Функція Гріна знаходить широке застосування при розв'язку крайових задач для диференціальних рівнянь, до яких зводяться багато математичних і фізичних задач. Зокрема, розв'язки диференціальних рівнянь з частинними похідними методом Фур'є зводяться до крайових задач для звичайних диференціальних рівнянь. За допомогою функції Гріна для однорідної задачі можна знайти розв'язок неоднорідного диференціального рівняння. Знання функції Гріна дає можливість розв'язувати цілий клас задач з знаходження власних значень в квантовій теорії поля.

Описана розроблена побудова функції Гріна крайових задач для звичайних лінійних диференціальних рівнянь. Представлені алгоритм і програма в системі Maple для обчислення функції Гріна крайових задач для диференціальних рівнянь другого та третього порядків в явному аналітичному вигляді. Наведені приклади обчислення функції Гріна для конкретних крайових задач. Необхідна для побудови функції Гріна фундаментальна система розв'язків звичайних диференціальних рівнянь з особливими точками обчислюється в вигляді узагальнених степеневих рядів за допомогою розроблених програм в середовищі Maple. Розроблено алгоритм побудови функції Гріна в вигляді степеневих рядів для диференціального рівняння другого та третього порядків з заданими крайовими умовами. Складено робочі програми в середовищі Maple для обчислення функції Гріна довільних крайових задач для диференціальних рівнянь другого та третього порядків. Наведено розрахунки функції Гріна для конкретних крайових задач третього порядку за допомогою розробленої програми. Проведено порівняння отриманої наближеної функції Гріна з відомими виразами точної функції Гріна і отримана дуже гарна згода

Ключові слова: функція Гріна, звичайні диференціальні рівняння, степеневі ряди, узагальнені степеневі ряди, крайові задачі

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CALCULATION OF THE GREEN'S FUNCTION OF BOUNDARY VALUE PROBLEMS FOR LINEAR ORDINARY DIFFERENTIAL EQUATIONS

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1. Introduction

The Green's function is widely used in solving boundary value problems for differential equations, to which many mathematical and physical problems are reduced. In particular, solutions of partial differential equations by the Fourier method are reduced to boundary value problems for ordinary differential equations. Let's note that using the Green's function for a homogeneous problem, it is possible to calculate the solution of an inhomogeneous differential equation. Also, using the Green's function, one can solve the problem of finding eigenvalues, which are very relevant in quantum field theory.

Actual and important in mathematical research are the problems of integrating linear ordinary differential equations of the third order, as well as constructing on their basis the Green's function of the boundary value problem for an ordinary differential equation of the third order.

2. Literature review and problem statement

The Green's function in one variable must satisfy the original differential equation of any boundary value problem. Therefore, the Green's function itself can be represented

as a linear combination of linearly independent solutions (fundamental system of solutions) of the original differential equation. The problem of finding linearly independent solutions of the original differential equation is considered in detail in [1]. The problem of solving the initial differential equation when studying the properties of a harmonic oscillator in an electromagnetic field is considered in [2]. The problem of finding solutions to the differential equation in quantum mechanics is considered in [3, 4] when finding the field potential in the Schrödinger equation. Finding solutions of inhomogeneous differential equations with a power law dependence of heterogeneity is considered in detail in [5]. The problem of finding the integrability conditions for second-order differential equations is considered in [6]. However, finding a fundamental system of solutions is a rather complicated, practically unexplored mathematical problem. Thus, researchers in this field have come to the conclusion that it is necessary to use the Green's function to find solutions of differential equations. This problem is considered in detail in [7–9], but it is not solved due to the nonlinearity of systems of differential equations. The authors of this article managed to partially solve this problem using the MAPLE computer system in calculating the eigenvalues and eigenfunctions of the Mathieu equation [10]. The authors of this article also used the MAPLE computer system to study the nonlinear Hamiltonian system by the Birkhoff – Gustavson method [11]. The need to obtain the Green's function for solving differential equations in an analytical form is shown in [12–14]. In [15], the problem of the nonlinearity of a system of solutions of differential equations is considered, which makes it impossible to obtain the Green's function in an analytical form. In addition, the search for a fundamental system of solutions is even more complicated if the differential equation has singular points.

This difficulty can be overcome if solutions of differential equations are sought in the form of power series, and if there are singular points, they are searched in the form of generalized power series, for example, according to the Frobenius method. However, this entails laborious tasks, such as substituting series in series, their differentiation, comparing coefficients at the same powers. In addition, when constructing the Green's function, problems arise for solving systems of high-order algebraic equations. Therefore, for the successful and accurate solution of such problems, the well-known computer system of symbolic-numerical transformations Maple is used, which allows to perform such necessary transformations in an analytical form with sufficient accuracy and speed.

Therefore, the development of algorithms and compilation of programs in the Maple system for explicitly calculating Green's functions and verifying the operation of these programs for specific boundary value problems is quite reasonable. The main properties and methods of constructing the Green's function, which we denote by $G(x, \xi)$, as well as a wide range of different classes of applied problems that can be used to solve the Green's function, are described in many classical textbooks on differential equations, as well as in special manuals [16] and monographs [17] on Green's functions.

3. The aim and objectives of research

The aim of this research is calculation of the Green's function of boundary value problems of ordinary differential equations, linearly independent solutions of which, even if there are singular points, can be effectively calculated using programs using Maple computer-aided symbolic-numerical computing systems.

To achieve the aim, the following objectives are set:

– to develop an algorithm for constructing the Green's function of boundary value problems for ordinary differential equations of the second and third orders;

– to develop an algorithm for finding the fundamental system of solutions of ordinary differential equations of the second and third orders;

– to carry out calculations of the Green's function for specific boundary value problems.

4. Green's function calculation method

Let's introduce the differential operator

$$\hat{L} \equiv p_0(x)\frac{d^n}{dx^n} + p_1(x)\frac{d^{n-1}}{dx^{n-1}} + \\ + p_2(x)\frac{d^{n-2}}{dx^{n-2}} + \cdots + p_n(x). \quad (1)$$

In the interval $x \in [a, b]$, let's consider the boundary-value problem for the ordinary differential equation

$$\hat{L}\{y(x)\} = 0 \quad (2)$$

with homogeneous boundary conditions:

$$U_\mu(y) \equiv \sum_{k=0}^{n-1} \alpha_{\mu k} \cdot y^{(k)}(a) + \\ + \sum_{k=0}^{n-1} \beta_{\mu k} \cdot y^{(k)}(b) = 0, \\ \mu = 1,2,3,4. \quad (3)$$

Here $y^{(k)}(x)$ is the k-th derivative of the function $y(x)$, and $y^{(0)}(x) \equiv y(x)$, α_{ik} and β_{ik} are numerical coefficients that are not equal to zero at the same time, i. e. $\alpha_{ik}^2 + \beta_{ik}^2 \neq 0$, $i, k = 0, 1, 2, 3$.

For convenience, the boundary conditions (3) are briefly written in the form

$$U_\mu(y) \equiv U_{\mu a}(y) + U_{\mu b}(y) = 0. \quad (4)$$

Let's give the main properties of the Green's function $G(x, \xi)$.

1) the function is continuous and has continuous derivatives with respect to x up to the $(n-2)$ order, inclusive, for all values of x and ξ from the interval $[a, b]$;

2) the derivative of the $(n-1)$ -order for $x = \xi$ has a jump equal to $1/p_0(\xi)$, i. e.

$$\frac{d^{(n-1)}}{dx^{(n-1)}} G(\xi + 0, \xi) - \frac{d^{(n-1)}}{dx^{(n-1)}} G(\xi - 0, \xi) = \frac{1}{p_0(\xi)};$$

3) in each of the intervals $[a, \xi)$ and $(\xi, b]$, the function $G(x, \xi)$ in the variable x satisfies the differential equation and the boundary conditions $U_\mu(G) \equiv 0$, $\mu = 1, 2, 3, 4$. The function $G(x, \xi)$ is called the Green's function or the influence function for a given boundary value problem.

The following theorem is proved in the theory of differential equations: "If the boundary value problem has only the trivial solution $y(x)\equiv 0$, then the operator $\hat{L}$, that is, the boundary value problem has one and only one Green's function. It is also equivalent that the number $\lambda\equiv 0$ is an eigenvalue of the operator $\hat{L}$".

Using the properties of the Green's function, let's present general formulas for its calculation. For the application of computer calculations, the Green's function is conveniently sought in the form [15]:

$$G(x,\xi)=\begin{cases}G_L(x,\xi), & a\le x\le\xi\le b,\\ G_R(x,\xi), & a\le\xi\le x\le b,\end{cases} \tag{5}$$

where

$$G_L(x,\xi)=\sum_{k=1}^{n}\left[A_k(\xi)+B_k(\xi)\right]y_k(x),$$

$$G_R(x,\xi)=\sum_{k=1}^{n}\left[A_k(\xi)-B_k(\xi)\right]y_k(x), \tag{6}$$

and $y_{(k)}(x)$ are linearly independent solutions of differential equation (2).

The conditions for the continuity of the Green's function (property 1) are written in the form of two equations

$$\begin{cases}\sum\limits_{k=1}^{n}B_k(x)y_k(x)=0,\\ \sum\limits_{k=1}^{n-2}B_k(x)\dfrac{dy_k(x)}{x}=0,\end{cases} \tag{7}$$

and property 3) – the jump of the (n–1)-th derivative at the point $x\equiv\xi$ is written in the form of the following equation

$$\sum_{k=1}^{n}B_k(\xi)y_k^{(n-1)}(\xi)=-\frac{1}{p_0(\xi)}. \tag{8}$$

As a result, let's obtain a linear system of algebraic equations with respect to the functions $B_k(\xi)$:

$$\begin{cases}B_1\cdot y_1(\xi)+B_2\cdot y_2(\xi)+B_3\cdot y_3(\xi)+\\ +B_4\cdot y_4(\xi)+\cdots+B_n\cdot y_n(\xi)=0,\\ B_1\cdot y_1^{(1)}(\xi)+B_2\cdot y_2^{(1)}(\xi)+B_3\cdot y_3^{(1)}(\xi)+\\ +B_4\cdot y_4^{(1)}(\xi)+\cdots+B_n\cdot y_n^{(1)}(\xi)=0,\\ B_1\cdot y_1^{(2)}(\xi)+B_2\cdot y_2^{(2)}(\xi)+B_3\cdot y_3^{(2)}(\xi)+\\ +B_4\cdot y_4^{(2)}(\xi)+\cdots+B_n\cdot y_n^{(2)}(\xi)=0,\\ B_1\cdot y_1^{(n-1)}(\xi)+B_2\cdot y_2^{(n-1)}(\xi)+B_3\cdot y_3^{(n-1)}(\xi)+\\ +B_4\cdot y_4^{(n-1)}(\xi)+\ldots+B_n\cdot y_n^{(n-1)}(\xi)=-\dfrac{1}{2p_0(\xi)}.\end{cases} \tag{9}$$

Since the determinant of this system is equal to the Wronskian of linearly independent solutions $y_k(\xi)$, $k=1, 2, 3, 4$, which is not equal to zero, system (9) is defined and has a unique solution $A_k(\xi)$, $k=1, 2, 3, 4$. To find the functions $A_k(\xi)$, $k=1, 2, 3, 4$, let's use the boundary conditions (3):

$$\sum_{k=0}^{n-1}\alpha_{\mu k}y^{(k)}(a)+\sum_{k=0}^{n-1}\beta_{\mu k}y^{(k)}(b)=0,\ \ (\mu=1,\ldots,n). \tag{10}$$

From this system, for known $B_k(\xi)$, let's find the solutions $A_k(\xi)$, knowing which let's calculate the Green's function according to expressions (5), (6).

If the determinant of system (10) is equal to zero, then the obtained equation for this determinant

$$\det\left(U_\mu\left(y_i(x,\lambda)\right)\right)=0$$

will determine the eigenvalues λ when solving the problem on the eigenvalues of operator (1).

As follows from the general scheme of constructing the Green's function described above, it is necessary to calculate the fundamental system of solutions for differential equation (2). Algorithms are developed and programs developed in the MAPLE environment for calculating all linearly independent solutions of differential equations of type (2) in the form of generalized power series [18, 19].

According to the general scheme for computing the Green's function, an algorithm has also been developed, the main steps of which are presented below, and the corresponding programs are compiled using the MAPLE programming system to construct the Green's function of some boundary value problems [20, 21].

5. The algorithm for constructing the Green's function for ordinary differential equations of the second and third order

Algorithm description [20]:

Input:

$P_k(x)$, $k=0,1,...$ are the coefficient-functions of a given differential equation; n is the maximum exponent of the power series used; x_0 is the singular point of equation (2), if any; α_{ik} and β_{ik} are the coefficients of the boundary conditions (3); a, b are the boundary points of the segment $[a, b]$;

Output:

$y_k(x)$ is the fundamental system of solutions of a given differential equation (2); *G_left*(x, ξ) is the Green's function on the interval $a\le x\le\xi\le b$; *G_right*(x, ξ) is the Green's function on the interval $a\le\xi\le x\le b$.

Description of algorithm steps:

1) calculation of linearly independent solutions $y_k(x)$ in the form of power series for differential equation (2);

2) verification of the solutions found by substitution;

3) calculation of the coefficients $B_k(\xi)$ from the system of equations (9);

4) verification of the found solutions of this system;

5) drawing up a system of equations (10), finding its solutions $A_k(\xi)$ and checking these solutions;

6) construction of the functions $G_L(x, \xi)$, $G_R(x, \xi)$, $G(x, \xi)$ according to expressions (5), (6);

7) verification of the main properties of the Green's function $G(x, \xi)$.

6. Examples of calculations of Green's functions of boundary value problems for ordinary differential equations of the second order

Let's present the results of calculating the Green's function of boundary value problems for second-order differential equations using the program [20]:

Example 1

Let's consider the differential equation

$$x^2y''-xy'+y=x^2\ln x+x \qquad (11)$$

with boundary conditions

$$y(1)=0,$$

$$y'(1)+y'(2)=0.$$

Using the program [20] for the corresponding homogeneous differential equation, the Green's function is obtained in the form:

$$G(x,\xi)=\begin{cases}G_L(x,\xi), & 1\le x\le\xi\le 2,\\ G_R(x,\xi), & 1\le\xi\le x\le 2,\end{cases} \qquad (12)$$

where

$$G_L(x,\xi)=\frac{(\ln\xi-\ln 2-1)x\ln x}{\xi^2(2+\ln 2)},$$

$$G_R(x,\xi)=\frac{x(\ln x\ln\xi-2\ln\xi-\ln 2\ln\xi+\ln x)}{\xi^2(2+\ln 2)}.$$

Knowing the Green's function (12) for a homogeneous differential equation, which corresponds to an inhomogeneous differential equation (11), by the formula

$$y(x)=\int_1^2 G(x,\xi)(\xi^2\ln\xi+\xi)d\xi$$

let's obtain the solution of the inhomogeneous equation

$$y(x)=\frac{x}{(4+2\ln 2)}\cdot\begin{bmatrix}2\ln^2 x+2x\ln 2\ln x+\\ +\ln 2\ln^2 x+4x\ln x-\\ -10\ln 2\ln x+10\ln x-\\ -\ln^2 2\ln x-8x-\\ -4x\ln 2+4\ln 2+8\end{bmatrix}.$$

Thus, a solution to the original differential equation is found in an analytical form.

Example 2

Let's consider the differential equation

$$x^2y''-3xy'+5y=3x^2$$

with boundary conditions

$$2y(1)+5y'(1)+y(2)-3y'(2)=0,$$

$$y(1)-5y'(1)-y(2)+3y'(2)=0.$$

For the corresponding homogeneous equation with the same boundary conditions, the Green's function is obtained from the program [20]

$$G(x,\xi)=\begin{cases}G_L(x,\xi), & 1\le x\le\xi\le 2,\\ G_R(x,\xi), & 1\le\xi\le x\le 2,\end{cases}$$

where

$$G_L(x,\xi)=\frac{2x^2\sin(\ln x)\{3\cos[\ln(\xi/2)]-4\sin[\ln(\xi/2)]\}}{\xi^3[5-8\sin(\ln 2)-6\cos(\ln 2)]},$$

$$G_R(x,\xi)=\frac{x^2\{5\sin[\ln(\xi/2)]+6\sin[\ln\xi]\cos[\ln(x/2)]-8\sin[\ln(\xi)]\sin[\ln(x/2)]\}}{\xi^3[5-8\sin[\ln 2]-6\cos(\ln 2)]}.$$

Thus, the Green's function is calculated for the original differential equation.

Example 3

Let's consider the differential equation

$$y''-2y/x^2=3x^3\sin x+\ln x$$

with boundary conditions

$$y(1)=0,$$

$$y(2)+y'(2)=0.$$

The following Green's function is obtained:

$$G(x,\xi)=\begin{cases}G_L(x,\xi), & 1\le x\le\xi\le 2,\\ G_R(x,\xi), & 1\le\xi\le x\le 2,\end{cases}$$

where

$$G_L(x,\xi)=\frac{(1-x)(x^2+x+1)}{3x\xi}, \quad 1\le x\le\xi\le 2,$$

$$G_R(x,\xi)=\frac{(1-\xi)(\xi^2+\xi+1)}{3x\xi}, \quad 1\le x\le\xi\le 2.$$

Similarly to the previous examples, from the given values of the Green's function let's find

$$y(x)=60x\sin x+120\cos x-$$
$$-3x^3\sin x+1/6\ln^2 x-1/9\ln x.$$

Thus, a solution to this inhomogeneous differential equation is obtained:

Example 4

Let's consider the differential equation

$$y''+\frac{1}{x}y'-\frac{m}{x^2}y=0, \quad m=1,2,\ldots$$

with boundary conditions

$$y(0)=y(1)=0.$$

The following Green's function is obtained:

$$G(x,\xi)=\begin{cases}G_L(x,\xi), & 0\le x\le\xi\le 1,\\ G_R(x,\xi), & 0\le\xi\le x\le 1,\end{cases}$$

where

$$G_L(x,\xi)=\frac{x^m(\xi^{-m}-\xi^m)}{2m},$$

$$G_R(x,\xi)=\frac{\xi^m\left(x^{-m}-x^m\right)}{2m},$$

which coincides with the known result [22].

Example 5

Let's consider the following boundary value problem [23]

$$\frac{d}{dx}\left\{(1+\alpha x)^4\frac{dy}{dx}\right\}+\lambda^2 y=0,$$

$$y(0)=y(L)=0. \qquad (13)$$

This problem is an eigenvalue problem that arises when studying the stability of a cone-shaped rod under the action of an external longitudinal force. The parameter α determines the geometric configuration of the truncated cone. In this problem, the critical force at which the rod loses stability is equal to the product of Young's modulus and the smallest eigenvalue λ.

Equation (13) has the following linearly independent solutions:

$$y_1(x)=\alpha\cos\left[\frac{\lambda}{\alpha(1+\alpha x)}\right]+$$
$$+\left(\frac{\lambda}{(1+\alpha x)}\right)\sin\left[\frac{\lambda}{\alpha(1+\alpha x)}\right],$$

$$y_1(x)=-\alpha\sin\left[\frac{\lambda}{\alpha(1+\alpha x)}\right]+$$
$$+\left(\frac{\lambda}{(1+\alpha x)}\right)\cos\left[\frac{\lambda}{\alpha(1+\alpha x)}\right].$$

Eigenvalues are found from the equation

$$\det\left(U_\mu\left(y_i(x,\lambda)\right)\right)=0,$$

which leads to the following transcendental equation

$$\lambda^2-\alpha^2 L ctg\left[\frac{\lambda L}{1+\alpha L}\right]\cdot\lambda+\alpha^2+\alpha^3 L=0. \qquad (14)$$

The value λ obtained by formula (14) differs by less than 2 % from the same value obtained in [23] in another way.

The examples of solving boundary value problems in this section are used by the authors in solving the equations of heat conduction and oscillations with partial derivatives.

7. Examples of calculations of the Green's functions of some boundary value problems for ordinary differential equations of the third order

We present the results of calculating the Green's function of boundary value problems for third-order differential equations using the program [20].

Example 1

Let's consider the boundary value problem for the differential equation

$$y'''=x\sin x+2x^2\cos x+3x^3$$

with boundary conditions

$$y(0)=0,\ \ y(1)=0,\ \ y'(0)-y'(1)=0.$$

For this homogeneous differential equation, the Green's function is obtained in the form:

$$G(x,\xi)=\begin{cases}G_L(x,\xi), & 1\le x\le\xi\le 2,\\ G_R(x,\xi), & 1\le\xi\le x\le 2,\end{cases}$$

where

$$G_L(x,\xi)=\frac{1}{2}x(\xi-1)(x-\xi),$$

$$G_R(x,\xi)=-\frac{1}{2}\xi(x-1)(\xi-x).$$

Next, a solution to the inhomogeneous equation is obtained:

$$y(x)=\frac{1}{40}x^6+\frac{197}{40}x^2-\frac{99}{20}x-2x^2\sin(x)-$$
$$-\frac{7}{2}x^2\sin(1)-4x^2\cos(1)-11x\cos(x)+$$
$$+21\sin(x)+15x\cos(1)-\frac{31}{2}x\sin(1).$$

Thus, a solution to the original differential equation is found in an analytical form.

Example 2

Let's consider the differential equation

$$y'''=0$$

with boundary conditions

$$y(0)=0,\ \ y(1)=0,\ \ y'(0)=y'(1).$$

Using the developed program, the Green's function is obtained:

$$G(x,\xi)=\begin{cases}G_L(x,\xi), & 0\le x\le\xi\le 1,\\ G_R(x,\xi), & 0\le\xi\le x\le 1,\end{cases}$$

where

$$G_L=\frac{1}{2}\left(x^2\xi-x\xi^2-x^2+x\xi\right),$$

$$G_R=\frac{1}{2}\left(x^2\xi-x\xi^2-x\xi+\xi^2\right).$$

The resulting expression coincides with the exact expression and is anti-self-adjoint.

Example 3

For a differential equation

$$y'''(x)+y'(x)=x\cos^2(x)$$

with uniform boundary conditions

$$y(0)=0,\ \ y\left(\frac{\pi}{2}\right)=0$$

and

$$y'(0)=y'\left(\frac{\pi}{2}\right)$$

the Green's function is found in the form

$$G(x,\xi)=\begin{cases}G_L(x,\xi), & 0\le x\le\xi\le 1,\\ G_R(x,\xi), & 0\le\xi\le x\le 1,\end{cases}$$

where

$$G_L(x,\xi)=\frac{1}{2}\begin{bmatrix}\cos x-\sin x+\cos x\cos\xi-\\ -\cos x\sin\xi+\sin x\cos\xi+\\ +\sin x\sin\xi-\cos\xi+\sin\xi-1\end{bmatrix},$$

$$G_R(x,\xi)=\frac{1}{2}\begin{bmatrix}\cos x-\sin x-\cos x\ \cos\xi-\\ -\cos x\ \sin\xi+\sin x\ \cos\xi-\\ -\sin x\ \sin\xi-\cos\xi+\sin\xi+1\end{bmatrix},$$

where let's find:

$$y(x)=-\frac{\pi^2}{32}\sin x-\frac{11}{72}\sin x+\frac{\pi}{6}\sin x-$$
$$-\frac{x}{6}\cos x\ \sin x+\frac{\pi^2}{32}\cos x-\frac{\pi}{6}-\frac{\pi^2}{32}+\frac{11}{72}+$$
$$+\frac{x^2}{4}-\frac{11}{36}\cos^2 x+\frac{\pi}{6}\cos x+\frac{11}{72}\cos x.$$

Thus, the solution of the initial inhomogeneous differential equation in the analytical form is obtained.

8. Construction of the Green's function for third-order differential equations in the form of power series

Let's consider a third order differential equation

$$p_0(x)y'''+p_1(x)y''+p_2(x)y'+p_3(x)y=0 \qquad (15)$$

with boundary conditions

$$\alpha_{1,0}y(a)+\alpha_{1,1}y'(a)+\alpha_{1,2}y''(a)+$$
$$+\beta_{1,0}y(b)+\beta_{1,1}y'(b)+\beta_{1,2}y''(b)=0,$$

$$\alpha_{2,0}y(a)+\alpha_{2,1}y'(a)+\alpha_{2,2}y''(a)+$$
$$+\beta_{2,0}y(b)+\beta_{2,1}y'(b)+\beta_{2,2}y''(b)=0,$$

$$\alpha_{3,0}y(a)+\alpha_{3,1}y'(a)+\alpha_{3,2}y''(a)+$$
$$+\beta_{3,0}y(b)+\beta_{3,1}y'(b)+\beta_{3,2}y''(b)=0, \qquad (16)$$

where $p_0(x)$, $p_1(x)$, $p_2(x)$, $p_3(x)$ are continuous functions together with continuous derivatives of the first and second orders on the interval $[a, b]$, $\alpha_{1,0}$, $\alpha_{1,1}$, $\alpha_{1,2}$, $\alpha_{2,0}$, $\alpha_{2,1}$, $\alpha_{2,2}$, $\alpha_{3,0}$, $\alpha_{3,1}$, $\alpha_{3,2}$, $\beta_{1,0}$, $\beta_{1,1}$, $\beta_{1,2}$, $\beta_{2,0}$, $\beta_{2,1}$, $\beta_{2,2}$, $\beta_{3,0}$, $\beta_{3,1}$, $\beta_{3,2}$ are the coefficients in the boundary conditions (16) for a particular boundary-value problem,

$$\sum_i\sum_k\alpha_{ik}^2\neq 0,\ \sum_i\sum_k\beta_{ik}^2\neq 0,\ i=1, 2, 3,\ k=0, 1, 2.$$

To construct the Green's function of the boundary value problem (15), (16), let's first solve the Cauchy problem at the point x_0, find linearly independent solutions for equation (15) in the form of series:

$$y_1(x)=1+\sum_{k=3}^{\infty}c_k^{(1)}\cdot(x-x_0)^k, \qquad (17)$$

$$y_2(x)=(x-x_0)+\sum_{k=3}^{\infty}c_k^{(2)}\cdot(x-x_0)^k, \qquad (18)$$

$$y_3(x)=(x-x_0)^2/2+\sum_{k=3}^{\infty}c_k^{(3)}\cdot(x-x_0)^k, \qquad (19)$$

where $c_k^{(1)}$, $c_k^{(2)}$, are the numerical coefficients.

Let's find the Green's function in the form

$$G(x,\xi)=\begin{cases}G_L(x,\xi), & a\le x\le\xi\le b,\\ G_R(x,\xi), & a\le\xi\le x\le b,\end{cases}$$

where

$$G_L(x,\xi)=\sum_{k=1}^{3}\left[A_k(\xi)+B_k(\xi)\right]\cdot y_k(x),$$

$$G_R(x,\xi)=\sum_{k=1}^{3}\left[A_k(\xi)-B_k(\xi)\right]\cdot y_k(x).$$

Using the properties of the Green's function given above, let's find the coefficient functions $A_k(\xi)$, $B_k(\xi)$ and with their help let's construct the Green's function according to formulas (17)–(19). Since linearly independent solutions are represented by power series, the Green's function is also in the form of power series.

In accordance with the above formulas, an algorithm is developed for constructing the Green's function in the form of power series for the boundary value problem (15) in the Maple environment.

9. An algorithm for constructing the Green's function for third-order equations in the form of power series

Using three linearly independent solutions found in the form of power series, the Green's function is constructed on the basis of the developed algorithm and its calculation program.

Input:

n is the desired maximum order of the power series;

$P0(x)\neq 0$, $P1(x)$, $P2(x)$, $P3(x)$ in general, the coefficient functions in a given differential equation of the third order (15);

a, b are the boundary points of the segment [a, b] on which the Green's function is sought;

$\alpha_{1,0}$, $\alpha_{1,1}$, $\alpha_{1,2}$, $\alpha_{2,0}$, $\alpha_{2,1}$, $\alpha_{2,2}$, $\alpha_{3,0}$, $\alpha_{3,1}$, $\alpha_{3,2}$, $\beta_{1,0}$, $\beta_{1,1}$, $\beta_{1,2}$, $\beta_{2,0}$, $\beta_{2,1}$, $\beta_{2,2}$, $\beta_{3,0}$, $\beta_{3,1}$, $\beta_{3,2}$ are the coefficients in the boundary conditions (16) for a particular boundary-value problem.

Output:

$y1(x)$, $y2(x)$, $y3(x)$ are the fundamental system of solutions for a given differential equation of the third order (15);

$G\_L(x, \xi)$ is the Green's function on the interval $a\le x\le\xi\le b$;

$G\_R(x, \xi)$ is the Green's function on the interval $a\le\xi\le x\le b$.

Description of algorithm steps:

1) procedure for calculating linearly independent solutions of a third-order differential equation in the form of power series;

2) verification of the found solutions;
3) calculation of coefficient-functions $B1(\xi)$, $B2(\xi)$, $B3(\xi)$ for the Green's function $G(x, \xi)$;
4) verification of the calculated coefficient-functions $B1(\xi)$, $B2(\xi)$, $B3(\xi)$;
5) specification of particular boundary conditions in the interval $[a, b]$;
6) solving a system of algebraic equations for determining coefficient-functions $A1(\xi)$, $A2(\xi)$, $A3(\xi)$;
7) verification of the calculated coefficient-functions $A1(\xi)$, $A2(\xi)$, $A3(\xi)$;
8) construction of the Green's function G_L(x, ξ), ($a \le x \le \xi \le b$) and $G\_R(x, \xi)$, ($a \le \xi \le x \le b$);
9) verification of all properties of the Green's function $G(x, \xi)$.

10. Examples of constructing the Green's function of ordinary differential equations of the third order in the form of power series

Example 1
Let's consider the differential equation

$$y''' - 6y'' + 11y' - 6y = 0 \tag{20}$$

with boundary conditions

$$y(0) = 0, \quad y(1) = 0, \quad y''(0) = 0. \tag{21}$$

For differential equation (20), the solutions

$$y_1 = e^x, \quad y_2 = e^{2x}, \quad y_3 = e^{3x} \tag{22}$$

are a fundamental system of decisions. According to the above, the Green's function $G(x, \xi)$ can be constructed from the general solutions of (22) using the formulas:

$$G(x,\xi) = \begin{cases} G_L(x,\xi), & a \le x \le \xi \le b, \\ G_R(x,\xi), & a \le \xi \le x \le b, \end{cases}$$

where

$$G_L(x,\xi) = \sum_{k=1}^{3} \left[A(\xi) + B(\xi) \right] y_k(x),$$

$$G_R(x,\xi) = \sum_{k=1}^{3} \left[A(\xi) - B(\xi) \right] y_k(x).$$

From the continuity conditions for the Green's function, its first derivative, and also the jump of the second derivative, let's obtain a system for determining the coefficients of the functions $B_1(\xi)$, $B_2(\xi)$, $B_3(\xi)$:

$$\begin{cases} B_1(\xi) y_1(\xi) + B_2(\xi) y_2(\xi) + B_3(\xi) y_3(\xi) = 0, \\ B_1(\xi) y_1'(\xi) + B_2(\xi) y_2'(\xi) + B_3(\xi) y_3'(\xi) = 0, \\ B_1(\xi) y_1''(\xi) + B_2(\xi) y_2''(\xi) + B_3(\xi) y_3''(\xi) = -1 / \left(2P_0(\xi) \right). \end{cases} \tag{23}$$

System (23) is always solvable and has a unique solution, because $P_0(\xi) \ne 0$, and therefore, the main determinant of this system is the Wronskian $W[y_1, y_2, y_3]$, which is not equal to zero.

From system (23) let's find solutions:

$$B_1(\xi) = -\frac{1}{4} e^{-\xi}, \quad B_2(\xi) = \frac{1}{2} e^{-2\xi}, \quad B_3(\xi) = -\frac{1}{4} e^{-3\xi}.$$

To find the coefficient functions $A_i(\xi)$, (i=1,2,3), let's use the boundary conditions (21) and as a result let's obtain the system

$$\begin{cases} A_1(\xi) + A_2(\xi) + A_3(\xi) = -B_1(\xi) - B_2(\xi) - B_3(\xi), \\ A_1(\xi) + 4A_2(\xi) + 9A_3(\xi) = -B_1(\xi) - 4B_2(\xi) - 9B_3(\xi), \\ A_1(\xi) + eA_2(\xi) + e^2 A_3(\xi) = B_1(\xi) + eB_2(\xi) + e^2 B_3(\xi). \end{cases}$$

From this system let's find solutions:

$$A_1(\xi) = -\frac{1}{4\left(3e^2 - 8e + 5\right)} \times$$
$$\times \left(-20e^{1+\xi} + 5e^{2\xi} + 10e^2 + 8e^{1-2\xi} - 3e^{2+2\xi} \right) e^{-3\xi},$$

$$A_2(\xi) = \frac{1}{2\left(3e^2 - 8e + 5\right)} \times$$
$$\times \left(-8e^{1+\xi} + 8e^{2\xi} + 8e^2 - 5e^{\xi} - 3e^{2+\xi} \right) e^{-3\xi},$$

$$A_3(\xi) = -\frac{1}{4\left(3e^2 - 8e + 5\right)} \times$$
$$\times \left(-12e^{1+\xi} - 5 + 6e^{2\xi} + 3e^2 + 8e \right) e^{-3\xi}.$$

Using the found expressions for $A_k(\xi)$, $B_k(\xi)$, (k=1,2,3), let's find the exact Green's function for the boundary value problem (20), (21) in the form:

$$G(x,\xi) = \begin{cases} G_L(x,\xi), & 0 \le x \le \xi \le 1, \\ G_R(x,\xi), & 0 \le \xi \le x \le 1, \end{cases}$$

where

$$G_L(x,\xi) = \frac{e^x \left(-5/2e^{-\xi} + 5e^{1-2\xi} - 5/2e^{2-3\xi} \right)}{3e^2 - 8e + 5} +$$
$$+ \frac{e^{2x} \left(4e^{-\xi} - 8e^{1-2\xi} + 4e^{2-3\xi} \right)}{3e^2 - 8e + 5} +$$
$$+ \frac{e^{3x} \left(-3/2e^{-\xi} + 3e^{1-2\xi} - 3/2e^{2-3\xi} \right)}{3e^2 - 8e + 5}, \tag{24}$$

$$G_R(x,\xi) = \frac{e^x \left(3/2e^{2-\xi} - 4e^{1-\xi} + 5e^{1-2\xi} - 5/2e^{2-3\xi} \right)}{3e^2 - 8e + 5} +$$
$$+ \frac{e^{2x} \left(4e^{-\xi} - 3e^{2-2\xi} - 5e^{-2\xi} + 4e^{2-3\xi} \right)}{3e^2 - 8e + 5} +$$
$$+ \frac{e^{3x} \left(-3/2e^{-\xi} + 3e^{1-2\xi} + 5/2e^{-3\xi} - 4e^{1-3\xi} \right)}{3e^2 - 8e + 5}. \tag{25}$$

Using the developed program for the boundary value problem (20), (21), the approximate Green's function is obtained in the form of power series, the first terms of which are given below

$$G(x,\xi) = \begin{cases} G_L(x,\xi), & 0 \le x \le \xi \le 1, \\ G_R(x,\xi), & 0 \le \xi \le x \le 1, \end{cases}$$

where

$$G_L^{(7)}=\frac{19663}{35489}x-\frac{82489}{35489}x\xi+\frac{308167}{70978}x\xi^2-$$
$$-\frac{216293}{212934}x^3-\frac{1059601}{212934}x\xi^3+\frac{907379}{212934}x^3\xi-$$
$$-\frac{98315}{70978}x^4+\frac{3462703}{851736}x\xi^4-\frac{3389837}{425868}x^3\xi^2+$$
$$+\frac{412445}{70978}x^4\xi-\frac{4699457}{4258680}x^5-\ldots$$

$$G_R^{(7)}=\frac{19663}{35489}x+1/2\xi^2-\frac{117978}{35489}x\xi+$$
$$+1/2x^2-\xi^3+\frac{521101}{70978}x\xi^2-3x^2\xi-$$
$$-\frac{3359}{212934}x^3+\frac{25}{24}\xi^4-\frac{324471}{35489}x\xi^3+$$
$$+\frac{25}{4}x^2\xi^2+\frac{3359}{35489}x^3\xi-\frac{292555}{851736}x^4-$$
$$-3/4\xi^5+\frac{6656713}{851736}x\xi^4-15/2x^2\xi^3-$$
$$-\frac{195827}{425868}x^3\xi^2+\frac{292555}{141956}x^4\xi-\frac{1505447}{4258680}x^5+\ldots.$$

Comparisons are calculated for different values ξ and for the values of the power series n=13 and n=16. From the calculations it follows that the obtained approximate Green's function differs from the exact one by 1.3 % and $6\cdot10^{-3}$ % at n=13 and n=16, respectively.

Example 2

Let's consider the differential equation

$$x^3y'''+xy'-y=0$$

with boundary conditions

$$y(1)+y'(1)+y''(1)+y(2)+y'(2)+y''(2)=0,$$
$$2y(1)+y'(1)-2y''(1)+y(2)+y'(2)-2y''(2)=0,$$
$$2y(1)+y'(1)+3y''(1)-2y(2)-y'(2)+y''(2)=0.$$

Using the developed program, the Green's function is obtained:

$$G(x,\xi)=\begin{cases}G_L(x,\xi), & 0\le x\le\xi\le 1,\\ G_R(x,\xi), & 0\le\xi\le x\le 1,\end{cases}$$

where the first terms of the Green's function are:

$$G_L(x,\xi)=\frac{-x}{2\xi^2\left(44+90\ln^2 2-248\ln 2\right)}\times$$
$$\times\left[180\ln^2\xi-120\ln\xi+\ldots+\right],$$
$$G_R(x,\xi)=\frac{-x}{2\xi^2\left(44+89\ln^2 2-248\ln 2\right)}\times$$
$$\times\left[136\ln^2\xi-120\ln\xi+\ldots+\right].$$

Thus, the Green's function of the initial homogeneous differential equation is obtained.

11. Discussion of the results obtained on the construction of the Green's function of ordinary differential equations

Methods for constructing the Green's function for linear ordinary differential equations of the second and third orders are developed. Knowing the Green's function [8] allows to calculate the solution of a linear inhomogeneous differential equation with given boundary conditions, as well as to find the eigenvalues and functions of the boundary value problem.

An algorithm is developed for constructing the Green's function in the case when in the Maple system it is possible to obtain in explicit form three linearly independent solutions of a given third-order differential equation with boundary conditions. The description of the algorithm for constructing the Green's function for ordinary differential equations of the third order in an explicit analytical form is given. The Green's function calculations for specific boundary value problems using the developed program are presented.

An algorithm has been developed for constructing the Green's function in the form of power series for a third-order differential equation with given boundary conditions. The description of the algorithm for constructing the Green's function for third-order equations in the form of power series is given. The Green's function is calculated for specific third-order boundary value problems using the developed program, and the obtained approximate Green's function is compared with the exact, if known, and the accuracy of their agreement is shown.

Based on the known properties of the Green's function, in this research, an algorithm is developed and a program of symbol-numerical calculations of the Green's function is developed using computer systems of analytical calculations; any boundary value problems for ordinary differential equations of the second and third orders can be stated. An essential and important node in the calculation of the Green's function is the search for a fundamental system of solutions for a given differential equation. In this paper, this problem is solved by calculation using working programs.

Calculations are carried out for a number of boundary value problems, and the corresponding Green's functions are obtained. For the answers of problems known from the literature [22], a very good agreement is found (generally exact coincidence) with the calculations of the Green's functions presented in this paper. This proves the efficiency of constructing Green's functions in the proposed approach. Similar calculations in the known literature have not been identified. It can and is important to note that the accuracy of the calculation of the Green's function is determined by the accuracy of the calculation of the fundamental system of solutions, which in the general case is automatically controlled by the number of terms in power series and the number of decimal places in decimal numbers.

In conclusion, it is possible to say that thanks to the methods and algorithms proposed in this article, any boundary-value problem for ordinary differential equations of the second and third orders can be solved. The only drawback of this work is that the calculation of the Green's function and finding all the linearly independent solutions of the fundamental system of solutions necessary for this is a difficult operation, manual calculations are practically impossible for homogeneous equations, and even more so for heterogeneous ones.

12. Conclusions

1. A method for constructing the Green's function for linear ordinary differential equations of the second and third orders having singular points is described in the form of generalized power series using computer systems of algebraic transformations.

2. The construction of a fundamental system of solutions in the form of convergent series allows, in subsequent numerical calculations, to obtain the desired accuracy by increasing the number of terms in the series and by increasing the number of digits after the decimal point, it means with such accuracy to calculate the Green's function itself.

3. Examples of calculations of the Green's functions of boundary value problems for ordinary differential equations of the second and third order in the form of power series in the Maple system are presented, which allows one to efficiently and accurately perform all the necessary transformations when constructing the Green's function. The calculated Green's functions are compared with those available in the literature and the accuracy of their agreement is shown. The exact coincidence of the calculated Green's functions with the known from other sources is obtained, which proves the effectiveness of the calculation method used and the developed program.

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Запропоновано генетичний метод для прогнозування показників здоров'я населення на основі нейромережевих моделей. Принципова відмінність запропонованого генетичного методу від існуючих аналогів полягає у використанні диплоїдного набору хромосом в особин популяції, яка еволюціонує. Така модифікація робить залежність фенотипу особини від генотипу менш детермінованою і, врешті, сприяє збереженню різноманітності генофонду популяції і варіабельності ознак фенотипу впродовж виконання алгоритму. Крім цього, запропоновано модифікацію генетичного оператору мутацій. На відміну від класичного методу, особини, які піддаються дії оператору мутації, обираються не випадковим чином, а у відповідності до їх мутаційної стійкості, що відповідає значенню функції пристосованості особини. Таким чином, мутують особини, що характеризуються гіршими значеннями цільової функції, а геном сильних особин залишається незмінним. У цьому випадку зменшується вірогідність втрати досягнутого впродовж еволюції екстремуму функції внаслідок дії оператора мутацій, а перехід до нового екстремуму здійснюється у випадку накопичення достатньої питомої ваги кращих ознак в популяції.

Порівняльний аналіз роботи моделей, синтезованих за допомогою розробленого генетичного методу, показав, що найкращі результати досягнуті у моделі на основі нейронної мережі довгої короткочасної пам'яті. Під час створення і навчання моделі на основі мережі довгої короткочасної пам'яті було досліджено можливість використання методу рою часток для оптимізації параметрів мережі. Результати експериментальних досліджень показали, що розроблена модель дає найменшу помилку передбачення кількості нових випадків туберкульозу – середня абсолютна помилка складає 6,139, що менше у порівнянні з моделями, побудованими за допомогою інших методів.

Практичне використання розроблених методів дасть можливість своєчасно коригувати плановані лікувально-діагностичні, профілактичні заходи, завчасно визначати необхідні ресурси для локалізації та ліквідації захворювань з метою збереження здоров'я населення

Ключові слова: нейронні мережі, генетичний алгоритм, фенотип, модифікований генетичний оператор мутації, прогнозування показників здоров'я населення

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CONSTRUCTION OF A GENETIC METHOD TO FORECAST THE POPULATION HEALTH INDICATORS BASED ON NEURAL NETWORK MODELS

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1. Introduction

Quality of life of population is determined by different indicators, in particular health indicators, whose condition is predetermined by environmental factors. According to medical research conducted in recent years [1], there is a close relationship between the anthropogenic air pollution in certain areas and the increased population morbidity. As estimated by the World Health Organization (WHO), air pollution is the biggest factor of environmental health risks at present [2]. Based on this assessment, about 3.7 million of additional deaths are related to ambient air pollution, 4.3 million – to air pollution indoors. Since many people are exposed to both indoor and outdoor polluted air, causes and deaths from various diseases caused by different sources cannot be determined through the usual generalization of data. The biggest health problems caused by direct influence of air pollution are related to diseases of blood circulation, respiratory diseases, cancer, neuro-mental disorders, as well as some others [3, 4].

Consequently, the health condition and population morbidity in a region can be considered as derivatives from the environment.

The use of known statistics methods for forecasting the dependence of health indicators, as well as mathematical