

*Роботу присвячено розробці інженерної методики теплотехнічного розрахунку огорожувальних конструкцій з утеплювачем, в якому організовані повітряні камери з тепловідбивним покриттям.*

*Інженерна методика полягає у визначенні приведеного опору теплопередачі огорожувальної конструкції на основі розрахунку температурного поля в ній. Для знаходження температурного поля розглядається одновимірною задачею теплопровідності в багатощаровій огорожувальній конструкції. Теплотехнічні неоднорідності, які викликані наявністю повітряних камер, що чергуються з прошарками матеріалу утеплювача, враховані в математичній моделі за рахунок ефективного коефіцієнта теплопровідності відповідного шару. Даний коефіцієнт враховує конвективну та променисту складові теплопередачі через повітряні камери. Отримано вираз для визначення його значення, який залежить від значень температури на стиках відповідного шару з сусідніми шарами огорожувальної конструкції. Запропоновано ітераційну процедуру, що дозволяє з використанням даного виразу знаходити температурне поле в огорожувальній конструкції, що розглядається. Як вихідні дані в цій процедурі використовуються геометричні та теплофізичні характеристики елементів огорожувальної конструкції, а також температури повітря всередині і зовні приміщення і коефіцієнти тепловіддачі на відповідних поверхнях.*

*Інженерна методика верифікована шляхом порівняння з результатами тривимірного CFD-моделювання, яке детально враховує вільноконвективний рух в повітряних камерах і теплообмін випромінюванням між термічно неоднорідними стінками повітряної камери. Показано, що застосування одновимірної математичної моделі замість детальної тривимірної призводить до погрешностей, що не перевищують 2,5 %.*

*В результаті порівняльного розрахункового аналізу показано ефективність застосування запропонованого утеплювача з повітряними камерами з тепловідбивним покриттям в порівнянні з традиційними підходами до утеплення огорожувальних конструкцій. Розрахунки виконані для випадку найбільш холодної п'ятиденки в умовах кліматичної зони Шимкента (Республіка Казахстан)*

*Ключові слова: утеплювач з повітряними камерами, тепловідбивне покриття, приведені опір теплопередачі огорожувальної конструкції*

# DEVELOPMENT OF THE ENGINEERING PROCEDURE FOR THE THERMOTECHNICAL CALCULATION OF A BUILDING ENVELOPE WITH AIR CHAMBERS AND A HEAT-REFLECTING COATING

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## 1. Introduction

The ever-growing prices of fossil hydrocarbon fuels and the possibility of their exhaustion in the near future make reasonable development of energy-saving technologies and sustainable energy. For post-Soviet countries, this problem is especially acute in the housing and communal sector. Most buildings designed and erected in the

Soviet period in the presence of bargain prices for energy carriers have turned out to be ineffective in the market economy realities from the point of view of energy conservation. Current regulatory documents impose tighter requirements on the thermal insulation characteristics of a building envelope. This must be taken into account when designing new and thermally modernizing existing buildings. In this regard, finding new highly effective ap-

proaches to reduce heat losses through building envelope is an urgent problem.

## 2. Literature review and problem statement

Numerous studies have been devoted to the problem of energy conservation in building engineering. For example, energy-saving issues of heating the buildings are discussed in [1–9]. One of the main ways to reduce heat losses in heating the buildings implies raising RSI-value of building envelope. Currently, the use of multilayer building envelope in which, along with the load-bearing layers, insulation layers are present is considered as the simplest approach to heat loss reduction.

For example, the energy efficiency of houses in the conditions of the Canadian climate is studied in [1]. High rates of RSI-value of building envelope in these houses are achieved through the use of multilayer structures with air chambers. However, the use of polyethylene in such structures can result in a violation of air exchange and condensate formation. The feasibility of applying heat-reflecting coatings was considered in [2]. At the same time, it is noted that the effectiveness of such coatings largely depends on climatic conditions. The lack of effective methods for calculating such structures is also pointed out. Authors of [3] have performed calculations to compare multilayer building envelope in different climatic conditions. However, the software package they used is a commercial product.

Issues of net zero and near net zero energy consumption houses are addressed in [4]. At the same time, the results of this study are largely based on experimental data but not on a detailed calculation of thermal-insulating structures taking into account all thermotechnical heterogeneities.

The author of [5] paid attention to finding ways to the reduction of heat losses in buildings taking into consideration various climatic zones of Turkey. However, he focused just on determining the optimal thickness of thermal insulation without analyzing the possible thermotechnical heterogeneities in it and their effect on heat loss. At the same time, as indicated in [6], the use of conventional thermal-insulating materials, especially in a sharply continental climate, often results in significant growth of the building envelope prime cost. In other words, the increased capital expenditure of thermal insulation do not pay off in acceptable terms by reducing energy consumption.

Results of a detailed three-dimensional simulation of heat and vapor transport in a building envelope in a zone of its thermal inhomogeneities caused by thermal bridges are presented by authors of [7]. However, their model was based on the use of two non-stationary convection-diffusion equations: for heat and vapor. As a result, the calculation becomes impossible for air interlayers of building envelope where there is air motion. The authors of [8] use the finite volume method to calculate the thermal state of not just the building envelope but the entire building as well. To estimate heat flux through a multilayer building envelope, authors of [9] solved the non-stationary heat conduction problem taking into account diurnal variations of outdoor temperature and insolation. The results were validated by comparison with infrared thermography data on the outer surface of the building envelope. At the same time, it should be noted that in [7–9], like in other studies using three-dimensional and/or non-stationary modeling, a rather complicated math-

ematical apparatus is used. This imposes very high qualification requirements on the end-user in the field of constructing models of thermophysical processes, applying numerical methods to solve them, creating, and verifying software and analyzing the results. Therefore, it is desirable for an ordinary designer of building envelope to have a simplified engineering procedure.

In the vast majority of cases, the low thermal conductivity of building materials, both those that used in load-bearing layers and those that form thermal-insulating layers is determined by the presence of air-filled micro spaces in their structure. The usage of air interlayers allows to reduce material consumption and, consequently, the cost of thermal insulation without prejudice to the insulating properties. The main problem in the design of thermal insulation, in this case, consists in minimizing the amount of heat transferred through the air interlayer. As is known [10], all three mechanisms, that is the conduction, convection, and radiation, take place during heat transfer through the air interlayer. The main way to reduce the convective component implies choosing the thickness of the air interlayer such as to minimize or eliminate free convective flow in it caused by the temperature difference on its opposite walls. As is known [10], there is no free convective flow in vertical flat air interlayers if the Rayleigh number does not exceed  $10^4$ . To reduce the radiant component of heat transfer through the air interlayer, it is recommended either to apply heat-reflecting coatings to its walls (aluminum foil is the most common variant in construction) or use a shielding system. In studies on construction heat engineering, for example, in [11], the inefficiency of using air layers more than 5 cm thick was substantiated and it was also noted that the use of a continuously ventilated air interlayer leads to significant heat loss because of formation of a developed free-convection flow in it. This can be avoided by dividing the air interlayer into separate air chambers.

It should be noted that quite a lot of new thermal-insulating materials have appeared recently in the market including those with air interlayers and heat-reflecting coatings. However, their manufacturers either do not provide concrete numerical characteristics reflecting the thermal-insulating properties of such materials or this information is overestimated.

The regulatory documents currently in force in construction practice provide recommendations regarding the calculation of average RSI-value of a building envelope. For the simplest case of building envelope in the form of a multilayer wall with homogeneous layers, the following formula of sequential thermal resistances is used:

$$R = \frac{1}{\alpha_{int}} + \sum_{i=1}^n \frac{\delta_i}{\lambda_i} + \frac{1}{\alpha_{ext}}, \quad (1)$$

where  $R$  is the average RSI-value of the multilayer building envelope,  $^{\circ}\text{C}\times\text{m}^2/\text{W}$ ;  $\alpha_{int}$  is the heat transfer coefficient on the inner surface of the building envelope,  $\text{W}/(\text{m}^2\cdot^{\circ}\text{C})$ ;  $\delta_i$  is the thickness of the  $i$ -th layer,  $\text{m}$ ;  $\lambda_i$  is the coefficient of thermal conductivity of the material's  $i$ -th layer,  $\text{W}/(\text{m}\cdot^{\circ}\text{C})$ ;  $\alpha_{ext}$  is the coefficient of heat transfer on the outer surface of the building envelope.

In many cases, formula (1) is inapplicable because of thermotechnical heterogeneities (a discontinuous air interlayer) or for other reasons (e. g., the problem non-linearity). Then, the same regulatory documents recommend to calculate the temperature field in a selected building envelope

fragment with subsequent calculation of average RSI-value using the following formula:

$$R=(T_{int}-T_{ext})A/Q, \quad (2)$$

where  $T_{int}$  is the rated indoor temperature, °C;  $T_{ext}$  is the rated temperature, °C;  $A$  is the surface area of the considered fragment of building envelope, m<sup>2</sup>;  $Q$  is the total heat flux passing through the fragment under consideration, W.

It should be noted that the calculation of the temperature field in a building envelope with air interlayers is connected with the solution to a nonlinear thermal conductivity problem. In the case of thermal heterogeneity or the presence of convective flow in air interlayers, this problem is two-dimensional or three-dimensional. The solution to such problems requires a sufficiently high qualification in the application of numerical methods of integrating partial differential equations or the use of commercial software for solving problems of hydrodynamics and heat and mass transfer. In addition, the process of solving such problems may require quite a lot of labor and/or usage of expensive commercial software products. At the same time, design organizations demand fairly simple calculation procedures which can give results in a fairly short time including the case of multivariate calculations.

Thus, we can conclude that the creation of highly effective thermal insulation materials for building envelope based on air interlayers with heat-reflecting coatings is still in demand in the construction market. So, the developer must have an engineering procedure for thermal engineering calculations. On the one hand, this procedure should be quite simple so that it can be used by employers who have no high scientific qualifications. On the other hand, it should not lead to significant errors because of simplifications introduced into the mathematical model.

### 3. The aim and objectives of the study

The study objective was to develop an engineering procedure of the thermotechnical calculation of a building envelope with insulation containing air chambers with a heat-reflecting coating and calculation substantiation of its effectiveness.

To achieve the objective, the following tasks were set:

- to form a mathematical model of thermophysical processes taking place in a multilayer building envelope in which heat insulation of the structure in question is used;
- to develop an engineering procedure of calculation of temperature fields in similar building envelope designs and their RSI-value based on the application of a one-dimensional heat conduction problem;
- to carry out comparative calculations of a building envelope with the considered insulation and a conventional building envelopes.

### 4. Mathematical model of a multilayer building envelope with air chambers and a heat-reflecting coating

Researchers from the South Kazakhstan State University named after M. Auezov have proposed principles of creating highly efficient insulation from extruded polystyrene foam (XPS) plates with air chambers and heat-re-

flecting coatings. Air chambers are formed in a XPS plate as parallelepiped-shaped recesses with a vertical size corresponding to the size of the XPS plate. The width and depth of all recesses are the same. Recesses are separated from each other by intervals of XPS of the same width. A heat-reflecting coating of aluminum foil is applied to one of the three recess surfaces parallel to the plate surface. When such a plate is mounted in a multilayer building envelope, the recesses are closed on their fourth side by the surface of the adjacent building envelope layer. As a result, the parallelepiped-shaped air chambers are formed with a vertical dimension far exceeding width and depth,  $\delta_4$  (Fig. 1). If necessary, a heat-reflecting coating can also be applied to the surface of the adjacent layer of the building envelope that closes recesses in the XPS plate and both opposite walls of the air chamber become heat shielded. Production of plates with such recesses and a heat-reflecting coating applied is quite easy to organize in conditions of an existing enterprise manufacturing heat-insulating materials from XPS.

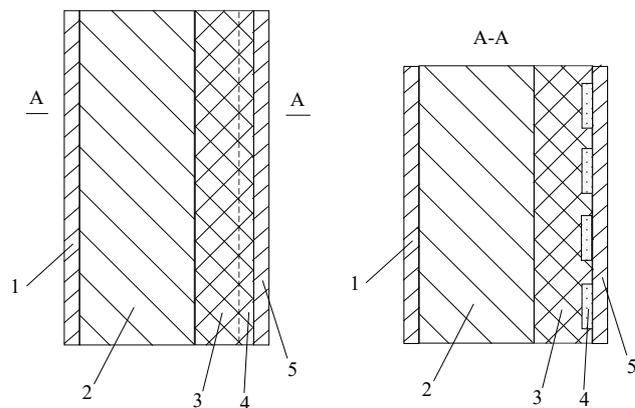


Fig. 1. Schematic view of a multi-layer building envelope with highly efficient insulation: 1 – internal finishing layer; 2 – load-bearing layer; 3 – solid cross-section of XPS; 4 – air chambers in XPS; 5 – external finishing layer

The engineering procedure of thermophysical calculation shall be created in a way based on the solution to a one-dimensional heat conduction problem in the building envelope using effective thermal conductivity taking into account the convective component of heat transfer in air chambers neglecting thermal heterogeneity caused by alternating air chambers and XPS dividers between them.

Let us enumerate the layers sequentially from the innermost to the outermost side starting from 1. In the mathematical model, the insulation layer of XPS in which rectangular air chambers are formed is divided into two layers. The first is the layer of solid XPS in which there are no air chambers, the second is the layer in which the air chambers alternating with dividers of XPS extend through its entire thickness.

The mathematical model of thermophysical processes for the multilayer building envelope under consideration will be constructed based on the following assumptions.

Assume that there is an ideal thermal contact between the layers. Neglect thickness of the heat-reflecting coating and consider the coating itself as a gray body [10] with a given emissivity.

The linear dimension of the air chambers in the vertical direction far exceeds linear dimensions in the other two directions and the width of the XPS dividers between the air chambers. In other words, assume that the thermal in-

sulation layer is thermally uniform in the vertical direction. Assume the remaining layers of the building envelope thermally uniform in all directions.

According to [10], the mathematical model of convective heat transfer in air chambers containing the Navier-Stokes motion equations will not be considered in detail. Describe these interlayers as a homogeneous isotropic body with effective thermal conductivity taking into account both conductive and convective components of heat transfer through this interlayer. The heat flux due to radiant heat exchange between the walls of the air chamber parallel to the outer and inner surfaces of the building envelope will be represented as a conductive heat flux due to the introduction of effective thermal conductivity. In this case, the chamber is considered as a homogeneous isotropic body. Re-reflection on the two remaining walls of the air chamber will be neglected.

We will also neglect the non-uniformity of the temperature field in the horizontal direction along the building envelope caused by the thermal heterogeneity in a form of alternating air chambers and XPS dividers between them. Under the last assumption, a layer consisting of alternating air chambers and XPS dividers will be considered as a homogeneous isotropic body with effective thermal conductivity. This effective thermal conductivity will take into account all three mechanisms of heat transfer (conduction through XPS and conduction, convection and radiation through the air chamber).

The described assumptions enable consideration of the problem of heat transfer through any of the studied building envelope in a one-dimensional stationary statement.

To determine the effective thermal conductivity of the layer with air chambers, choose a building envelope fragment that has a linear dimension  $h$  corresponding to the repeating geometry of thermal heterogeneity in the horizontal direction along the building envelope. In other words, one air chamber with an adjacent XPS divider between the chambers will be considered in this direction. Thus,

$$h = h_{air} + h_{XPS}, \tag{3}$$

where  $h_{air}$  is the width of the air chamber;  $h_{XPS}$  is the width of the XPS divider between adjacent air chambers.

Let us orient the spatial axis  $x$  in a direction from the room to the outside placing the reference point on the inner building envelope surface. Then, introducing notation  $\delta_i$  for the thickness of the  $i$ -th building envelope layer, the following expressions can be written:

$$x_0 = 0, \quad x_k = \sum_{i=1}^k \delta_i, \quad x_n = \sum_{i=1}^n \delta_i, \tag{4}$$

where  $x_0$  is the coordinate of the inner building envelope surface;  $x_k$  is the coordinate of the junction of layers with numbers  $k$  and  $k+1$ ;  $x_n$  is the coordinate of the outer building envelope surface;  $n$  is the number of layers.

Write the one-dimensional linear stationary heat conduction equation for each layer

$$\lambda_k \frac{\partial^2 T}{\partial x^2} = 0, \quad x_{k-1} < x < x_k, \quad k = 1, \dots, n, \tag{5}$$

where  $\lambda_k$  is the thermal conductivity (material one for the layers without air interlayers or effective one for the layers

with air interlayers);  $T=T(x)$  is the spatial change in the building envelope temperature.

At the layer junctions, relations describing equality of temperature and heat flow are fulfilled due to ideal thermal contact:

$$T|_{x=x_k-0} = T|_{x=x_k+0}, \quad k = 1, \dots, n-1, \tag{6}$$

$$\lambda_k \frac{\partial T}{\partial x} \Big|_{x=x_k-0} = \lambda_{k+1} \frac{\partial T}{\partial x} \Big|_{x=x_k+0}, \quad k = 1, \dots, n-1. \tag{7}$$

Convective heat exchange of the building envelope with the surrounding air is described by the following boundary conditions:

$$\lambda_1 \frac{\partial T}{\partial x} \Big|_{x=0} = \alpha_{int} (T|_{x=0} - T_{int}), \tag{8}$$

on the inner surface of the building envelope and

$$\lambda_n \frac{\partial T}{\partial x} \Big|_{x=x_n} = \alpha (T|_{x=x_n} - T), \tag{9}$$

on the outer surface of the building envelope.

To simplify further entries, introduce the following notation for temperatures of the layer junctions and the outer and inner surfaces of the building envelope:  $T_k=T(x_k); k=0, 1, \dots, n$ .

Find an expression for effective thermal conductivity  $\lambda_{eff,k}$  of the layer consisting of alternating air chambers and XPS dividers.

Since the air chamber is considered as a homogeneous isotropic body included in the layer  $k$ , then according to the Fourier law, the heat flux passing through it will take the following form:

$$q_{air} = -\lambda_{eff,air,k} \frac{\partial T}{\partial x}, \tag{10}$$

where  $\lambda_{eff,air,k}$  is the effective thermal conductivity of the air chamber.

Due to the one-dimensional nature of the problem and the absence of internal heat sources, partial temperature derivative with respect to the spatial coordinate in expression (10) can be precisely written in the form of a finite difference:

$$q = -\lambda_{eff,air,k} \frac{T_k - T_{k-1}}{\delta_k} = \lambda_{eff,air,k} \frac{T_{k-1} - T_k}{\delta_k}. \tag{11}$$

On the other hand, when considering the air chamber, the heat flux can be represented as a sum of two components: conductive-convective and radiant ones:

$$q_{air} = q_c + q_r, \tag{12}$$

where  $q_c$  is the conductive-convective component of the heat flow;  $q_r$  is the radiant component of the heat flux.

According to [10], conductive-convective heat transfer in vertical air interlayers between flat walls during natural convection can be adequately described by mathematical models for fixed layers using the concept of effective thermal conductivity coefficient  $\lambda_c$  defined as follows:

$$\lambda_c = \lambda_{air} \text{ at } Ra < 10^4,$$

$$\lambda_c = \lambda_{air} \times 0,062 \times Ra^{1/3} \text{ at } 10^4 < Ra < 10^4, \\ \lambda_c = \lambda_{air} \times 0,22 \times Ra^{1/4} \text{ at } 10^7 < Ra < 10^{10}, \quad (13)$$

where  $\lambda_{air}$  is the air thermal conductivity;

$$Ra = \frac{g \delta^3 \beta \Delta T}{\nu} Pr,$$

where Ra is the Rayleigh number;  $g$  is the acceleration of gravity;  $\delta$  is the thickness of the air interlayer which in the case under consideration is equal to the layer thickness  $\delta_k$ ;  $\beta$  is the thermal expansion coefficient which for air under normal conditions can be taken equal to  $1/273$  °C;  $\Delta T$  is the difference of temperatures on opposite walls of the air interlayer ( $\Delta T = T_{k-1} - T_k$  in the case under consideration);  $\nu$  is the kinematic viscosity of air at averaged values of temperature and pressure in the air interlayer; Pr is the Prandtl number.

When using expressions (13), the values of  $\lambda_{air}$ ,  $\nu$ , and Pr are taken at averaged values of pressure and air temperature in the air chamber. Pressure can be taken equal to 1 bar and the arithmetic mean of temperature values on the air chamber walls can be considered as a determining temperature  $(T_{k-1} + T_k)/2$ .

Thus,

$$q_c = -\lambda_c \frac{\partial T}{\partial x}. \quad (14)$$

According to [10], the radiant component of heat flux can be found from expression

$$q_r = \epsilon \sigma_0 (T_{k-1}^4 - T_k^4), \quad (15)$$

where  $\sigma_0 = 5.67 \times 10^{-8} \text{ W}/(\text{m}^2 \times \text{K}^4)$  is the Stefan-Boltzmann constant;  $\epsilon$  is the reduced emissivity which in the case of two parallel plane walls of great length can be calculated from expression:

$$\frac{1}{\epsilon} = \frac{1}{\epsilon_{k-1}} + \frac{1}{\epsilon_k} - 1,$$

where  $\epsilon_{k-1}$  and  $\epsilon_k$  are the emissivities of the walls (in this case, solid surfaces at the corresponding junctions of the building envelope layers); temperatures  $T_{k-1}$  and  $T_k$  are taken in Kelvin.

Substituting (11), (14) and (15) into (12) and using a finite-difference representation of partial temperature derivative for (14) the same as in (11), the following is obtained:

$$\lambda_{eff,air,k} \frac{T_{k-1} - T_k}{\delta_k} + \frac{T_{k-1} - T_k}{\delta_k} \sigma_0 (T_{k-1}^4 - T_k^4), \quad (16)$$

from where

$$\lambda_{eff,air,k} = \lambda + \frac{\delta_k \epsilon \sigma_0 (T_{k-1}^4 - T_k^4)}{T_{k-1} - T_k}. \quad (17)$$

Value of the effective thermal conductivity,  $\lambda_{eff,k}$ , for the entire layer consisting of alternating air chambers and XPS dividers will be found by considering the total heat flux passing through the selected fragment of the layer under consideration of arbitrary height  $L$  and width described by formula (3). It can be represented as a sum of two fluxes:

$$Q = Q_{air} + Q_{XPS}, \quad (18)$$

where  $Q_{air}$  is the total heat flux through the air chamber in the selected layer fragment;  $Q_{XPS}$  is the total heat flux through the XPS divider between adjacent air chambers.

The heat flow passing through the air chamber can be represented as

$$Q_{air} = q_{air} \times h_{air} \times L, \quad (19)$$

where  $q_{air}$  is determined from (11).

The total heat flux passing through the XPS divider can be written as

$$Q_{XPS} = q_{XPS} \times h_{XPS} \times L, \quad (20)$$

where

$$q_{XPS} = -\lambda_{XPS} \frac{\partial T}{\partial x},$$

where  $q_{XPS}$  is the heat flux passing through the XPS divider;  $\lambda_{XPS}$  is the thermal conductivity of XPS.

As in (11), represent  $q_{XPS}$  in the form of a finite difference:

$$q_{XPS} = \lambda_{XPS} \frac{T_{k-1} - T_k}{\delta_k}. \quad (21)$$

Substitute (21) in (20), (11) in (19) and the obtained expressions in (18) to have the following:

$$Q = \lambda_{eff,air,k} \frac{T_{k-1} - T_k}{\delta_k} \cdot h_{air} \cdot L + \lambda_{XPS} \frac{T_{k-1} - T_k}{\delta_k} \cdot h_{XPS} \cdot L. \quad (22)$$

On the other hand, if we consider the heat transfer through a homogeneous isotropic layer with effective heat conductivity  $\lambda_{eff,k}$ , the total heat flux through it can be written as follows, taking into account the finite-difference representation of the partial temperature derivative with respect to the spatial coordinate,

$$Q = -\lambda_{eff,k} \frac{\partial T}{\partial x} \cdot h \cdot L = \lambda_{eff,k} \frac{T_{k-1} - T_k}{\delta_k} \cdot h \cdot L. \quad (23)$$

Comparing (22) and (23) considering (3), the following is obtained:

$$\lambda_{eff,k} \frac{T_{k-1} - T_k}{\delta_k} \cdot (h_{air} + h_{XPS}) \cdot L = \\ = \lambda_{eff,air,k} \frac{T_{k-1} - T_k}{\delta_k} \cdot h_{air} \cdot L + \lambda_{XPS} \frac{T_{k-1} - T_k}{\delta_k} \cdot h_{XPS} \cdot L, \quad (24)$$

from where

$$\lambda_{eff,k} = \frac{\lambda_{eff,air,k} \cdot h_{air} + \lambda_{XPS} \cdot h_{XPS}}{h_{air} + h_{XPS}}, \quad (25)$$

where  $\lambda_{eff,air,k}$  is calculated according to (17) taking into account (13).

These mathematical models make it possible to find temperature distribution over the building envelope thickness  $T = T(x)$ ,  $0 < x < x_n$ .

## 5. Engineering procedure of calculating temperature fields in the building envelope

Since the problem is considered in a stationary statement, then, due to the absence of internal sources or sinks of heat in a multilayer building envelope, the heat flux passing through it does not depend on the spatial coordinate. Therefore, knowing the RSI-value, the heat flux  $q$  passing through the building envelope can be determined either directly from the definition of RSI-value:

$$q = \frac{T_{\text{int}} - T_{\text{ext}}}{R} \quad (26)$$

or according to the Fourier law:

$$q = -\lambda \frac{\partial T}{\partial x}. \quad (27)$$

By substituting expression (26) into the right-hand side of (8) considering (27), the following is obtained:

$$-\frac{T_{\text{int}} - T_{\text{ext}}}{R} = \alpha_{\text{int}} (T|_{x=0} - T_{\text{int}}). \quad (28)$$

Similarly, (9) gives

$$\frac{T_{\text{int}} - T_{\text{ext}}}{R} = \alpha_{\text{ext}} (T|_{x=x_n} - T_{\text{ext}}). \quad (29)$$

The following expression is obtained from (28):

$$T|_{x=0} = T_0 = T_{\text{int}} - \frac{T_{\text{int}} - T_{\text{ext}}}{\alpha_{\text{int}} R}, \quad (30)$$

which gives the following, considering (26):

$$T_0 = T_{\text{int}} - q / \alpha_{\text{int}}. \quad (31)$$

From the definition of thermal resistance  $R_k$  for the  $k$ -th layer,

$$T_k = T_{k-1} - q \times R_k, \quad (32)$$

where  $R_k$  is calculated as

$$R_k = \frac{\delta_k}{\lambda_k}. \quad (33)$$

For a layer consisting of alternating air chambers and XPS dividers, the effective thermal conductivity  $\lambda_{\text{eff},k}$  determined according to (25) should be used in (33). The values in the right side of expression (25) depend on the Rayleigh number and the radiant component of the heat flux. In turn, these two values are determined by the temperature difference on the air chamber walls (in the first or fourth degree, respectively). Since values of these temperatures are also sought quantities of the problem, direct application of formula (25) to calculate the final value of the effective thermal conductivity of the layer is impossible. To solve this problem, an approach shall be used that will make it possible to refine the mentioned temperatures and effective thermal conductivity of the layer in the process of iterations.

At the beginning of each current iteration, the current value of thermal resistance of a layer consisting of alternating air chambers and XPS dividers is determined from formula (33). In this case, current value of effective thermal conductivity of this layer is used as  $\lambda_k$ . To find it, calculations shall be conducted proceeding from the temperature values  $T_{k-1}$  and  $T_k$  which were found at the previous iteration. For the first iteration, the value of thermal conductivity of XPS can be used as an initial approximation for  $\lambda_k$ . As a result of such an approach, the current value of total RSI-value  $R$  of the entire building envelope can be determined according to (1). Knowing it, the current value of heat flux  $q$  passing through the building envelope is determined from formula (26) and then the current temperatures on the building envelope surfaces and at layer junctions are found according to (31) and (32).

After that, the condition for stopping iterations is checked and, if necessary, the transition to the next iteration is performed. Iterations stop when discrepancies between temperature values in characteristic points at previous and current iterations become less than the predetermined error. Inner and outer surfaces of the building envelope and all layer junctions can be used as such characteristic points. However, one can take into account the fact that nonlinearity which has the main influence on the convergence rate of the iterative process is inherent only in a layer consisting of an air interlayer or containing air chambers. Therefore, the number of checked conditions for stopping iterations can be reduced by confining oneself to only two points corresponding to the boundaries of the layer where this nonlinearity is present.

Thus, the above-described procedure allows us to determine the temperature on the building envelope surfaces and at junctions of its layers, the heat flux through the building envelope and its total RSI-value. After finding these values, one can find temperature distribution at any point in the building envelope using linear interpolation, if necessary.

Note that the proposed engineering procedure can be used to solve problems of thermotechnical analysis and design of building envelope with other insulating materials or in which air chambers are formed in the load-bearing layer.

## 6. Discussion of the results: verification of the procedure and comparative thermotechnical calculations of various building envelope designs

In order to substantiate the effectiveness of a proposed thermal insulation material with air chambers and heat-reflecting coatings, a comparison with conventional building envelope designs (Fig. 2) was made. Five building envelope schemes were considered. Scheme 1 is a three-layer building envelope with homogeneous load-bearing and outer and inner finishing layers without thermal insulation (Fig. 2, *a*). A solid layer of XPS insulation is added between the bearing and the outer finishing layers (Fig. 2, *b*) in Scheme 2. In contrast to Scheme 2, a continuous closed air interlayer was considered for thermal insulation (Fig. 2, *c*). A layer of the proposed thermal insulation with air chambers and heat-reflecting coatings was used between the bearing and the outer finishing layers in Scheme 4. The air chambers were closer to the outer finishing layer (Fig. 1). Scheme 5 differed from Scheme 4 in that the insulation layer was turned so that the air chambers were adjacent to the bearing layer.

The geometric and thermophysical characteristics of these layers are given in Table 1.

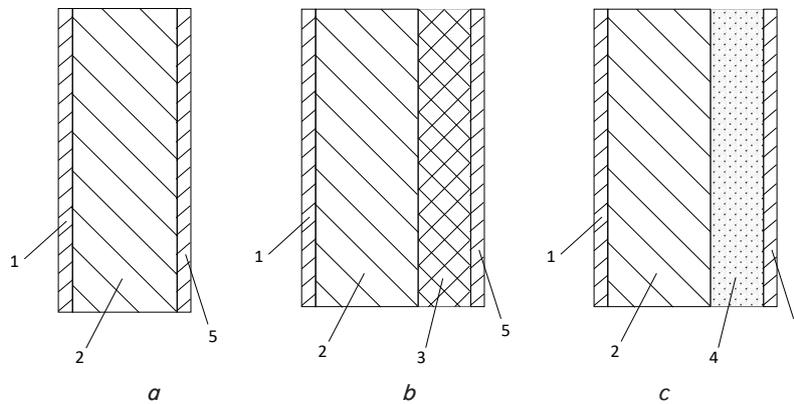


Fig. 2. Conventional building envelope designs: *a* – Scheme 1; *b* – Scheme 2; *c* – Scheme 3; 1 – inner finishing layer; 2 – load-bearing layer; 3 – XPS; 4 – air interlayer; 5 – outer finishing layer

Table 1

Characteristics of the building envelope layers

Description	Thickness, cm	Width, cm	Thermal conductivity, W/(m×K)	Emissivity		
				with no heat-reflecting coating	with heat reflecting coating*	
Inner finishing layer: cement-sand plaster	1	–	0.76	–	–	
Bearing layer: ceramic brickwork	38	–	0.58	0.93	0.03	
Thermal insulation: solid XPS	5	0.03	–	–	–	
Thermal insulation: continuous air interlayer	5	0.03	–	–	–	
Proposed thermal insulation	Solid XPS	3.5	–	0.03	0.9	0.03
	Alternating bands of XPS and air	1.5	1/5	–	–	–
Cement-sand plaster	0.5	–	0.76	0.91	0.03	

Note: \* – aluminum foil

Calculations for Schemes 3–5 were performed for different variants of applying a heat-reflecting coating: without coating, a coating applied to one wall of the air chamber, a coating applied to two opposite walls of the air chamber.

Before comparative calculations, the proposed engineering procedure and the created software that implements it were verified by comparing the calculated data obtained for Scheme 4 with the results of CFD simulation. In the latter case, the constructed three-dimensional computer model took into detailed account the free-convective motion of air in air chambers by using the Reynolds average Navier–Stokes equations with the Boussinesq approximation. The three-dimensional CFD model also included radiant heat transfer between all four walls of the air chamber taking into account spatial temperature changes on each wall. The results of this comparison showed that the temperature variations obtained during three-dimensional CFD simulation within each joint of the layers arising because of thermal heterogeneity did not exceed 1 °C. Moreover, the difference in the mean temperature values at each joint of the layers resulting from CFD simulation and the proposed engineering procedure did not exceed 0.8 °C. In this case, the difference in the calculated values of the average RSI-value did not exceed 2.5 %. Thus, we can conclude that the proposed calculation procedure based on the solution to the one-dimensional heat conduction problem for building envelope does not give significant errors in comparison with detailed three-dimensional CFD simulation. As a result, this engineering procedure can be used for engineering calculations in designing thermal insulation systems of the proposed type.

Using this engineering procedure, the above five building envelope designs were compared. In this case, the conditions of the steady-state temperature regime were considered. The values obtained on the inner and outer surfaces of the building envelope were selected as initial data in accordance with the State Standards in the field of architecture, urban planning and construction being in force in the Republic of Kazakhstan:

- indoor air temperature: 20 °C;
- outdoor temperature: –15 °C (which corresponds to the coldest five-day period with confidence levels of 0.92 in the climatic zone of Shymkent);
- heat transfer coefficients on the inner and outer building envelope surfaces: 8.7 and 23 W/(m<sup>2</sup>·°C).

Table 2 gives the calculation results.

As can be seen from the calculation results, the building envelope made of a high-performance thermal insulation material (XPS) in which air chambers are arranged (Schemes 4, 5) ensures a significant heat loss reduction compared to Schemes 1, 3. Compared to the building envelope in which solid (XPS) plates are used for thermal insulation (Scheme 2), XPS insulation with air chambers reduces heat loss only if a heat-reflecting coating is applied to the walls of the air chambers. In this case, the total RSI-value of the building envelope grew by 2.8 % compared to Scheme 2. Additional computational studies have shown that in the case of using two XPS layers with air chambers, the total RSI-value of the building envelope is increased by more than 6 % compared to the variant of thermal insulation with solid XPS plates of a similar thickness.

Note that the application of a second heat-reflecting coating did not give a significant gain in reducing heat losses compared to a single-sided coating (the total RSI-value increased by only 0.4 %). Obviously, this is because the calculations considered a coating variant with a rather low emissivity (0.03). In real designs, an emissivity may be greater than this ideal value. Therefore, in the future, it is necessary to carry out additional studies on the influence of the emissivity of the coating on thermal-insulating properties of the corresponding building envelope layer. Based on these studies, a conclusion can be drawn on the advisability of applying a second heat-reflecting coating on the opposite walls of the air chamber.

In the case of Schemes 4 and 5, the temperature difference between of indoor air and the inner surface of the building envelope was 1.8 °C for the insulation variant without a heat-reflecting coating and 1.6 °C for the variant with this coating. These values did not exceed the standardized value of this parameter (4.0 °C for external walls of residential buildings, medical treatment, and children’s institutions) specified in State Standards of the Republic of Kazakhstan in the field of architecture, urban planning, and construction.

Analysis of the study results shows that applying a heat-reflecting coating to the air chamber walls leads to an increase in the Rayleigh number in it. For example, in the case of Scheme 4, applying a heat-insulating coating to one side of the air chamber at the considered structural and operating parameters results in an increase in the Rayleigh number by 115 % (from 1,615 to 3,475). In the case of applying a heat-insulating coating to two opposite walls of the air chamber, the corresponding figure is 120 % (from 1,615 to 3,553). Obviously, this is due to an increase in temperature difference on the walls. This fact must be taken into account so that when designing effective thermal isolation materials with heat-reflecting coatings, their applying to the walls of air cavities will not result in exceeding critical Rayleigh number value of 10<sup>4</sup>. At high Rayleigh numbers, in addition to the radiant-conductive heat transfer, heat transfer effects caused by free-convection air movement begin to appear.

The temperature values given in Table 2 confirm the physically explainable fact of an increase in thermal resistance of the thermal insulating layer as a result of applying

a heat-reflecting coating. This concerns namely the temperature in the building envelope layers between the air chamber and the indoor space in the case of the application of a heat-reflecting coating to walls of air chambers is higher than in the case without the heat-reflecting coating. For the building envelope layers facing from the air chambers to the outside, an opposite feature was observed: applying of a heat-reflecting coating reduced layer temperature.

It should also be noted that in contrast to Scheme 4, Scheme 5 is characterized by positive air temperatures in air chambers. However, air chambers in Scheme 5 are closer to the inner building envelope surface which results in that the relative air humidity there may be higher than in air chambers of the building envelope made according to Scheme 4. Therefore, the issue of comparing building envelope designs according to Schemes 4 and 5 in terms of calculating the dew point and identifying possible condensation zones requires further elaboration.

### 7. Conclusions

1. A mathematical model of thermophysical processes taking place in a multilayer building envelope in which insulation is provided through the use of XPS plates having air chambers with reflecting coating applied to their walls. To construct a mathematical model in a one-dimensional statement, an approach was used in which thermal heterogeneity caused by alternating air chambers and dividers of XPS was described by the effective thermal conductivity of corresponding layers of the multilayer building envelope. The expression for dependence of the effective thermal conductivity on the ratio of temperature values on the air chamber walls was obtained.

2. The engineering procedure has been developed for calculating temperature fields in similar building envelope designs and their total RSI-value. This procedure is based on an iterative recalculation of the effective thermal conductivity for a thermal insulating layer with air chambers. The proposed procedure can also be used to solve the problems of thermotechnical analysis and develop building envelope designs with other insulating materials or with air chambers formed in a load-bearing layer.

Table 2

Values of thermophysical parameters of building envelope

Parameter	Scheme 1	Scheme 2	Scheme 3			Scheme 4			Scheme 5			
			V0*	V1*	V2*	V0*	V1*	V2*	V0*	V1*	V2*	
Total RSI-value $R$ , (°C×m <sup>2</sup> )/W	0.67	2.50	1.04	1.34	1.36	2.23	2.56	2.57	2.20	2.55	2.56	
Heat flux $q$ , W/m <sup>2</sup>	42.00	14.00	33.69	26.07	25.80	15.73	13.68	13.60	15.93	13.75	13.65	
Effective thermal resistance of thermal insulation layer $R_3+R_4$ , (°C×m <sup>2</sup> )/W	–	–	0.21	0.51	0.52	1.39	1.73	1.74	1.36	1.71	1.73	
Rayleigh number	–	–	1.17×10 <sup>5</sup>	2.24×10 <sup>5</sup>	2.27×10 <sup>5</sup>	1615	3475	3553	1322	3179	3264	
Temperature of the inner surface, °C	15.17	18.39	16.13	17.00	17.03	18.19	18.43	18.44	18.17	18.42	18.43	
Temperature at layer junction	$T_1$ , °C	14.62	18.21	15.68	16.66	16.69	17.99	18.25	18.26	17.96	18.24	18.25
	$T_2$ , °C	–12.90	9.03	–6.39	–0.42	–0.31	7.68	9.28	9.35	7.52	9.23	9.31
	$T_3$ , °C	–	–14.30	–13.31	–13.69	–13.71	–10.66	–6.68	–6.51	4.38	1.73	1.61
	$T_4$ , °C	–	–	–	–	–	–14.21	–14.32	–14.32	–14.20	–14.31	–14.32
Temperature of the outer surface, °C	–13.17	–14.39	–13.54	–13.87	–13.88	–14.32	–14.41	–14.41	–14.31	–14.40	–14.41	

Notes: \* – the calculation variant depending on presence/absence of a heat-reflecting coating; V0 – no heat-reflecting coating; V1 – a heat-reflecting coating applied to one wall of the air chamber; V2 – a heat-reflecting coating applied to two opposite walls of the air chamber

3. The developed engineering procedure was verified by comparing the calculation results with the results obtained in three-dimensional CFD simulation of heat transfer through the corresponding building envelope. In the latter case, free-convective air movement in the air chambers and radiation heat exchange between thermally heterogeneous walls of the air chamber were taken into account in detail. Verification results showed that the application of the proposed engineering procedure based on the solution to the one-dimensional heat conduction problem does not lead to significant errors in determining

the total RSI-value of the building envelope. Deviation from the total RSI-value obtained by calculating temperature fields using three-dimensional CFD simulation did not exceed 2.5 %. Comparative calculations showed that the presence of air chambers with heat-reflecting coatings in a 5 cm thick XPS plate reduced heat loss by 2.8 % compared to the XPS insulation without air chambers. When applying thermal insulation with air chambers in two layers, energy savings increased up to 6 % compared to the variant of thermal insulation with solid XPS plates of a similar thickness.

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