Розроблена стохастична модель водопровідної мережі з витоками, яка в порівнянні із запропонованими раніше моделями (без урахування витоків) більш адекватно описує процеси транспорту і розподілу води в системах водопостачання. При математичному моделюванні водопровідних мереж виникають труднощі, пов'язані з величезною розмірністю реальних водопровідних мереж, обмеженістю інформаційних ресурсів і оперативних даних, що не дозволяє досить адекватно оцінити параметри технологічного обладнання і структуру водопровідної мережі. Тому для реальної водопровідної мережі по її диктуючих точках будується еквівалентна схема, для якої проводяться всі подальші розрахунки. Задача побудови схеми еквівалентної водопровідної мережі складається з трьох задач: ідентифікації структури, параметрів і стану водопровідної мережі. Запропонований метод виявлення витоків заснований на порівнянні зміни величини напору на насосних станціях і в диктуючих точках водопровідної мережі. На базі стохастичної моделі водопровідної мережі з витоками розроблений метод розрахунку величини витоків, який полягає в наступному: знаючи напір води у вузлах еквівалентної водопровідної мережі і приблизні діаметри витоків у вузлах проводиться розрахунок нових значень напорів у вузлах еквівалентної водопровідної мережі. Далі знову розраховується величина витоку, знаючи новий напір у вузлі і діаметр витоків. Зробивши кілька таких ітерацій приходимо до висновку, що починаючи з деякого кроку величина витоків і напори у вузлах еквівалентної водопровідної мережі перестають змінюватися. Знаючи величину витоку і напір у кожному вузлі еквівалентної водопровідної мережі, визначаємо дійсний діаметр свищів у кожному вузлі. Запропонований метод виявлення розрахунку величини витоків не вимагає фінансових витрат або використання додаткового обладнання, він може застосовуватися у Водоканалах для виявлення і розрахунку величини витоків

Ключові слова: водопровідна мережа, витоки, еквівалентна водопровідна мережа, стохастична модель, насосна станція

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CONSTRUCTION OF A STOCHASTIC MODEL FOR A WATER SUPPLY NETWORK WITH HIDDEN LEAKS AND A METHOD FOR DETECTING AND CALCULATING THE LEAKS

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1. Introduction

One of the most acute issues in water supply systems is water leaks from damaged pipelines and shut-off fittings due to their wear. Most of the leaks account for hidden leaks that do not come to the soil surface. Fistulas, cracks, and minor defects in a water pipeline are difficult to detect, and even more difficult to eliminate, without a major overhaul. In addition to the loss of water, leaks often destroy underground communications, cause failures of bridges and pavements, washing the foundations of buildings, washing up cable networks, result in subsidence of tram tracks, etc. This leads to accidents and, in turn, causes damage repair costs. Electricity costs are increasing to compensate consumers for the undelivered amount of water and reagents to purify it.

Another negative effect of the leak is a pressure drop in a water supply system. Forced increase in head in a water pipeline to compensate for such losses leads to the increased electricity consumption. At the same time, increasing the head only increases the leak.

The best way to reduce water leaks is to replace worn pipes, shut-off fittings, and other plumbing equipment. However, given the high cost of this technique, various organizations and specialists were engaged in studying the volumes of water leaks from water supply networks and methods for determining the sites of leaks, in the development of technical tools and techniques for detecting leaks. Research in this field is carried out by national and international organizations and many companies in the United States, Japan, England, Germany, Sweden, Finland, France, Austria, etc.

Detecting a leak using mathematical methods makes it possible to save financial and water resources. Therefore, there is reason to consider it relevant to undertake studies into modeling of water supply networks with leaks.

2. Literature review and problem statement

Paper [1] models leaks under elastic deformation; however, issues related to the leak detection method have not been resolved.

Non-standard devices are used for research in [2], which are difficult to apply under actual conditions.

Work [3] considers a procedure for arranging measuring devices; the authors, however, do not take into consideration the fact that actual water supply networks employ a limited number of measuring devices.

Comparison of the measured pressure vectors with a leak sensitivity matrix by using the least square method [4] to detect leaks is effective under ideal conditions, without taking into consideration the uncertainty of node consumers. In the cited work, a stochastic model of quasi-stationary modes of operation of a water supply network takes into consideration the stochastic nature of water consumption processes.

Paper [5] considers a change in the flow rate at the joints of a water pipeline, which requires additional measurement tools.

Detection of leaks based on a water network hydraulic model using the Monte Carlo model [6] works well for linear networks, but is poor when branched networks are considered.

In study [8], the difference between actual and simulated pressure measurements is minimized in order to detect a leak. This is possible by constantly measuring pressure in a network. However, not all networks constantly measure pressure and, therefore, they cannot use these approaches.

Work [9] reports a method for analyzing the data on the size of a leak and its location, acquired from a set of pressure sensors. Actual "Water utilities" employ a limited number of pressure sensors, while leaks abound, so a given approach is not applicable in practice.

Study [10] explores sensitivity of the magnitude of a leak to the leak site and the distance to the sensor. The leaks themselves are measured by minimizing the difference between actual and simulated measurements. In order to implement this approach, a water supply network must be provided with a sufficient number of metering devices.

In [11], the method for detecting leaks is based on image processing algorithms, however, to obtain an image, appropriate equipment is needed, which is not available at real "Water utilities".

Papers [1–11] address the construction of leak detection methods, all of which are based on measuring pressure at the nodes of a water supply network and analyzing the difference between measured value and evaluation. The application of these methods in practice for actual water supply networks is difficult due to the limited number of accounting devices. Leak detection methods work well under ideal conditions, but in actual water supply networks water consumption processes are random processes; also random are the magnitudes of assessed parameters of technological equipment. Deterministic models describe a water supply network at a particular point in time. Therefore, even a slight change in the parameters of the model or boundary conditions could not only significantly change the optimal solution, but also take it beyond the acceptable area.

All of this suggests that it is appropriate to construct a method for detecting and calculating leaks whose application would not require additional equipment to acquire initial data. The stochastic nature of water consumption processes should also be taken into consideration.

3. The aim and objectives of the study

The aim of this study is to build a stochastic model of quasi-stationary modes of operation of a water supply network with leaks, which would take into consideration the random character of water consumption processes and the estimated parameters of technological equipment. This model could be used to describe processes in water supply systems, to optimize the operational modes of water supply networks with leaks.

To achieve the set aim, the following tasks have been solved:

– to build a mathematical model of an equivalent water supply network;

– to build a mathematical model of an equivalent water supply network with leaks;

– to describe methods to detect hidden leaks in water supply networks and to calculate the magnitude of a leak.

4. Construction of a stochastic model of a water supply network with hidden leaks and a method for detecting and calculating the magnitude of a leak

4. 1. Stochastic model of equivalent water supply network

All existing water supply networks have undetectable and irreparable leaks, so the methods of hydraulic calculation of WN are not adequate as the magnitude of flow rate is a function of head [12].

Mathematical modeling of water supply networks (WN) is associated with difficulties related to the huge dimensionality of actual WN, limited information resources and operational data, which does not make it possible to adequately assess parameters for the technological equipment and the structure of WN.

An actual WN is assigned with the graph reflecting its structure. Consider a WN, which employs N PSs. To represent the structure of WN in the form of a directed graph $G(V, E)$, where V is the set of vertices, E is the set of arcs ($e=Card(E)$, $v=Card(V)$), the actual WN is supplemented with a zero vertex and fictitious chords connecting the zero vertex to all the inputs and outputs of the network. To mathematically state the problem, the following WN encoding is executed: the graph tree is chosen so that the fictitious sections of the network become chords. At the same time, the actual sections will be partially chords, and partly – branches of the tree. M is the set of branches and actual chords of the graph tree, N is the set of fictitious arcs corresponding to the outputs from WN.

A branch of the tree with a pump is assigned umber 1, the rest of the branches – from 2 to $v-1$, the chords of actual sections – from v to $v+\eta_2-1$, the fictitious ones with the assigned nodal flow rates – from $v+\eta_2$ to $v+\eta_2+\xi_1-1$, the fictitious ones with the predetermined heads – from $v+\eta_2+\xi_1$ to e, where η_2 is the number of chords of actual sections, ξ_1 is the number of outputs with the assigned nodal flow rates.

The stochastic model of quasi-stationary operational modes of WN take the form [13]:

$$\Omega: \underset{\omega}{M}\left(\begin{array}{l}\operatorname{sgn} q_r(\omega) S_r\left(q_i(\omega)\right) q_r^2(\omega)+ \\ +\sum_{i=1}^{v-1} b_{1ri} \operatorname{sgn} q_i(\omega)) S_i\left(q_i(\omega)\right) q_i^2(\omega)\end{array}\right)=0,$$

$$\left(r=v,\ldots,v+\eta_2-1\right), \qquad (1)$$

$$\underset{\omega}{M}\left(\begin{array}{l} h_r^c(\omega)-h_{NS1}(\omega)+ \\ +\sum_{i=1}^{v-1} b_{1ri}\left(sgn\, q_i(\omega) S_i\left(q_i(\omega)\right) q_i^2(\omega)+h_i^g\right) \end{array}\right)=0,$$

$$\left(r=v+\eta_2,\ldots,v+\eta_2+\xi_1-1\right), \tag{2}$$

$$\underset{\omega}{M}\left(\begin{array}{l} \operatorname{sgn} q_r(\omega) S_r\left(q_i(\omega)\right) q_r^2(\omega)+ \\ +h_r^g-h_{NSn}(\omega)+h_{NS1}(\omega)+ \\ +\sum_{i\in M_1} b_{1ri}\left(\operatorname{sgn} q_i(\omega) S_i\left(q_i(\omega)\right) q_i^2(\omega)+h_i^g\right) \end{array}\right)=0,$$

$$\left(r=v+\eta_2+\xi_1,\ldots,e; n=1,\ldots,N\right), \tag{3}$$

$$\underset{\omega}{M}\left(q_i(\omega)\right)=\underset{\omega}{M}\left(\sum_{r=v}^{v+\eta_2-1} b_{1ri} q_r(\omega)+\sum_{r=v+\eta_2}^{e} b_{1ri} q_r(\omega)\right),$$

$$\left(i=1,\ldots,v-1\right), \tag{4}$$

$$P\left(h_i^c(\omega)\geq h_i^+\right)\geq\alpha,\quad (\alpha\cong 1),$$

$$i=\left(v+\eta_2,\ldots,\ v+\eta_2+\xi_1-1\right), \tag{5}$$

$$P\left(q_i(\omega)>0\right)\cong\beta,\quad (\beta\cong 1),\quad i\in N, \tag{6}$$

where random quantities characterize: $q_i(\omega)$ is the water flow rate along the i-th section of a water pipeline; $h_r^c(\omega)$ is the free head at the r-th node in WN ($r=v+\eta_2,\ldots, v+\eta_2+\xi_1-1$); h_r^+ is the minimum allowable head at the r-th node of WN; $S_i(q_i(\omega))$ is the assessment of hydraulic resistance of the i-th section of a pipeline ($i\in M$); h_i^g is the geodesic marker of the i-th section of a pipeline ($i\in M$); b_{1ri} is the element of a cyclomatic matrix; $h_{NSn}(\omega)$ is the head at the output from the n-th PS.

In actual WNs, it is almost impossible to operatively control its parameters in all vertices and arcs of the network graph due to physical constraints on remote monitoring and control tools. Therefore, we shall control mode parameters only at the dictating points of WN, which are the vertices of the graph for an equivalent model of WN. The dictating points that we shall accept are those several nodes within a WN, at which a maximum excess head is observed.

The parameters of quasi-stationary modes of the equivalent water supply network (EWN) operation should be the same as the actual parameters at the dictating points of the WN.

Consider the EWN whose structure is represented in the form of a direct graph $G^*(V^*,E^*)$, where V^* is the set of vertices (dictating points), E^* is the set of arcs.

In a WN with leaks, water flow rate depends on head, that is the greater the head the greater the magnitude of a leak, and, therefore, water flow rate.

When optimizing the operational modes of a water supply network with hidden leaks, one solves the problem on constructing a stochastic model of quasi-stationary modes of WN operation, which includes the identification of the state, parameters, and structure of the WN.

The initial data for constructing a stochastic model of quasi-stationary EWN operational modes are:

– an EWN graph;

– heads at EWN nodes (WN dictating points);

– flow rates and heads at the outputs from WN.

Mathematical statement of the problem on constructing an adequate EWN model: it is required to minimize in the time interval [0, T] the sum of squares of head deviations at EWN nodes and their corresponding nodes at the initial WN, the sum of squares of deviations of heads and flow rates at the outputs from pumping stations for EWN and the initial WN under constraints Ω.

The problem on identifying the structure of EWN:

$$\underset{\omega}{M}\left(\begin{array}{l} \sum_{j=v+\eta_2}^{v+\eta_2+\xi_1-1}\left(h_j^c(\omega)-h_j^{c*}(\omega)\right)^2+ \\ +\sum_{i=1}^{n}\left(h_{NSi}(\omega)-h_{NSi}^*(\omega)\right)^2+ \\ +\sum_{i=1}^{n}\left(q_{NSi}(\omega)-q_{NSi}^*(\omega)\right)^2 \end{array}\right)\rightarrow\min_{G(V,E)\in\Omega}, \tag{7}$$

where $h_j^{c*}(\omega)$ are the heads at EWN nodes; $h_{NSi}^*(\omega)$, $q_{NSi}^*(\omega)$ are the heads and flow rates at the outputs from EWN pumping stations.

The problem on identifying the EWN parameters:

$$\underset{\omega}{M}\left(\begin{array}{l} \sum_{j=v+\eta_2}^{v+\eta_2+\xi_1-1}\left(h_j^c(\omega)-h_j^{c*}(\omega)\right)^2+ \\ +\sum_{i=1}^{n}\left(h_{NSi}(\omega)-h_{NSi}^*(\omega)\right)^2+ \\ +\sum_{i=1}^{n}\left(q_{NSi}(\omega)-q_{NSi}^*(\omega)\right)^2 \end{array}\right)\rightarrow\min_{S_i^*\in E^*,E^*\in\Omega}. \tag{8}$$

The problem on identifying the state of EWN:

$$\underset{\omega}{M}\left(\begin{array}{l} \sum_{j=v+\eta_2}^{v+\eta_2+\xi_1-1}\left(h_j^c(\omega)-h_j^{c*}(\omega)\right)^2 \\ +\sum_{i=1}^{n}\left(h_{NSi}(\omega)-h_{NSi}^*(\omega)\right)^2+ \\ +\sum_{i=1}^{n}\left(q_{NSi}(\omega)-q_{NSi}^*(\omega)\right)^2 \end{array}\right)\rightarrow\min_{q_i\in V^*}, \tag{9}$$

where $q_i\in V^*$ is the set of water end users.

Problem (1) to (7) belongs to the class of non-linear problems of stochastic programming of M-type under statistical (1) to (4) and probabilistic (5), (6) conditions.

4. 2. Stochastic model of equivalent water supply network with hidden leaks

Model (1) to (9) could be used to plan the modes of operation of a WN over days:

$$\underset{\omega}{M}\sum_{k=1}^{24}\left(\underset{\omega}{M}\left(\begin{array}{l} \sum_{j=v+\eta_2}^{v+\eta_2+\xi_1-1}\left(h_j^c(\omega,k)-h_j^{c*}(\omega,k)\right)^2+ \\ +\sum_{i=1}^{n}\left(h_{NSi}(\omega,k)-h_{NSi}^*(\omega,k)\right)^2+ \\ +\sum_{i=1}^{n}\left(q_{NSi}(\omega,k)-q_{NSi}^*(\omega)\right)^2 \end{array}\right)\right)\rightarrow\min_{\sum_{i=1}^{n}q_{NSi}\in\Omega}. \tag{10}$$

The result of solving the problem on load distribution among pumping stations for the WN with leaks is the derived heads and flow rates at the PS outputs, at which electricity costs and the sum of squares of deviations of free heads at EWN nodes are minimal.

The mathematical notation of the problem on distributing the load among pumping stations for the WN with leaks takes the form:

$$\underset{\omega}{M}\left(\begin{array}{l}\lambda\sum_{i=1}^{n}\left(h_{NSi}(\omega)-h_{NSi}^{*}(\omega)\right)^{2}+\\+(1-\lambda)\sum_{i=1}^{n}\left(q_{NSi}(\omega)-q_{NSi}^{*}(\omega)\right)^{2}\end{array}\right)\rightarrow\min_{\sum_{i=1}^{n}q_{NSi}\in\Omega}, \quad (11)$$

$$\Omega:\underset{\omega}{M}\left(\begin{array}{l}\operatorname{sgn}q_r(\omega)S_r\left(q_i(\omega)\right)q_r^2(\omega)+\\+\sum_{i=1}^{v-1}b_{1ri}\operatorname{sgn}q_i(\omega)S_i\left(q_i(\omega)\right)q_i^2(\omega)\end{array}\right)=0,$$

$$(r=v,\ldots,v+\eta_2-1), \quad (12)$$

$$\underset{\omega}{M}\left(\begin{array}{l}h_r^c(\omega)-h_{NS1}(\omega)+\\+\sum_{i=1}^{v-1}b_{1ri}\left(\operatorname{sgn}q_i(\omega)S_i\left(q_i(\omega)\right)q_i^2(\omega)+h_i^g\right)\end{array}\right)=0,$$

$$(r=v+\eta_2,\ldots,v+\eta_2+\xi_1-1), \quad (13)$$

$$\underset{\omega}{M}\left(\begin{array}{l}\operatorname{sgn}q_r(\omega)S_r\left(q_i(\omega)\right)q_r^2(\omega)+h_r^g-h_{NSn}(\omega)+\\+h_{NS1}(\omega)+\sum_{i\in M_1}b_{1ri}\left(\operatorname{sgn}q_i(\omega)S_i\left(q_i(\omega)\right)q_i^2(\omega)+h_i^g\right)\end{array}\right)=0,$$

$$(r=v+\eta_2+\xi_1,\ldots,e;\ n=1,\ldots,N), \quad (14)$$

$$\underset{\omega}{M}\left(q_i(\omega)\right)=\underset{\omega}{M}\left(\sum_{r=v}^{v+\eta_2-1}b_{1ri}q_r(\omega)+\sum_{r=v+\eta_2}^{e}b_{1ri}q_r(\omega)\right),$$

$$(i=1,\ldots,v-1), \quad (15)$$

$$P\left(h_i^c(\omega)\geq h_i^{+}\right)\geq\alpha,\quad(\alpha\cong1),$$

$$i=\left(v+\eta_2,\ldots,\ v+\eta_2+\xi_1-1\right), \quad (16)$$

$$P\left(q_i(\omega)>0\right)\cong\beta,\quad(\beta\cong1),\quad i\in N, \quad (17)$$

$$\underset{\omega}{M}\left(q_{i\,ut}(\omega)-\frac{\sqrt{2gh_i^c(\omega)}\cdot\pi d_{i\,ut}^2}{4}\right)=0,$$

$$\left(i=v+\eta_2,\ldots,\ v+\eta_2+\xi_1-1\right), \quad (18)$$

$$\underset{\omega}{M}\left(q_i(\omega)-q_{i\,ut}(\omega)-q_{i0}(\omega)\right)=0,$$

$$\left(i=v+\eta_2,\ldots,\ v+\eta_2+\xi_1-1\right), \quad (19)$$

where $q_{i\,ut}(\omega)$ is the magnitude of a leak at the i-th WN node, $q_{i0}(\omega)$ is the magnitude of water flow rate without leaks at the i-th WN node; $d_{i\,ut}$ is the diameter of a leak at the i-th WN node, the factor $\lambda\in[0,1]$.

4. 3. Method to detect hidden leaks in a water supply network

To apply the considered method for detecting hidden leaks in a water supply network, we shall consider EWN and several PS that it employs. If at increasing the head on all PSs by 1 meter the head at a dictating point would also increase by 1 meter, then there are no leaks. If at increasing the head on all PSs by 1 meter the head at a dictating point would increase by less than 1 meter, then there is a leak.

Suppose that in each EWN node has a hole of known diameter (the diameter is specified in calculations) from which the water leaks. By knowing the head of water at the node and the diameter of the leak, the Torricelli formula (18) may come in handy to determine the mathematical expectation of the magnitude of the leak at each node. By substituting the consumption by consumers $q_{i0}(\omega)$, $(iv+\eta_2, \ldots, v+\eta_2+\xi_1-1)$ with a magnitude of consumer consumption plus a leak in (19) and by solving the problem on flow distribution, we shall derive new values of heads at EWN nodes.

Next, we again calculate the magnitude of the leak by knowing the new head at the node and the diameter of the leak. Upon completion of several such iterations, we conclude that starting at a certain step the magnitude of leaks ceases to change.

By knowing the magnitude of the leak and head at each EWN node, one could determine the actual diameter of fistulas at each node.

5. Example of building a stochastic model of an equivalent water supply network with hidden leaks and the application of the method for detecting and calculating the magnitude of a leak

As an example, we shall consider a water distribution network, which consists of 33 nodes (end consumers). There are two pumping stations employed by the WN – PS1 and PS2; its scheme is shown in Fig. 1.

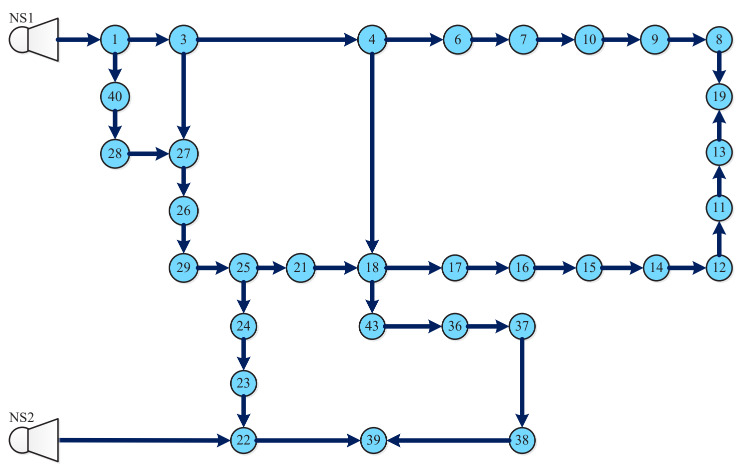

Fig. 1. A water supply network scheme

Let the following data for WN be known: lengths, diameters, and geodesic markers of the pipeline sections, flow rates at nodes (end-users) and minimally allowable values of heads at the nodes (Table 1).

For a given WN, we calculate flow distribution in accordance with the stochastic model of quasi-stationary modes of WN operation and solving the problem on optimal load redistribution among pumping stations.

Results of the flow distribution for the examined WN at q_{PS1}=0.405; q_{PS2}=0.4 are shown in Fig. 2.

Table 1

Initial data for WN pipeline sections and nodes

Pipeline sections					Nodes		
Node 1	Node 2	l, m	*d*, m	*dh*, m	Node No.	$q0$, m^3/s	Nmd, m
0	1	0	0.8	0	0	–	–
1	3	478	0.8	0.79	1	0.194	52.6
3	4	1036	0.8	1.19	3	0.092	51.309
4	6	600	0.6	–1.99	4	0.084	49.464
6	7	686	0.6	–4	6	0.074	48.048
7	10	1693	0.8	–6	7	0.01	51.368
10	9	566	0.25	–3	10	0.031	46.887
9	8	302	0.3	–1	9	0.002	45.432
8	19	500	0.3	–1	8	0.005	46.401
13	19	442	0.3	–1	19	0.039	48.281
11	13	313	0.3	–2	13	0.004	49.835
12	11	454	0.3	–1	11	0.003	69.5
14	12	445	0.3	–1	12	0.002	50.919
15	14	471	0.3	–1	14	0.001	52.915
16	15	895	0.3	–3	15	0.004	54.946
17	16	290	0.3	–3	16	0.004	56.379
18	17	209	0.3	–4.3	17	0.001	52.8
1	40	181	0.6	0.28	18	0.001	49.986
40	28	212	0.6	–4.23	40	0.037	52.124
28	27	432	0.6	–0.74	28	0.054	56.198
27	26	375	0.6	1.15	27	0.017	56.783
26	29	250	0.5	2.63	26	0.008	55.526
29	25	266	0.4	–2.1	29	0.006	52.74
25	21	300	0.4	3	24	0.002	49.5
25	24	217	0.5	–5	23	0.003	51.568
24	23	215	0.5	–6	22	0.014	54.288
23	22	305	0.7	–1	39	0.064	48.063
22	39	671	0.7	–3	38	0.006	55.277
38	39	497	0.5	–1	37	0.005	53.842
37	38	525	0.45	–4	36	0.01	51.418
36	37	274	0.4	–5	43	0.011	50.848
43	36	219	0.3	–5	25	0.001	49.825
3	27	400	0.6	–5.48	21	0.016	49.876
4	18	700	0.6	–0.69	–	–	–
21	18	350	0.4	1.3	–	–	–
18	43	214	0.4	–4.3	–	–	–
0	22	0	0.8	0	–	–	–

As one could see from Fig. 2, we derived for each WN section the values of flow rates q and losses of heads h; we obtained, for each WN node, the value of head hc and excess head $hizb$.

Let the heads hc be known only for several WN nodes, which are the vertices of the graph of the equivalent model of WN. These nodes are marked in Fig. 3.

Next, we shall build EWN. To this end, we solve the problems on identifying the structure of EWN, identifying the EWN parameters, and identifying the EWN states.

The initial data to solve the problem on identifying the EWN structure: heads at the selected nodes of WN, heads and flow rates at the outputs from WN. The structure of EWN is shown in Fig. 4.

The problem of identifying the EWN parameters is then addressed. Consumer consumption is redistributed in some way across all nodes. One determines the hydraulic resistances of pipeline sections, at which heads at the nodes of WN and at the outputs from WN are retained, as well as the magnitude of water supply from PS. Fig. 5 shows results of solving a given problem.

Table 2 gives values of the parameters to be identified (columns 3–5).

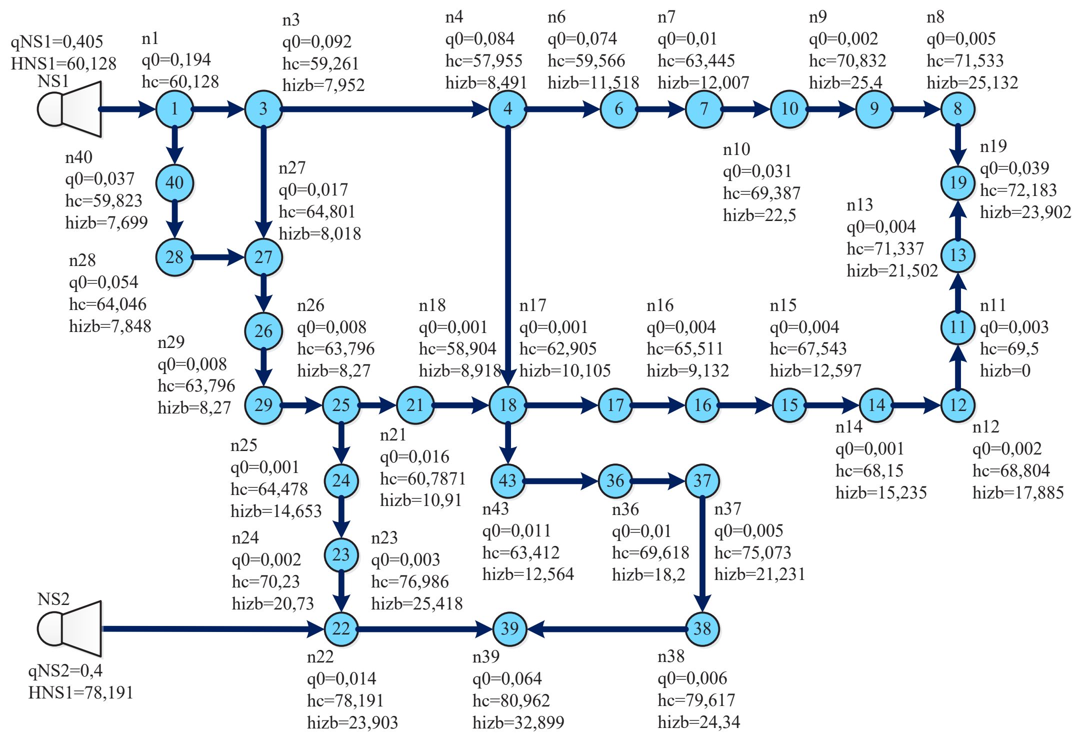

Fig. 2. Flow distribution in WN

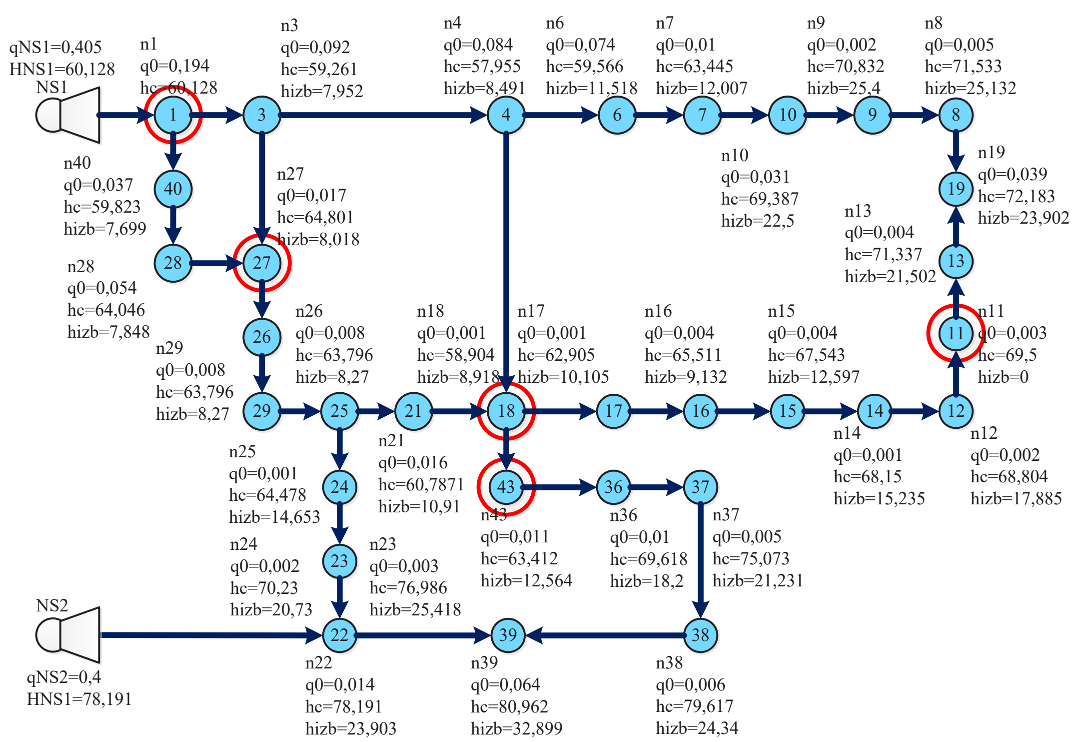

Fig. 3. Nodes with known heads

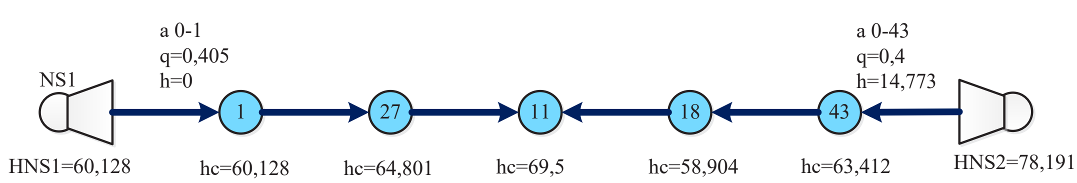

Fig. 4. EWN structure

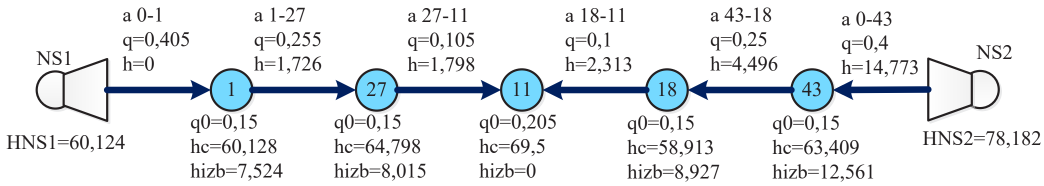

Fig. 5. Results of solving the problem on the identification of EWN parameters

Table 2

EWN identified parameters

Node 1	Node 2	l, m	*d*, m	*dh*, m
0	1	0	0.8	0
1	27	3,329	0.75	–6.4
27	11	20,450	0.75	–6.5
18	11	29,000	0.75	–12.9
43	18	12,700	0.8	0
0	43	16,300	0.8	0

The result of the EWN construction is the defined structure and parameters of EWN, at which, at the same values of heads and flow rates at the PS outputs, the heads at EFA nodes coincide with the heads at the nodes of the original WN. Next, we add leaks to the EWN.

In order to determine whether there is a leak in the EWN (un-authorized water flow rate), we apply the following method: increase the head at a dictating point in the EWN (node 11) by 1 meter. To this end, it would suffice to increase the minimum allowable head at a node 11 by 1 meter, that is there was a minimum allowable head of 69.5 m, now we obtain 70.5 m. If there are no leaks in the EWN, the heads at both pumping stations would also grow by the same magnitude, by 1 meter (Fig. 6). If there are leaks in the EWN, the heads at both pumping stations would also grow, not by 1 meter but by a smaller magnitude.

Comparing Fig. 5, 6, one could see that when the head increases at a dictating point by 1 meter, the head at the pumping stations also increases by exactly 1 meter. It could be concluded that there are no leaks in the examined EWN.

Next, we shall consider not the original WN, but the EWN. Suppose there is a hole at each EWN node that has a known diameter, from which water leaks. By knowing the head of water at the node and the diameter of the leak, the Torricelli formula (18) could be used to determine the mathematical expectation of the magnitude of a leak at each node. By substituting the consumption by consumers $q_{i0}(\omega)$, $(i=v+\eta_2,..., v+\eta_2+\xi_1-1)$ with the magnitude of consumption by consumers plus a leak (19), and by solving the flow distribution problem, we derive new values of heads at EWN nodes.

Results of calculations for EWN with leaks are shown in Fig. 7, 8.

Table 3

Results of solving the problem about flow distribution in EWN with leaks

Parameters	Node 1	Node 27	Node 11	Node 18	Node 43
Leak diameter, m	0.02	0.012	0.02	0.01	0.013
Flow rate, m^3/s	0.15	0.15	0.205	0.15	0.15
Iteration 1					
Head, m	60.124	64.798	69.5	58.913	63.409
Leak, m^3/s	0.010779078	0.004028478	0.01158911	0.00266749	0.00467692
Flow rate + leak, m^3/s	0.1607791	0.1540285	0.2165891	0.1526675	0.1546769
Iteration 2					
Head, m	60.100	64.741	69.5	59.567	64.654
Leak, m^3/s	0.010776926	0.004026706	0.01158911	0.00268226	0.00472261
Flow rate + leak, m^3/s	0.160777	0.154027	0.216589	0.152682	0.154723
Iteration 3					
Head, m	60.103	64.744	69.5	59.563	64.649
Leak, m^3/s	0.010777195	0.004026799	0.01158911	0.00268217	0.00472243
Flow rate + leak, m^3/s	0.160777195	0.154026799	0.216589110	0.152682168	0.154722428
Iteration 4					
Head, m	60.103	64.743	69.5	59.563	64.649
Leak, m^3/s	0.010777195	0.004026768	0.01158911	0.00268217	0.00472243
Flow rate + leak, m^3/s	0.1607772	0.1540268	0.2165891	0.1526822	0.1547224

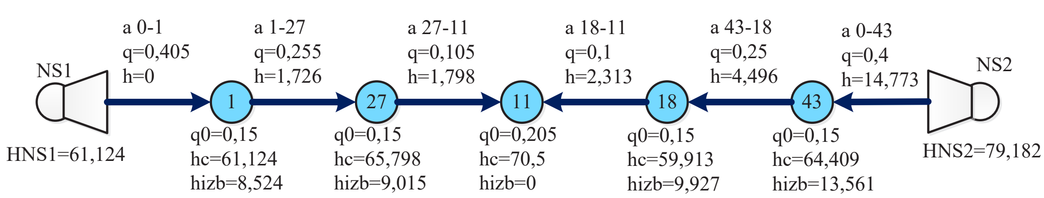

Fig. 6. Change in the head at pumping stations as the head increases at a dictating point by 1 meter

Comparing Fig. 7, 8, one could see that for EWN with leaks at the increase in head at a dictating point by 1 meter, the head at PS1 increases by 0.978 meters, the head at PS2 increases by 3.813 meters. This confirms the presence of leaks in the EWN.

Next, we again calculate the magnitude of a leak by knowing the head at the node and the diameter of the leak. Upon completion of several such iterations, we come to the conclusion that starting at a certain step the magnitude of leaks ceases to change.

Results of solving the problem about flow distribution in EWN with leaks are given in Table 3.

Results of solving the problem about flow distribution in EWN with leaks based on iterations are shown in Fig. 9.

By knowing the magnitude of a leak and head at each EWN node, one could determine the diameter of fistulas at each node (Table 4).

Thus, the result of three iterations is the determined magnitudes of the established heads and leaks at EWN nodes.

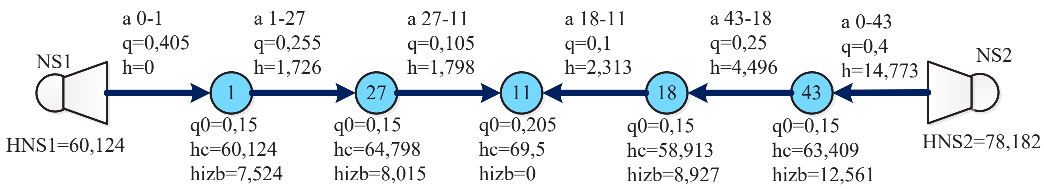

Fig. 7. Flow distribution for EWN with leaks

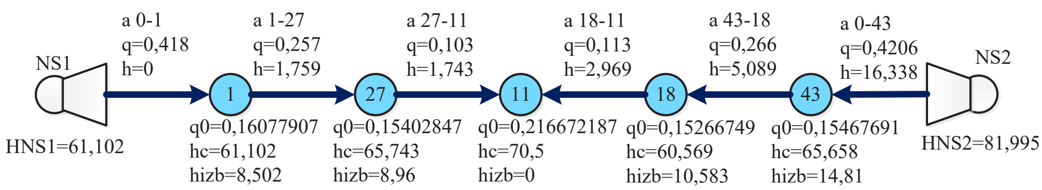

Fig. 8. Change in the heads at pumping stations as the head increases at a dictating point by 1 meter

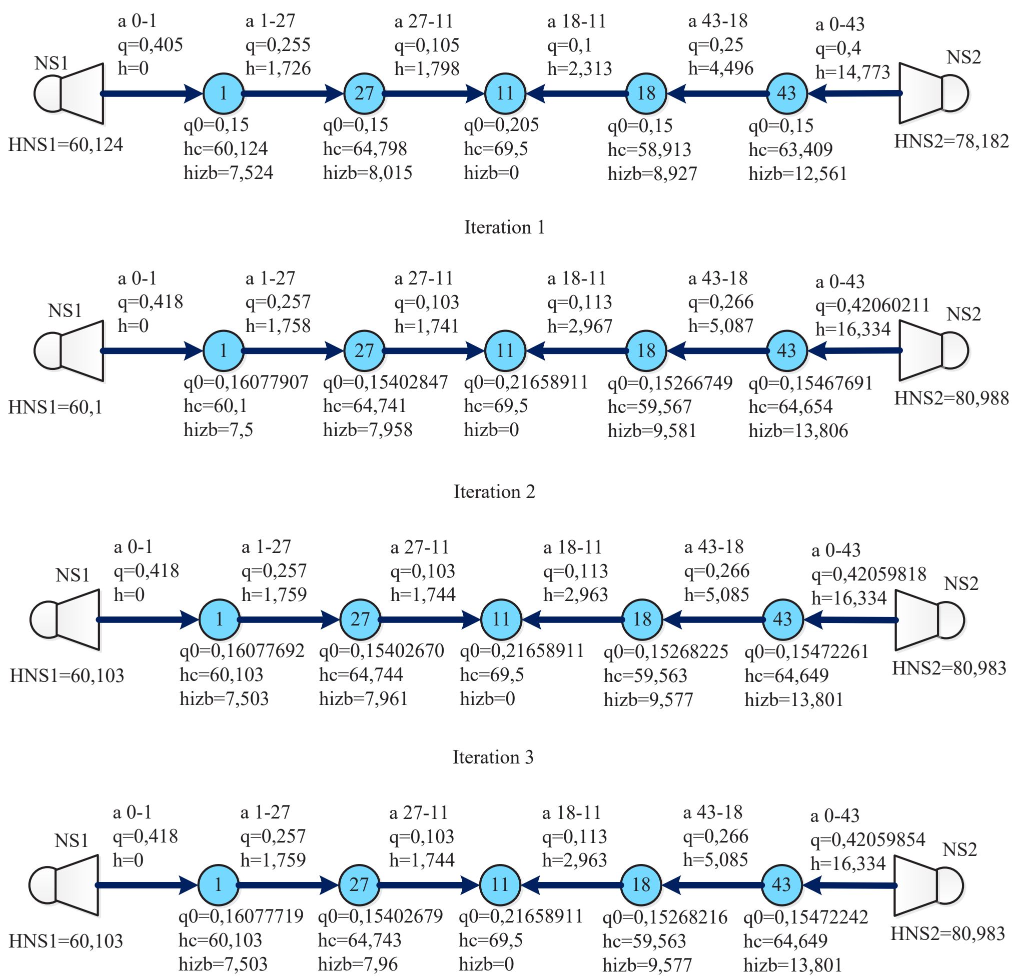

Fig. 9. Results of solving the problem about flow distribution in EWN with leaks based on iterations

Table 4

Determining the diameter of fistulas at each node

Node No.	Head, m	Leak, m^3/s	Fistula d, m
1	60.103	0.010777	0.02
27	64.743	0.004027	0.012
11	69.5	0.011589	0.02
18	59.563	0.002682	0.01
43	64.649	0.004722	0.013

6. Discussion of results of applying the stochastic model of water supply networks with leaks to calculate leaks

A mathematical model of the equivalent water supply network (7) to (9) has been built. Actual water supply networks have large dimensionality, a huge number of elements (pipeline section, nodes, consumers, pumping units, fittings). Because of this, and given the limited number of accounting devices, as well as the operational information provided for "Water utilities", is not enough to build an adequate mathematical model of a water supply network with leaks. To solve this issue, it is proposed to move from the scheme of an actual water supply network to the scheme of an equivalent water supply network. EWN is built based on checkpoints where accounting devices are installed. A technology for constructing EWN model has been devised: problems (7) to (9) are consistently solved for this purpose. A modified Newton method is used to solve them. The result of problem solving (7) to (9) is the structure (Fig. 4), parameters (Fig. 5, Table 2), and states of EWN, at which, in terms of control parameters (heads and flow rates at the outputs from PS, heads at nodes), EWN is identical to the original water supply network.

Based on the stochastic model of quasi-stationary modes of operation of WN (1) to (6) for the EWN whose model was built in section. 4. 1., by adding the Torricelli formula (18), a mathematical model of EWN with leaks is constructed (11) to (19).

A new method for detecting hidden leaks in water supply networks has been proposed. It is based on increasing the head at pumping stations and monitoring the magnitude of head changes at EWN nodes (Fig. 6). The advantage of the method is that no additional accounting tools are required to detect a leak; there are, however, limitations the site of a leak is determined with an accuracy to the district. To improve the accuracy of identification of the leak site, one needs to increase the number and location of head sensors. Such activities involve organizational rather than computational difficulties.

Special feature of the proposed method for detecting and calculating the magnitude of leaks in an equivalent water supply network is that the method makes it possible to perform calculations only for linear circuits, as shown in our example (Fig. 4).

The calculation of schemes of another structure is associated with computational difficulties and is the subject of the further research.

Increasing the number of checkpoints makes it possible to build more adequate models that preserve the circular structure of a water supply network.

The use of the proposed stochastic model and method to calculate the magnitude of leaks has allowed us to derive, in 3 iterations, the established head values at nodes of the equivalent water supply network and to refine the magnitudes of leaks (Fig. 9). Such a rapid convergence suggests that the proposed approach could be used to calculate leaks by "Water utilities".

7. Conclusions

1. A mathematical model of water supply networks with leaks has been built, which could be used to describe processes in water supply systems, to optimize the modes of operation of water supply networks with leaks. The following approach was used: instead of the original water supply network, an equivalent scheme of the water supply network was built. The task on constructing a scheme of an equivalent water supply network included the problem of identifying the structure, parameters, and condition of a water supply network.

2. A method for detecting hidden leaks in water supply networks has been developed, which is based on comparing a change in the magnitude of head at pumping stations and at the dictating points of the water supply network.

3. Based on the stochastic model of a water supply network with leaks, a method for calculating the magnitude of leaks has been constructed. By knowing water heads at the nodes of an equivalent water supply network and the supposed diameters of leaks at the nodes, the calculation of new values of head at the nodes of the equivalent water supply network is performed. Next, we again calculate the magnitude of a leak by knowing the new head at the node and the diameter of the leak. Upon completion of several such iterations, we come to the conclusion that starting at a certain step the magnitude of leaks and the heads at the nodes of the equivalent water supply network cease to change. By knowing the magnitude of a leak and head at each node in the equivalent water supply network, we determine the actual diameter of fistulas at each node.

Thus, a new approach to building a mathematical model of a water supply network with leaks implies moving from an actual scheme of a water supply network to an equivalent one. This makes it possible to significantly simplify calculations due to the reduced dimensionality of problems; it does not affect the adequacy of the model. The advantage of the constructed method for calculating the magnitude of leaks is that its application for actual water supply networks does not require additional financial costs and additional equipment.

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