

36. Analiz stanu bezpeky rukhu, sudnoplavstva ta avariynosti na transporti v Ukraini za 2017 rik. Available at: [http://dsbt.gov.ua/sites/default/files/imce/Bezpeka\\_DTP/2018/analiz\\_avariynosti\\_2017.compressed.pdf](http://dsbt.gov.ua/sites/default/files/imce/Bezpeka_DTP/2018/analiz_avariynosti_2017.compressed.pdf)
37. Dovidnyk osnovnykh pokaznykiv roboty Rehionalnykh filiy PAT «Ukrainska zaliznytsia» (2005–2015 roky) (2016). Ukrainska zaliznytsia, Upravlinnia statystyky. Kyiv: VTs «Prydniprovia», 60.
38. Samsonkin, V. M., Martyshko, A. M. (2015). Praktychne zastosuvannya vyznachennia «vuzkykh mist» v ubezpechenni rukhu na pidpryemstvakh zaliznychnoho transportu dlia profilaktyky transportnykh podii. Zaliznychnyi transport Ukrainy, 1, 3–10.
39. Samsonkin, V., Druz', V., Feldman, A. (2018). Applying of activities management based on self-learning. EUREKA: Physics and Engineering, 1, 29–38. doi: <https://doi.org/10.21303/2461-4262.2018.00530>

Пропонується алгоритм урахування динаміки судна, що оперує, для методу попередження зіткнень «Velocity Obstacle». Цей алгоритм забезпечує основу для вибору спільних маневрів курсом і швидкістю із заданим початком для розходження з декількома «цілями» шляхом визначення методом перебору представницької множини допустимих варіантів маневру. Для застосування методу перебору виділяються діапазони зміни параметрів маневру (курсу і швидкості) і проводиться їх дискретизація з досить малим кроком. Для всіх пар дискретних значень зміни курсу і швидкості з урахуванням динаміки судна знаходиться траєкторія і тривалість маневру з визначенням на момент його закінчення місця судна і «цілей», а також встановлюється, чи буде він супроводжуватися перетином доменів небезпеки «цілей». Якщо немає пересічення жодного з таких доменів, то варіант маневру вважається допустимим. Отримана при переборі сукупність таких спільних змін курсу і швидкості утворює множини допустимих варіантів маневру. При знаходженні цієї множини динаміка судна враховується спрощено. Вважається, що повороти виконуються з постійною кутковою швидкістю, зміну лінійної швидкості при гальмуванні можна представити степеневим поліномом другого порядку, а зміни курсу і швидкості в спільному маневрі незалежні. У «цілей» використовуються кругові домени небезпеки, центр яких зміщений від центру маси «ціль» в бік носа на 1/3 частину радіуса домену. В цей радіус внесена поправка на розміри «ціль» і судна, що оперує.

Для перевірки отриманого алгоритму була складена програма на мові «Borland Delphi». Розрахунки по ній підтвердили працездатність алгоритму. Він дозволяє в реальному часі знаходити множини векторів швидкостей для розходження з урахуванням динаміки судна, що дозволяє підвищити точність прогнозу і безпеку маневрів. Використання у «цілей» зміщених, кругових доменів небезпеки дає можливість враховувати неоднакову ступінь ризику при пересіченні їх курсу по носі і по кормі

Ключові слова: попередження зіткнень, метод перебору, множина допустимих варіантів, алгоритм урахування динаміки

UDC 656.61.052

DOI: 10.15587/1729-4061.2019.185319

## IMPROVEMENT OF THE ANTI-COLLISION METHOD "VELOCITY OBSTACLE" BY TAKING INTO CONSIDERATION THE DYNAMICS OF AN OPERATING VESSEL

A. Aleksishin

PhD, Associate Professor\*

E-mail: avg1951safonova@gmail.com

A. Vagushchenko

Second Mate

LLC Nordic Hamburg Shipmanagement Ltd.  
Yoghana Ghena str., 19, Odessa, Ukraine, 65098

O. Vagushchenko

PhD, Associate Professor\*

Y. Kalinichenko

PhD, Associate Professor\*\*

E-mail: kyevgeniy@ukr.net

\*Department of Navigation\*\*\*

\*\*Department of Shiphandling\*\*\*

\*\*\*National University "Odessa Maritime Academy"

Didrikhsona str., 8, Odessa, Ukraine, 65029

Received date 26.09.2019

Accepted date 22.11.2019

Published date 13.12.2019

Copyright © 2019, A. Aleksishin, A. Vagushchenko, O. Vagushchenko, Y. Kalinichenko

This is an open access article under the CC BY license

<http://creativecommons.org/licenses/by/4.0>

### 1. Introduction

Among the components of scientific and technological progress at present is the development and introduction of unmanned vehicles, including autonomous marine vessels (ASV). To be operational on sea routes, such a vessel must be equipped with a collision avoidance system (CAS) responsible for divergence from other vessels in accordance with International Rules for Preventing Collisions at Sea-72. This task is complex, as the proper level of safety has not yet been

achieved even for conventional, not autonomous, vessels. About 150 large ships collide in the world each year, about 3 of them sink. Technical losses from collisions are enormous. Among the measures taken to reduce the number of such accidents, the following are worth mentioning. The International Rules for Preventing Collisions at Sea (COLREG) have been introduced, which are mandatory for all ships of varying affiliation. In coastal areas of heavy navigation, they have been tightened by national requirements. Vessels are equipped with powerful integrated bridge systems that

use electronic mapping navigation and information systems to facilitate the selection of divergence maneuvers based on mapping information. Practical all entries to all major ports and transit areas of heavy navigation are equipped with ship traffic control and monitoring systems using satellite navigation and communications. The E-Navigation development program is being implemented, which would make it possible for all participants of maritime transportation process to contact each other at any time. And yet, despite constant progress, the issue on preventing collisions of ships remains unresolved. There are many reasons: unfavorable weather conditions, navigation obstacles, heavy traffic; subjectivity of assessing the situation and making decisions. Therefore, it is still a relevant task to undertake a research aimed at developing methods to improve the efficiency of ship divergence processes at sea.

---

## 2. Literature review and problem statement

---

The general characteristic of autonomous collision prevention in shipping, with the provision of a series of its theoretical foundations, is given in work [1]. It discusses both classical methods based on mathematical models and algorithms, as well as methods based on artificial intelligence. One of the methods proposed to solve the task on ASV divergence is VO – Velocity Obstacle. Underlying it is determining the set of a robot's velocity vectors that result in a collision against obstacles given the permanent character of their motion parameters. The region of unacceptable velocity vectors, corresponding to a single obstacle, is a sector (a cone in 3-dimensional space) of dangerous relative robot courses. By selecting the end of the velocity vector outside the cones that match all the obstacles, one can avoid collision.

Paper [2] proposes an algorithm for avoiding collisions of ships, which makes it possible, by using the VO method, to find the maneuvers of divergence that meet the requirements by COLREG. A solution to the problem is based on splitting the velocity space of OS (own ship) into four areas, and determining in which of them one should not choose the ends of velocity vectors. There are the unresolved issues on taking into consideration the dynamics of a maneuvering vessel and determining the regions of acceptable maneuver parameters values, in rapprochement situations with several ships. The ways of taking into consideration national requirements were also not considered.

Note that some authors termed the VO method differently. Thus, in a series of studies it is given as “collision cones”, in article [3] – as “prohibited speed maps”, and as the “generalized speed obstacles” – in paper [4]. These studies addressed various aspects in applying the VO method and were conducted without taking into consideration the inertia of an operating object.

Work [5] describes one of the experimental samples of an on-board collision prevention support system. It uses a model of visualization of the convergence of ships to select the course and speed maneuvers. The main requirement to this model was to facilitate the evaluation and selection of waterway divergence maneuvers. The dynamics of an operating vessel were also not taken into consideration in its development.

A procedure to prevent collisions with multiple vessels based on the VO method is described in [6]. This procedure is used to draw up a divergence plan, followed by determin-

ing a route to the port of destination. The dynamics of an operating vessel is also not taken into consideration.

The VO method is also used in a multi-agency approach to solving divergence problems. An example is paper [7], which, routinely, does not deal with issues related to accounting for dynamics.

Thus, various options are proposed to use the VO anti-collision method to facilitate assessing ship convergence situations and to select divergence maneuvers. However, the dynamics of vessels are not taken into consideration in solving the tasks set. Therefore, the proposed procedures do not make it possible to ensure the required accuracy in the calculation of maneuver parameters, especially for large-capacity vessels. The margin of error in the predicted distance of the shortest approach could reach 0.5 miles due to neglect of inertia for these vessels. In addition, target danger domains are used to form sectors of unacceptable relative courses, which do not take into consideration the greater risk of crossing their course on the bow than on the stern.

---

## 3. The aim and objectives of the study

---

The aim of this work is to improve the anti-collision method “Velocity Obstacle” in relation to maritime navigation by taking into consideration the dynamics of an operating vessel.

To accomplish the aim, the following tasks have been set:

- to analyze the possibility of deriving a region of acceptable maneuver options, taking into consideration the dynamics of a vessel by determining its boundaries;
- to devise a numerical method for defining a set of speed vectors that are safe for divergence;
- to verify the validity of the results obtained on the basis of the proposed method.

---

## 4. Methods and materials for solving the set problems

---

### 4.1. Deriving a region of acceptable maneuver options based on the dynamics of a vessel by determining its boundaries

In order to be able to choose controlling influences, it is often necessary to obtain regions of their acceptable variants. The VO method is also based on determining the regions of unacceptable velocity vectors (RUVV) for divergence. When there is a single “target”, this region is a sector of relative dangerous courses, whose boundaries depend on the shape taken by a “target” danger domain. “Targets” can use target's domains of danger (TDD) that can be of different types [8]. The option of changing a course together with velocity is considered permissible in terms of divergence at a distance at the closest point of approach (DCPA) if an OS does not cross any of the “target” domains. The principle of deriving a sector of dangerous relative courses (SDRC) for TS with a circular non-displaced domain without taking into consideration the OS dynamics is illustrated in Fig. 1. The divergence would be safe if one chooses the end of the OS velocity vector outside of SDRC.

When using the VO method, the region of unacceptable velocity vectors for diverting a single vessel is a sector of dangerous relative courses (SDRC). This sector is determined by its peak and boundary courses, which are calculated analytically through simple expressions [2]. When

there are several vessels, RUVV is derived as a combination of such regions for individual vessels. RUVV for several vessels can be represented by the polar diagram (Fig. 2) that matches the beginning of an OS maneuver, which includes parts of SDRC that enter a circle with the radius equal to the maximum OS velocity. The angular scale of the diagram shows the changes in the course to the right and left, and the linear scale – the OS velocity values. This diagram makes it easier for the operator to choose the right maneuver for divergence. An example of a polar diagram with a range of course changes from  $-90^\circ$  to  $+90^\circ$  for an event with 5 TSs is shown in Fig. 2.

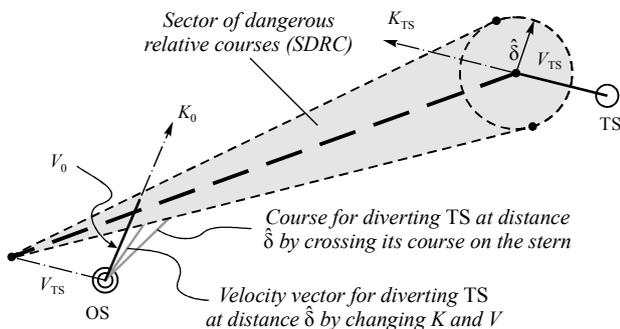


Fig. 1. A relative dangerous course sector for selecting divergence maneuvers:  $K_0$ ,  $V_0$  – OS course and velocity before a maneuver;  $K_{TS}$ ,  $V_{TS}$  – TS course and velocity;  $\hat{\delta}$  – the limit of permissible values for the distance at the closest point of approach

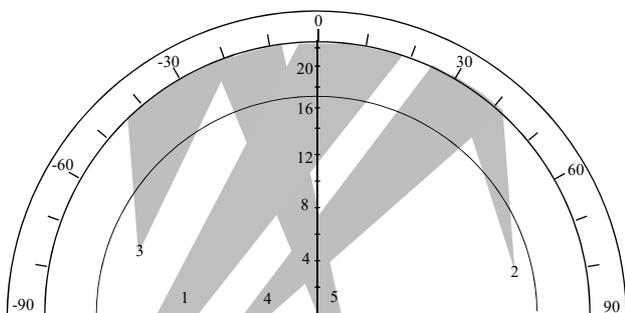


Fig. 2. Polar diagram of velocities at a range of course changes from  $-90^\circ$  to  $+90^\circ$

Until now, when solving problems on diverging, the regions of maneuvers that were unacceptable for divergence were represented only by their boundaries. Therefore, in the beginning, to take into consideration the inertia of OS, we analyzed the option of deriving RUVV by correcting the boundaries of sectors of dangerous true courses (SDTC), calculated without taking into consideration the inertia of OS. Let us consider the principle of solving this problem using a single “target” as an example (Fig. 3).

According to SDRC for this “target” for a specific value of  $V_h$  the sector of dangerous true courses can be defined as shown in Fig. 3. To clarify the boundaries of SDRC, it is possible for a series of velocity values to find SDTC taking into consideration the OS dynamics [9]. In Fig. 3, such SDTC are marked on the circles corresponding to  $V_0$  and  $V_h$  with bold arcs. By calculating, for several OS velocity values, the positions of the ends of such arcs and by connecting the related ones to one SDRC boundary, we shall derive it taking into consideration the OS dynamics. However, this line will no longer be straight.

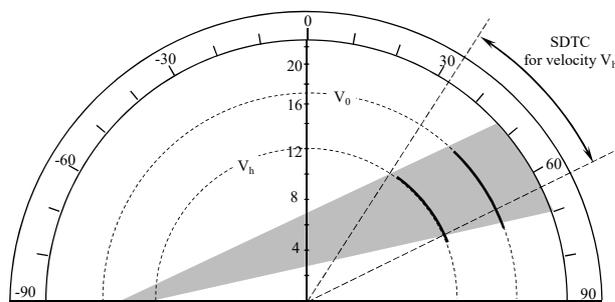


Fig. 3. Considering the inertia of OS

The developed algorithm for determining the boundaries of RUVV for a single “target” includes the following operations.

One finds a moment when an OS comes to the starting point of a maneuver. At this point, the coordinates of the “target” are calculated. Based on the start point of the OS maneuver, the course and velocity of the “target” (Fig. 1), the coordinates of the SDRC peak are calculated.

Based on the position of this peak, the coordinates of the “target” and the radius of its domain, one determines the directions of SDRC boundaries. The coordinates of the peak and direction of the boundaries completely determine SDRC without taking into consideration the OS dynamics.

The dynamics of an operating vessel are then taken into consideration. In the interval of possible OS velocities at divergence, four equally spaced velocity values  $V_i$  are selected. For each of them, by using the SDRC parameters, one finds the directions of SDTC boundary (Fig. 3). The OS course before a maneuver determines the turning angles that correspond to these directions. They are considered to be the initial approximations of SDTC when calculating the boundaries of this sector, taking into consideration the inertia of OS.

$V_i$  and the resulting turning angles are used, in line with a simplified OS dynamics model, to find the increments of coordinates during the maneuver and the time spent on it. These data are applied to calculate the end point of a maneuver when OS moves relative to TS. The coordinates of this point and the coordinates of TS are employed to calculate the direction of tangents to the domain and the first approximation of directions of the SDTC boundaries. If necessary, these directions and the OS course are used again to calculate turning angles, and a second approximation for the boundaries of SDTC is derived. Two approximations would typically suffice to derive these boundaries with an accuracy of up to  $1^\circ$ .

The values of  $V_i$  and the corresponding SDTC boundaries are employed to find the points (the ends of bold arcs in Fig. 3) of SDRC boundaries, taking into consideration the dynamics of a vessel. By connecting the SDRC peak and the neighboring points derived for the SDRC boundaries in straight segments, we obtain the boundaries of SDRC taking into consideration the dynamics of the vessel.

The given procedure can be used to represent RUVV on a CAS screen to select maneuvers in a dialogue mode with this system. However, this procedure does not make it possible to highlight, using a computer, options in the regions of acceptable parameters of the maneuver that, in a varying degree, correspond to IRPCS. Typically, this would require determining options with a high, medium, and low priority for divergence. In addition, when applying the proposed

procedure, the RUVV boundary expressions for events involving multiple “targets” become cumbersome, rendering them impossible to use when calculating maneuvers using a computer.

#### 4.2. A numerical method for calculating a set of velocity vectors that are safe for divergence

Another method has been proposed to derive RUVV, free from the shortcomings described above. This method implies mathematical notation of the region of velocity vectors, unacceptable for divergence, via a representative set of its discrete values. To derive a given set, we sample the ranges of changes in the OS velocity and course at a small enough step ( $\Delta_V$  – for velocity,  $\Delta_\theta$  – for course). The number of discrete values within these ranges shall be  $n_W$  and  $n_\theta$ , the randomly selected values in them –  $W_i$ ,  $\theta_j$ , and the maneuver option corresponding to them –  $\mu(i, j)$ .

To determine the set of valid options  $\mu(i, j)$  for all pairs of values  $W_i$ ,  $\theta_j$ , taking the OS dynamics in consideration, one finds the trajectory and duration of the joint change in course and speed. This procedure calculates the position of OS and “targets” at the end of the maneuver, and determines whether it would be accompanied by an excessive rapprochement with the “targets.” If a variant  $\mu(i, j)$  is safe in relation to all “targets”, it is assigned a value of “1”, or otherwise –  $\mu(i, j)=0$ .

Each of the options identified at sorting, permissible by DCPA, can be checked for compliance with COLREG. This is performed by using some formative version of these rules [10, 11]. The result is the selected variants with a high, medium, and low priority for application. A generalized block diagram of the algorithm to obtain a set of valid maneuver options is shown in Fig. 4, where  $\bar{\theta}$  is the limit to a change in course to the left.

To reduce the time for sorting maneuver options, it is advisable to use analytical expressions to predict it. The situations with an actual threat of collision, considered in this paper, are not extreme and do not require “strong” maneuvers to be resolved. It is believed that for divergence, the turns will be performed at a specified radius, roughly corresponding to the clutch of the steering wheel of 15°, while the speed will decrease under a “rear small move” mode. Under such a condition, the assumption about the independence of these processes can be used to simplify the prediction of the turn together with a change in velocity. Analytical expressions to calculate the parameters of these processes separately can be borrowed from many sources.

#### 4.3. Verifying the validity of results obtained based on the proposed numerical method

To verify the results of our study, a program in the Borland Delphi language was developed to simulate the processes of divergence. DCPA-permissible collision-prevention maneuvers are derived in it by using circular “target” danger domains, whose center is shifted from the TS mass center toward the stern by 1/3 of the TDD radius. To determine this radius, the following expression is used

$$r = \delta_p + \Delta_L.$$

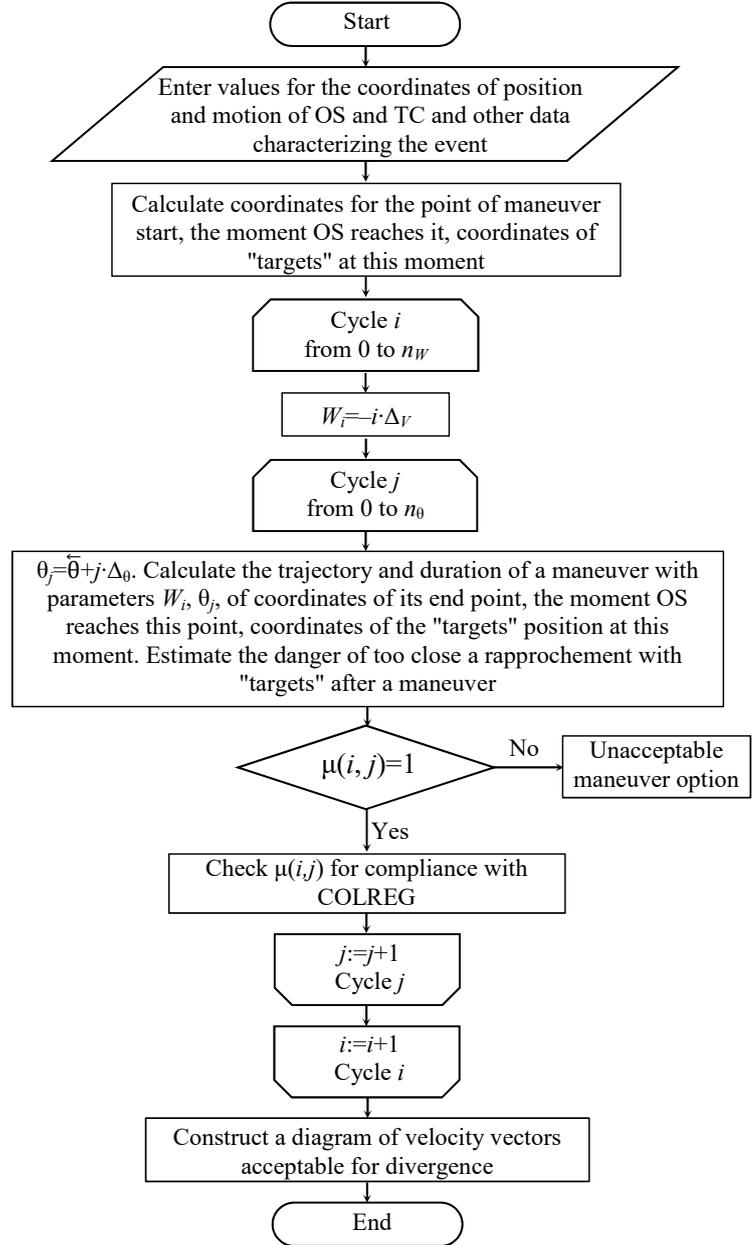


Fig. 4. Block diagram of the algorithm for determining a set of velocity vectors acceptable for divergence

Here,  $\delta_p$  is the safe distance limit  $\delta_w$ , introduced to CAS, along water surface between the nearest OS and TS points;  $\Delta_L = (L_{OS} + L_{TS})/2$  is the adjustment to the size of vessels, where  $L_{OS}$ ,  $L_{TS}$  is the length of OS and TS. Such TDD is determined only by its radius and takes into consideration the greater risk of the TS crossing at the stern at a divergence.

The OS dynamics are taken into consideration simplistically. It is believed that the OS turn takes place at a predefined radius, and the process of changing  $V$  at braking is described by expression

$$V = V_0 + a_1 t_\tau + a_2 t_\tau^2,$$

where  $t_\tau$  is the braking time.

In this case, the distance that is traveled at braking is

$$S = V_0 t_\tau + a_1 t_\tau^2 / 2 + a_2 t_\tau^3 / 3.$$

The  $a_1, a_2$  coefficients can be found from the data on field tests, given on the OS form of maneuver characteristics – the time ( $t_{AB}$ ) and distance ( $S_{AB}$ ) of braking from velocity  $V_A$  to velocity  $V_B$ . By substituting these data in expressions that describe the velocity and traveled distance, we obtain a system of two linear equations from which the values  $a_1$  and  $a_2$  are determined.

$$\left. \begin{aligned} t_{AB}a_1 + t_{AB}^2a_2 &= V_A - V_B, \\ \frac{t_{AB}^2}{2}a_1 + \frac{t_{AB}^3}{3}a_2 &= V_0t_{AB} - S_{AB}. \end{aligned} \right\}$$

The program was used to define RUVV in situations with multiple “targets” at different types of OS convergence with dangerous TS: on opposite courses, on intersecting courses, when overtaking. The OS in these events acted both as a ship giving way and a vessel that should maintain course and speed. The program-based calculations have proven the effectiveness of the approach used. Below are the results from simulating only a single process of divergence of ships, where a rectangular diagram of divergence velocities (Fig. 5) is defined in the situation involving six vessels. The parameters of their mutual location and rapprochement are given in Table 1, where  $L$  is the length of the ship;  $B, D$  are the “target” bearing and the distance to it. Dangerous in this situation is  $TS_1$ . In calculating this diagram, it is accepted:  $\delta\rho=5$  kb; steps in change of  $\theta$  and  $W$  are  $1^\circ$  and 1 knot, respectively. The cells that meet the DCPA-compliant maneuver parameters have been crossed out.

Table 1

Parameters of mutual location and rapprochement of ships

Parameter \ Vessels	OS	TS <sub>1</sub>	TS <sub>2</sub>	TS <sub>3</sub>	TS <sub>4</sub>	TS <sub>5</sub>
$L, m$	220	250	140	175	330	80
$B, degrees$	–	78	99	287	16	358
$D, kb$	–	69.1	58.7	18.5	49.4	66.3
$K, degrees$	35	293	302	116	217	26
$V, knots$	17.0	19.1	17.3	14.8	10.1	15.8

The accepted generalized quality indicators of the proposed method are an estimate of the reduction in the error of DCPA forecast compared to determining RUVV without taking into consideration the OS dynamics, and the time spent to derive RUVV. The second indicator is important because a divergence maneuver must be performed in real time. Based on the results of RUVV calculation, taking into consideration and disregarding the dynamics of the vessel, it was found that the error of DCPA in the first case does not

exceed 15 % of the error in the second. The time to calculate the velocity diagrams on a personal computer at  $n_0 \leq 200, n_W \leq 30$  for situations in which the number of “targets” did not exceed ten, was no more than 0.5 s. This time indicates the possibility of obtaining RUVV in real time.

**5. Discussion of results of studying the improvement of the Velocity Obstacle method by taking into consideration the dynamics of an operating vessel**

Procedures for obtaining regions of acceptable options for maneuvers, joint in course and velocity, taking into consideration the dynamics of an operating vessel for situations with several “targets” have not yet been proposed. Here are two variants for solving task. A first method is based on determining the boundaries of the region of unacceptable maneuver options in relation to one “target.” The principle of determining the points of these boundaries is shown in Fig. 3. Limitations on this variant of RUVV determination are noted in chapter 4. 1. This method can be used to select maneuvers by skippers under a mode of dialogue with CAS.

A second method is based on determining a change in course and speed within a discrete set of possible pairs of values for its parameters taking the vessel’s dynamics into consideration. The algorithm of this method is shown in Fig. 4. When determining RUVV, one takes into consideration the size of a vessel and “targets” and the lower risk of crossing their course on bow at divergence. The result of our numerical experiment, according to the developed algorithm, is the sets of acceptable variants of joint maneuvers were, one of which is shown in Fig. 5. The generalized quality indicators for the proposed method are given in chapter 4. 3. Each variant of the action for divergence that is predicted during a sorting process, can be analyzed to the extent of compliance with COLREG. Such an operation makes it possible, by computation, to determine effective maneuvers in CAS. It has been established that the use of the developed method makes it possible to determine maneuvers to evade a collision with several vessels in real time. Let us emphasize that the task accomplished is aimed only at facilitating the choice of action to evade danger. This task is an important part of identifying strategies for divergence with joint course and velocity changes. Such strategies should include not only a collision evasion, but also actions to return to the route to a port of destination. When they are synthesized, one should establish the sequence of actions necessary for the divergence, the values of the parameters, and the initiation points of these actions. Identifying divergence strategies that include changes in course along with velocity is the goal of our further work. At its first stage, it is expected to construct methods for the synthesis of two step-by-step strategies optimal in terms of the loss of running time.

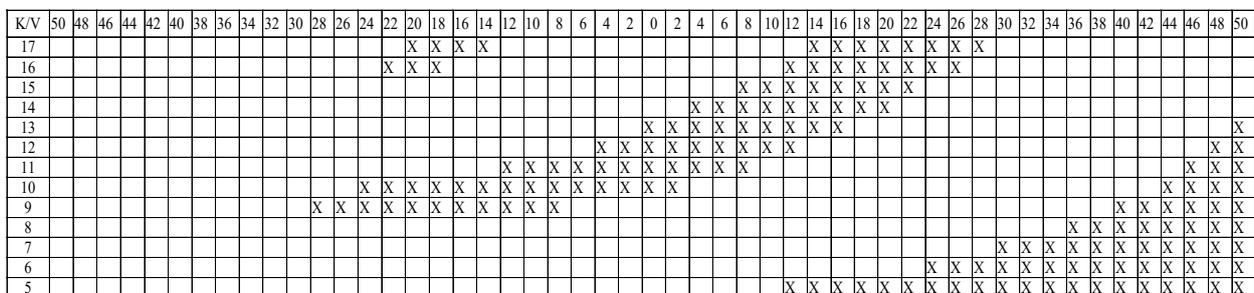


Fig. 5. Rectangular velocity diagram with a range of changes in course from  $-50^\circ$  to  $+50^\circ$

Such methods should be built considering the dynamics of a vessel, the COLREG requirements for situations with multiple “targets” and navigational obstacles. A detailed assessment of results from solving a general problem, using the method proposed in this paper, is planned after constructing the methods for the synthesis of two step strategies optimal in terms of the loss of running time.

---

## 6. Conclusions

---

1. A mathematical procedure has been proposed to determine the boundaries of the region of values, acceptable at divergence, for parameters of a joint maneuver by course and velocity, taking into consideration the dynamics of an operating vessel. Displaying this region on a CAS screen would make it easier for the watch assistant to choose such maneuvers under a mode of dialogue with the system and

to avoid the errors associated with disregarding the ship’s dynamics.

2. We have developed a numerical method for finding, taking into consideration the inertia of a vessel, a set of acceptable variants for a joint maneuver by sorting its possible options. At sorting, each of the valid options can be analyzed for the degree of compliance with COLREG. Executing such an operation makes it possible to make recommendations in CAS on divergence considering these rules.

3. The Borland Delphi-language program has been developed in order to simulate ships divergence processes, making it possible to obtain, based on the constructed numerical method, a set of possible variants of maneuvers for events involving several ships. The calculations that were performed by applying this program for a different number of “targets” in different situations of OS convergence with dangerous TS have confirmed the effectiveness of the developed method and a possibility to obtain real-time results when using it.

---

## References

1. Statheros, T., Howells, G., Maier, K. M. (2007). Autonomous Ship Collision Avoidance Navigation Concepts, Technologies and Techniques. *Journal of Navigation*, 61 (1), 129–142. doi: <https://doi.org/10.1017/s037346330700447x>
2. Kuwata, Y., Wolf, M. T., Zarzhitsky, D., Huntsberger, T. L. (2014). Safe Maritime Autonomous Navigation With COLREGS, Using Velocity Obstacles. *IEEE Journal of Oceanic Engineering*, 39 (1), 110–119. doi: <https://doi.org/10.1109/joe.2013.2254214>
3. Damas, B., Santos-Victor, J. (2009). Avoiding moving obstacles: the forbidden velocity map. 2009 IEEE/RSJ International Conference on Intelligent Robots and Systems. doi: <https://doi.org/10.1109/iros.2009.5354210>
4. Wilkie, D., van den Berg, J., Manocha, D. (2009). Generalized velocity obstacles. 2009 IEEE/RSJ International Conference on Intelligent Robots and Systems. doi: <https://doi.org/10.1109/iros.2009.5354175>
5. Pietrzykowski, Z., Borkowski, P., Wołajsza, P. (2012). NAVDEC - navigational decision support system on a sea-going vessel. *Maritime University of Szczecin*, 30 (102), 102–108.
6. Yong, X., Yixiong, H., Liwen, H. (2015). Multi-ship automatic collision avoidance control method based on speed obstacle. *China maritime*, 38 (3), 144–153.
7. Kim, D.-G., Hirayama, K., Park, G.-K. (2014). Collision Avoidance in Multiple-Ship Situations by Distributed Local Search. *Journal of Advanced Computational Intelligence and Intelligent Informatics*, 18 (5), 839–848. doi: <https://doi.org/10.12716/1001.09.01.03>
8. Hayama, I. (2014). Computation of OZT by using Collision Course. *Navigation*, 188, 78–81. doi: [https://doi.org/10.18949/jinnavi.188.0\\_78](https://doi.org/10.18949/jinnavi.188.0_78)
9. Vagushchenko, A. A., Vagushchenko, A. L. (2017). Definition of safe z-maneuvers areas for collision avoidance in confined waters. *Ekspluatatsiya morskogo transporta*, 2 (83), 73–75.
10. Banas, P., Breitsprecher, M. (2011). Knowledge Base in the Interpretation Process of the Collision Regulations at Sea. *International Journal on Marine Navigation and Safety of Sea Transportation*, 5 (3), 359–364.
11. Burmeister, H., Bruhn, W. (2014). Designing an autonomous collision avoidance controller respecting COLREG. *Maritime-Port Technology and Development*, 83–88. doi: <https://doi.org/10.1201/b17517-11>