Визначено умови настання динамічного автобалансування у випадку ротора на двох пружно-в'язких опорах, що балансується двома і більше пасивними автобалансиром будь-якого типу.

Застосовано модернізований енергетичний метод в припущенні, що маса вантажів автобалансирів набагато менше маси ротора. Метод розроблений для роторів на ізотропних пружно-в'язких опорах, коли до ротора приєднані тіла, відносному руху яких перешкоджають пружні і в'язкі сили опору. Метод дозволяє знаходити стаціонарні рухи роторної системи, оцінювати їх стійкість. На стаціонарних рухах відносні рухи приєднаних тіл припиняються, і система обертається як одне ціле навколо осі обертання, утвореної опорами.

Описана механіко-математична модель системи. Знайдений узагальнений потенціал на стаціонарних рухах і дисипативна функція, що відповідає опорам. Для узагальнених координат ротора складені рівняння стаціонарних рухів системи. Узагальнений потенціал досліджений на умовний екстремум в припущенні, що виконуються рівняння стаціонарних рухів, що відповідають узагальненим координатам ротора.

Встановлено, що динамічне балансування ротора можливо тільки в разі довгого ротора, двох і більше автобалансирів будь-якого типу, встановлених в різних площинах корекції і тільки на зарезонансного швидкостях обертання ротора. Встановлено, що сили опору в опорах явно не змінюють умови настання автобалансування, однак можуть змінювати ці умови неявно – шляхом зміни області існування стаціонарних рухів.

Отриманий результат збігається з результатом, отриманим із застосуванням узагальненого емпіричного критерію настання автобалансування при неврахування демпфірування в опорах. Показано, що модернізований енергетичний метод (як і узагальнений емпіричний критерій настання автобалансування) дозволяє знаходити узагальнені умови настання автобалансування, придатні для будь-яких типів автобалансирів. Тому обидва методи придатні для побудови загальної теорії пасивних автобалансирів – застосовної для автобалансирів будь-якого типу

Ключові слова: ротор, ізотропна опора, автобалансир, стаціонарний рух, стійкість руху, рівняння усталеного руху

ESTABLISHING CONDITIONS FOR THE OCCURRENCE OF DYNAMIC AUTO-BALANCING IN A ROTOR ON TWO ELASTIC-VISCOUS SUPPORTS

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1. Introduction

Passive auto-balancers are used to balance the rapidly rotating rotors [1–3]. Over time, the motion of a rotor with auto-balancers becomes steady state. At the so-called main (steady state) motions, loads balance the rotor, while at the side ones they do not [3]. For auto-balancers to work, it is required that the main motions should be stable [1–15] while

it would suffice that the side motions are unstable. The most general information on the performance of auto-balancers is provided by the results of analytical studies. In this regard, there is a common problem of building the theory of passive auto-balancing.

Passive auto-balancers can be classic (pendulum, roller, ball, ring, etc.) [1–3] and non-classic [3, 9, 10]. A large variety of both different types of auto-balancers and different rotary machines makes it much more difficult to build an analytical theory of passive auto-balancing by traditional methods based on the search for steady state motions of the rotary system and on studying their stability [2–10, 15]. Such analytical studies are much more difficult, or impossible at all, for cases of auto-balancers with many loads, multi-row auto-balancers, several auto-balancers, with an increase in the number of degrees of freedom of the rotary system, etc. Importantly, such cases are absolutely relevant to practice.

In order to construct a general theory of passive auto-balancing, empirical criteria for auto-balancing occurrence were developed [3, 12, 13]. The analytical results obtained from their application are general in nature as they are applicable for auto-balancers of any type [3, 11–13]. The criteria make it possible to answer the question under what conditions and over which range of rotation speeds it becomes feasible to balance a rigid or flexible rotor, fixed in a certain way by one or more passive auto-balancers.

Energy methods [14, 15] were devised to find and assess the stability of all stationary steady state motions (at which loads rotate in sync with the rotor).

It is a relevant task to show that under certain additional assumptions energy methods also make it possible to build a general theory of passive auto-balancing. At the same time, it is important to find analytical conditions for the occurrence of dynamic auto-balancing for a rotor on two isotropic supports, formed both by elastic bodies (springs) and viscous bodies (dampers). The specified mechanical system can simulate balancing, by passive auto-balancers, of washing machine drums, centrifuges, extractors, separators, rotating parts in the assemblies of axial and centrifugal fans, etc.

The existence of two different approaches (empirical and energy) makes it possible, at the lowest labor cost, to build a general theory of passive auto-balancing, to test correctness of the results obtained by solving a single problem by different methods.

2. Literature review and problem statement

We shall take a closer look at the results obtained for the case of dynamic balancing of a two-support rotor by two passive auto-balancers with two identical loads (pendulums, balls, etc.).

It was assumed in [4] that the mass of auto-balancers was much smaller than the mass of the rotor, that there were no resistance forces in the supports of the rotor; the rotational motion of balls rolling along running tracks was not taken into consideration (the balls were modeled by mathematical pendulums). It was established that balancing was possible only for the case of a long rotor at over-the-resonance speeds of rotation.

Similarly to [4], the conditions for auto-balancing occurrence were obtained in [5], but the authors considered the rotation of balls during rolling without sliding along the running track.

A mathematical model, similar to the model from work [4], was built in paper [6]; it, however, took into consideration

viscous resistance in the rotor supports. The numerical methods established that auto-balancing sets in at speeds slightly above the largest resonance speed of rotor rotation, provided that the rotor is long. The result is specific in nature as it was obtained at certain values for the system parameters. The authors did not examine a pattern of the effect of damping in supports on the conditions for auto-balancing occurrence.

Paper [7] reports a mathematical model under whose framework balls are considered to be particles (mathematical pendulums). The model takes into consideration the forces of dry and viscous friction, which prevent the motion of balls relative to the rotor, the forces of damping in the supports and the forces of gravity. Computational experiments found that the dynamic auto-balancing occurs for the case of a long rotor at the over-the-resonance speeds of rotor rotation. The result is specific in character. The authors did not study the patterns of influence exerted by damping in supports, forces of gravity, dry friction on the conditions for auto-balancing occurrence.

Article [8] describes a study, similar to that reported in [6,] the difference being the consideration of supports anisotropy. Numerical methods established that the dynamic auto-balancing was possible only for the case of a long rotor at the over-the-resonance speeds of rotation. The authors did not investigate the pattern of the effect of damping in supports on the conditions for auto-balancing occurrence.

The balancing of the rotor by a new type of the passive auto-balancer, ball-rod-spring-type, was explored in [9], and the ball-spring-type was explored in [10]. The research was conducted mainly by numerical methods. Each auto-balancer had two identical loads. At certain parameters of the system it was found that the new auto-balancers had parameter spaces, within which auto-balancing occurs, that were no less than those in a conventional ball-type auto-balancer.

The approaches applied in papers [4–10] are labor-intensive because they are implemented for a particular type of an auto-balancer and are based on finding the system's steady state motions and studying their stability. Analytical results were obtained only if the forces of viscous resistance in supports were not taken into consideration and only for auto-balancers with two loads.

Let us consider those approaches that make it possible to build a general theory of passive auto-balancers. Such approaches provide analytical results suitable for any type of auto-balancers.

An engineering (empirical) criterion for auto-balancing occurrence was proposed in [3] when balancing the rotor with a single auto-balancer of any type in a single plane of correction. The authors established conditions for auto-balancing occurrence when balancing the rotor with one auto-balancer of any type at the different kinematics of rotor motion.

In [11], the engineering (empirical) criterion was used to determine the conditions for auto-balancing occurrence when balancing the rotor on two isotropic elastic supports with by a single auto-balancer of any type. The authors demonstrated applicability of the criterion at the mass of auto-balancers' loads and rotor imbalance comparable to the mass of the rotor.

The engineering (empirical) criterion for auto-balancing occurrence was modernized in [12] to obtain the auto-balancing occurrence conditions when balancing the rotor by any number of auto-balancers of any type. The application of the new criterion and its effectiveness were illustrated by the problem on balancing, by several auto-balancers (excessive quantity), a solid axisymmetric rotor with a fixed point and an isotropic elastic support.

In [13], the modernized empirical criterion for the occurrence of auto-balancing was applied to the axisymmetric rotor on two isotropic elastic supports. As a result, it was found that it was possible to dynamically balance only a long rotor, by two or more passive auto-balancers of any type and only at the over-the-resonance speeds of rotation. The results obtained in [13] are the most general, as they cover the results reported in studies [4–10]. However, the influence of resistance forces in the supports on the conditions for auto-balancing occurrence was not studied.

It should be noted that the application of empirical criteria is complicated when the forces of viscous resistance in the supports are taken into consideration [3, 11–13]. The criteria themselves require a validation of their operation correctness. Therefore, it is advisable to obtain conditions for the occurrence of auto-balancing through an alternative method. An alternative method that mighty be used is the generalized energy method for studying the stationary motions of rotors with passive auto-balancers, outlined in [14]. This method is a generalization of the approach used in [15] in order to search for all stationary motions and to assess their stability within a flat rotor model on isotropic elastic supports, balanced by a two-ball auto-balancer.

The generalized energy method is applicable for rotors on isotropic elastic-viscous supports, for cases when the rotor carries attached bodies, on which viscous and elastic forces act when moving relative to the rotor. The method makes it possible to find conditions for the emergence, existence, and disappearance of all stationary motions of the rotor system, as well as to assess the stability of these motions. The results reported in [14] suggest that the generalized energy method is applicable in order to obtain conditions for the occurrence of auto-balancing suitable for any type of passive auto-balancers.

3. The aim and objectives of the study

The aim of this study is to find analytical conditions for the occurrence of dynamic auto-balancing for a rotor on two isotropic elastic-viscous supports and to assess at the same time: the effect of damping in supports on the stability of stationary motions of the system; applicability of the modernized energy method for building a general theory of passive auto-balancing.

To accomplish the aim, the following tasks have been set:
– to find, by using the modernized energy method, conditions for the occurrence of auto-balancing for the examined rotor system;
– to assess the impact of damping in support on the stability of stationary motions of the system;
– to verify suitability of the method to build a general theory of passive auto-balancers (applicable to auto-balancers of any type).

4. Method of determining conditions for the occurrence of auto-balancing

We shall use a generalized energy method [14]. The rotor is mounted on isotropic elastic-viscous supports and rotates at a constant angular velocity w. The auto-balancers' loads move relative to the rotor. The relative motion of loads is hindered by the Newton's viscous resistance forces.

The generalized coordinates of the rotor are denoted as z_i, $/i = \overline{1, n_r}/$, where n_r is the number of the rotor's degrees of freedom. The generalized coordinates of the attached bodies are denoted as ψ_j, $/j = \overline{1, n_b}/$, where n_b is the number of degrees of freedom of the attached bodies. At stationary motions, the generalized coordinates are constant:

$$z_i = \text{const}_i, \ /i = \overline{1, n_r}/; \ \psi_j = \text{const}_{j+n_r}, \ /j = \overline{1, n_b}/. \quad (1)$$

Equations of stationary motions are divided into two groups:

$$\frac{\partial \Pi}{\partial z_j} + \frac{\partial D_1}{\partial \dot{z}_j} = 0, \ /j = \overline{1, n_r}/; \ \frac{\partial \Pi}{\partial \psi_j} + \frac{\partial D_1}{\partial \dot{\psi}_j} = 0, \ /j = \overline{1, n_b}/, \quad (2)$$

where Π is the generalized potential, D_1 is the linear part of the dissipative function, constructed for the generalized coordinates of the rotor.

In accordance with the generalized energy method, the following constraints are imposed on rotor motions:

$$\frac{\partial \Pi}{\partial z_j} + \frac{\partial D_1}{\partial \dot{z}_j} = 0, \ /j = \overline{1, n_r}/. \quad (3)$$

In accordance with them, the rotor instantly enters a position corresponding to the total imbalance. After that, the loads tend to some equilibrium position.

For the stability of some stationary motion (1) of the rotor system, it is required that the generalized potential Π on it should have at least an uninsulated conditional minimum. In this case, the conditions are equations (3).

To obtain the generalized conditions for auto-balancing occurrence, suitable for any type of auto-balancers, we intend neither specify nor use the second group of equations in (2). Special features related to the implementation of this idea are outlined below using a rotor on two isotropic elastic-viscous supports as an example.

5. Results of determining the generalized conditions for the occurrence of dynamic auto-balancing

5. 1. Description of the system model
Fig. 1 shows a diagram of the rotor on two supports, Fig. 2 demonstrates its motion pattern. The rotor is balanced, rotating at a constant angular velocity w around the axis, passing through the longitudinal axis of the rotor shaft at undeformed supports. The masses that create imbalances are rigidly connected to it. The rotor is equipped with passive auto-balancers in order to balance the imbalance. The auto-balancers' bodies are rigidly connected to the rotor. Therefore, let us relate them to the rotor. Unbalanced masses are considered separately from the rotor.

The rotor is held by isotropic elastic-viscous supports, with the coefficients of rigidity and viscosity k_1, b_1 and k_2, b_2 respectively. The action of gravity forces is not taken into consideration.

We shall set the motion of the rotor with the help of two threes of the $OXYZ$ and $P\Xi HZ$ axes. The $P\Xi HZ$ axes are the main central axes of rotor inertia. In the static equilibrium position of the stationary rotor, the two axis systems are the same, with the Z, Z axes directed along the axis of the rotor shaft. In the process of moving, the $P\Xi HZ$ axes move in the following way. First, the $P\Xi HZ$ axes move progressively

along x, y relative to the $OXYZ$ axes, and, therefore, move into an intermediate position $PX_pY_pZ_p$ – Fig. 2, *a*. Then, the $PX_pY_pZ_p$ axes are rotated at angles *a*, *b*, as shown in Fig. 2, *b*, then they coalesce to the $P\Xi HZ$ axes. Then the $P\Xi HZ$ axes and the $OXYZ$ axes turn around the Z axis an angular velocity w.
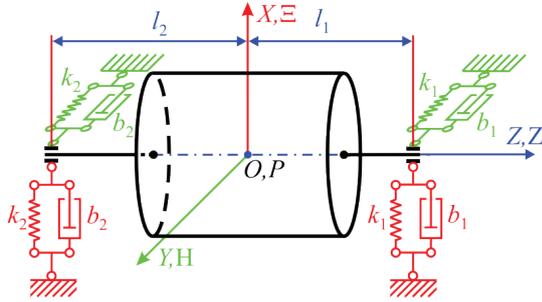
Fig. 1. Schematic of a rotor
on two elastic-viscous supports
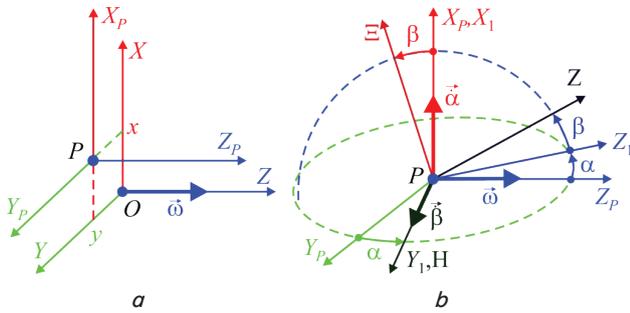
Fig. 2. Schematic of a rotor motion

Note that at the steady motion the system rotates like a rigid whole around the Z axis at a constant angular velocity w.

5. 2. Generalized potential, dissipative function, and equations of stationary motions

The kinetic energy of the system at steady state motion. Relative to the $P\Xi HZ$ axes, the inertia momentum of the system is formed by two components – a rotor and an unbalance with loads in auto-balancers. Denote the tensor of rotor inertia through $J_P^{(r)}$, the imbalances with loads – through $J_P^{(S)}$:

$$J_P^{(r)} = \begin{pmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{pmatrix};$$

$$J_P^{(S)} = \begin{pmatrix} J_\xi & -J_{\xi\eta} & -J_{\xi\zeta} \\ -J_{\xi\eta} & J_\eta & -J_{\eta\zeta} \\ -J_{\xi\zeta} & -J_{\eta\zeta} & J_\zeta \end{pmatrix}. \qquad (4)$$

The system's inertia tensor with respect to the $P\Xi HZ$ axes is $I_P = J_P^{(r)} + J_P^{(S)}$, hence:

$$I_\xi = A + J_\xi, \quad I_\eta = B + J_\eta, \quad I_\zeta = C + J_\zeta,$$

$$I_{\xi\eta} = J_{\xi\eta}, \quad I_{\xi\zeta} = J_{\xi\zeta}, \quad I_{\eta\zeta} = J_{\eta\zeta}. \qquad (5)$$

For passive auto-balancers with solid loads $J_\zeta = $ const [3]. Therefore:

$$I_\zeta = C + J_\zeta = \text{const}. \qquad (6)$$

Let the system have coordinates of the center of mass (point G, not shown in the diagram) ξ_G, η_G, ζ_G relative to the $P\Xi HZ$ axes. Then the system's inertia tensor relative to the central axes of the system $G\Xi_G H_G Z_G$ (not shown in the diagram), parallel to the $P\Xi HZ$ axes:

$$\mathbf{I}_G =$$
$$= \begin{pmatrix} I_\xi - M_\Sigma\left(\eta_G^2 + \zeta_G^2\right) & -I_{\xi\eta} + M_\Sigma\xi_G\eta_G & -I_{\xi\zeta} + M_\Sigma\xi_G\zeta_G \\ -I_{\xi\eta} + M_\Sigma\xi_G\eta_G & I_\eta - M_\Sigma\left(\xi_G^2 + \zeta_G^2\right) & -I_{\eta\zeta} + M_\Sigma\eta_G\zeta_G \\ -I_{\xi\zeta} + M_\Sigma\xi_G\zeta_G & -I_{\eta\zeta} + M_\Sigma\eta_G\zeta_G & I_\zeta - M_\Sigma\left(\xi_G^2 + \eta_G^2\right) \end{pmatrix},$$

hence

$$I_{\xi_G} = I_\xi - M_\Sigma\left(\eta_G^2 + \zeta_G^2\right), \quad I_{\xi_G\eta_G} = I_{\xi\eta} - M_\Sigma\xi_G\eta_G,$$

$$I_{\xi_G\zeta_G} = I_{\xi\zeta} - M_\Sigma\xi_G\zeta_G,$$

$$I_{\eta_G} = I_\eta - M_\Sigma\left(\xi_G^2 + \zeta_G^2\right), \quad I_{\eta_G\zeta_G} = I_{\eta\zeta} - M_\Sigma\eta_G\zeta_G,$$

$$I_{\zeta_G} = I_\zeta - M_\Sigma\left(\xi_G^2 + \eta_G^2\right). \qquad (7)$$

Here, M_Σ is the mass of the entire system.

Note that the products of inertia $I_{\xi\zeta}$, $I_{\eta\zeta}$ and coordinates of the center of mass ξ_G, η_G are the parameters that characterize the rotor unbalance.

We shall assume that the masses of imbalance and loads are much less than the mass of the rotor. Given this, we shall consider the following to be the magnitudes of the first order of smallness:

– coordinates of the center of masses ξ_G, η_G, ζ_G and components of the inertia tensor $J_P^{(S)}$ of imbalance with loads;

– coordinates a, b, x, y of the rotor.

We shall search for the reduced potential with an accuracy to the magnitudes of the second order of smallness inclusive.

According to the König's theorem (kinetics) [15], the kinetic energy of a system at steady state motion is the sum of two components of kinetic energy: T_{tr} – translational motion of the system together with the center of masses; T_r – rotational motion of the system around the center of masses. In this case,

$$T = T_{tr} + T_r,$$

$$T_{tr} = \frac{1}{2}M_\Sigma r_G^2\omega^2, \quad T_r = \frac{1}{2}\omega_{G\xi_G\eta_G\zeta_G}^B I_G \omega_{G\xi_G\eta_G\zeta_G}, \qquad (8)$$

where r_G is the distance from the center of mass of the system to the rotation axis Z; $\omega_{G\xi_G\eta_G\zeta_G}$ is the vector column of the angular velocity of system rotation, found in projections onto the $G\Xi_G H_G Z_G$ axes.

With an accuracy to the magnitudes of the first order of smallness inclusive, the displacement of the center of rotor mass relative to the $OXYZ$ axes:

$$x_G = x + \xi_G + \zeta_G\beta \approx x + \xi_G,$$

$$y_G = y + \eta_G - \zeta_G\alpha \approx y + \eta_G, \quad z_G = 0. \qquad (9)$$

Then, with an accuracy to the magnitudes of the second order of smallness inclusive:

$$T_{tr} = \frac{1}{2}M_\Sigma\left[\left(x + \xi_G\right)^2 + \left(y + \eta_G\right)^2\right]\omega^2 =$$

$$= \frac{1}{2}M_\Sigma\left[x^2 + y^2 + 2\left(x\xi_G + 2y\eta_G\right) + \xi_G^2 + \eta_G^2\right]\omega^2. \qquad (10)$$

Projections of the system rotation angular velocity onto the $G\Xi_G H_G Z_G$ axes (Fig. 2, b):

$$\omega_{\xi_G} = -\omega\cos\alpha\sin\beta = -\omega\beta + O(\beta^3),$$

$$\omega_{\eta_G} = \omega\sin\alpha = \omega\alpha + O(\alpha^3),$$

$$\omega_{\zeta_G} = \omega\cos\alpha\cos\beta = \omega\left[1 - (\alpha^2 + \beta^2)/2\right] + O(\alpha^4, \beta^4, \alpha^2\beta^2). \quad (11)$$

In turn,

$$T_r = \frac{1}{2}\left(I_{\xi_G}\omega_{\xi_G}^2 + I_{\eta_G}\omega_{\eta_G}^2 + I_{\zeta_G}\omega_{\zeta_G}^2\right) - I_{\xi_G\eta_G}\omega_{\xi_G}\omega_{\eta_G} - I_{\xi_G\zeta_G}\omega_{\xi_G}\omega_{\zeta_G} - I_{\eta_G\zeta_G}\omega_{\eta_G}\omega_{\zeta_G}.$$

Then, with an accuracy to the magnitudes of the second order of smallness inclusive:

$$T_r = \frac{\omega^2}{2}\left[\begin{array}{l}(B-C)\alpha^2 + (A-C)\beta^2 + \\ +2(I_{\xi\zeta}\beta - I_{\eta\zeta}\alpha) + I_\zeta - (\xi_G^2 + \eta_G^2)M_\Sigma\end{array}\right]. \quad (12)$$

Kinetic energy of the system with an accuracy to the magnitudes of the second order of smallness inclusive:

$$T = \frac{\omega^2}{2}\left\{\begin{array}{l}(B-C)\alpha^2 + (A-C)\beta^2 + \\ +2(I_{\xi\zeta}\beta - I_{\eta\zeta}\alpha) + \\ +M_\Sigma\left[x^2 + y^2 + 2(x\xi_G + y\eta_G)\right]\end{array}\right\} + I_\zeta\omega^2/2. \quad (13)$$

The system's potential energy:

$$V = (k_1\Delta l_1^2 + k_2\Delta l_2^2)/2 + \text{const}_1, \quad (14)$$

where Δl_1, Δl_2 is the module of deformation of supports' springs, const_1 is the undefined constant. In projections onto the $Kxyz$ axes:

$$\Delta l_{1x} \approx x + l_1\beta. \quad (15)$$

With an accuracy to the magnitudes of the second order of smallness inclusive:

$$V = \left[\begin{array}{l}k_{11}(x^2 + y^2) + 2k_{14}(\beta x - \alpha y) + \\ + k_{33}(\alpha^2 + \beta^2)\end{array}\right]\Big/2 + \text{const}, \quad (16)$$

where

$$k_{11} = k_1 + k_2, \ k_{14} = k_1 l_1 - k_2 l_2, \ k_{33} = k_1 l_1^2 + k_2 l_2^2. \quad (17)$$

The generalized potential at the steady motion $\Pi = V - T$. Assume:

$$\text{const}_1 = -I_\zeta\omega^2/2.$$

Then, with an accuracy to the magnitudes of the second order of smallness inclusive:

$$\Pi = \frac{1}{2}\left\{\begin{array}{l}\left[k_{33} - (B-C)\omega^2\right]\alpha^2 + \left[k_{33} - (A-C)\omega^2\right]\beta^2 + \\ + (k_{11} - M_\Sigma\omega^2)(x^2 + y^2) + 2k_{14}(\beta x - \alpha y)\end{array}\right\} +$$

$$+ \left[I_{\eta\zeta}\alpha - I_{\xi\zeta}\beta - M_\Sigma(x\xi_G + y\eta_G)\right]\omega^2. \quad (18)$$

Introduce designations:

$$v_{11} = (B-C)\omega^2 - k_{33}, \quad v_{22} = (A-C)\omega^2 - k_{33},$$

$$v_{33} = v_{44} = M_\Sigma\omega^2 - k_{11}. \quad (19)$$

Introduce vectors and matrices:

$$q^{(r)} = \begin{pmatrix}\alpha \\ \beta \\ x \\ y\end{pmatrix}, \quad S = \begin{pmatrix}-I_{\eta\zeta} \\ I_{\xi\zeta} \\ M_\Sigma\xi_G \\ M_\Sigma\eta_G\end{pmatrix},$$

$$\Pi = \begin{pmatrix}v_{11} & 0 & 0 & k_{14} \\ 0 & v_{22} & -k_{14} & 0 \\ 0 & -k_{14} & v_{33} & 0 \\ k_{14} & 0 & 0 & v_{33}\end{pmatrix}. \quad (20)$$

Then the generalized potential in the vector-matrix form:

$$\Pi = 0.5(q^{(r)})^T \Pi q^{(r)} - \omega^2(q^{(r)})^T S. \quad (21)$$

A dissipative function that corresponds to the rotor supports:

$$D^{(r)} = \frac{b_1}{2}\left\{\begin{array}{l}\left[\dot{x} + l_1\dot{\beta} - \omega(y - l_1\alpha)\right]^2 + \\ +\left[\dot{y} - l_1\dot{\alpha} + \omega(x + l_1\beta)\right]^2\end{array}\right\} +$$

$$+ \frac{b_2}{2}\left\{\begin{array}{l}\left[\dot{x} - l_2\dot{\beta} - \omega(y + l_2\alpha)\right]^2 + \\ +\left[\dot{y} + l_2\dot{\alpha} + \omega(x - l_2\beta)\right]^2\end{array}\right\} = D_0^{(r)} + D_1^{(r)} + D_2^{(r)}. \quad (22)$$

In (22):

$$D_2^{(r)} = 0.5\left[b_{11}(\dot{x}^2 + \dot{y}^2) + 2b_{14}(\dot{\beta}\dot{x} - \dot{\alpha}\dot{y}) + b_{33}(\dot{\alpha}^2 + \dot{\beta}^2)\right],$$

$$D_1^{(r)} = \left[\begin{array}{l}b_{11}(x\dot{y} - y\dot{x}) + b_{14}(\alpha\dot{x} - x\dot{\alpha} + \beta\dot{y} - \dot{\beta}y) + \\ +b_{33}(\dot{\beta}\alpha - \beta\dot{\alpha})\end{array}\right]\omega,$$

$$D_0^{(r)} = 0.5\left[\begin{array}{l}b_{11}(x^2 + y^2) + 2b_{14}(\beta x - \alpha y) + \\ + b_{33}(\alpha^2 + \beta^2)\end{array}\right]\omega^2; \quad (23)$$

– quadratic, linear and constituents independent on the generalized velocities, and in (23):

$$b_{11} = b_1 + b_2, \quad b_{14} = b_1 l_1 - b_2 l_2, \quad b_{33} = b_1 l_1^2 + b_2 l_2^2. \quad (24)$$

Introduce a skew-symmetric matrix:

$$B_1^{(r)} = \begin{pmatrix}0 & -b_{33} & -b_{14} & 0 \\ b_{33} & 0 & 0 & -b_{14} \\ b_{14} & 0 & 0 & -b_{11} \\ 0 & b_{14} & b_{11} & 0\end{pmatrix}. \quad (25)$$

Then, in the vector-matrix form:

$$D_1^{(r)} = 0.5\omega\dot{q}^T B_1^{(r)} q. \quad (26)$$

Equations of stationary motions of the system, constructed for generalized coordinates of the rotor:

$$\frac{\partial \Pi}{\partial q^{(r)}} + \frac{\partial D_1^{(r)}}{\partial \dot{q}^{(r)}} = \Pi q^{(r)} - \omega^2 S + \omega B_1^{(r)} q^{(r)} = 0. \qquad (27)$$

Hereafter, we consider equalities (27) as conditions whose satisfaction implies investigating the generalized potential for an extremum (21).

5. 3. Investigating the generalized potential for a conditional extremum
5. 3. 1. Transforming the generalized potential
We shall investigate Π for a conditional extremum. From (27) we find:

$$S = \left(\Pi q^{(r)} + \omega B_1^{(r)} q^{(r)}\right)\big/\omega^2; \qquad (28)$$

$$q^{(r)} = \omega^2 \left(\Pi + \omega B_1^{(r)}\right)^{-1} S. \qquad (29)$$

1. Exclude imbalances S from Π. Substituting S in Π, we obtain:

$$\Pi = 0.5\left(q^{(r)}\right)^T \Pi q^{(r)} - \omega^2 \left(q^{(r)}\right)^T S =$$

$$= 0.5\left(q^{(r)}\right)^T V q^{(r)} - \omega^2 \left(q^{(r)}\right)^T \left(\Pi q^{(r)} + \omega B_1^{(r)} q^{(r)}\right)\big/\omega^2 =$$

$$= -0.5\left(q^{(r)}\right)^T \Pi q^{(r)}. \qquad (30)$$

The transforms take into consideration that the skew-symmetric matrix $\mathbf{B}_1^{(r)}$ generates a zero quadratic form $((q^{(r)})^T B_1^{(r)} q^{(r)} = 0)$.

The transformed reduced potential does not implicitly depend on the forces of viscous resistance in the supports.

2. Exclude the generalized coordinates of rotor $\mathbf{q}^{(r)}$ from Π.

Substitute the generalized coordinates of the rotor from (29) in (30), we obtain:

$$\Pi = -0.5\left(q^{(r)}\right)^T \Pi q^{(r)} =$$

$$= -0.5\left[\omega^2 \left(\Pi + \omega B_1^{(r)}\right)^{-1} S\right]^T \Pi \omega^2 \left(\Pi + \omega B_1^{(r)}\right)^{-1} S =$$

$$= -0.5\omega^4 S^T \left\{\left[\left(\Pi + \omega B_1^{(r)}\right)^{-1}\right]^T \Pi \left(\Pi + \omega B_1^{(r)}\right)^{-1}\right\} S. \qquad (31)$$

According to the property of linear transformations of quadratic forms, quadratic forms (30) and (31) simultaneously accept extreme values, under the same conditions they are sign-defined, etc. [17]. Therefore, hereafter we shall investigate stability of the main motion in quadratic form (30).

5. 3. 2. Conditions for the occurrence of dynamic auto-balancing
We assume that the rotor is dynamically unbalanced. Therefore, it is balanced by two or more auto-balancers in two or more different planes of correction. Therefore, the parameters of imbalance $I_{\xi\zeta}, I_{\eta\zeta}, \xi_G, \eta_G$ are independent of each other and expressed at least through four independent coordinates, setting the positions of loads relative to the rotor.

Let us evaluate stability of the main motions. According to Sylvester's criterion [17], the necessary and sufficient conditions for a minimum of function of Π from (30) at the main motion:

$$v_{ii} > 0, \quad /i = \overline{1,4}/;$$

$$\Delta_2 = \begin{vmatrix} v_{11} & 0 \\ 0 & v_{22} \end{vmatrix} = v_{11} v_{22} > 0;$$

$$\Delta_3 = \begin{vmatrix} v_{11} & 0 & 0 \\ 0 & v_{22} & -k_{14} \\ 0 & -k_{14} & v_{33} \end{vmatrix} = v_{11}\left(v_{22} v_{33} - k_{14}^2\right) > 0;$$

$$\Delta_4 = \begin{vmatrix} v_{11} & 0 & 0 & k_{14} \\ 0 & v_{22} & -k_{14} & 0 \\ 0 & -k_{14} & v_{33} & 0 \\ k_{14} & 0 & 0 & v_{33} \end{vmatrix} = \left(v_{11}v_{33} - k_{14}^2\right)\left(v_{22}v_{33} - k_{14}^2\right) > 0. \qquad (32)$$

The first four conditions in (32) can be satisfied under the following condition:

$$A > C, \quad B > C \qquad (33)$$

at speeds that exceed:

$$\omega^* = \max\left(\sqrt{c_{11}/M_\Sigma}, \ \sqrt{c_{33}/(A-C)}, \ \sqrt{c_{33}/(B-C)}\right). \qquad (34)$$

In accordance with condition (33), it is possible to dynamically balance only a long rotor.

Assume $v_{ii} > 0, /i = \overline{1,4}/$. The condition $\Delta_2 > 0$ is then met automatically. Conditions $\Delta_3 > 0$ and $\Delta_4 > 0$ are met when the following conditions are satisfied:

$$\Delta_{41} = a_{11}a_{33} - c_{14}^2 > 0, \quad \Delta_{42} = a_{22}a_{33} - c_{14}^2 > 0. \qquad (35)$$

Equation $\Delta_4 = 0$ is an equation for finding the resonance frequencies of rotor vibrations in the absence of resistance forces [3]. Equation $\Delta_4 = 0$ is split into two equations $\Delta_{41} = 0$, $\Delta_{42} = 0$. Conditions (35) are met only at the over-the-resonance speeds of rotor rotation. Thus, dynamic balancing is only possible for a long rotor at the over-the-resonance rotation speeds.

Let the rotor be mounted on supports so that $k_{14} = 0$. Then conditions (35) take the form:

$$a_{11}a_{33} > 0, \quad a_{22}a_{33} > 0. \qquad (36)$$

In this case, auto-balancing can occur at speeds exceeding ω^* rom (34). For actual rotor systems [3]:

$$\sqrt{c_{11}/M_\Sigma} \le \sqrt{c_{33}/(B-C)} \le \sqrt{c_{33}/(A-C)},$$

which is why the necessary conditions for the occurrence of auto-balancing:

$$\omega > \sqrt{c_{33}/(A-C)}, \quad A > C. \qquad (37)$$

The resulting conditions for the occurrence of auto-balancing summarize the results obtained earlier in [13] by applying the results for the case of viscous resistance forces in the supports.

6. Discussion of the derived conditions for the occurrence of dynamic auto-balancing

It follows from the analysis of conditions (32) that the rotor that executes a spatial motion and which is mounted on two isotropic elastic-viscous supports can be dynamically balanced by two or more auto-balancers only for the case of a long rotor (33) at the over-the-resonance speeds of rotation.

The result obtained coincides with the result derived in [13] with the application of a generalized empirical criterion of auto-balancing occurrence, while not accounting for damping in supports. This confirms correctness of the results obtained by using energy and empirical methods.

Damping in supports does not affect the existence and the domain of stability of the main motions. Note that damping can affect both the side motions themselves and their domains of existence. However, this influence can only be investigated for specific types of auto-balancers using the second group of stationary motion equations in (2).

The modernized energy method makes it possible to find the necessary conditions for the occurrence of auto-balancing without:

– searching for and assessing the stability of side stationary motions;

– constructing differential equations of system motion.

The type and number of auto-balancers are not taken into consideration in such studies. Therefore, the resulting conditions are suitable for auto-balancers of any type, and the method itself is suitable for building a general theory of passive auto-balancing (applicable for auto-balancers of any type).

The method has flaws inherent in the approximate methods for investigating motion stability by Lyapunov. The method yields approximated boundaries of the regions where auto-balancing sets in. In addition, the method does not make it possible to study the non-stationary steady state motions of the system and transitional processes.

In the future, it is planned to obtain, with the help of the modernized energy method, conditions for the occurrence of single-plane auto-balancing for a rotor on isotropic elastic-viscous supports.

7. Conclusions

1. A rotor that executes a spatial motion and which is mounted on two isotropic elastic-viscous supports can be dynamically balanced by two or more auto-balancers of any type only for the case of a long rotor at the over-the-resonance speeds of rotation.

2. Damping in supports:

– does not affect the existence and the region of stability of the main motions;

– can affect both the side motions themselves and the regions of their existence.

3. The modernized energy method makes it possible to find the necessary conditions for the occurrence of auto-balancing without:

– searching for and assessing the stability of side stationary motions;

– constructing differential equations of system motion.

The type and number of auto-balancers are not taken into consideration in such studies. Therefore, the resulting conditions are suitable for auto-balancers of any type, and the method itself is suitable for building a general theory of passive auto-balancing (applicable for auto-balancers of any type).

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В теперішній час одна з основних проблем, що виникає під час тривалої експлуатації односекційних електровозів, пов'язана із необхідністю підтримання їх справного технічного стану. При цьому нерідко визначальним аспектом є оперативне виявлення наявних дефектів і пошкоджень основних несучих конструктивних елементів кузовів машин, а також недопущення їх розвитку в більш серйозні конструктивні відхилення.

Метою проведених досліджень є розробка спеціалізованого методу, що дозволяє виявити виникаючі в процесі експлуатації односекційних електровозів дефекти їх основних несучих конструктивних елементів кузовів на ранніх стадіях виникнення й розвитку. Даний метод динамічної інтегральної оцінки оснований на аналізі парційного динамічного спектру конструкції електровозу. За величиною відхилень від спектру по відношенню до теоретичного, отриманого на основі моделювання методом скінчених елементів, є можливим встановити приблизний характер і місце розташування наявного пошкодження, особливо скритого типу.

Отриманий в ході досліджень частотний спектр основних несучих конструктивних елементів кузовів односекційних електровозів є досить щільним і знаходиться в діапазоні частот до 20 Гц. Наявність пошкоджень знижує його значення, причому для найбільш розповсюджених в практиці експлуатації типів дефектів подібне зниження становить 25–30 %.

Ефективність практичного застосування метода динамічної інтегральної оцінки проілюстрована на прикладі машини ДСЗ-008. Метод дозволив виявити скрите пошкодження одного із елементів несучого каркасу передньої поверхні кабіни, не встановлене при стандартній процедурі технічного обслуговування машини. Застосування методу динамічної інтегральної оцінки технічного стану для електровозів є доволі універсальним та може бути рекомендовано також і для інших одиниць залізничного рухомого складу. В практиці впровадження подібного підходу дозволить ефективно попереджувати розвиток аварійних ситуацій

Ключові слова: односекційний електровоз, технічне обслуговування, метод динамічної інтегральної оцінки, технічний стан

DEVELOPMENT OF DYNAMIC INTEGRAL EVALUATION METHOD OF TECHNICAL STATE OF ONE-SECTION ELECTRIC LOCOMOTIVE BODY

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1. Introduction

At present, a whole fleet of one-section electric locomotives of various types and manufacturers is widely used on railways. They were produced in bulk in the 20th century and therefore have a fairly significant life. Most machines of this type are designed for standard-gauge railways (1,524/1,520 mm). The most structurally successful