Розроблено метод визначення функції розподілу часу стоянці судна під навантаженням згідно прямого варіанту (тобто без участі складу) виконання навантажувальних робіт на портовому терміналі в умовах нерівномірності завезення вантажу залізничним транспортом. Враховуються фактори невизначеності та ризику (випадкові моменти прибуття вагонів з вантажем на термінал та обсяг вантажів у вагонах). Запропоновано використання апарату лінійчатих марковських процесів, які описують динаміку прибуття вагонних партій та навантаження судна. При цьому вважається, що інтервали між сусідніми моментами часу, в які вагоні з вантажем прибувають на термінал, є випадковими величинами, що розподілені згідно експоненційного закону. Перевантаження вантажу із вагонів на судно виконується із постійною інтенсивністю. Детально розглянуто випадки, коли обсяг вантажу у вагонах є випадковою величиною або фіксований (постійний). Для знаходження щільностей ймовірностей та ймовірностей станів відповідного марковського процесу виведено систему лінійних диференціальних рівнянь та початкових умов. Знайдено вирішення цієї системи рівнянь в термінах перетворення Лапласу, зокрема функцію розподілу часу стоянці судна з урахуванням можливих перерв в очікуванні підвозу вантажу вагонами. Для випадку постійного розміру вантажних партій на вагонах також знайдено відповідну функцію розподілу часу стоянці судна та знайдена її асимптотика при великої вантажопідйомності судна на підставі центральної граничної теореми. На основі одержаних результатів сформульовано задачу знаходження критерію доцільності страхування ризику перевищення сталійного (договірного) часу стоянці судна. Доведено, що отримані результати важливі для практики роботи портового оператора та судноплавних компаній, оскільки дозволяють зменшити ризик перевищення часу стоянці судна під вантажними операціями. Наведено числову ілюстрацію методу, що пропонується

Ключові слова: судно, портовий термінал, вагони з вантажем, ризик перевищення сталії, страхування ризику

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DEVELOPMENT OF A METHOD FOR DETERMINATION OF SHIP'S LOADING TIME DISTRIBUTION UNDER IRREGULAR CARGO ARRIVAL

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1. Introduction

The quality of stevedoring activities in a port is largely determined by the organization of coordinated interaction of vehicle flows for loading and unloading. For example, the charterer of the ship arriving at a port terminal to load cargo delivered by ground transport should not be at risk of exceeding the so-called laytime (i. e. contractual) due to waiting for cargo delivery during loading. This risk is reduced to a certain extent if cargo is loaded on the ship from the warehouse (i. e., under the warehouse option), but the rate of cargo loading is reduced. During cargo transshipment directly from loaded vehicles on the ship (i. e., under the direct option), transshipment rate increases. However, there are risks of downtime of ground transport vehicles due to waiting for the arrival of ships in violation of the schedule. Given the specifics of maritime transport (objectively existing irregular arrival of ships in the port), these risks cannot be completely avoided, so they should be taken into account when organizing stevedoring operations in the port. In addition, the loading rate of the fleet and rolling stock depends on many factors. These include cargo transportability, weather conditions, fluctuations in the output of port workers, organization of transshipment operations, as well as organization of technical operation of transshipment equipment in the event of sudden failures of the latter. For these reasons, the real rate is variable and may deviate at any time from the level established by contractual obligations, for example, gross ship handling rate.

Therefore, in the described situation, there are risks of the charterer (or port operator) associated with the possibility that the actual berthing time of the ship exceeds the laytime. This kind of risk can be quantified by the probability that the actual time of ship loading exceeds the laytime. Finding the indicated probability as a function of some controlled parameters will reduce this risk. However, finding the mentioned probability presents certain analytical difficulties, which require a special study.

From the foregoing, the problem described is relevant to the theory and practice of stevedoring in the port.

2. Literature review and problem statement

This problem has been given considerable attention in the scientific literature during the last decade [1–8]. This is due to the increased interest of businesses and researchers in risk management in maritime transport as a whole and organizing multimodal (intermodal) transportation, in particular.

So, in [1], the functioning of the "port terminal – transshipment fleet" logistics system is investigated by the methods of Markov processes. This system is presented as some kind of service system with randomly fluctuating service capacity. For it, the problem of determining the optimum volume of warehouse replenishment with spare parts (SP) for the repair of failed transshipment machines is formulated. The goal of the "port terminal – transshipment fleet" logistics system is achieved by performing logistics functions related to transshipment, storage of cargo, maintenance of port transshipment equipment, etc. For this service system, a system of algebraic equations for stationary state probabilities of the introduced Markov process is derived. A method is developed to control the operational reliability of transshipment fleet of the port terminal based on determining the supply volume of SP. As a criterion of optimality, minimum average total costs per unit of time for the purchase, delivery and storage of SP, machine repair, as well as losses due to ship demurrage caused by machine failures, is chosen. However, the factor of cargo delivery (pickup) for the ship is not taken into account.

In [2], a probabilistic model of port terminal operation is developed, taking into account the irregular cargo delivery by ground transport and pickup by ships in order to find a number of indicators of port terminal capacity. At the same time, the task of assessing the increase in the berthing time of individual ships, taking into account the possible lack of cargo in the warehouse is not considered there.

Since the studied problems relate to the organization of effective interaction of different means of transport within intermodal transportation, it should be noted that the theoretical level of research in this area of transport science is still not sufficient [3–5].

The review papers on this issue [3, 4] discuss the problems associated with the operational and strategic planning of multimodal hubs and classify them in terms of logistics principles. It is noted that perfect coordination of intermodal transport modes, especially maritime and land, cannot be achieved in practice because of the complexity of organizing a strict traffic schedule. At the same time, the cited works hardly address such an important problem as management of risks encountered in multimodal systems. When solving certain problems, either scheduling theory (which implies a clear traffic schedule of vehicles, which is usually not true) [5], or simulation is used [6]. In the latter case, a reliable forecast of possible additional demurrage of the ship under loading operations, taking into account the operation of intermodal transport means cannot be made. In [7], the problem of incurring additional costs of the shipowner caused by the relocation of ships between the port berths in order to reduce the total berthing time of ships, is considered close to the main topic of this work. But the work does not take into account the possible interaction of ships and land transport.

The monograph [8] describes the mathematical apparatus and some physical concepts that can be used to create (modernize) an integrated intelligent transport system. However, these concepts do not take into account random factors typical for the operation of port terminals, where there is an interaction of different transport modes, as well as the objectively inherent irregular arrival of vehicles.

3. The aim and objectives of the study

The aim of the study is to develop a method for finding the probability that the actual loading time of the ship exceeds the laytime for the case of the direct option of the transshipment process, as well as recommendations for reducing the financial consequences of this excess. The practical result of finding the probability distribution of the actual ship berthing time is that before cargo loading the charterer (or shipowner) can predict possible ship demurrage and associated losses, as well as take appropriate measures to reduce the corresponding losses, for example, by insuring the specified risk.

To achieve the aim, the following tasks were set:

– to develop a probabilistic model of port terminal operation to account for irregular cargo delivery by rail to the port terminal using linear Markov processes;

– to find an analytical expression for calculating the probability of ship demurrage under loading operations caused by the late cargo delivery by rail organized under the direct option (i. e., without cargo warehousing);

– to develop an asymptotic formula for the specified demurrage probability for large deadweight tonnage values and to conduct numerical analysis;

– to develop a criterion of expediency of insuring the risk of additional ship demurrage due to excessive waiting for cargo delivery to the terminal.

4. Prerequisites for building a probabilistic model of the port terminal

First, we consider the formal statement of the problem. Suppose that at the initial moment of time, at the terminal berth there is a ship for cargo loading and its net deadweight tonnage is a random variable with the distribution function $H(x)$. The ship is loaded under the direct option, that is, directly from railcars, at a predetermined rate W. At the initial moment of time, there are no loaded cars at the terminal, so the ship is waiting for their arrival. It is considered that loaded trains arrive at the terminal regularly in time, moreover, the train flow is described by the recovery-accumulation process model [9, 10]. This assumption means the following:

a) time intervals between arrivals of trains are mutually independent random variables τ_1, τ_2,..., subordinate to the same distribution law $A(t)$;

b) train capacities are mutually independent random variables distributed according to the same law $G(x)$ (and independent of random time intervals between train arrivals).

It follows from the assumptions that the ship loading time (denoted by θ) is a random variable, since it may be interrupted with a positive probability several times for a random period due to waiting for cargo delivery by trains. In addition, because train capacities can also fluctuate randomly. Thus, there is a risk that the ship berthing time exceeds the laytime stipulated by the contract with the stevedoring company. The problem is to find an analytical expression for the distribution function of the ship loading time.

5. Solution to the problem of finding the probability of ship demurrage under loading using linear Markov processes

Consider the case when

$$A(t)=P\{\tau_1<t\}=1-e^{-\lambda t}, t>0,$$

where $1/\lambda$ is the average time interval between arrivals of loaded trains to the terminal.

We also assume that

$$G(x)=1-e^{-x/g}\sum_{k=0}^{m-1}\frac{(x/g)^k}{k!}, \quad x>0,$$

i. e., train capacities are subordinate to the m-th order Erlang distribution with an average value of mg. Note that in this case, the time of cargo loading from the train on the ship is also distributed according to the m-th order Erlang law with an average value of m/μ, where $\mu=W/g$. Only one train can be unloaded at a time. The queue length of the trains waiting for unloading is limited by the number R.

To solve the formulated problem, we use the fact that the ship loading process, by virtue of the above assumptions, can be reduced to the linear Markov process [9] if we use the Erlang pseudo-phase method [10].

In what follows, we limit ourselves to studying the case of $R=0$ (i. e., only one train can be at the terminal at any time). This assumption can also be interpreted as follows. Cargo is brought to the terminal periodically by one train, wherein the time interval from the end of unloading to the next moment of arrival with cargo is distributed exponentially with the parameter λ.

We introduce the following symbols:

– $p_0(x, t)\mathrm{d}x$, $x>0$ – the probability that at time t the ship is under loading, but waiting for the train to arrive at the terminal, and the amount of cargo loaded on the ship is within $(x, x+\mathrm{d}x)$;

– $p_{i1}(x, t)\mathrm{d}x$, $x>0$, $i=0, 1, ..., m-1$, – the probability of the next event. At time t, the ship is under loading, the number of the current phase of the Erlang distribution of the amount of cargo in the cars is i, and from the beginning of loading, the amount of cargo within $(x, x+\mathrm{d}x)$ is loaded;

– $p(t)$ – the probability that by the time t ship handling is completed (this state of the Markov process under consideration is absorbing).

It's obvious that

$$P\{\theta\le t\}=p(t).$$

Denote

$$\pi_0(x,t)=\frac{p_0(x,t)}{1-H(x)}, \quad \pi_{i1}(x,t)=\frac{p_{i1}(x,t)}{1-H(x)}, \quad i=0,1,...,m-1,$$

where $H(x)=\mathbf{P}\{\gamma\le x\}$, γ is the deadweight tonnage of the ship.

To determine the functions $\pi_i(x, t)$, $p(t)$ using standard probabilistic reasoning [9], the following system of differential equations can be derived:

$$(\partial/\partial t)\pi_0(x,t)=-\lambda\pi_0(x,t)+\mu\pi_{m-1,1}(x,t),$$

$$(\partial/\partial t+W\partial/\partial x)\pi_{01}(x,t)=-\mu\pi_{01}(x,t)+\lambda\pi_0(x,t), \quad (1)$$

$$(\partial/\partial t+W\partial/\partial x)\pi_{i1}(x,t)=-\mu\pi_{i1}(x,t)+\mu\pi_{i-1,1}(x,t),$$

$$x>0,\ i=0, 1, ..., m-1,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}p(t)=W\int_0^\infty(\pi_0(x,t)+\sum_{i=0}^{m-1}\pi_{i1}(x,t)\,\mathrm{d}H(x), \quad t>0. \quad (2)$$

Initial and boundary conditions for the system (1), (2) are as follows:

$$\pi_0(x,0)=\delta(x),\ \pi_{i1}(x,0)=0,\ i=0, 1, ..., m-1, \quad (3)$$

$$p(0)=0,\ W\pi_0(0,t)=\delta(t),\ \pi_{i1}(0,t)=0,\ i=0, 1, ..., m-1, \quad (4)$$

where $\delta(x)$ is the Dirac delta function. According to (3), at the initial time $t=0$, there is no loaded train at the terminal and the ship is waiting for arrival.

Applying the Laplace transform of the variable t to the system of equations (1), we obtain the following system of ordinary differential equations:

$$0=-(s+\lambda)\pi_0^*(x,s)+\mu\pi_{m-1,1}^*(x,s),$$

$$W\partial/\partial x\pi_{01}^*(x,s)=-\mu\pi_{01}^*(x,s)+\lambda\pi_0^*(x,s), \quad (5)$$

$$W\partial/\partial x\pi_{i1}^*(x,s)=-\mu\pi_{i1}^*(x,s)+\mu\pi_{i-1,1}^*(x,s),$$

$$x>0, \quad i=0,1,...,m-1,$$

$$sp^*(s)=W\int_0^\infty\sum_{i=0}^{m-1}\pi_{i1}^*(x,s)\,\mathrm{d}H(x), \quad (6)$$

where

$$\pi_{i1}^*(x,s)=\int_0^\infty e^{-st}\pi_{i1}(x,t)\,\mathrm{d}t, \quad i=0,1,...,m;$$

$$p^*(s)=\int_0^\infty e^{-st}p(t)\,\mathrm{d}t, \quad Res>0.$$

The system (5) shall be solved under the initial conditions ((3), (4)):

$$W\pi_{i1}^*(0,s)=\delta_{i0}, \quad i=0,1,...,m-1, \quad (7)$$

where δ_{i0} is the Kronecker delta.

The solution of the system of equations (5) under (7) and arbitrary m can be found by standard methods for solving systems of first-order linear differential equations, for example, by the Laplace transform of the variable x. For small values of m, the solution is quite simple. We give this solution for two special cases $m=1$ and $m=2$.

If $m=1$, then from (5) we get

$$0=-(\lambda+s)\ \pi_0^*(x,s)+\mu\pi_{01}^*(x,s),$$

$$W\partial/\partial x\pi_{01}^*(x,s)=$$
$$=-(\mu+s)\pi_{01}^*(x,s)+\lambda\pi_0^*(x,s), \quad x>0.$$

Integration of this system with allowance for (7) gives

$$\pi_{01}^*(x,s)=\frac{1}{W}\exp\left[\left(-s-\mu+\frac{\lambda\mu}{\lambda+s}\right)x/W\right], \quad Res>0.$$

Using these equalities, from (6) we find

$$sp^*(s)=W\int_0^\infty \pi_{01}^*(x,s)\,\mathrm{d}H(x)=$$
$$=\int_0^\infty e^{-xs(1+\mu/(s+\lambda))/W}\mathrm{d}H(x)=h\big(s(1+\mu/(s+\lambda))/W\big), \tag{8}$$

where $h(s)$ is the Laplace-Stieltjes transform of $H(x)$.

From (8), in particular, it follows that the first two initial distribution moments of the random variable θ are equal to

$$M\theta=-\frac{\mathrm{d}}{\mathrm{d}s}\big(sp^*(s)\big)\Big|_{s=0}=\frac{h_1}{W}\left(1+\frac{\mu}{\lambda}\right),$$
$$M\theta^2=\frac{\mathrm{d}^2}{\mathrm{d}s^2}\big(sp^*(s)\big)\Big|_{s=0}=\frac{2h_1\mu}{\lambda^2 W}+\frac{h_2}{W^2}\left(1+\frac{\mu}{\lambda}\right), \tag{9}$$

where h_1, h_2 are the first two moments of the distribution function $H(x)$.

With the help of (9), the lower bound of the distribution of the random variable θ can be found using one of the modifications of the Chebyshev inequality [10]:

$$P\{\theta\le t\}\ge\frac{(M\theta-t)^2}{(M\theta-t)^2+M\theta^2-(M\theta)^2}=$$
$$=\frac{\left[h_1(1+\mu/\lambda)-Wt\right]^2}{\left[h_1(1+\mu/\lambda)-Wt\right]^2+2h_1\mu W/\lambda^2++h_2(1+\mu/\lambda)-h_1^2(1+\mu/\lambda)^2}. \tag{10}$$

If the deadweight tonnage of the ship is known in advance and is D, then

$H(x)=0$, if $x<D$, $H(x)=1$ otherwise,

and from (8) we get

$sp^*(s)=\exp[-sT(1+\mu/(s+\lambda))]$,

where $T=D/W$. This expression is the Laplace transform of the following function

$$p(t)=\begin{cases} e^{-\mu T}\left[1+\sum\limits_{n=1}^{\infty}\frac{(\mu T)^n}{n!}E_n(t-T)\right], & t>T,\\ 0, & t\le T,\end{cases} \tag{11}$$

where

$$E_n(t)=1-e^{-\lambda t}\sum_{k=0}^{n-1}\frac{(\lambda t)^k}{k!}.$$

Note that in the formula (11), the parameter $\mu T=D/g$.

For $m=2$, the system (5) takes the following form:

$$0=-(\lambda+s)\pi_0^*(x,s)+\mu\pi_{11}^*(x,s), \tag{12}$$
$$W\partial/\partial x\pi_{01}^*(x,s)=-(\mu+s)\pi_{01}^*(x,s)+\lambda\pi_0^*(x,s),$$
$$W\partial/\partial x\pi_{11}^*(x,s)=-(\mu+s)\pi_{11}^*(x,s)+\mu\pi_{01}^*(x,s), \tag{13}$$

Expressing $\pi_0^*(x,s)$ through $\pi_{11}^*(x,s)$ from (12) and substituting in the first equation of the system (13), we come to the following system of two first-order linear differential equations:

$$W\partial/\partial x\pi_{01}^*(x,s)=-(\mu+s)\pi_{01}^*(x,s)+$$
$$+\lambda\mu\pi_{11}^*(x,s)/(\lambda+s),$$
$$W\partial/\partial x\pi_{11}^*(x,s)=$$
$$=-(\mu+s)\pi_{11}^*(x,s)+\mu\pi_{01}^*(x,s),\quad x>0. \tag{14}$$

To solve the system of equations (14), we use the Laplace transform of the variable x. After applying this transform, taking into account (7), we come to the following system of algebraic equations with respect to the functions

$$\pi_{i1}^{**}(z,s)=\int_0^\infty e^{-zx}\pi_{i1}^*(x,s)\,\mathrm{d}x,\quad Rez>0;\quad i=0,1;$$

$$(Wz+s+\mu)\pi_{01}^{**}(x,s)-\frac{\lambda\mu}{\lambda+s}\pi_{11}^{**}(x,s)=1,$$

$$\mu\pi_{01}^{**}(z,s)-(Wz+s+\mu)\pi_{11}^{**}(x,s)=0,\quad Rez>0,\quad Res>0.$$

The solution to the last system of equations is:

$$\pi_{01}^{**}(z,s)=\frac{(\lambda+s)(s+\mu+Wz)}{\lambda\mu^2-(Wz+s+\mu)^2(\lambda+s)}, \tag{15}$$

$$\pi_{11}^{**}(z,s)=\frac{\mu}{Wz+s+\mu}\pi_{01}^{**}(z,s). \tag{16}$$

Using the rules of recovery of the original time function from its transform for the Laplace transform of the variable z, from (15) we find:

$$\pi_{01}^*(x,s)=\frac{1}{2\mu W}\sqrt{\frac{\lambda+s}{\lambda}}\times$$
$$\times\left[(s+\mu+z_1W)e^{z_1x}-(s+\mu+z_2W)e^{z_2x}\right], \tag{17}$$

where

$$z_1\equiv z_1(s)=\frac{1}{W}\left[\mu\sqrt{\frac{\lambda}{\lambda+s}}-s-\mu\right],$$

$$z_2\equiv z_2(s)=-\frac{1}{W}\left[\mu\sqrt{\frac{\lambda}{\lambda+s}}+s+\mu\right].$$

From the last equalities, in particular, it follows that

$$z_1(0)=0,\quad z_2(0)=-\frac{2\mu}{W}, \tag{18}$$

$$z_1'(0)=-\frac{1}{W}\left(1+\frac{\mu}{2\lambda}\right),$$

$$z_2'(0)=\frac{1}{W}\left(3+\frac{\mu}{2\lambda}\right).$$

From (16), by virtue of the theorem of Laplace convolution transform, the equality follows

$$\pi^*_{11}(x,\ s)=\frac{\mu}{W}\int_0^x \pi^*_{01}(y,s)e^{-(\mu+s)(x-y)/W}\mathrm{d}y.$$

Therefore, taking into account (15), (16), after integration, we find

$$\pi^*_{11}(x,s)=\frac{1}{2W}\sqrt{\frac{\lambda+s}{\lambda}}\left(e^{z_1x}-e^{z_2x}\right),\quad x\geq 0. \tag{19}$$

Now, using the expressions (17), (19) from the relation (6), we find the Laplace transform of the desired distribution function of the ship berthing time (θ) under loading operations, taking into account possible loading interruptions due to the lack of cargo at the terminal:

$$sp^*(s)=W\int_0^\infty\left[\pi^*_{01}(x,s)+\pi^*_{11}(x,s)\right]\mathrm{d}H(x)=$$
$$=\frac{1}{2\mu}\sqrt{\frac{\lambda+s}{\lambda}}\int_0^\infty\begin{bmatrix}(2\mu+s+z_1W)e^{z_1x}-\\-(2\mu+s+z_2W)e^{z_2x}\end{bmatrix}\mathrm{d}H(x)=$$
$$=\frac{1}{2\mu}\sqrt{\frac{\lambda+s}{\lambda}}\begin{bmatrix}(2\mu+s+z_1W)h(-z_1)-\\-(2\mu+s+z_2W)h(-z_2)\end{bmatrix}. \tag{20}$$

The formula (20) gives the desired solution for m=2. The expression (20) can also be reversed using the appropriate technique of recovery of the original time function from the Laplace transform [11]. However, the corresponding original function will have a very complex look.

From (20), by differentiating the variable s at the point s=0, it is possible to find the moments of distribution of the random variable θ. For example, for mathematical expectation, taking into account the equations (18), we have

$$M\theta=-\frac{\mathrm{d}}{\mathrm{d}s}\left(sp^*(s)\right)\Big|_{s=0}=\frac{h_1}{W}\left(1+\frac{\mu}{2\lambda}\right)+$$
$$+\left(4+\frac{\mu}{2\lambda}\right)\frac{h(2\mu/W)}{2\mu}-\frac{1}{4\lambda}. \tag{21}$$

Using (21), it is possible to determine the upper bound of the probability that θ exceeds the given time t using the Markov inequality:

$\mathbf{P}\{\theta\geq t\}\leq\mathbf{M}\theta/t.$

However, the upper bound obtained is very rough.

6. Finding an asymptotic formula of demurrage probability for large values of ship deadweight tonnage. Numerical illustration of the results

As a numerical illustration, we consider two examples:

a) The case of constant train capacity.

The results obtained above (11) are inconvenient for numerical calculations, since it becomes necessary to find the sum of an infinite functional series. Therefore, it is desirable to have an approximate formula to simplify the calculations. This can be done if, for example, cargo is assumed to be periodically delivered to the terminal by the same train with a constant capacity (i. e. load) equal to d. Let τ_1, τ_2, ... be random time intervals between the arrivals of the loaded train to the terminal. We assume, as above, that the random variables τ_1, τ_2, ... are mutually independent and obey the exponential distribution law with the parameter λ. Thus, the only risk factor is the duration of intervals between train arrivals.

Let the ship deadweight tonnage be fixed and equal to D, then exactly N trains will be required for full loading, where

$N=\langle D/d\rangle,$

$\langle z\rangle=z$, if z – integer, $[z]+1$, if – fractional; $[z]$ is the integer part of z.

In this case, the ship berthing time θ will be equal to

$$\theta=\tau_1+...+\tau_N+T. \tag{22}$$

In practical cases, the number N can be considered quite large. For example, for D=30 thousand tons, d=1 thousand tons, N=30, i. e., the sum of 30 random variables should be considered in (22). In such a situation, it is natural to use the central limit theorem, according to which the random variable $\tau_1+...+\tau_N$ for large N is approximately normal. Therefore, according to (22), we can write

$$p(t)=P\{\theta\leq t\}\approx\Phi\left(\frac{\lambda(t-T)-N}{\sqrt{N}}\right),\quad N\to\infty,\quad t>T, \tag{23}$$

where

$$\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}e^{-u^2/2}\mathrm{d}u.$$

In (23), the parameter $1/\lambda=\mathbf{M}\tau_1$ is the average time interval between adjacent arrivals of the loaded train to the terminal. It follows from (23) that for large values of N, the most probable values of the random variable θ belong to the interval (the three sigma rule for the normal distribution law):

$$\left(T+\frac{N-3\sqrt{N}}{\lambda},\ T+\frac{N+3\sqrt{N}}{\lambda}\right).$$

Note that the middle of this interval is equal to the average value of the random variable θ, i. e.

$\mathbf{M}\theta=T+N/\lambda.$

If we take the last expression as laytime, then it follows from (23) that

$p(t_l)=\Phi(0)=0.5,$

i. e., the probability of exceeding it is very high (=0.5). Given this, the following value can be taken as laytime

$$t_l=T+\frac{N+3\sqrt{N}}{\lambda}. \tag{24}$$

Taking into account (24), from (23) we find the probability that the ship berthing time does not exceed t_l:

$$p(t_l)\approx\Phi\left(\frac{\lambda(t_l-T)-N}{\sqrt{N}}\right)=\Phi(3)\approx 0.9999,\quad N\to\infty,$$

i. e., the probability of exceeding the laytime is almost zero. However, the value of t_l, calculated by (24), significantly exceeds the duration of actually loading operations T. Therefore, to determine t_l, additional economic justification is required.

Table 1 shows the values of the confidence limits of the ship berthing time for different values of N, λ, T. The data correspond to the actual reporting data on bulk carriers for transporting bulk cargo handled at berths 5, 6, 7, 8 of the Yuzhny sea trade port in 2019. Loaded cargo – iron ore concentrate and pellets.

Table 1

Confidence limit values

N	T, days	λ, 1/day	$T+(N-3\sqrt{N})/\lambda$, days	$T+(N+3\sqrt{N})/\lambda$, days	$\mathbf{M}\theta$, days
9	3	1	3	21	12
9	3	2	3	12	7.5
9	3	3	3	9	6
16	5	2	7	19	13
16	6	2	8	20	14
16	4	3	5.3	13.3	9.3
25	5	2	10	25	17.5
25	6	2	11	26	18.5
25	7	3	10.3	20.3	15.3
36	4	2	13	31	22
36	6	2	15	33	24
36	6	3	12	24	18

Table 1 shows that with an increase in N with a fixed value of the parameter λ (rate of trains arrival to the terminal), the confidence limits also increase. Moreover, the width of the interval grows as $\sqrt{N}$. In addition, the upper limit of the confidence interval and $\mathbf{M}\theta$ with increasing N significantly exceed the net loading time of the ship T. This is a consequence of a significant irregularity of the arrival of loaded trains to the port (time intervals between arrivals of loaded trains are distributed exponentially). With a greater degree of regularity, these intervals can be distributed, for example, by Erlang's law.

The data in Table 1 indicate that the risk of exceeding $\mathbf{M}\theta$ increases with increasing N and certain organizational and economic measures to reduce it need to be taken. This issue is addressed in the problem below.

b) Insurance of the laytime exceeding risk.

Due to the possible additional demurrage of the ship due to waiting for cargo delivery by ground transport, the charterer of the ship runs the risk of exceeding the laytime and paying the corresponding fines (so-called demurrage) by the shipowner. In fact, these financial losses can be reduced by insuring the specified risk on certain conditions. In such a situation, it is necessary to be able to quantify possible (expected) gain of the charterer from insurance using models similar to those discussed above. Below we demonstrate such a possibility.

Let us first estimate the shipowner's possible losses upon the occurrence of an insured event, i. e., for $\theta>t_l$. If we designate daily berthing costs of the ship by e_b, then the indicated losses will amount to

$$e_b \max(0, \theta-t_l). \tag{25}$$

The charterer can insure himself against these losses by paying an insurance premium in the amount of c to the insurer. The problem is to compare c with the random variable (25). The simplest criterion for insurance expediency is to compare the charterer's expected benefits in case of insurance (P_{ins}) and non-insurance (P_{nins}) of the indicated losses. Obviously

$$\mathbf{M}P_{\text{ins}}=-c+e_{\text{ins}}\mathbf{M}\max(0, \theta-t_{ins}),$$

$$\mathbf{M}P_{\text{nins}}=c-e_{\text{ins}}\mathbf{M}\max(0, \theta-t_{ins}). \tag{26}$$

Note that the variances of the random variables P_{ins} and P_{nins} coincide, which gives the basis when deciding on insurance to be limited only to the average gain (26).

Further, from (26) it can be seen that

$$M\max\left(0,\theta-t_l\right)=\int_{t_l}^{\infty}\left(\tau-t_l\right)\mathrm{d}p\left(\tau\right). \tag{27}$$

Thus, risk insurance is advisable if the following condition holds

$$\mathbf{M}P_{\text{ins}}>\mathbf{M}P_{\text{nins}}$$

or, taking into account (27),

$$\int_{t_l}^{\infty}\left(\tau-t_l\right)\mathrm{d}p\left(\tau\right)>c/e_b. \tag{28}$$

For the practical use of the criterion (28), it is necessary to know the explicit form of the probability $p(t)$. For example, for (11) from (28) we get the following condition of insurance expediency

$$e^{-\mu T}\int_{t_l}^{\infty}\left(\tau-t_l\right)\sum_{n=1}^{\infty}\frac{(\mu T)^n}{n!}E_n'\left(\tau-T\right)\mathrm{d}\tau>c/e_b.$$

However, as noted above, practical calculations of the left side of the last inequality involve significant computational difficulties with real values of the parameter μT. Therefore, for the numerical illustration of the criterion (28), we use the asymptotic formula (23).

Substituting the expression (23) into the left side of the inequality (28), we get

$$\frac{1}{\sqrt{2\pi}}\int_{t_l}^{\infty}\left(\tau-t_l\right)\exp\left\{-\frac{1}{2N}\left[\lambda\left(\tau-T\right)-N\right]^2\right\}\mathrm{d}\tau=$$
$$=\frac{1}{\sqrt{2\pi}}\int_{0}^{\infty}x\exp\left\{-\frac{1}{2N}\left[\lambda\left(x+t_l-T\right)-N\right]^2\right\}\mathrm{d}x,$$

$$N\to\infty.$$

It can be shown that after some transformations, the last integral is reduced to the following form:

$$I=\frac{\sqrt{N}}{\lambda\sqrt{2\pi}}\left\{\begin{aligned}&\exp\left[-\frac{\left(\lambda\left(t_l-T\right)-N\right)^2}{2N}\right]+\\&+\frac{N-\lambda\left(t_l-T\right)}{\sqrt{N}}\left[1-\Phi\left(\frac{\lambda\left(t_l-T\right)-N}{\sqrt{N}}\right)\right]\end{aligned}\right\}. \tag{29}$$

Table 2 shows numerical values of the expression (29) for different values of the parameters N, λ, t_l-T. Here

$$\rho=[\lambda(t_l-T)-N]/\sqrt{N}.$$

Table 2

Numerical values of the expression (29)

N	λ, 1/day	$(1/\lambda)\sqrt{N/2\pi}$	T_l–T, days	ρ	I, days
9	0.5	2.394	2	–2.667	5.854
9	1.0	1.197	1	–2.667	3.214
9	2.0	0.599	1	–2.333	0.132
16	1.0	1.596	1	–3.500	5.584
16	1.5	1.064	2	–3.250	3.460
16	2.0	0.798	2	–3.000	2.400
20	1.0	1.772	2	–4.025	7.132
20	1.5	1.189	2	–3.081	4.519
20	2.0	0.892	2	–2.530	2.257

From Table 2 it can be seen that for the fixed values of N, the value of I decreases with increasing flow rate of loaded trains λ. Comparison of I with the ratio c/e_b leads to the conclusion about the expediency of insuring the risk that the total ship loading time exceeds the laytime t_l. If, for example, e_b=5 thousand c. u. per day, then for the last row of Table 2 insurance will be appropriate if the insurance premium is less than 5·2, 257=11,285 thousand c. u. For these reasons, the charterer may take 2+T as laytime in the contract of carriage.

Although only the simplest case is considered above, the approach used allows solving similar problems in a more general formulation (several ships, limited reliability of transshipment equipment, etc.). The practical implementation of the developed method will reduce charterer's risks associated with the laytime excess by actual berthing time.

The proposed approach for estimating the ship demurrage probability can be used in further studies on this problem. For example, to take into account varying regularity of cargo delivery by land transport, the Erlang distribution to simulate time intervals between adjacent arrivals of loaded trains to the terminal can be used. This will allow taking into account the varying regularity of cargo delivery to the terminal – from completely random (for example, Poisson) to completely regular (i. e., at constant intervals between car arrivals) flow.

An important factor is also possible random fluctuations in the rate of cargo transshipment from the trains to the ship, caused by weather conditions or sudden failures of transshipment equipment. This circumstance can also be taken into account in further studies using the above approach.

A more accurate method for assessing the laytime exceeding probability will allow the charterer and port operator to reduce losses associated with additional unproductive demurrage of the ship.

7. Discussion of the developed method for finding the distribution of the ship berthing time under loading

As a result of the study, a practically important and theoretically difficult problem is solved. It concerns finding the time distribution function of ship berthing under loading operations, taking into account possible increase in berthing time under handling due to the irregular cargo delivery by trains when organizing transshipment operations under the direct option. Knowing the specified distribution function allows the charterer (or shipowner) to predict the possible excess of laytime and resulting losses. To solve this problem, a method based on the use of linear Markov processes, which describe random variation of cargo delivery to the terminal and volume of the cargo lot is developed. The proposed approach allows quantifying the feasibility of insuring financial losses of the charterer. As a result of the study, exact (in terms of the Laplace transform) and asymptotic formulas for calculating the distribution function of the total ship berthing time are obtained, taking into account possible interruptions in loading due to the lack of loaded trains at the terminal. For this purpose, a system of differential equations of the probability densities of the corresponding Markov process is derived and solved. For the practical use of the described method, simple asymptotic formulas are found and the criterion for the feasibility of insuring the risk of ship demurrage due to waiting for cargo delivery is developed.

8. Conclusions

1. To study the transshipment process, the model of port terminal operation that takes into account irregular cargo delivery to the port terminal by rail is developed. This model is built using linear Markov processes, which allows taking into account objectively existing irregular arrival of loaded trains to the terminal. This irregularity is caused by the impossibility of perfect coordination of the dates of cargo delivery by rail and ship readiness for loading due to many objective and subjective factors.

2. In order to find an analytical dependence of the probability of ship demurrage under loading operations due to untimely cargo delivery by trains, the system of differential equations of the probability densities and state probabilities of the specified Markov process is derived and solved. The solution found to the indicated system of equations made it possible to determine the desired time distribution of ship berthing under loading operations and waiting for cargo delivery.

3. The asymptotic formula for the indicated ship demurrage probability for large values of deadweight tonnage is obtained and numerical analysis is carried out. The numerical illustration of this formula on the basis of realistic initial data shows that it gives acceptable practical results.

4. The quantitative criterion for the economic feasibility of insuring the risk of additional ship demurrage due to excessive waiting for cargo delivery to the terminal is developed.

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