Аналітично досліджені усталені режими руху системи, складеної зі збалансованого ротора на ізотропних пружно-в'язких опорах, і вантажу (кулі, ролика, маятника), встановленого усередині ротора з можливістю відносного руху. При цьому маятник вільно насаджений на вал ротора, а куля чи ролик котяться без ковзання по кільцевій доріжці з центром на подовжній осі ротора.

Описана фізико-математична модель системи. Записані диференціальні рівняння руху системи щодо системи координат, що обертається з постійною швидкістю обертання у безрозмірному вигляді.

Знайдено всі усталені режими руху системи, в яких вантаж обертається з постійною кутовою швидкістю. В системі координат, що синхронно обертається з вантажем, ці рухи стаціонарні.

Проведені теоретичні дослідження показують, що на усталених режимах руху:

– за відсутністю сил опору в системі вантаж синхронно обертається з ротором;

– за наявністю сил опору в системі вантаж відстає від ротора.

Режими застрягання вантажу є однопараметричними сім'ями усталених рухів. Кожен режим застрягання характеризується відповідною частотою застрягання.

В залежності від параметрів системи можуть існувати одна чи три можливі швидкості застрягання вантажу. Якщо на будь-якій швидкості обертання ротора існує тільки одна кутова швидкість застрягання вантажу, то відповідний режим руху (однопараметрична сім'я) глобально асимптотично стійкий. Якщо кількість швидкостей застрягання змінюється в залежності від кутової швидкості обертання ротора, то асимптотично стійкими є:

– єдиний існуючий режим застрягання (глобально асимптотично стійкий, коли інших немає);

– режими застрягання з найменшою і найбільшою швидкостями.

Режим застрягання вантажу з найменшою кутовою швидкістю (близька до резонансної) можна використовувати для збудження резонансних коливань в вібраційних машинах. Найбільша частота застрягання вантажу близька до швидкості обертання ротора. Цей режим можна використовувати для збудження нерезонансних коливань в вібраційних машинах

Ключові слова: пасивний автобалансир, ефект Зоммерфельда, інерційний віброзбудник, резонансна вібромашина, біфуркація рухів

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STUDYING THE EXCITATION OF RESONANCE OSCILLATIONS IN A ROTOR ON ISOTROPIC SUPPORTS BY A PENDULUM, A BALL, A ROLLER

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1. Introduction

In [1], it is proposed to use passive auto-balancers – pendulum-, ball-, roller-type – as vibration exciters. It is assumed that the new technique is applicable for one- and multi-mass vibration machines at different kinetics of vibratory platform motion. In this regard, a relevant problem is the substantiation of feasibility of the new technique for vibration excitation and the development of a theory of new vibration machines.

The use of a known device for the new purpose becomes possible due to the fact that a rotary machine with the specified auto-balancers can execute various steady motion modes that correspond to:

– auto-balancing or a synchronous rotation of loads together with a rotor [2];

– loads' jamming at the resonance rotational speed of the rotor (caused by the Sommerfeld effect) [3–12];

– parametric and other oscillations of loads [4, 13].

Therefore, the proper choice of system parameters can provide for the excitation of resonance oscillations.

To create purely resonant vibration machines, it is necessary that the rotor should be balanced and the loads should be stuck at the resonance rotor speed. The simplest resonance vibration exciter is a single load (a pendulum, a ball, a roller), mounted on the balanced rotor with a possibility for the center of mass of a load to move in a circle centered on the longitudinal axis of the rotor. It is an actual task, within the framework of a flat model of such a vibration exciter, to analytically find all the steady motion modes and explore their stability. This is important both for constructing an analytical theory of new resonance vibration machines and for the development of effective methods for studying the dynamics of the specified machines for cases of several loads, one- and multi-mass vibration machines, etc.

2. Literature review and problem statement

The most complete information on the origin, disappearance, conditions of existence and resistance of different motion modes of a dynamic system is provided by a bifurcation theory. In rotary machines, a bifurcation parameter that is typically adopted is the rotor's rotation speed.

As of today, the most analytical results were obtained in the framework of a flat model of the rotor balanced on isotropic elastic-viscous supports, which carries an auto-balancer with the same loads. Thus, all the stationary modes, under which loads rotate synchronously to the rotor [2], or lag behind the rotor [3], have been defined; the stability of these regimes has been partially investigated.

It should be noted that the most difficult in terms of analytical studies is to investigate the stability of steady motions, in particular, the families of steady motions. Difficulties are due to a large number of degrees of freedom of the system, a significant nonlinearity of the problem, known problems related to the approximate methods of a small parameter, computational methods and experiments, etc.

Consider the disadvantages and advantages of some methods for studying jam modes.

Paper [4] gives an analytical statement of the problem on building a nonlinear bifurcation theory for the considered system. However, the bifurcation analysis is carried out by numerical methods, for the case of two loads. As a result, at the system's specific parameters, it was revealed that, along with the stationary motions, the system has limit cycles and chaotic motions. The advantage of numerical methods for a bifurcation analysis is the possibility to detect all possible motion modes of the system and assess their stability. In this case, the load jam frequencies are determined accurately. The disadvantage is a particular character of the results obtained (for specific numerical values for system parameters), significant difficulties in applying the method with an increase in the number of loads, etc.

Study [5] experimentally established the modes of pendulum jams in the system rotor – a pendulum auto-balancer. Under these modes, pendulums are bound, they cannot accelerate and get stuck at one of the resonance speeds of rotor rotation. Work [6] investigated experimentally a phenomenon of jamming the pendulum freely mounted onto the shaft of an electric motor. It was found that the pendulum is stuck at one of the natural frequencies of system oscillations. The advantage of field experiments is that these experiments make it possible to identify persistent steady motions (implemented in practice). Disadvantages are that if the system has several simultaneously stable steady motions, the motion that the system would eventually execute significantly depends on the starting conditions. Therefore, based on the results from experiments, it is difficult to assess the stability regions of different motion modes of the system. In addition, the results obtained are of particular character (for a specific machine). Field experiments are labor-intensive and require large resources, they do not make it possible to find at great accuracy the frequencies of load jams, nor the relative positions of loads with respect to the rotor, etc.

The load jam modes were examined by modelling the system's dynamics at a PC:

– for a rotor on isotropic supports, which executes spatial motion and is balanced by one or two two-pendulum auto-balancers [7];

– for a rotor on anisotropic supports, which executes a flat motion and is statically balanced by a two-ball auto-balancer [8];

– for a rotor mounted on isotropic supports at a platform that moves steadily in a straight line, while balancing the rotor with a two-ball auto-balancer [9].

Simulating the machine's dynamics at a PC is the least laborious method of research, which makes it possible to identify steady motion modes, to more accurately find the speeds of load jams. However, the results obtained are of particular character. It is difficult to determine the stability limits of various simultaneously stable modes of system motion by simulation, because the motion that would eventually be executed depends on the initial conditions.

The most general results are provided by analytical research methods. The loads jam modes were analytically examined by approximate methods for:

– a rotor on isotropic supports, which executes a spatial motion and is statically balanced by a two-ball auto-balancer, using the theory of synchronization of mechanical systems [10];

– a rotor on isotropic supports, which executes a flat motion and is balanced by a two-ball auto-balancer, using the modified incremental harmonic balance method [11];

– a rotor on isotropic supports that executes a flat motion and is balanced by a two-ball auto-balancer, using a Limit-Cycle Analysis [12].

It was established in [10–12] that loads are stuck at one of the resonance speeds of rotor rotation; the authors found the approximate boundaries for the jam modes' stability regions. The advantages of analytical approximate methods include the generality of results obtained, a possibility to derive analytical results. The disadvantages of approximate methods include the asymptomatic character of solutions, the complexity of using methods for the case of many loads. In addition, the methods do not produce a precise statement of the problem on determining the frequencies and modes of load jams. Therefore, they were employed to analytically explore only those jamming modes under which loads are combined (whose existence was known).

In [3], the load jam modes were analytically examined in a precise statement of the problem for a balanced rotor on isotropic supports, which executes a flat motion and is balanced by an auto-balancer with many identical loads. The authors applied the elements of a bifurcation analysis by finding all possible jamming modes, determining the conditions of their existence, origin, and disappearance.

The research revealed that:

– there are other jamming modes within the system, under which the loads are not combined;

– each jam mode is characterized by a certain configuration of loads relative to the rotor, which changes with altering a rotor speed;

– each load configuration is matched with a single or three possible frequencies of load jams;

– for the case of three possible frequencies of load jams, two are close to the resonance speed, and one – to the rotational speed of the rotor.

Thus, only the precise statement of the problem, as well as its exact solution, make it possible to identify and theoretically investigate all possible jamming modes, to find the bifurcation values for a rotor speed in the transition of which the jamming modes acquire or lose stability, emerge, or disappear.

It should be noted that in the system examined in [3] the authors had discovered and investigated, in the precise statement, the steady motions at which loads synchronously rotate with the rotor [2]. Because of the existence of a large number of steady motions, it is difficult to investigate their stability and build a complete bifurcation diagram. On the other hand, for the case of a single load, one obtains a full-fledged vibration exciter of the simplest design. At the same time, the problem on exploring the stability of motions is greatly simplified. Thus, there is an opportunity both for solving the problem and the development of approaches and procedures, which are applicable for the case of many loads.

3. The aim and objectives of the study

The aim of this study is to find and estimate the stability of all steady motion modes of the system composed of a rotor on the isotropic elastic and viscous supports and a load (a pendulum, a ball, or a roller), installed in the rotor with a possibility of relative motion. That would make it possible to identify the ways to use such loads in vibration machines in order to excite resonance oscillations, as well as produce research methods and approaches, applicable for the case of many loads.

To accomplish the aim, the following tasks have been set:

– to find all steady motion modes of the specified system, conditions for their origin, existence, and disappearance;

– to study analytically the stability of steady motion modes;

– to complement the results from an analytical study with a computational experiment.

4. Methods for finding all possible stationary modes of system motion

To construct a mechanical-mathematical model of the system rotor – load, we use the results from work [3], the elements of classical mechanics [14], the perturbation theory [15] and the bifurcation of motions [16].

Differential equations of the system motion are recorded with respect to the coordinate system, rotating at a constant angular velocity. In such a coordinate system:

– the motion of a mechanical system is described by a system of regular nonlinear autonomous differential equations;

– all the steady modes are stationary motions, provided that the rotation speed of the movable coordinate system coincides with the angular speed of load rotation.

A search for all possible stationary modes of system motion is reduced to solving a nonlinear system of algebraic equations. In this case, all possible speeds of load rotation (angular velocities of a moving coordinate system's rotation) are to be found at the same time, as well as the position of a load with respect to the rotating coordinate system, and the corresponding deviation of the rotor.

To solve the system of nonlinear algebraic equations, we shall apply a method of decomposition of the roots of equations by the degrees of a small parameter [16]. In this case, different ratios of a smallness between the system parameters shall be considered.

A bifurcation parameter to be accepted is the angular velocity of rotor rotation. The load jam modes are to be found depending on the angular velocity of rotor rotation. The occurrence and disappearance of different jamming modes shall be studied in terms of a bifurcation theory of motions [16].

Stationary motion stability shall be studied based on the first Lyapunov method [16].

The results obtained from a theoretical study will be supplemented with and verified by a computational experiment.

5. Load jamming modes in an auto-balancer – a flat model of the rotor on isotropic supports

5. 1. Mechanical-mathematical model of the system

5. 1. 1. Description of a mechanical-mathematical model of the system

A flat system model was adopted to investigate the system's dynamics [3]. Within its framework, a rotor is a symmetrical flat disk of mass M, mounted onto a completely rigid shaft, perpendicular to its plane (Fig. 1). The rotor is arranged vertically, it moves flat parallel in the horizontal plane and rotates at a constant angular velocity ω. For the case of a ball (a roller) (Fig. 1, *b*), the ball (the roller) rolls without slip along a circular path. The ball's mass (the roller) is m, its radius is R, the distance from the axis of the shaft to the center of the ball (roller) is l. For the case of a pendulum (Fig. 1, *c*), the rotor shaft hosts a pendulum whose mass is m, physical length – l, and the main central axial momentum of inertia – I_C.

At a stationary rotor, the shaft coincides with the axis of rotation. In the motion process, the shaft is point O, which deviates from the axis of rotation, point K, and it is exposed to a restoring force, and the force of viscous resistance of a medium. Coefficients of stiffness and damping in the shaft supports are c, $\tilde{b}$. To describe the system motion, we use the following systems of axes:

– $O\Xi H$ – the right-hand system of fixed rectangular axes;

– OXY – the right-hand system of moving rectangular axes, rotating around the rotation axis (point K) at constant angular velocity Ω;

– OX_OY_O – the right-hand system of moving rectangular axes, originating from the center of the disc and parallel to the system of axes OXY.

The rotation angle of the system of OXY axes around point K equals Ωt, where t is time. The rotor rotation angle is ωt. Position of a load is determined, with respect to the system of OX_OY_O axes, by angle α. When a ball (a roller) moves along a path, it is exposed to the force of viscous resistance $\tilde{\beta} l(\omega-\Omega-\alpha')$, where $\tilde{\beta}$ is the coefficient of viscous resistance forces, and $l(\omega-\Omega-\alpha')$ is the motion speed of a ball's center (a roller) along the path relative to the rotor. When a pendulum rotates around the shaft, it is exposed to the momentum of force of viscous resistance $\tilde\beta l^2(\omega-\Omega-\alpha')$, where $\tilde\beta$ is the coefficient of viscous resistance forces, $(\omega-\Omega-\alpha')$ is the angular velocity of pendulum rotation around the shaft (the rotor) and a bar behind the magnitude denotes a time derivative.

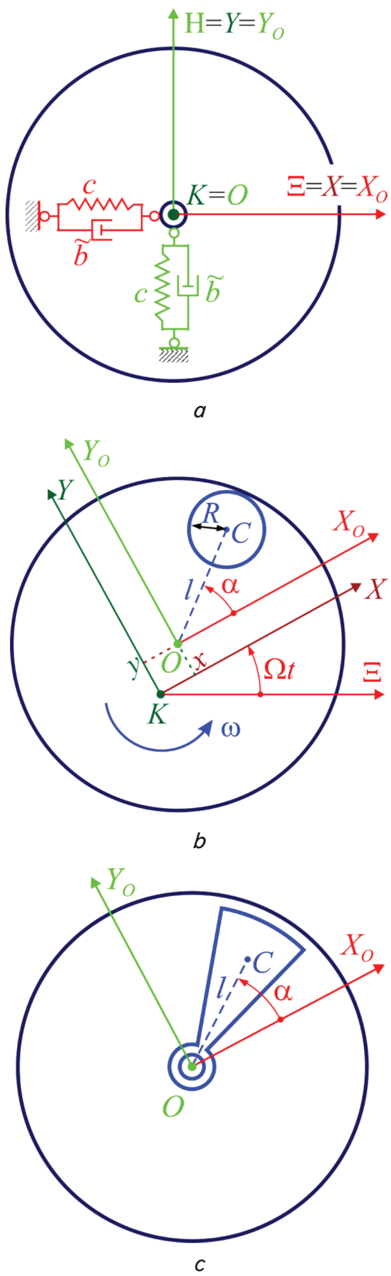

Fig. 1. A flat model of the system:
a — rotor on isotropic elastic and viscous supports;
b — kinematics of the motion of a rotor and a ball or a roller;
c — kinematics of the pendulum motion

For the examined system:

$$M_\Sigma = M + m,\quad \omega_0 = \sqrt{c/M_\Sigma}. \tag{1}$$

where M_Σ is the system's mass, ω_0 is the resonance rotor speed.

5. 1. 2. Differential equations of system motion in the dimensionless form, equations of stationary motions

Differential equations of system motion in the dimensionless form are:

$$L_v = \ddot v + 2\nu\dot u - \nu^2 v + b(\dot v + \nu u) + v -$$
$$-\varepsilon\left[-\ddot\alpha\cos\alpha + (\dot\alpha+\nu)^2\sin\alpha\right] = 0, \tag{2}$$

where dimensionless time, variables, and parameters are introduced:

$$\tau = \Omega t,\quad u = \frac{x}{\kappa l},\quad v = \frac{y}{\kappa l},\quad n = \frac{\omega}{\omega_0},\quad \beta = \frac{\tilde\beta}{\kappa m\omega_0},$$
$$\nu = \Omega/\omega_0,\quad b = \tilde b/(\omega_0 M_\Sigma),\quad \varepsilon = m/(\kappa M_\Sigma), \tag{3}$$

a point above the magnitude indicates the dimensionless time derivative for a ball, a roller, or a pendulum, respectively:

$$\kappa = \frac{7}{5},\quad \kappa = \frac{3}{2},\quad \kappa = 1 + I_C/(ml^2). \tag{4}$$

Note that for the mathematical pendulum $I_C = 0$, $\kappa = 1$.

At stationary steady motions, the dimensionless generalized coordinates are stable: $\tilde\alpha, \tilde u, \tilde v = \text{const}$. By substituting it in (2), we obtain:

$$\tilde L^{(0)} = \beta(\nu - n) + \nu^2(\tilde u\sin\tilde\alpha - \tilde v\cos\tilde\alpha) = 0,$$
$$L_u^{(0)} = \tilde u(1-\nu^2) - b\nu\tilde v - \varepsilon\nu^2\cos\tilde\alpha,$$
$$L_v^{(0)} = (1-\nu^2)\tilde v + b\nu\tilde u - \varepsilon\nu^2\sin\tilde\alpha \tag{5}$$

– a system of algebraic equations to derive stationary motions.

5. 2. Finding the system's stationary motions at which a load rotates synchronously with the rotor

At a simultaneous rotation of the load with the rotor $\nu = n$ and equations (5) take the form:

$$\tilde L^{(0)} = n^2(\tilde u\sin\tilde\alpha - \tilde v\cos\tilde\alpha) = 0,$$
$$L_u^{(0)} = \tilde u(1-n^2) - bn\tilde v - \varepsilon n^2\cos\tilde\alpha,$$
$$L_v^{(0)} = (1-n^2)\tilde v + bn\tilde u - \varepsilon n^2\sin\tilde\alpha. \tag{6}$$

We derive from the second and third equations (6):

$$\tilde u = \varepsilon n^2\frac{(1-n^2)\cos\tilde\alpha + bn\sin\tilde\alpha}{(n^2-1)^2 + b^2n^2},$$
$$\tilde v = \varepsilon n^2\frac{(1-n^2)\sin\tilde\alpha - bn\cos\tilde\alpha}{(n^2-1)^2 + b^2n^2}. \tag{7}$$

Substitute it in the first equation in (6) and obtain:

$$\frac{\varepsilon bn^3}{(n^2-1)^2 + b^2n^2} = 0. \tag{8}$$

It follows from (8) that in the presence of resistance forces, the supports do not execute stationary motions at which a load rotates synchronously with the rotor.

In the absence of resistance forces in supports, $b=0$, and equation (8) is performed automatically, while equations (7) take the form:

$$\tilde{u}=-\frac{\varepsilon n^2\cos\tilde{\alpha}}{n^2-1},\quad \tilde{v}=-\frac{\varepsilon n^2\sin\tilde{\alpha}}{n^2-1}. \qquad (9)$$

Equations (9) determine a one-parameter family of stationary motions, where angle $\tilde{\alpha}$ is the parameter. It is evident that at the pre-resonance speeds of rotor rotation the center of the rotor's mass deviates in the direction of the load's center of mass, and at the resonance speeds – in the opposite direction.

5. 3. Finding the modes of load jamming

5. 3. 1. General sequence of problem solving

Introduce angle $\tilde{\vartheta}$ between vector $\overrightarrow{KO}$ (the rotor's displacement vector) and the X axis. Then:

$$\cos\tilde{\vartheta}=\tilde{u}/\tilde{\rho},\quad \sin\tilde{\vartheta}=\tilde{v}/\tilde{\rho},\quad \tilde{\rho}=\sqrt{\tilde{u}^2+\tilde{v}^2}, \qquad (10)$$

And the equations of steady motions (5) are transformed to the form:

$$\tilde{L}^{(0)}=\beta(\nu-n)-\nu^2\tilde{\rho}\sin\tilde{\varphi}=0,$$

$$\tilde{u}\tilde{L}_u^{(0)}+\tilde{v}\tilde{L}_v^{(0)}=\left(1-\nu^2\right)\tilde{\rho}^2-\varepsilon\tilde{\rho}\nu^2\cos\tilde{\varphi}=0,$$

$$\tilde{u}\tilde{L}_u^{(0)}-\tilde{v}\tilde{L}_v^{(0)}=\tilde{b}\nu\tilde{\rho}^2+\varepsilon\tilde{\rho}\nu^2\sin\tilde{\varphi}=0, \qquad (11)$$

where

$$\tilde{\varphi}=\tilde{\vartheta}-\tilde{\alpha}. \qquad (12)$$

is the derived system of three nonlinear algebraic equations relative to three unknowns $\tilde{\rho}$, ν, $\tilde{\varphi}$.

Solve the system of equations (12). We represent it in the form:

$$\tilde{\rho}^2=-\frac{\varepsilon\nu}{b}\tilde{\rho}\sin\tilde{\varphi},\quad \tilde{\rho}\cos\tilde{\varphi}=\frac{\left(1-\nu^2\right)}{\varepsilon\nu^2}\tilde{\rho}^2,$$

$$\tilde{\rho}\sin\tilde{\varphi}=\frac{\beta}{\nu^2}(\nu-n). \qquad (13)$$

We find from the third and fourth equations:

$$\tilde{\rho}^2=-\frac{\varepsilon\nu}{b}\frac{\beta}{\nu^2}(\nu-n)=\frac{\varepsilon\beta}{\nu b}(n-\nu). \qquad (14)$$

It follows from (14) that a load can only lag behind the rotor ($\nu<n$). Then it follows from the first equation in (13) that $\sin\tilde{\varphi}<0$, therefore $\tilde{\varphi}\in(-\pi,0)$.

By using the first and second equations in (13), introduce the angle:

$$\gamma=\arctan\left(\frac{\sin\tilde{\varphi}}{\cos\tilde{\varphi}}\right)=\arctan\left(\frac{b\nu}{\nu^2-1}\right),$$

$$\gamma\in(-\pi/2,\pi/2). \qquad (15)$$

Then, find $\tilde{\varphi}\in(-\pi,0)$:

$$\tilde{\varphi}=\begin{cases}\gamma, & \gamma\le 0;\\ \gamma-\pi, & \gamma>0.\end{cases} \qquad (16)$$

Apply an identity:

$$\tilde{\rho}^2=(\tilde{\rho}\sin\tilde{\varphi})^2+(\tilde{\rho}\cos\tilde{\varphi})^2=\frac{\beta^2}{\nu^4}(\nu-n)^2+\frac{\left(1-\nu^2\right)^2}{\varepsilon^2\nu^4}\tilde{\rho}^4.$$

Substitute this equation with $\tilde{\rho}^2$ from (14), we obtain:

$$\frac{\varepsilon\beta}{\nu b}(n-\nu)=\frac{\beta^2}{\nu^4}(\nu-n)^2+\frac{\left(1-\nu^2\right)^2}{\varepsilon^2\nu^4}\frac{\varepsilon^2\beta^2}{\nu^2 b^2}(n-\nu)^2.$$

This equation can be satisfied in the following two cases:

1) $n-\nu=0$ – a load rotates synchronously to the rotor;

2) $\frac{\varepsilon}{b}=\frac{\beta}{\nu^3}(\nu-n)+\frac{\left(1-\nu^2\right)^2}{\nu^5}\frac{\beta}{b^2}(n-\nu)$ – a load lags behind the rotor.

Above, it was found that in the presence of damping in the supports, the motion modes under which a load rotates synchronously to the rotor do not exist. We then find the load jamming modes.

The second case is provided from the following equation:

$$P(\nu)=\chi\nu^5-(n-\nu)\left[\left(1-\nu^2\right)^2+b^2\nu^2\right]=$$

$$=a_0\nu^5+a_1\nu^4+a_2\nu^3+a_3\nu^2+a_4\nu+a_5=0, \qquad (17)$$

where

$$\chi=\varepsilon b/\beta,$$

$$a_0=1+\chi,\quad a_1=-n,\quad a_2=-\left(2-b^2\right),$$

$$a_3=n\left(2-b^2\right),\quad a_4=1,\quad a_5=-n. \qquad (18)$$

We find from (17) frequencies ν_i, at which a load can be jammed. Then, from equation (14), we derive:

$$\tilde{\rho}=\sqrt{\varepsilon\beta(n-\nu)/(\nu b)}. \qquad (19)$$

Then, from (15), (16), we find:

$$\tilde{\varphi}=\begin{cases}\gamma, & \gamma\le 0;\\ \gamma-\pi, & \gamma>0,\end{cases}\quad \gamma=\arctan\left(\frac{b\nu}{\nu^2-1}\right). \qquad (20)$$

We derive from (12):

$$\tilde{\alpha}=\tilde{\vartheta}-\tilde{\varphi}. \qquad (21)$$

We find from (10):

$$\tilde{u}=\tilde{\rho}\cos\tilde{\vartheta},\quad \tilde{v}=\tilde{\rho}\sin\tilde{\vartheta}. \qquad (22)$$

Quantities (19) to (22) are calculated for the specific frequency of a load jam and at a specific (any) value of parameter ϑ. The derived dimensionless coordinates and the angle can be used as the initial conditions in a computational experiment. These initial conditions (a motion starts at a certain jam mode) will make it possible to investigate the stability of a jam mode in a small (by Lyapunov) – at small deviations of the disturbed motion from the jam mode whose stability is examined.

5. 3. 2. Investigation of the number and conditions of the existence of load jam modes

We consider a dimensionless angular velocity of the rotor n as a bifurcation parameter. Consider changing n from 0 to $+\infty$. A change in n would alter the roots of equation (17). We shall search for valid roots and characteristic angular velocities of the rotor rotation (bifurcation points), at which various jamming modes emerge or disappear.

One can see from (17) that $\forall\nu\le 0\ P(\nu)<0, \forall\nu\ge n\ P(\nu)>0$. Therefore, all valid roots of a polynomial are in the interval $(0, n)$, and in this interval there is always a single root.

It follows from a Descartes theorem (a Descartes sign rule) that polynomial (17) may possess:

– $\forall b<\sqrt{2}$ – 1 or 3 valid roots;

– $\forall b>\sqrt{2}$ – 1, 3, or 5 valid roots.

Since finding the roots from a polynomial of the fifth degree is difficult, then we solve the problem parametrically. A parameter to accept is the frequency of a load jam. Then a solution to equation (17) in the parametric form takes the form:

$$n(\nu)=\nu\frac{\chi\nu^4+\left(1-\nu^2\right)^2+b^2\nu^2}{\left(1-\nu^2\right)^2+b^2\nu^2},\ \ \nu\in(0,+\infty). \qquad (23)$$

Fig. 2 shows, in the plane (n, ν), the constructed graph of function $n(\nu)$ at different ratio of smallness between parameters χ and b.

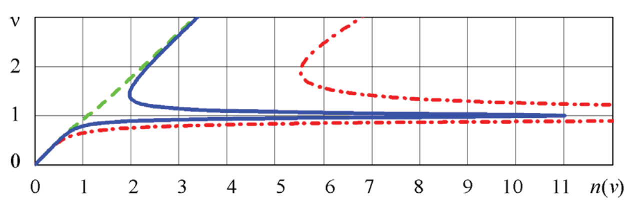

Fig. 2. Dependence of the number and conditions of the existence of load jamming frequencies on ratios of a smallness between parameters χ and b: –·–·– χ~1, $b\ll 1$; – – – b~1 ($\forall\chi$); ——— $\chi\ll 1$, $b\ll 1$

One can see from Fig. 2 that at χ~1, $b\ll 1$ or $\chi\ll 1$, $b\ll 1$ in the system, depending on the rotor speed, there are a single or three possible load jamming frequencies. At b~1, the system has the only possible frequency of a load jam, which is close to the rotor speed.

Next, we shall consider the case of small forces of viscous resistance in the supports. We shall introduce three characteristic rotor speeds for this case. Their transition changes the number or properties of possible frequencies of a load jam. In this case, $1<n_1\ll n_2<n_3$ and at the rotor speeds:

– lower than n_1 $(0<n<n_1)$, there is a single frequency of load jam ν_1, with $0<\nu_1<1$;

– that exceed n_1, but are less than n_2 $(n_1<n<n_2)$, there are three frequencies of load jam $\nu_{1,2,3}$, such that $0<\nu_1<1<\nu_2<\nu_3<n$;

– that exceed n_2, but are less than n_3 $(n_2<n<n_3)$, there are three frequencies of load jam $\nu_{1,2,3}$, such that $1<\nu_1<\nu_2\ll\nu_3<n$;

– that exceed n_3 $(n>n_3)$, there is a single frequency of load jamn_3, such that $1\ll\nu_3<n$.

Fig. 3 shows the numbering of jam frequencies, and characteristic speeds.

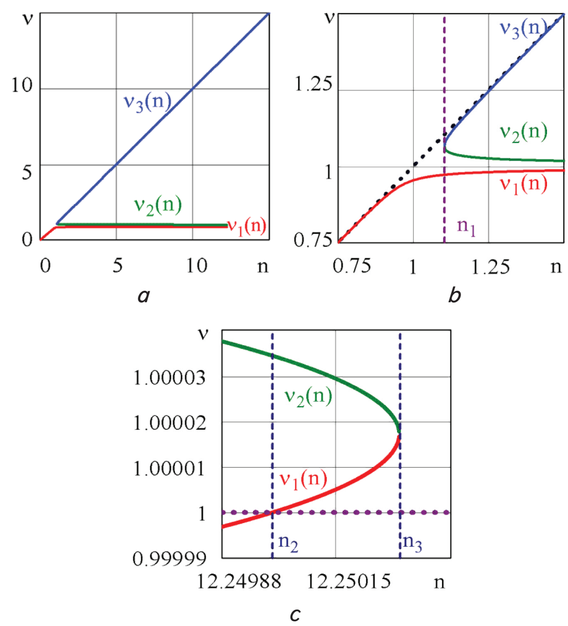

Fig. 3. Standard diagrams of dependences of possible angular velocities of a load jam on a rotor speed [3]: *a* — general view; *b* — in the vicinity of characteristic angular velocity n_1; *c* — in the vicinity of characteristic angular velocities n_2, n_3

The characteristic rotor speeds n_1, n_3 are the bifurcation points. At point n_2, jam modes ν_2 and ν_3 emerge, and at point n_3 – jam modes n_1 and n_2 disappear. According to the theory of bifurcation of motions, different jamming modes can acquire or lose stability only when passing through the points of bifurcation of motions [16].

5. 3. 3. Finding characteristic rotor speeds and critical frequencies of load jamming

Characteristic speed n_2 shall be found from the condition that root $\nu=1$ appears in polynomial (17):

$$P(1)=\chi-(n-1)b^2=0.$$

Hence, we find:

$$n_2=\frac{\chi}{b^2}+1=\frac{\varepsilon}{b\beta}+1. \qquad (24)$$

In critical cases, frequencies merge and thus become multiples. Therefore, polynomial (17) and its derivative for ν produce multiple roots:

$$\frac{dP(\nu)}{d\nu}=5a_0\nu^4+4a_1\nu^3+3a_2\nu^2+2a_3\nu+a_4=0.$$

Hence, we find a rotor speed n as a function of the load jam frequency ν:

$$n(\nu)=\frac{5(1+\chi)\nu^4-3\nu^2\left(2-b^2\right)+1}{2\nu\left[2\left(\nu^2-1\right)+b^2\right]}. \qquad (25)$$

Substitute (25) in (17). Upon transforms, we obtain:

$$P(\nu)=\frac{(1+\chi)\nu^8-\left(2-b^2\right)(2+3\chi)\nu^6+\left[6+5\chi-b^2\left(4-b^2\right)\right]\nu^4-2\left(2-b^2\right)\nu^2+1}{2\nu\left[2\left(1-\nu^2\right)-b^2\right]}.$$

Hence, we derive the following equation to find the critical (multiple) frequencies of a load jam:

$$F(\nu)=(1+\chi)\nu^8-\left(2-b^2\right)(2+3\chi)\nu^6+$$
$$+\left[6+5\chi-b^2\left(4-b^2\right)\right]\nu^4-2\left(2-b^2\right)\nu^2+1=0. \qquad (26)$$

A procedure for finding critical frequencies of load jamming and corresponding characteristic rotor speeds:

1) find the root of a polynomial (26) in the form of a truncated series by the degrees of a small parameter;

2) the found expansion is substituted in (25), and the resulting expression is expanded by the degrees of a small parameter; the result is the corresponding characteristic speed in the form of a truncated series;

3) check whether the accuracy was lost when using, instead of the root of polynomial (26), a truncated series.

One can see from (26) see that $n(\nu)$ is an odd function. One can see from (25) that if ν is the root of equation (26), then $-\nu$ is the root of this equation. Therefore, hereafter only positive critical frequencies of load jamming will be searched. In this case, a rotor speed will be positive.

Zero approximation. At $\chi=0$, $b=0$, we obtain the following equation for finding the critical frequencies of load jamming in a zero approximation:

$$F_0(\nu)=(\nu-1)^4(\nu+1)^4.$$

Thus, in a zero approximation:

$$\nu_{1-4}^{(0)}=1,\ \nu_{5-8}^{(0)}=-1. \qquad (27)$$

Find characteristic speeds in a zero approximation. From (25), at $\chi=0$, $b=0$, we derive the following equation for finding characteristic speeds in a zero approximation:

$$n^{(0)}(\nu)=\frac{\left(\nu^2-1\right)\left(5\nu^2-1\right)}{4\nu\left(\nu^2-1\right)}=\frac{\left(5\nu^2-1\right)}{4\nu},\quad n_{1-4}^{(0)}(1)=1. \qquad (28)$$

Subsequent approximations. To determine the ratios of a smallness between the parameters, we find:

$$F(1)=b^2\left(3\chi+b^2\right).$$

In order to account for both b and χ at expansions, assume:

$$b\ll 1,\ \chi=\mathrm{X}b^2\ \left(\chi\sim b^2,\ X\sim 1\right). \qquad (29)$$

Find critical frequencies of load jam in the form:

$$\nu_i^{(c)}\approx 1+\nu_i^{(2/3)}b^{2/3}. \qquad (30)$$

Substitute (30) in (26). Substitute the resulting expression in (29). We collect the coefficients at the lowest degree b we obtain:

$$b^{8/3}:\ -8\nu_i^{(2/3)}\left(\mathrm{X}-2\left(\nu_i^{(2/3)}\right)^3\right)=0.$$

Hence, we derive the following two valid solutions:

$$\nu_{2,3}^{(2/3)}=\sqrt[3]{\mathrm{X}/2}=\sqrt[3]{\chi/\left(2b^2\right)},$$
$$\nu_{1,2}^{(2/3)}=0. \qquad (31)$$

Note that a greater quantity corresponds to the emergence of regimes ν_2 and ν_3, and a lower one – the disappearance of regimes ν_1 and ν_2. This is accounted for in the new lower indexes.

By following such an algorithm, we find a first multiple frequency of jamming with an accuracy to the magnitude of a second order smallness, including:

$$\nu_1\approx n\left(1-\chi n^4\right);$$
$$\nu_{2,3}^{(c)}\approx 1+\nu_{2,3}^{(2/3)}b^{2/3}+\nu_{2,3}^{(4/3)}b^{4/3}+\nu_{2,3}^{(2)}b^2=$$
$$=1+\sqrt[3]{\frac{\chi}{2}}-\frac{b^2\left(b^2-2\chi\right)}{6}\sqrt[3]{\frac{2}{\chi}}-\frac{b^2\left(b^2+7\chi\right)}{24\chi}. \qquad (32)$$

Substitute (32) in (25). Substitute the resulting expression in (29). The obtained is expanded into a series by degrees b, we obtain the appropriate characteristic rotor speed:

$$n_1\approx 1+\frac{3}{4}\sqrt[3]{4b^2\mathrm{X}}+\frac{b^2(8\mathrm{X}-1)}{8}\sqrt[3]{\frac{2}{b^2\mathrm{X}}}-$$
$$-\,b^2\frac{\left(53\mathrm{X}^2-14\mathrm{X}-1\right)}{48\mathrm{X}}=$$
$$=1+\frac{3}{4}\sqrt[3]{4\chi}+\frac{\left(8\chi-b^2\right)}{8}\sqrt[3]{\frac{2}{\chi}}+\frac{53}{48}\chi-\frac{7}{24}b^2-\frac{b^4}{48\chi}. \qquad (33)$$

A first characteristic rotor speed is determined with an accuracy to the magnitudes of a second order smallness, inclusive. One can check that the accuracy of determining a characteristic speed is not lost in this case.

Similarly, we find a second multiple frequency of load jamming with an accuracy to the magnitudes of a fourth order smallness, including:

$$\nu_{1,2}^{(c)}\approx 1+\nu^{(2)}b^2+\nu^{(4)}b^4=$$
$$=1+b^2\frac{3\chi+b}{8\chi}+b^4\frac{18\chi^3+15b\chi^2+6b^2\chi+b^3}{64\chi^3}. \qquad (34)$$

Substitute (34) in (25). Substitute the resulting expression in (29). The obtained is expanded into a series by degrees b, we obtain the appropriate characteristic rotor speed:

$$n_3\approx \mathrm{X}+1+b^2\frac{(3\mathrm{X}+1)^2}{16\mathrm{X}}+b^4\frac{\begin{pmatrix}999\mathrm{X}^5+1332\mathrm{X}^4+\\+\,900\mathrm{X}^3+378\mathrm{X}^2+\\+\,93\mathrm{X}+10\end{pmatrix}}{256\mathrm{X}^3(1+5\mathrm{X})}=$$

$$=\frac{\chi}{b^2}+1+\frac{\left(3\chi+b^2\right)^2}{16\chi}+\frac{b^2\begin{pmatrix}999\chi^5+1332\chi^4b^2+\\+\,900\chi^3b^4+378\chi^2b^6+\\+\,93\chi b^8+10b^{10}\end{pmatrix}}{256\chi^3\left(b^2+5\chi\right)}. \qquad (35)$$

A third characteristic rotor speed is determined with an accuracy to the magnitudes of a fourth order smallness, inclusive. One can check that the accuracy of determining a characteristic speed is not lost in this case.

5. 3. 4. Finding the load jamming speeds

Table 1 gives formulae intended for the approximate computation of load jamming frequencies at different ratios of a smallness between the system parameters. These formulae were derived using the results obtained in [3].

Table 1

Dependence of dimensionless load jamming frequencies ν_i on a rotor speed (n) and ratios of a smallness between the system parameters

No. of entry	Ratios of smallness between parameters	Frequency of load jamming – expanding the roots of polynomial (39)
1	$n \ll 1$	
2	$n \gg 1$	$\nu_3 \approx \frac{n}{1+\chi} - \frac{2\chi\left(1-b^2/2\right)}{n}$
3	$n>1$, $\lvert n-1\rvert \sim 1$, $\chi \ll 0$, $h \ll 0$	$\nu_{1/2} \approx 1 \mp \frac{1}{2}\sqrt{\frac{\chi}{n-1}} + \frac{\chi(4n-3)}{8(n-1)^2}$, $\nu_3 \approx n - \frac{\chi(k) n^5}{\left(n^2-1\right)^2}$
4	$n \approx n_1$: $(n-1) \sim \sqrt[3]{\delta}$, $\chi, h \sim \delta$	$\nu_1 \approx 1 - \frac{1}{4}\sqrt[3]{4\chi}\left[1-\frac{w}{3}\right]$, $\nu_{2/3} \approx 1 + \frac{1}{2}\sqrt[3]{4\chi}\left[1 \mp \sqrt{w} + 2\frac{w}{3}\right]$, $w = \frac{4}{3}\frac{(n-1)}{\sqrt[3]{4\chi}} - 1$
5	$n \approx n_2$: $n \sim 1/\delta^2$, $h \sim \delta$, $\sigma \sim 1$ – parameter	$n \approx \frac{\chi}{b^2} + 1 + \frac{9}{16}\chi + \sigma\frac{b^2}{4}$, $\nu_{1/2} \approx 1 + \frac{3}{8}b^2 \mp \frac{b^3}{32\chi}\sqrt{96+81\chi-64\sigma}$, $\nu_3 \approx \frac{n}{1+\chi} - \frac{2\chi}{n}$

In Table 1, δ is a dimensionless positive quantity that is much less than 1 ($0<\delta\ll 1$). It is introduced to define the ratios of a smallness between the system parameters.

5. 4. Investigation of stability of jamming modes

5. 4. 1. Linearization of differential equations of motion in the vicinity of stationary motion

Introduce a disturbed motion:

$$\alpha = \tilde{\alpha} + \psi, \quad u = \tilde{u} + \xi, \quad v = \tilde{v} + \eta, \quad |\psi|, \ |\xi|, \ |\eta| \ll 1. \qquad (36)$$

Then

$$\dot{\alpha} = \dot{\psi}, \quad \dot{u} = \dot{\xi}, \quad \dot{v} = \dot{\eta}; \quad \ddot{\alpha} = \ddot{\psi}, \quad \ddot{u} = \ddot{\xi}, \quad \ddot{v} = \ddot{\eta};$$

$$|\dot{\psi}|, \ |\dot{\xi}|, \ |\dot{\eta}|, \ |\ddot{\psi}|, \ |\ddot{\xi}|, \ |\ddot{\eta}| \ll 1.$$

and with an accuracy to the magnitudes of a first order smallness, including:

$$L^{(1)} \approx \ddot{\psi} + \beta\dot{\psi} + \nu^2\psi\left(\tilde{u}\cos\tilde{\alpha} + \tilde{v}\sin\tilde{\alpha}\right) -$$
$$-\left(\ddot{\xi} - 2\nu\dot{\eta} - \nu^2\xi\right)\sin\tilde{\alpha} + \left(\ddot{\eta} + 2\nu\dot{\xi} - \nu^2\eta\right)\cos\tilde{\alpha} = 0,$$
$$L_u^{(1)} \approx \ddot{\xi} - 2\nu\dot{\eta} - \nu^2\xi + b\left(\dot{\xi} - \nu\eta\right) + \xi -$$
$$-\varepsilon\left(\ddot{\psi}\sin\tilde{\alpha} + 2\nu\dot{\psi}\cos\tilde{\alpha} - \nu^2\psi\sin\tilde{\alpha}\right),$$
$$L_v^{(1)} = \ddot{\eta} + 2\nu\dot{\xi} - \nu^2\eta + b\left(\dot{\eta} + \nu\xi\right) + \eta +$$
$$+\varepsilon\left(\ddot{\psi}\cos\tilde{\alpha} - 2\nu\dot{\psi}\sin\tilde{\alpha} - \nu^2\psi\cos\tilde{\alpha}\right). \qquad (37)$$

By applying (10) and (12), we transform:

$$\tilde{u}\cos\tilde{\alpha} + \tilde{v}\sin\tilde{\alpha} = \tilde{\rho}\left(\cos\tilde{\vartheta}\cos\tilde{\alpha} + \sin\tilde{\vartheta}\sin\tilde{\alpha}\right) =$$
$$= \tilde{\rho}\cos\left(\tilde{\vartheta} - \tilde{\alpha}\right) = \tilde{\rho}\cos\tilde{\varphi}. \qquad (38)$$

Find from (13) and (14):

$$\tilde{\rho}\cos\tilde{\varphi} = \frac{\left(1-\nu^2\right)}{\varepsilon\nu^2}\tilde{\rho}^2 = \frac{\left(1-\nu^2\right)}{\varepsilon\nu^2}\frac{\varepsilon\beta}{\nu b}(n-\nu) =$$
$$= \frac{\left(1-\nu^2\right)}{\nu^3}\frac{\beta}{b}(n-\nu). \qquad (39)$$

We obtain from (17):

$$(n-\nu) = \chi\nu^5 \Big/ \left[\left(1-\nu^2\right)^2 + b^2\nu^2\right]. \qquad (40)$$

Then

$$\tilde{\rho}\cos\tilde{\varphi} = \frac{\left(1-\nu^2\right)}{\nu^3}\frac{\beta}{b}(n-\nu) =$$
$$= \frac{\left(1-\nu^2\right)}{\nu^3}\frac{\beta}{b}\chi\nu^5 \Big/ \left[\left(1-\nu^2\right)^2 + b^2\nu^2\right] = \varepsilon\frac{\nu^2\left(1-\nu^2\right)}{\left(1-\nu^2\right)^2 + \nu^2 b^2}. \qquad (41)$$

Ultimately, we obtain:

$$\tilde{L}^{(1)} = \ddot{\psi} + \beta\dot{\psi} + \varepsilon\frac{\nu^4\left(1-\nu^2\right)}{\left(1-\nu^2\right)^2 + \nu^2 b^2}\psi -$$
$$-\left(\ddot{\xi} - 2\nu\dot{\eta} - \nu^2\xi\right)\sin\tilde{\alpha} + \left(\ddot{\eta} + 2\nu\dot{\xi} - \nu^2\eta\right)\cos\tilde{\alpha} = 0,$$
$$L_u^{(1)} = \ddot{\xi} - 2\nu\dot{\eta} - \nu^2\xi + b\left(\dot{\xi} - \nu\eta\right) + \xi -$$
$$-\varepsilon\left(\ddot{\psi}\sin\tilde{\alpha} + 2\nu\dot{\psi}\cos\tilde{\alpha} - \nu^2\psi\sin\tilde{\alpha}\right),$$
$$L_v^{(1)} = \ddot{\eta} + 2\nu\dot{\xi} - \nu^2\eta + b\left(\dot{\eta} + \nu\xi\right) + \eta +$$
$$+\varepsilon\left(\ddot{\psi}\cos\tilde{\alpha} - 2\nu\dot{\psi}\sin\tilde{\alpha} - \nu^2\psi\cos\tilde{\alpha}\right) \qquad (42)$$

– differential equations of first approximation.

5. 4. 2. Characteristic equation and conditions of stability

The characteristic equation of system (42) in the form of a determinant takes the form:

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \qquad (43)$$

where

$$a_{11}=\lambda^2+\beta\lambda+\varepsilon z,\quad z=\frac{\nu^4\left(1-\nu^2\right)}{\left(1-\nu^2\right)^2+\nu^2 b^2},$$

$$a_{12}=-\left(\lambda^2-\nu^2\right)\sin\tilde{\alpha}+2\nu\lambda\cos\tilde{\alpha}=0,$$

$$a_{13}=2\nu\lambda\sin\tilde{\alpha}+\left(\lambda^2-\nu^2\right)\cos\tilde{\alpha}=0,$$

$$a_{21}=-\varepsilon(\lambda^2\sin\tilde{\alpha}+2\nu\lambda\cos\tilde{\alpha}-\nu^2\sin\tilde{\alpha}),$$

$$a_{31}=\varepsilon(\lambda^2\cos\tilde{\alpha}-2\lambda\nu\sin\tilde{\alpha}-\nu^2\cos\tilde{\alpha}),$$

$$a_{22}=a_{33}=\lambda^2+b\lambda+1-\nu^2,$$

$$a_{32}=-a_{23}=\nu\left(2\lambda+b\right). \quad (44)$$

From (43), (44), we obtain the following characteristic equation in the form of a polynomial:

$$D\left(\lambda\right)=\left(b_0\lambda^5+b_1\lambda^4+b_2\lambda^3+b_3\lambda^2+b_4\lambda+b_5\right)\lambda, \quad (45)$$

where

$$b_0=1-\varepsilon,\ b_1=b\left(2-\varepsilon\right)+\beta,$$

$$b_2=\left(1+\nu^2\right)\left(2-\varepsilon\right)+z\varepsilon+b\left(b+2\beta\right),$$

$$b_3=2\left(\beta+b\right)\left(1+\nu^2\right)+2\varepsilon\left(\nu^2+z\right)b+\beta b^2,$$

$$b_4=\left(1-\nu^2\right)^2+b^2\nu^2+2\beta b\left(1+\nu^2\right)+$$
$$+\varepsilon\left\{\left[2\left(1+\nu^2\right)+b^2\right]z+\nu^2\left(6+\nu^2\right)\right\},$$

$$b_5=\beta\left[\left(1-\nu^2\right)^2+b^2\nu^2\right]+\varepsilon b\left[2\left(1+\nu^2\right)z+3\nu^4\right]=$$
$$=\left\{\left(1-\nu^2\right)^2+b^2\nu^2+\chi\left[2\left(1+\nu^2\right)z+3\nu^4\right]\right\}\beta=$$
$$=\beta F\left(\nu\right)\Big/\left[\left(1-\nu^2\right)^2+b^2\nu^2\right], \quad (46)$$

Note that the presence of a single zero root in polynomial (45) is due to the fact that the stability of a one-parameter family of steady motions is examined, rather than the fact that this case is critical [16].

According to the Raus-Hurwitz criterion, the conditions for asymptomatic stability (the negative of the real parts of nonzero roots of characteristic equation (45)) take the form [16]:

$$b_i>0,\ /\ i=\overline{0,5}/,\quad \Delta_2>0,\ \Delta_3>0,\ \Delta_4>0,$$

where

$$\Delta_2=\begin{vmatrix} b_1 & b_3\\ b_0 & b_2\end{vmatrix}>0,\quad \Delta_3=\begin{vmatrix} b_1 & b_3 & b_5\\ b_0 & b_2 & b_4\\ 0 & b_1 & b_3\end{vmatrix}>0,$$

$$\Delta_4=\begin{vmatrix} b_1 & b_3 & b_5 & 0\\ b_0 & b_2 & b_4 & 0\\ 0 & b_1 & b_3 & b_5\\ 0 & b_0 & b_2 & b_4\end{vmatrix}>0. \quad (47)$$

Note that the coefficients (46) of the polynomial and the conditions of stability (47) do not include angle $\tilde{\alpha}$, which is the parameter that distinguishes a certain motion from a one-parameter family of steady motions. Therefore, (47) are the conditions of stability (instability) of the entire one-parameter family of motions. Consider the required condition for stability $b_5>0$. It will be met when and only when $F(\nu)>0$. Since $F(0)=1>0$, $F(+\infty)>0$, then:

$$\forall \nu_1,\nu_2,\nu_3\ F\left(\nu_1\right)>0,\ F(\nu_2)<0,\ F(\nu_3)>0. \quad (48)$$

Therefore, the jamming modes ν_1, ν_3 can be locally asymptomatically stable in the region of their natural existence, while a jamming mode ν_2 is unstable. Since, in critical cases, b_5 takes a zero value, that might be the critical cases of a single zero root and the character of a stability loss is the aperiodic distance from the non-disturbed motion. Should all other conditions for stability be met, a complete bifurcation diagram of steady motions could be obtained, as the conditions for stability would not yield other bifurcation points [16].

5. 5. Computational experiment

Computations are performed for the following values of coefficients for the dimensionless differential equations of motion (2):

$$b=0.1;\ \beta=0.01;\ \varepsilon=0{,}01;\ \chi=\varepsilon b/\beta=0.1.$$

Table 2 gives the characteristic rotor speeds and the corresponding multiple speeds of load jamming, calculated from precise and approximate formulae.

Table 2

Characteristic (bifurcation) rotor speeds and critical velocities of load jamming

No.	Quantity	Precise value	Approximate value
1	$\nu_{2,3}^{(c)}$	1.433660015274	1.45140137439
2	n_1	1.967573850609	1.928132631101
3	ν_1	1	
4	n_2	11	
5	$\nu_{1,2}^{(c)}$	1.003905905625	1.003905564063
6	n_3	11.060410678753	11.60936877604

The greatest error of 2 % is produced by formulae (32) and (33) when computing the approximate values for quantities $\nu_{2,3}^{(c)}$ and n_1.

Our computations demonstrated that if $b_5>0$, all other stability conditions (47) are met automatically.

Differential equations of motion (2) were recorded in the normal form and integrated at different initial conditions and various rotor speeds.

The stability or instability of jamming modes in the vicinity of characteristic speed n_1 is illustrated in Fig. 4.

The stability or instability of jamming modes in the vicinity of characteristic speed n_3 is illustrated in Fig. 5.

Our experiments demonstrated the following:

1) in the range of angular velocities of rotor rotation $(0, n_1)$, jamming mode ν_1 is globally asymptomatically stable;

2) in the range of angular velocities of rotor rotation (n_1, n_3), jamming modes ν_1, ν_3 are locally asymptomatically stable, and:

– when a rotor speed approaches n_3, the region of attraction by jamming mode ν_1 decreases, and ν_3 – increases;

– at the acceleration of an initially immovable load, it gets stuck at frequency ν_1, if $0<n<6.2$, and gets stuck at frequency ν_3, if $n>6.3$;

3) in the range of angular velocities of rotor rotation $(n_3, +\infty)$, jamming mode ν_3 is globally asymptotically stable;

4) jamming mode n_2 is unstable;

5) no other steady modes of motion were identified.

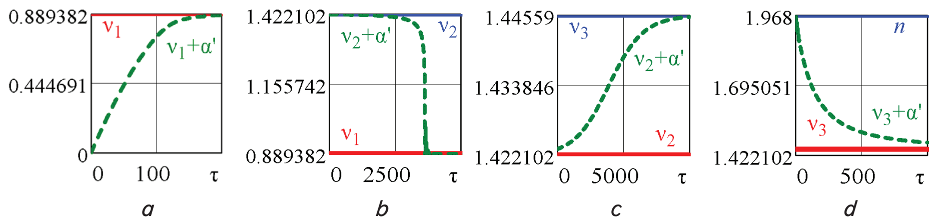

Fig. 4. Stability or instability of jamming modes in the vicinity of characteristic velocity n_1 ($n=1.968$; $\nu_1=0.889382$; $\nu_2=1.4221015$; $\nu_3=1.4455905$):
a — local asymptomatic stability of jamming mode ν_1 at load acceleration; *b* — instability of jamming mode ν_2 and a system's transition to jamming mode ν_1 (the initial load rotation speed is slightly less than ν_2); *c* — instability of jamming mode ν_2 and a system's transition to jamming mode ν_3 (the initial load rotation speed is slightly greater than ν_2); *d* — local asymptomatic stability of jamming mode ν_3 when the rotation speed of a load, initially accelerated to a rotor speed, is falling

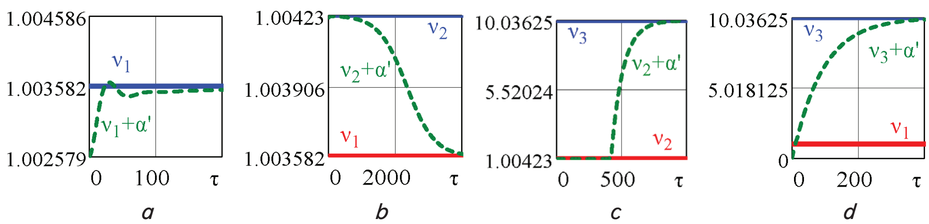

Fig. 5. Stability or instability of jamming modes in the vicinity of characteristic velocity n_2 ($n=11.06$; $\nu_1=1.003582$; $\nu_2=1.00423$; $\nu_3=10.03625$):
a — local asymptomatic stability of jamming mode ν_1 when the initial speed of load rotation is slightly less than ν_1; *b* — instability of jamming mode ν_2 and a system's transition to jamming mode ν_1 (the initial load rotation speed is slightly less than ν_2); *c* — instability of jamming mode ν_2 and a system's transition to jamming mode ν_3 (the initial load rotation speed is slightly greater than ν_2); *d* — local asymptomatic stability of jamming mode ν_3 even at load acceleration (due to a small region of attracting by mode ν_1)

Since no other steady motion modes of the system were identified, it is possible, for the considered system, to build a complete bifurcation diagram, in which the bifurcation parameter chosen was a rotor speed. Such a diagram can be represented in the form of charts, shown in Fig. 3.

6. Discussion of the results obtained from studying a system's steady motions

Our theoretical study shows that the system a rotor – a load demonstrates steady motion modes at which a load's center of mass rotates around the rotor longitudinal axis at a constant angular velocity. In the coordinate system that rotates synchronously to a load, these motions are stationary.

In the absence of resistance forces, a load during such motions rotates synchronously to the rotor.

The emergence of any small forces of viscous resistance radically changes the steady motion modes of the system. Synchronous rotation regimes disappear and there appear the modes (one-parameter families of steady motions) at which a load lags behind the rotor.

Consequently, the mechanical system is not rough in relation to the resistance forces [16]. That is why the theories of such systems (rotors with passive auto-balancers) that are built without considering resistance forces do not reflect the actual properties of such systems and cannot be used for practical purposes.

In cases that are important in terms of practice, in particular, when the forces of external and internal resistance are small, the mass of a load is much smaller than the rotor mass, etc. $(b,\chi \ll 1)$, there are three characteristic rotor speeds n_1, n_2, n_3. In this case, they are all above the resonance $(1<n_1 \ll n_2 < n_3)$ and, at the rotor speeds:

– smaller than n_1 $(0<n<n_1)$, there is a single frequency of load jamming ν_1, with $0<\nu_1<1$;

– larger than n_1, but smaller than n_2 $(n_1<n<n_2)$, there are three frequencies of load jamming $\nu_{1,2,3}$, such that $0<\nu_1<1<\nu_2<\nu_3<n$;

– larger than n_2, but smaller than n_3 $(n_2<n<n_3)$, there are three frequencies of load jamming $\nu_{1,2,3}$, such that $1<\nu_1<\nu_2\ll\nu_3<n$;

– exceeding n_3 $(n>n_3)$, there is a single frequency of load jamming ν_3, such that $1\ll\nu_3<n$.

The characteristic rotor speeds n_1, n_3 are the points of bifurcation of motions. When a rotor speed passes these velocities, jamming modes can acquire or lose stability.

By applying the first Lyapunov method, it was established that the second load jamming mode is always unstable, and the first and third modes can be stable (in the regions of natural existence). Our computational experiment for the case when $\tilde{b},\chi \ll 1$ shows that:

– in the range of angular velocities of rotor rotation $(0, n_1)$, jamming mode ν_1 is globally asymptotically stable;

– in the range of angular velocities of rotor rotation (n_1, n_3), jamming modes ν_1, ν_3 are locally asymptotically stable, and the type of a mode to be set depends on the initial conditions;

– in the range of angular velocities of rotor speed $(n_3, +\infty)$, jamming mode ν_3 is globally asymptotically stable;

– jamming mode ν_2 is unstable;

– no other steady motion modes of the system were not identified;

– for the system under consideration, it is possible to construct a complete bifurcation diagram, in which the bifurcation parameter chosen is a rotor speed, in particular, it can be represented in the form of charts, shown in Fig. 3.

Our studies have found a drawback in the technique for excitation of resonance oscillations by a load (passive auto-balancers). The technique cannot be used over the entire range of the existence and stability of jamming mode ν_1, due to a reduction in the attraction region by this mode. However, the range of resonance velocities from 1 to 6.2, identified for a particular case, is large enough for practical application.

The solved problem can be considered to be a model. In it, we found all possible steady motions of the system and explored their stability. It is shown that in practical terms not only the stability (by Lyapunov) of a certain motion mode is important, but also the region of attraction of this mode, if there are several steady modes. The disadvantage of this work is that the analytical studies were conducted at different depths.

In the future, it is planned to investigate the steady motion modes of two-mass and three-mass resonance vibration machines with a vibration exciter in the form of a pendulum, a ball, or a roller.

7. Conclusions

We have analytically investigated the steady motion modes of the system, composed of a balanced rotor on isotropic elastic and viscous supports, and a load (a ball, a roller, a pendulum), installed inside the rotor with a possibility for a relative motion. In this case, the pendulum is freely mounted onto the rotor shaft, while the ball or roller roll without slipping along a circular path centered on the longitudinal axis of the rotor:

1. We have found all the steady motion modes of the system under which a load rotates at a constant angular velocity. In the coordinate system that synchronously rotates with the load, these motions are stationary and:

– in the absence of resistance forces in the system, the load rotates synchronously to the rotor;

– in the presence of resistance forces in the system, the load lags behind the rotor.

The load jamming regimes are the one-parameter families of steady motions. Each jamming mode is characterized by the corresponding jam frequency.

Depending on the system parameters and a rotor speed, there may exist one or three possible speeds of a load jam.

2. If, at any rotor speed there is only one angular velocity of a load jam, then the corresponding motion mode (a one-parameter family) is globally asymptomatically steady. If the number of jamming speeds varies depending on the angular velocity of rotor rotation, the asymptomatically stable are:

– the only existing jamming mode (globally stable when there are no others);

– the jamming modes with the lowest and greatest speeds.

3. A load jamming mode with the lowest angular velocity (close to resonance) can be used in order to excite resonance oscillations in vibration machines. The highest frequency of a load jam is close to a rotor speed. This mode can be used in order to excite the non-resonance oscillations in vibration machines.

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ESTABLISHING THE EFFECT OF A DECREASE IN POWER INTENSITY OF SELF-OSCILLATING GRINDING IN A TUMBLING MILL WITH A REDUCTION IN AN INTRACHAMBER FILL

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Оцінено вплив величини ступеня заповнення камери завантаженням на ефективність автоколивного процесу подрібнення в барабанному млині.

За допомогою наближеного аналітико-експериментального методу встановлено динамічний ефект підвищення автоколивної ударної дії мольного завантаження на подрібнюваний матеріал порівняно із традиційним усталеним режимом руху. Виявлено суттєве зростання середніх сум вертикальних складових автоколивних ударних імпульсів та середніх сум потужностей таких складових зі зменшенням заповнення камери. Прояв ефекту зумовлено збільшенням розмаху автоколивань при зменшенні заповнення. Виявлено зростання максимальних значень імпульсів приблизно у 2,4 рази при ступені заповнення κ***=0,45, у 3,1 рази при*** κ***=0,35 та у 5,8 рази при*** κ***=0,25. Встановлено зростання максимальних значень потужностей імпульсів у 5,7 рази при*** κ***=0,45, у 9,6 рази при*** κ***=0,35 та у 45,5 рази при*** κ***=0,25.***

Експериментально встановлено технологічний ефект суттєвого спадання питомої енергоємності та зростання продуктивності інноваційного автоколивного процесу подрібнення, порівняно із характеристиками традиційного усталеного процесу, зі зменшенням заповнення камери.

Було розглянуто процес помелу цементного клінкера при повному заповненні частинками подрібнюваного матеріалу проміжків між кульовими мольними тілами із відносним розміром 0,026. Встановлено, що під час самозбудження автоколивань енергоємність подрібнення спадає, а продуктивність зростає. Виявлено зниження відносної питомої енергоємності на 27 % при κ***=0,45, на 42 % при*** κ***=0,35 та на 55 % при*** κ***=0,25. Встановлено підвищення відносної продуктивності на 7 % при*** κ***=0,45, на 30 % при*** κ***=0,35 та на 46 % при*** κ***=0,25.***

Встановлені в роботі ефекти дозволяють прогнозувати раціональні параметри автоколивного процесу подрібнення в барабанному млині при варіації ступеня заповнення камери

Ключові слова: барабанний млин, ступінь заповнення камери, автоколивання завантаження, питома енергоємність подрібнення

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1. Introduction

Due to a series of their operational advantages, the tumbling type mills remain to be the main equipment in many industries for small- and large-tonnage fine grinding of solid materials.

Replacement of the conventional steady-state grinding process with a novel self-oscillating process improves existing equipment of relatively low power efficiency. Use of the phenomenon of excitation of self-oscillations makes it possible to apply conventional solutions to designing the tumbling mills with a smooth working chamber surface without additional activating elevators in a form of protruding elements which undergo rapid abrasive wear.

On the other hand, significant variability of the self-excited pulsation behavior of the rotating chamber fill de-