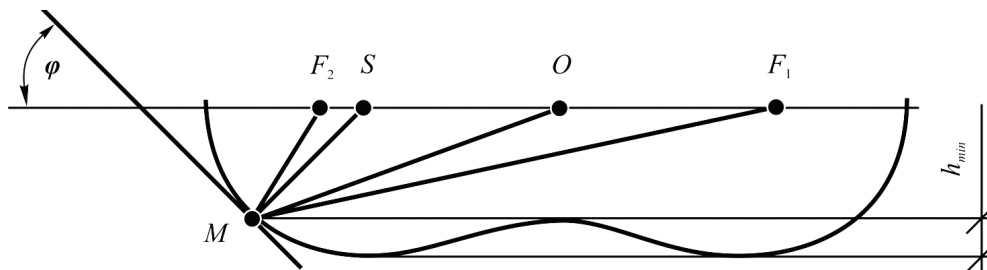


Fig. 12. Geometrical determination of track depth  $h_{\min}$  under a mover of the mobile wheeled vehicle:  $OS$  – median;  $MS$  – symmedian of triangle  $\Delta F_1MF_2$

The final data from comparative estimation of the adequacy of results of the model experiment on the deformation of soil in the contact area «soil – the surface of an elastic wheeled mover of the mobile vehicle» are given in Table 3.

Table 3

Level of adequacy of simulation results (%) by the criteria of Genkie and Swinger

Shape change parameter $(a/c) \cdot (1/n)^{-1}$	Deformation measure by Genkie	Deformation measure by Swinger	Deformation measure from experiment	Level of adequacy of simulation results, %	
				by Genkie	by Swinger
1.0	0.66	0.50	–	–	–
0.455			0.545	82.6	91.0
0.208			0.543	82.3	91.4
0.089			0.572	86.7	85.6
0.039			0.562	85.2	87.6
0.018			0.539	82.7	92.2
0.008			0.556	84.2	89.8

Table 3 gives the analysis of data on estimating the adequacy of a model experiment on soil deformation in the contact zone «soil – the surface of an elastic wheeled mover of the mobile vehicle». The assessment results indicate an adequate level of confidence probability (82.3–86.7% by the Genkie criterion  $\epsilon^H$  and 87.6–92.2% by the Swinger criteria  $\epsilon^S$ ) of the results from a graph-analytical study into the nonlinearity of soil deformation. In this case, it should also be noted that the indicators of a soil deformation measure, obtained based on the experimental results, are somewhat smaller than the Genkie criterion  $\epsilon^H$  and do not substantially exceed the Swinger criteria  $\epsilon^S$ . This indicates the reliability of the obtained results.

**6. Discussion of results from studying the minimization of technogenic pressure on soils by wheeled movers of mobile technical vehicles**

A special feature of the proposed methodology for the graph-analytical modelling of layer-wise shape change in soil under movers with an elastic periphery of the mobile wheeled agricultural machinery is the possibility to study the dynamics of soil compaction and the process of forming a track in a viscous-plastic deformed environment. This is the difference between the above results and the results from earlier studies that registered information only about the final result of the soil compaction process and the properties of a micro-relief in the presence of an already formed track. All the results in the graph-analytical modelling of soil deformation in the contact zone «soil – the surface of an elastic wheeled mover of the mobile vehicle» were obtained in the intensive form. This makes it possible to generalize the results obtained for a wide range of wheeled movers of mobile agricultural machinery without a limitation imposed by a specific dimension of a pneumatic tire for different movers.

In the process of implementation of the graph-analytical study of the character of deformation of a viscous-plastic soil environment we have established three characteristic, different and distinct, stages in the shape change of an elastic deformer. The stages of shape change are described with a high level of confidence by an algebraic line of fourth order in the form of a Cassini oval (Fig. 13).

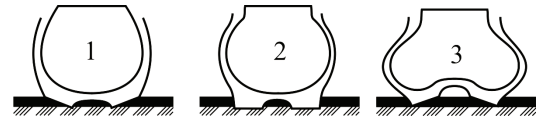


Fig. 13. Three characteristic stages in the deformation of an elastic sheath of the pneumatic tire of a mover of the mobile technical vehicle: 1, 2, 3 – stages in the deformation of an elastic sheath of the tire with a distinctive distinction of the shape of a contact line's profile «deformer – deformed environment»

A change in the shape of the contact line of the profile «deformer – deformed environment» in the deformation of soil by a pneumatic elastic mover of the wheeled mobile vehicle occurs at the following values of the geometrical parameter  $(a/c)$  for a Cassini oval:

- a transition from the shape characteristic of stage (1) to the shape characteristic of stage (2) –  $(a/c) \approx \sqrt{3} \approx 1.73$ ;
- a transition from the shape characteristic of stage (2) to the shape characteristic of stage (3) –  $(a/c) \approx \sqrt{2} \approx 1.41$ .

It was established that the most dangerous shape in terms of soil overcompaction in a track is the shape that is typical of the third stage of deformation of the elastic sheath of the pneumatic tire of a mover and is characterized by the value of geometrical parameter  $(a/c) < \sqrt{2} < 1.41$  (Fig. 13).

Thus, it is important to comply with such structural and operational parameters that would ensure operating conditions for the tire of a mover with the geometrical parameter of its elastic sheath  $(a/c) < \sqrt{2} < 1.41$  (Fig. 3). In this case, the requirement to minimize technogenic pressure on agricultural soils from the wheeled movers of mobile technical vehicles equipped with pneumatic tires (Fig. 14) will be fulfilled.

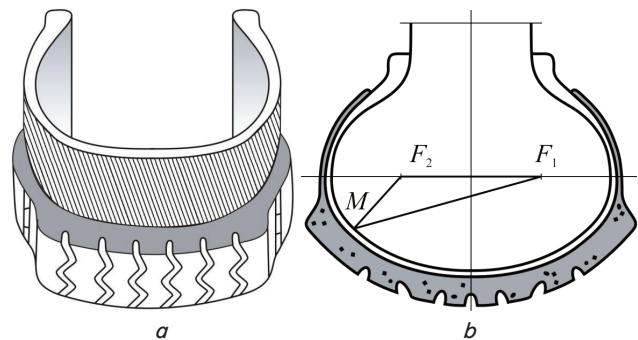


Fig. 14. Tire of a wheeled mobile technical vehicle: a – tire design; b – profile of a tire's elastic sheath, simulated by a Cassini oval

This requirement should be observed both at the stage of development and design of pneumatic tires for wheeled movers of mobile agricultural machinery, and at the stage of their operation when compiling normative documents.

The proposed graph-analytical method to study the character of soil deformation under the movers of mobile vehicles makes it possible to obtain the generalized parameters for wheeled movers. In order to relate them to a specific dimension of a pneumatic tire, further studies are required aimed at optimizing the radial, tangential, and transverse tire profile. In this case, it should be taken into account that the choice of a working pressure in tires for different types of soils is regulated by appropriate physical-mechanical and agrotechnological properties of soils.

## 7. Conclusions

1. The proposed procedure for the graph-analytical staged modelling of the process of soil deformation under pneumatic tires of movers of mobile agricultural machinery makes it possible to derive graphic models of the transverse vertical cross-section of contact area «soil – the surface of an elastic wheeled mover of the mobile vehicle». The resulting graphic models were constructed based on the changing shape of the elastic sheath of an ovaloid of equal pressure. That meets the condition for its «fold-free» character, and makes it possible to study, in stages, the process of soil compaction, including in the profile of the formed track.

2. It was established that in the contact area «soil – the surface of an elastic wheeled mover of the mobile vehicle» the highest level of soil compaction is observed in the soil layer, which is in direct contact with a deformer (an elastic mover). The depth of the overcompacted soil layer at the «track bottom» is dependent on the dimension of a tire for a wheeled mover and does not exceed the value of 0.075 of the tire width. The highest level of soil overcompaction is

observed in the area adjacent to the track's axis. The process of track formation is directly connected with a change in the character of shape of the contact line «soil – the surface of an elastic wheeled mover of the mobile vehicle». It was determined that the track formation depends not only on the specific physical-mechanical and agro-technological properties of soil, which determine the angle of its internal friction, but on the structural and operational characteristics of a mover's tire as well.

3. We have found the most dangerous shape of the profile of a pneumatic tire, in terms of soil overcompaction in a track that is formed by the tire of a wheeled mover of the mobile technical vehicle. This is the shape described by the Cassini oval with a certain ratio «the product of distances from the fixed point of the periphery of the tire's elastic sheath to the points that are the focus of the oval» to the «focal distance» of the same oval. The above ratio should not be greater than  $\sqrt{2}=1.41$ . A given ratio is the defining characteristic in the design of pneumatic tires for wheeled movers of mobile agricultural machinery and the development of recommendations to determine operational indicators of tires' working pressure.

## References

1. Karayel, D., Sarauskis, E. (2019). Environmental impact of no-tillage farming. *Environmental Research, Engineering and Management*, 75 (1), 7–12. doi: <https://doi.org/10.5755/j01.ere.m.75.1.20861>
2. González Cueto, O., Iglesias Coronel, C. E., Recarey Morfa, C. A., Urriolagoitia Sosa, G., Hernández Gómez, L. H., Urriolagoitia Calderón, G., Herrera Suárez, M. (2013). Three dimensional finite element model of soil compaction caused by agricultural tire traffic. *Computers and Electronics in Agriculture*, 99, 146–152. doi: <https://doi.org/10.1016/j.compag.2013.08.026>
3. Makharoblidze, R. M., Lagvilava, I. M., Basilashvili, B. B., Makharoblidze, Z. K. (2018). Interact of the tractor driving wheels with the soil by considering the rheological properties of soils. *Annals of Agrarian Science*, 16 (1), 65–68. doi: <https://doi.org/10.1016/j.aasci.2017.12.010>
4. Golub, G., Chuba, V., Kukharets, S. (2017). Determining the magnitude of traction force on the axes of drive wheels of self-propelled machines. *Eastern-European Journal of Enterprise Technologies*, 4 (7 (88)), 50–56. doi: <https://doi.org/10.15587/1729-4061.2017.107192>
5. Taghavifar, H., Mardani, A. (2014). Effect of velocity, wheel load and multipass on soil compaction. *Journal of the Saudi Society of Agricultural Sciences*, 13 (1), 57–66. doi: <https://doi.org/10.1016/j.jssas.2013.01.004>
6. Kutzbach, H. D., Bürger, A., Böttinger, S. (2019). Rolling radii and moment arm of the wheel load for pneumatic tyres. *Journal of Terramechanics*, 82, 13–21. doi: <https://doi.org/10.1016/j.jterra.2018.11.002>
7. Mudarisov, S. G., Gabitov, I. I., Lobachevsky, Y. P., Mazitov, N. K., Rakhimov, R. S., Khamaletdinov, R. R. et. al. (2019). Modeling the technological process of tillage. *Soil and Tillage Research*, 190, 70–77. doi: <https://doi.org/10.1016/j.still.2018.12.004>
8. Gubiani, P. I., Pértile, P., Reichert, J. M. (2018). Relationship of precompression stress with elasticity and plasticity indexes from uniaxial cyclic loading test. *Soil and Tillage Research*, 180, 29–37. doi: <https://doi.org/10.1016/j.still.2018.02.004>
9. Recuero, A., Serban, R., Peterson, B., Sugiyama, H., Jayakumar, P., Negrut, D. (2017). A high-fidelity approach for vehicle mobility simulation: Nonlinear finite element tires operating on granular material. *Journal of Terramechanics*, 72, 39–54. doi: <https://doi.org/10.1016/j.jterra.2017.04.002>
10. Ai, Z. Y., Zhao, Y. Z., Song, X., Mu, J. J. (2019). Multi-dimensional consolidation analysis of transversely isotropic viscoelastic saturated soils. *Engineering Geology*, 253, 1–13. doi: <https://doi.org/10.1016/j.enggeo.2019.02.022>
11. Rubinstein, D., Shmulevich, I., Frenckel, N. (2018). Use of explicit finite-element formulation to predict the rolling radius and slip of an agricultural tire during travel over loose soil. *Journal of Terramechanics*, 80, 1–9. doi: <https://doi.org/10.1016/j.jterra.2018.09.002>
12. Milkevych, V., Munkholm, L. J., Chen, Y., Nyord, T. (2018). Modelling approach for soil displacement in tillage using discrete element method. *Soil and Tillage Research*, 183, 60–71. doi: <https://doi.org/10.1016/j.still.2018.05.017>
13. Schjønning, P., Lamandé, M. (2018). Models for prediction of soil precompression stress from readily available soil properties. *Geoderma*, 320, 115–125. doi: <https://doi.org/10.1016/j.geoderma.2018.01.028>
14. Sharma, G., Tiwary, S., Kumar, A., Suresha Kumar, H. N., Keshava Murthy, K. A. (2018). Systematic design and development of a flexible wheel for low mass lunar rover. *Journal of Terramechanics*, 76, 39–52. doi: <https://doi.org/10.1016/j.jterra.2017.12.002>
15. Ruinskii, A. (2003). Cassini Curve and an Equilateral Hyperbola. *Mathematical Education*, 2 (25), 80–88.
16. Martini, H., Wu, S. (2014). Classical curve theory in normed planes. *Computer Aided Geometric Design*, 31 (7-8), 373–397. doi: <https://doi.org/10.1016/j.cagd.2014.03.003>