Запропонований метод структурного функціонально-вартісного моделювання складної ієрархічної системи. Визначені вихідні дані для проведення розрахунків за безпосередньо функціонально-вартісною моделлю. Запропонований та обґрунтований вартісний опис складної системи та її складових за допомогою аналітичних апроксимуючих залежностей. Приклад функціонально-вартісного алгоритму із застосуванням методу множників Лагранжа наведений для складних систем з послідовним сполученням окремих її частин. Вирішенням прикладу є розподіл між бажаними ймовірностями ефективного функціонування окремих частин з точки зору мінімальної вартості. Отримання такого розподілу не потребує абсолютних значень вартості як частин, так і всієї системи. До питань, що вирішуються при вартісній раціоналізації, відносяться такі: забезпечення заданого рівня функціональної досконалості системи при її мінімальній вартості; визначення мінімально необхідного рівня функціональної досконалості однієї ланки при відомих рівнях функціональної досконалості системи та всіх інших ланок крім тої, що досліджується; визначення необхідної кількості паралельно діючих ланок однакового призначення; з'ясування необхідного рівня функціональної досконалості ланок (датчиків інформації, ланок обробки інформації, каналів зв'язку), що мають паралельне сполучення; структурне удосконалення складної системи за рахунок вибору ланки системи, для якої підвищення функціональної досконалості може здійснитися з мінімальною вартістю. Запропоновані правила структурної раціоналізації складної системи. Першим з них є правило раціональної структурної будови складної системи. Воно дозволяє отримати достатню користь від складної системи при мінімальних витратах. Другим є правило доцільності ускладнення складної системи. Згідно нього, ускладнення складної системи доцільне тільки у випадку, коли при цьому зростає функціональна досконалість всієї складної системи. Третє правило – правило правильної будови – показує, що у складній системі відсутні зайві ланки, тобто такі ланки, які не виконують функціонально необхідних для даної системи дій

Ключові слова: структурні функціонально-вартісні моделі, складні ієрархічні системи, функціонально-вартісні розрахунки, апроксимуючі функції

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CONSTRUCTION OF A METHOD FOR THE STRUCTURAL FUNCTIONAL-COST MODELING OF A COMPLEX HIERARCHIC SYSTEM

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1. Introduction

The creation, operation, and improvement of complex man-machine hierarchical structures is an extremely costly and ambiguous enterprise. To take a reasonable decision about the required activities when solving these problems, it is important at least approximately to imagine the peculiarities of functioning and the generalized characteristics of such systems, their components at each stage of operation. This would make it possible to predict the required human, material, technical-technological, and financial resources for the modeling of possible options for the further functioning of a particular system.

The results of such a forecast could make it possible to timely choose the most effective variant of its structure regarding the specific requirements and the stage of operation. Achieving this is the aim of existing methods for the functional-cost modeling of a complex hierarchical system. However, a significant limitation of existing methods is the difficulty in deriving a parametric description of both the complex system itself and its components. This narrows the ability to search for the compromise options for defining the values for the functional parameters and the characteristics of the system and its components. Therefore, it is a relevant task to devise methods and techniques for the functional-cost analysis, which could define the requirements for clarifying the rational values for the parameters and characteristics of both the system and its components.

2. Literature review and problem statement

Study [1] provides the most complete review of approaches to evaluating the efficiency of complex systems. Papers [2, 3] describe current variants of approaches to deriving logical conclusions concerning the content and order of functional and technological support to the development of complex systems. They propose making decisions based on the results of the analysis of specially organized data arrays. The physical heterogeneity of such systems, the dependence of the specific values of their functioning values on a large number of influential factors, including stochastic, necessitated the development of methods for the probabilistic assessments of efficiency [4]. The difficulties and, more often, the impossibility of analytical (algorithmic) formalization of complex systems functioning gave rise to the dissemination of expert evaluation methods [5]. The modern variant of their implementation is considered in [6]. However, the transition to the parametric description of the complex system in such methods is ambiguous.

Since in addition to the functional load, complex systems must meet the requirements for their economic feasibility, the important part of the stages of operation involving a complex system is the development and use of its functional-cost models, such as in [7]. The purpose of such models is to search for such a variant of the structure of a complex system at whose implementation the specified level of functionality is achieved through the minimum cost. However, work [7], as well as other ones that we analyzed, focus on the improvement of the highly specialized complex systems [8, 9]. Generalizing approaches, directions of their improvement were not considered.

An option to overcome such difficulties may be the use of neural algorithms. This approach is used in papers [10, 11]. However, even in this case, the derivation of the parametric description of a complex system in the analytical form was not considered.

Note that enhancing the functionality of a system is typically associated with an increase in its cost. The functionality and cost of a system are its competing, contradictory properties. The attempt to find a rational approach to their ratio is realized by the functional-cost modeling, as it was reported in [12]. However, the cited work examined a separate case of using the functional-cost rationalization using an example of improving the educational process as a complex system.

However, the substantiates approach to determining the parametric influence of a separate component on the generalized indicators of the cost of a complex system with the help of existing functional-cost methods is difficult. This is predetermined by almost the inability to strictly analytically relate the system cost and its level of functional perfection. In addition, the following tasks are not solved comprehensively: identifying the components of the system, the improvement of which is the most rational from the cost point of view on condition of maintaining the predefined level of its functionality; demands to the minimum required level of functional perfection of a component, planned to be included in the system, without changing the specified level of functional perfection of the system itself. And all this – under the condition of the minimum possible cost of the system or its components.

Thus, it can be argued that it is useful to undertake research implying the introduction of a polynomial approximating dependence of the complex system cost on the level of its perfection. This may make it possible to more reasonably approach determining the parametric effect of a subsystem on the generalized indicators of functional perfection and cost of the entire complex system. The use of polynomial approximating dependences translates the qualitative level of substantiation of the rational existence of a complex system into quantitative analysis. When statistical data are available, this makes it possible to maximally precisely parametrically control the functional suitability of a complex system at its minimum cost. In the absence of such data, it helps more consciously approach the parametric and structural improvement of the complex system.

Practical application of the method for the structural functional-cost modeling of a complex hierarchical system involving a polynomial approximating dependence of the cost of complex systems on the level of their functional perfection implies the formalization of mechanisms of its implementation. The unaddressed problematic components related to the application of such a method are the peculiarities of assigning the level of functional perfection of a complex system, its cost, and the order of structural functional-cost calculations.

3. The aim and objectives of the study

The aim of this study is to devise the mechanisms for implementing the method of structural functional-cost modeling of a complex hierarchical system involving the polynomial approximation of the dependence of the cost of a complex system and its components on the level of functional suitability of the system. This would enable the practical application of a given method for the functional-cost analysis of complex systems of various kinds.

To accomplish the aim, the following tasks have been set:

– to define the initial data for calculating, based on the functional-cost model, a complex system involving the polynomial approximation of the dependence of the cost of a complex system and its components on the level of the functional suitability of the system;

– to devise the order of structural functional-cost calculations of a complex system involving the polynomial approximation of the dependence of the cost of a complex system and its components on the level of functional suitability of the system in order to build its rational structure.

4. Initial data for calculations that are directly based on the functional-cost model

We consider the following data to be the initial data to perform calculations based on a functional-cost model:

– the analysis of types of the functional-cost models;

– the selection of a generalized indicator of the structural functional perfection of a system;

– the development of an approach to the polynomial approximation of the dependence of the cost of a complex system on the generalized indicator of its functional perfection.

4. 1. Analysis of types of functional-cost models

We shall categorize the functional-cost models in accordance with the technique for including information about the cost of the system, its components to the criterion of performance estimation of the system functioning. That is, it remains the same assessment of the system's efficiency but under the condition of assigning its cost to the most important limiting partial criterion. In this case, three types of the structure of the mathematical functional-cost models are identified: direct functional-cost models; relative functional-cost models, integrated functional-cost models (Fig. 1).

Note the peculiarities of their description.

The direct functional-cost models include those which employ, as the generalized criterion, the direct cost of the examined system. The main assumption when using these models is the additive cost C_i of the individual parts of the system relative to the total cost C_Σ of the entire system [13]:

$$C_\Sigma = \sum_{i=1}^{n} C_i, \tag{1}$$

where n is the total number of components of the system.

The decisive rule of the system structure in terms of reducing its cost is the following requirement:

$$C_\Sigma = \sum_{i=1}^{n} C_i = \min. \tag{2}$$

Relation (2) shows that the most effective, in terms of cost, is a simple ($n \to 0$) and cheap ($C_i \to 0$) system.

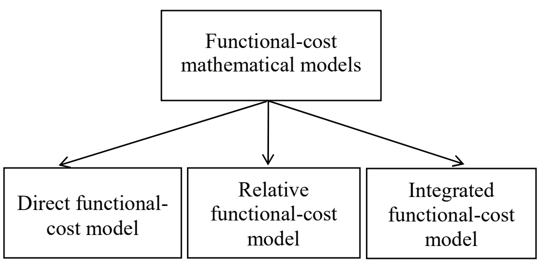

Fig. 1. Functional-cost mathematical models

However, each system performs certain functional tasks. While being sufficiently cheap, such a system may fail to fully satisfy the necessary level of functional purpose. Therefore, ratio (2) is supplemented with a condition for fulfilling the functional perfection P of the system. Since the cost of the system and its functional perfection are unconditionally related, it is possible, for the optimal case, to record the following requirement:

$$P_\Sigma = P[C_i] \to \max,$$

$$C_\Sigma = \min.$$

In a rational form, it will take the following form [12]:

$$P_\Sigma = P[C_i] \ge P_{pr}, \quad C_\Sigma = \min, \tag{3}$$

where P_Σ is the generalized indicator of the functional perfection of the system, for example, its efficiency, functionality, probability of task execution; $P[C_i]$ is the functionality that shows the dependence of the task execution by a system on the cost of its each component; P_{pr} is the predetermined level of functional perfection.

Relations (3) can be considered as a rational cost (functional-cost) mathematical model of the examined system. To apply it, it is necessary to determine the desired level of functional perfection P_{pr} of the system. It is a normative or assigned quantity. It is important to find out the dependence of the cost of each part (link) of the system on its parameters and to derive actual numerical estimates of the cost of these parts.

The relative functional-cost model is one of the variants of using the generalized utility ratio k_Σ [1]. The coefficient k_Σ is introduced as the ratio of the criterion responsible for the functional perfection P_Σ to the total cost C_Σ. The generalized utility ratio k_Σ would accept a larger value in proportion to the higher functional perfection of the system, its links at smaller cost expenses:

$$k_\Sigma = \frac{P_\Sigma}{C_\Sigma}. \tag{4}$$

Thus, the following ratio can be considered a rule of the system cost optimization based on a relative functional-cost model:

$$k_\Sigma = \frac{P_\Sigma}{C_\Sigma} = \max. \tag{5}$$

It should be noted that the values for a coefficient k_Σ do not have any generalized physical content and specific measurement units. If the P_Σ and C_Σ quantities are normalized, the values for a coefficient k_Σ are correlated among the systems within a single class. At the non-normalized P_Σ and C_Σ coefficients the current value k_Σ characterizes the level of perfection only of the examined system.

For the system optimal in terms of cost, it is necessary to find the only correlation between the criterion of functional perfection of the system and its cost at which it reaches a maximum. However, the actual technological, organizational, physical, and other constraints do not make it possible to infinitely improve the level of the system functional perfection. Therefore, a rational approach is more expedient.

A pragmatic (rational) approach to the use of expression (5) implies setting the required level of P_{pr} and minimizing the expenses C_Σ. The rational approach is actually used in practice. Expression (5) is transformed into a rationalization rule:

$$k_\Sigma = \frac{P_{pr}}{C_\Sigma} = \max. \tag{6}$$

The use of ratio (6) implies the preliminary construction of algorithms for calculating the P_{pr} and C_Σ values. Let us consider them.

The P_{pr} quantity is probabilistic. This is a consequence of the dependence of the resulting functional efficiency of each complex system on a sufficiently large number of random factors. Then we can assume:

$$P_\Sigma = \prod_{j=1}^{k} \left(1 - P_j\right) \ge P_{pr}, \tag{7}$$

where k is the number of independent influential factors that reduce the functional perfection of the system.

The better they are accounted for, the greater the magnitude k and the closer expression (7) to reality. The probabilistic characteristic P_j estimates the influence of the j-th factor.

Proceed to the order of assigning the magnitude C_Σ. The expenses C_Σ depend on the complexity of the system, which can be characterized by the correctness of its structure, the number of components n, the combinations l among them, as well as the operational costs for supporting the system feasibility, including the disposal of it).

If $\Psi(n, l)$ is the cost function of the combinations among system parts and operating costs, then:

$$C_\Sigma = \sum_{i=1}^{n} C_i + \Psi(n,l) = \sum_{i=1}^{n} C_i + \Psi_1(l) + \Psi_2(l+n), \tag{8}$$

where $\Psi_1(l)$ and $\Psi_2(l+n)$ are the functions of the cost of combinations and operating costs, respectively.

However, if the links among the parts of the system are to be considered as individual elements, one can write:

$$\Psi_1(l) = \sum_{i=1}^{l} C_j. \tag{9}$$

It is advisable to formalize the $\Psi(l+n)$ function as well. To this end, in the simplest case, the level of expenses per unit of operation time (depending on the examined task – a day, year, cycle, period of operation, etc.) is assumed to be proportional to the cost of the element. Then:

$$\Psi_2(l+n) = a\sum_{i=1}^{l+n} C_j, \tag{10}$$

where a estimates the average coefficient of proportionality.

Expression (8), considering (9), (10), takes the form:

$$C_\Sigma = \sum_{i=1}^{l} C_i + \sum_{i=1}^{n} C_i + a\sum_{i=1}^{l+n} C_i = \\ = \sum_{i=1}^{l+n} C_i + a\sum_{i=1}^{l+n} C_i = (1+a)\sum_{i=1}^{l+n} C_i. \tag{11}$$

Thus, the system of ratios (5), (7), (11) is a relative functional-cost model of the rational structure of a complex system. Its generalized form is recorded as follows:

$$\begin{cases} k_\Sigma = \dfrac{P_\Sigma}{C_\Sigma} = \max, \\ P_\Sigma = \sum_{j=1}^{k} (1-P_j) \geq P_{pr}, \\ C_\Sigma = (1+a)\sum_{i=1}^{l+n} C_i. \end{cases} \tag{12}$$

System (12) makes it possible, in addition to the operations that are characteristic of the direct functional-cost model, to perform another activity. It implies the following.

The proper structure of the system must ensure that the introduction of additional funds, efforts improves its functionality. However, when the system is enlarged, such additions lead to decreasing functionality. Therefore, the first equation in system (12) can be termed as a cost efficiency equation. If the increased investment, with a growth in the functional efficiency indicator, is accompanied by an increase in the value of coefficient k_Σ, it is possible to consider such additions appropriate. If the increased investment in the system is not accompanied in the increase in coefficient k_Σ (the quantity P_Σ grows with less intensity compared to C_Σ), then such contributions are not appropriate.

The last level of the examined categorization is an integrated functional-cost model. It implies the consideration of the system cost as a separate partial criterion among a series of other criteria. The evaluation of the system cost effectiveness in this case is implemented in multivariate space.

Each coordinate within this space is a partial criterion of system effectiveness: the weight, operational reliability, dimensions, mandatory probability of task execution, the cost, and other criteria that are important for a given case of the system examination. The problem to be solved comes down to selecting the best variant of the system existence, taking into consideration the influence of all partial criteria (Fig. 2).

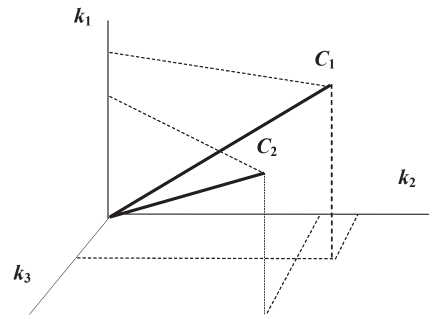

Fig. 2. Example of a three-dimensional existence space of the C_1 and C_2 systems in the coordinates of partial criteria k_1, k_2 and k_3

One point corresponds to each variant of the system's existence within the coordinate space. It can be connected to the coordinate origin by a radius-vector whose size would be equivalent to the overall usefulness of the variant of the examined system structure. This size would determine the magnitude of the generalized system performance criterion.

The generalized criterion has no physical essence. It is possible to compare absolute values of the generalized criteria only for systems that are examined in the identical coordinate systems and when applying the same algorithms.

Analytically, the procedure for determining the generalized criterion k_Σ is close to the previous case. It is believed that for the j-th variant of the system structure it is the better (higher) the higher the consumer cost CB_j of this variant of the system at a given level of expenditure B_j on it. Thus, the j-th variant of the system structure will be characterized by expression:

$$k_\Sigma = \frac{CB_j}{B_j} = \frac{f_j(k_k; k=1,n)}{\varphi_j(k_k; k=1,n)}, \tag{13}$$

where $CB_j=f_j(...)$, $B_j=\varphi_j(...)$ are considered to be the functionals of consumer value and expenses, respectively, with respect to n of partial criteria of k_k.

A rule for the optimum choice of system structure variants is the condition:

$$k_\Sigma = \frac{CB_j}{B_j} = \frac{f_j(k_k; k=1,n)}{\phi_j(k_k; k=1,n)} \to \max_{k_k \in Q;\, j=1,m}, \tag{14}$$

that is, one explores m variants of the structure of the system on the condition of belonging of all partial criteria to the region Q of their existence.

Equation (14) is the ultimate condition in the problems of mathematical programming. Its exact solution implies the knowledge of analytical dependences $f_j(...)$ and $\varphi_j(...)$. However, the partial criteria are physically and logically dissimilar indicators. Some of them are simply difficult to formalize (for example, modernity, ergonomics, operational convenience, etc.), let alone combine into a single ratio.

It is possible to simplify the problem by assuming the additive character of the generalized criterion relative to partial ones. Under such a precondition, the total benefit of the correctly made decision consists of the benefits obtained for each partial criterion.

Analytically, the rule of choice optimality in this case is described by the basic equation of linear programming:

$$k_{\Sigma}=\frac{CB_j}{B_j}=\sum_{k=1}^{n}\beta_{kj}k_k\rightarrow\max_{k_k\in Q,\ j=1,m}, \qquad (15)$$

where β_{kj} is the significance of partial criterion k_k; k_k is already the relative value of the partial criterion.

Equation (15) is free of a series of constraints. There is no need to adjust the measurement units for its components; that makes it possible to analyze the contribution of each partial criterion to the general one, but requires that in each case one should resolve the following two difficulties.

First, this is the presence of an alternative direction of the rational values of partial criteria. Some criteria improve along with an increase in them (quality, reliability, the probability of performing a functional purpose). Other criteria, on the contrary, must be reduced to improve their rationality (cost, complexity, maintenance costs). There are criteria for which the level of rationality is determined by the task set (the set weight, size, time of execution).

Second, this is the task to assign the significance β_{kj} of the partial criteria. They can be set based on the results of previous tests, by sorting possible variants, by using some analytical tools.

The expert variant of the integrated functional-cost model [14] is devoid of these shortcomings. Its structure is based on the method of a pairwise comparison implying determining the priorities among the partial criteria and system structure variants. In this case, the calculations involve not the specific physical values for the criteria but the experts-assigned abstract numerical values, the results of comparisons. Assigning the abstract numerical values (comparative estimations) is performed when an expert compares such quantities pairwise (partial criteria or system variants).

Note that in this case we no longer deal with the optimum choice but only with the rational one, that is the choice close to the optimal. This provision is the consequence of setting the priorities by individual experts whose viewpoint is always subjective. To a certain extent, this limitation is compensated for by the qualified selection of experts, as well as by a statistical approach to processing the results of the comparison.

Formally, a rule of rational choice in this case can be written in the form similar to (15). Taking into consideration the multivariate existence of the generalized criterion and in order to simplify the mathematical treatment of the comparison results, such expression is written in a vector form:

$$k_{\Sigma}=\frac{CB_j}{B_j}=\beta_{kj}k_k\rightarrow\max_{k_k\in Q,\ j=\overline{1,m}}, \qquad (16)$$

where β_{kj} is the priority of the j-th system variant based on the k-th criterion; k_k is the priority of the k-th criterion.

The use of rule (16) to all m models assigns to each variant of the structure of the examined system its value of factor $k_{\Sigma j}$. The list of coefficients is compiled by vector k_{Σ}.

The multi-criteria approach makes it possible, in each case, to consider the cost not as a dominant of the system structure but to weigh it among other influential factors on the existence of the system [14]. In this case, there is a possibility to analyze the contribution of each partial criterion to the generalized criterion. Thus, those factors are identified that require the greatest attention in terms of improving the level of system existence.

The mechanisms to calculate comparative priorities, similarly to previous cases, make it possible to investigate systems in the parametric domain and to develop the means to manage partial criteria.

The directly functional-cost model is based on the assumption about a direct dependence between the cost of the examined system and the degree of fulfillment of its functional purpose. This applies to both the entire system and its every separate part. Therefore, the process of the functional-cost optimization in this case is described by the following system of equations:

$$\begin{cases} C_{\Sigma}=\sum_{j=1}^{m}C_j=\min, \\ P_{\Sigma}=P_{\Sigma}\left[C_j\right]=\max. \end{cases} \qquad (17)$$

It is possible to solve system (17) only in the case when the P_{Σ} and C_j functions are set analytically and are fully adequate to the physical, public, and other processes that accompany the existence of the examined system. However, the exact expressions for such equations are not used in practice because of the difficulties and ambiguity in algorithmizing them. Therefore, it is accepted to construct approximating algorithms to describe the functions of cost and probabilistic dependence of the functional perfection of a system and its components. That once again stresses that each type of the functional-cost model considers not the optimization but rather the rationalization of the system structure.

4. 2. Selecting the generalized indicator for the system structural functional perfection

We shall proceed to select the generalized indicator for the system structural functional perfection. The functional perfection of the system (physical orientation, the generalized qualitative and quantitative requirements) is determined by the correspondence of results of its operation to the expected values, levels. Qualitative and quantitative indicators, which are approved in the normative and monographic literature, are developed for the common assessment of the level of the system functional excellence. For a system of higher education, for example, this implies the possibility to implement certain forms of training at a particular institution. For surveillance systems, these are the depth, the amount of information about an object considering the possibilities in the stages of planning, observation, results from processing, and their presentation.

Enlarging any system significantly diversifies the complexity and the number of such parameters (and each of them depends on several parameters); it becomes more difficult to describe the exact connection among the system-specific parameters and its functional parameters. Therefore, in the description of large systems, the probabilistic indicators are more often used, which, first, provide an opportunity to describe the effect exerted on the functional suitability by all system parameters that form its functional perfection, second, they reflect the stochastic behavior of a large system and, third, they make it possible not to search for precise functional dependences but to identify trends in such dependences [15].

The unconditional advantage of using the probabilistic indicators is the possibility of creating and the relative simplicity in constructing the generalized criterion of functional perfection that in essence corresponds to the generalized criterion of system efficiency.

It may denote the probability P_Σ of the system executing the set task at the current value of its parameters. As a rule, the generalized criterion will depend on a series of other probabilistic lower-level indicators – the partial criteria P_j.

They are determined by the probability of execution, by the system components, of their functional assignments. Partial criteria in this case reflect the probability of the system fulfilling the individual requirements of the generalized criterion.

While the partial criteria P_j may be considered independent relative to the generalized criterion P_Σ and of each other, the second equation of system (17), when the parts of the system sequentially perform their functional assignments, takes the following form:

$$P_\Sigma = \prod_{j=1}^{k} P_j = \max, \tag{18}$$

where k is the number of partial criteria.

In a given case, the partial criteria evaluate the functional suitability of certain parts of the system. The equality $k=m$ holds: – the number of separate links of a complex system equals the number of the partial criteria.

Equation (18) demonstrates that the perfection of system operation depends on the quality of its components and their arrangement within the system. The specified components are determined by the cost of the system, that is $P_j=P_j[C_j]$. The higher the costs (funds, various tools, efforts) to maintain the system, the more beneficial the expected result of the functioning of the system should be. The same conclusion can be reached when considering partial criteria in the form of the components of the generalized criterion.

Thus, the cost of the system or its separate part is also a generalized criterion. However, this is true only when the system structure is proper when increasing the investment (tools, effort) in the existence of the system leads to an increase in the probability of it fulfilling the set task.

Functional efficiency and cost-effectiveness are always competing. As a rule, when we improve the system's functional efficiency, we reduce its cost-effectiveness. In essence, functional efficiency forms the cost efficiency, although the latter is parametrically described by its factors, parameters: the cost and number of parts, nodes in the examined system, the number of system instances in the series, the required skills of producers and consumers, etc. Such parameters are not directly dependent on the functional parameters – accuracy, reliability, continuity, etc.

In addition, it is possible to introduce other types of efficiency, for example, the ergonomic, operational, ecological efficiency, etc. The specified concepts of efficiency are also complex, that is, they are determined by their own, independent groups of parameters. However, there is one common property in these components. Typically, the increase in them leads to a decrease in cost efficiency.

Probably, there is a concept of the overall efficiency of the system as a function of all its independent components. The rationalization of the system's overall efficiency can be carried out in parallel for all components, which extremely complicates both the mathematical and technological processes of its implementation. It can be conducted consistently, for example, in pairs: functional and cost, ergonomic and cost, ecological and cost, and so on. The presence of cost efficiency at all stages is justified by its dependence on all other components of the general system efficiency.

It is important to predict that, under such a sequential approach, the next stage of rationalization of the overall efficiency should not lead to the loss of products at the previous stages. To this end, the specified rational values of the efficiency of the previous stages can be considered as the restriction, the initial conditions for the implementation of all subsequent stages.

The desired level of the system functional efficiency is $P_\Sigma=1$. However, the probabilistic approach to the description of the generalized criterion makes it possible only to strive to the desired value. For existing systems, the required minimum level of such a criterion is assigned by the quantity P_{pr}. Therefore, equation (18) is rewritten as a rational condition:

$$P_\Sigma = \prod_{j=1}^{m} P_j \geq P_{pr}. \tag{19}$$

Condition (19) not only specifies the required level of efficiency of the entire system. It can be used to determine the satisfactory level of effectiveness of a separate part $P_{j,pr}$ if the actual efficiency of other parts of the system is known:

$$P_{j,\,pr} \geq \frac{P_{pr}}{P_1 P_2 \ldots P_{j\,1} P_{j+1} \ldots P_m}. \tag{20}$$

Expression (20) is required in assessing the feasibility of further complication of the system. Its use answers the question: is it possible to implement a part of the system with a satisfactory value P_j.

Note once again that determining all values P_j makes it possible to proceed to the parametric description of the system and its parts, that is, to proceed from its generalized characteristics to specific parameters of the individual components, the system links. Each probability P_j has a correlation that relates it as a functional perfection criterion to the parameters and characteristics of an entire system or its parts. This ratio can be derived empirically or analytically.

There are two main requirements for such analytical or empirical ratios. First, they must change in the range (0, 1). Second, the arguments to them should include all technical (technological) parameters, which would significantly affect the formation of a particular value of P_j.

Our example is the ratio for the probability P_0 of detecting an object based on its static image during observation. It takes the form [16]:

$$P_0 = \exp\left[-\left(\frac{Bm'}{2LR\sqrt{\Delta D}}\right)^2\right], \tag{21}$$

where B is the form factor of the identification attribute with a linear size L and tone contrast ΔD of an object.

The object is observed with a typical tool with resolution R at the denominator of scale m'. The probability P_0 can be considered as a partial criterion for the functional suitability of a large system, which solves the task of composing an object image of the terrain (for example, environmental monitoring, mapping) based on the results from its typical observation. Other partial criteria for this task will be the likelihood of the observation tool to reach the area over which it is carried out, the likelihood of such an instrument being triggered, the likelihood of the image processing equipment being triggered, the likelihood of correctly processing the image decryption results obtained, etc.

Note that each of the above probabilities characterizes the next stage (a separate part of the large system) in compiling a terrain map and is a function of the parameters of such a stage. Deriving the parametric dependences translates the investigation of the system by probabilistic indicators to its specific parameters and conditions of application.

For example, expression (21) shows that increasing P_0 implies increasing the resolution of an image (to improve the quality of an observing tool, to maintain its necessary modes), a tone contrast (to choose the modes and parameters of the receiver according to the contrast characteristics of an object, background, environment). The denominator m' must be reduced (to select an observing tool with a greater focal length or to reduce an observation distance).

Expressions such as (18), (19), (21) confirm the possibility of an analytical approach to assessing the probability of a functional perfection of a separate part of the system or the entire system together and their parametric control. Let us concentrate on introducing a cost parameter to such control.

4. 3. Devising an approach to the polynomial approximation of the dependence of the cost of a complex system on the generalized indicator of its functional perfection

The cost of a separate part (link) depends on many factors: the number and perfection of components, the technology of their combination, maintenance, etc. It is almost impossible to combine all these factors in a single functional (analytical) dependence and, in many cases, it is impractical (in terms of the required accuracy of calculations).

The system cost description can be carried out in two ways. The first way is to assign such a dependence based on a tabular value. In this case, each state of a complex system or its link is assigned with its cost or the cost range. Such data can be represented as tables, matrices. The disadvantages of the technique are the difficulties of obtaining accurate cost data and forecasting previous probable results.

A more flexible way of cost description is to use the approximating dependences. Here are some general conditions for the choice of a type of approximating cost-functional dependence.

First, the cost of a system part consists of the “cost” of the parameters of this part – the funds spent on the parameters to achieve the required value. In accordance with equation (21), Each combination of parameters corresponds to its own value of probability P_0. Thus, it can be argued that

$$C_j=C_j\left(P_j\right), \qquad (22)$$

where C_j is the cost of the j-th component of a part with a level of perfection P_j.

Expression (22) is the solution to the second equation of system (17) relative to the cost of the system or the cost of the system part.

Second, the proper structure of the system or its part implies an increase in its functional perfection (efficiency) with an increased amount of funds invested in them. Consequently, the cost equation should meet the following limitations and conditions for the existence of complex systems of the single type and purpose:

1. $C(P)\geq 0.$

2. $P_1\geq P_2\rightarrow C(P_1)\geq C(P_2).$

3. $\lim\limits_{P\to 0} C(P)=0.$

4. $\lim\limits_{P\to 1} C(P)=\infty. \qquad (23)$

The analytical constraints and conditions, given in (23), are the consequence of the possibility of the physical implementation of a complex system in the case when its cost is a set of socially significant expenses for achieving a certain goal. The first equation characterizes the current existence of a complex system. The second one emphasizes the direction of its rational improvement. The initial status of system creation is formalized in the third equation. In the fourth – the prospects for its final improvement. Separately, we emphasize the additive property of the general cost function C_Σ relative to its components C_j:

$$C_\Sigma=\prod_{j=1}^{m}C_j. \qquad (24)$$

Requirements (23) and the usability of equation (24) are satisfied, for example, by the following ratios:

$$C_j(P)=A_jP\exp\left(\frac{B_j}{1-P}\right), \qquad (25)$$

$$C_j(P)=\frac{A_jP^{B_j}}{\ln\left(\frac{1}{P}\right)}, \qquad (26)$$

$$C_j(P)=\frac{A_jP}{1-P}, \qquad (27)$$

in which A_j and B_j are the constants that can be chosen empirically.

The approximating expressions are selected based on the intensity (proportionality) of change in the system cost along with a change in the level of its functional perfection. Thus, for example, for the simplest case, when using ratio (27), the cost gain ΔC with an increase in the level of functional suitability by ΔP would equal:

$$\Delta C=A\frac{\Delta P}{(1-P)(1-P-\Delta P)}.$$

The presence of two constants in ratios (25) and (26) allows for more flexible and accurate use of these expressions to describe valid physical dependences. The A_j coefficient has a cost dimensionality. It can be interpreted as the cost, which a customer can pay for fulfilling the function by the entire system or a separate link.

The B_j coefficient characterizes the intensity of the approximation of the system cost to a maximum value at $P_j\to 1$. The choice of a specific ratio depends on the desired accuracy of calculations, the presence of relevant *a priori* data, the requirements for the analytical apparatus. Examples of calculations based on equations (25) to (27) are shown in Fig. 3.

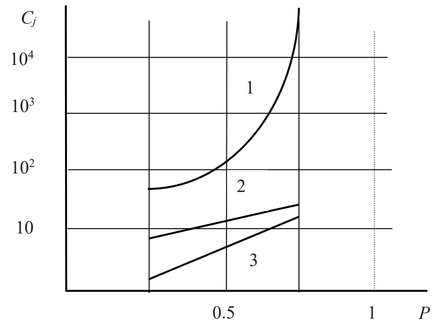

Fig. 3. Charts of calculations based on equations: 25 – 1; 26 – 2; 27 – 3

To select the most appropriate dependence from (25) to (27), a base of the *a priori* data is required for each constituent part of the examined system. For example, at $P=P_1$, the cost of a link was C_1 and, at $P=P_2$, the cost of the link changed to C_2. Moreover, according to Fig. 3, of importance is not only the chosen form of a ratio but also the range of change in the probability P, within which the system is examined.

Such statistical data perform the functions of points along a curve that approximates the dependence $C(P)$ (they are applied to choose the coefficients A and B, or only A). The chosen curve should accurately describe the real data near a solution point. Over other intervals of the curve, such a convergence is not needed because they are not used in calculations.

The application of ratios (25) to (27) in equation (24) yields the following expressions for the total system cost:

$$C_\Sigma\left(P_j\right)=\sum_{j=1}^{m} A_j P_j \exp\left(\frac{B_j}{1-P_j}\right), \qquad (28)$$

$$C_\Sigma\left(P_j\right)=\sum_{j=1}^{m} \frac{A_j P_j^{B_j}}{\ln\left(\dfrac{1}{P_j}\right)}, \qquad (29)$$

$$C_\Sigma\left(P_j\right)=\sum_{j=1}^{m} \left(\frac{A_j P_j}{1-P_j}\right). \qquad (30)$$

If we confine ourselves to the approximating cost-functional dependences of the (25) to (27) type, all three equations in (28) to (30) may be reduced to one:

$$C_\Sigma\left(P_j\right)=\sum_{j=1}^{m} A_j P_j \exp\left(\frac{B_j}{1-P_j}\right)+\sum_{j=m+1}^{n} \frac{A_j P_j^{B_j}}{\ln\left(\dfrac{1}{P_j}\right)}+\sum_{j=n+1}^{k} \frac{A_j P_j}{1-P_j}, \qquad (27)$$

where m is the number of links within a system with the approximation of the cost dependence by equation (25); $(n-m)$ is the number of parts within a system with the approximation of the cost dependence by equation (26); $(k-n)$ is the number of parts within a system with the approximation of the cost dependence by equation (27); k is the total number of links in the system.

Expressions (28) to (30) can be used in the system of cost rationalization (17). Taking into consideration the preliminary condition (19) as regards the use of an equation of the system functional conformity or its part, and an expression for the total cost, for example, (30), ratio (17) can be written as:

$$\begin{cases} C_\Sigma\left(P_j\right)=\sum_{j=1}^{m} C_j\left(P_j\right)= \\ =\sum_{j=1}^{m} A_j P_j \exp\left(\dfrac{B_j}{1-P_j}\right)=\min\limits_{0\le P_j\le 1}, \\ P_\Sigma=\prod_{j=1}^{m} P_j \ge P_{pr}. \end{cases} \qquad (32)$$

The system of equations the type of (32) can be used for the numerical cost rationalization of complex systems. The rule of rationalization in this case will be stated in the following way: a complex system should provide the predetermined level of functional perfection P_{pr} at its minimum total cost C_Σ.

We shall consider a procedure for using the system of equations (32) in order to perform a functional-cost study of the complex system with the consistent combination of its individual parts.

5. The order of structural functional-cost calculations in the sequential combination of individual parts

The simplest structure is a consistent combination of the individual parts in a complex system. This case corresponds to the systems whose ultimate target (goal) is achieved by the consistent execution of logically related individual tasks. Thus, the output value in the preceding part is the input value for the next part.

Consider a system of three sequential elements (Fig. 4).

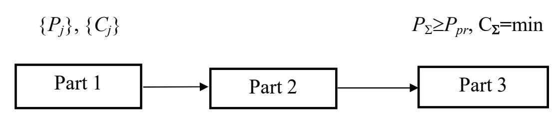

Fig. 4. A complex system of three sequential elements

If the cost dependence for all three parts can be described using the ratios of type (27), and the total cost is derived from expression (28), the system of equations (32) will take the following form:

$$\begin{cases} C_\Sigma\left(P_j\right)=\sum_{j=1}^{3} \left(\dfrac{A_j P_j}{1-P_j}\right)=\min\limits_{0\le P_j\le 1}, \\ P_\Sigma=\prod_{j=1}^{3} P_j \ge P_{pr}. \end{cases} \qquad (33)$$

The solution to system (33) means finding an extremum (minimum) of its first equation provided meeting the conditions relative to P_Σ and P_j.

A classical method to search for a conditional extremum is the method of Lagrange multipliers. To this end, one selects the Lagrange function in the form [17]:

$$f\left(P_j,\lambda\right)=\sum_{j=1}^{m} \left(\frac{A_j P_j}{1-P_j}\right)+\lambda\left(\prod_{j=1}^{m}\left(P_j-P_{pr}\right)\right), \qquad (34)$$

where λ is the Lagrange multiplier.

After finding the partial derivatives relative to variables P_j, by equating them to zero and by excluding the intermediate variables, we obtain the following system of algebraic equations:

$$\begin{cases} \dfrac{A_1P_1}{1-P_1}=\dfrac{A_2P_2}{1-P_2}, \\ \dfrac{A_1P_1}{1-P_1}=\dfrac{A_3P_3}{1-P_3}, \\ P_\Sigma = P_1P_2P_3 \geq P_{pr}. \end{cases} \quad (35)$$

System (35) includes three algebraic equations and three variables: P_1, P_2, P_3. This system has a unique solution. At small m (in this case, m=3), the solution is found through a direct substitution; with an increase in m (m>4), system (35) is solved by numerical methods using computer equipment.

The solution to system (35) yields the distribution among the desired probabilities of the effective operation of individual parts in terms of a minimal cost. It does not require the cost values either for parts and the entire system.

However, preliminary information on the cost of existing analogs of the system and its parts is extremely needed. Its presence will make it possible to choose the justified values for the A_j coefficients. The more accurately they are determined, the more significant the recommendations on the distribution of values for P_1, P_2, P_3 are. One can once again emphasize that in this case the cost is not necessarily money. This may be a conventional unit (point, estimate, numerical judgment). Of importance is their ratio only, for example: A_j is by many times (percent) more appropriate (more expensive) than A_{j+1}.

The devised procedure for the structural functional-cost calculations was tested when defining promising directions for the improvement of an educational process (EP) at a higher educational institution. The components considered were the actual organizational forms of mastering an educational material: training sessions (TS), independent work of students (IW), practical training (PT), control measures (CM). Accordingly, EP as a complex system includes four consecutive links, each of which is characterized by its functional relevance P_i and conditional cost C_i.

The general structural scheme of this system is shown in Fig. 5.

TS: P_1, C_1 → IW: P_2, C_2 → PT: P_3, C_3 → CM: P_4, C_4

Fig. 5. Structural scheme of the forms in an educational process

Determine the distribution of levels of functional conformity among individual links to achieve the purpose of EP at its minimum conditional cost.

Expression (27) is chosen as the approximating dependence of the cost of a particular link C_i on the probability P_i of it fulfilling the assigned functions. This choice is predetermined by the lack of statistical data on their interdependence. The distribution of the functional levels of conformity would be the solution to the following system of equations:

$$\begin{cases} C_\Sigma(P_i)=\sum_{i=1}^{4}\left(\dfrac{A_iP_i}{1\,P_i}\right)=\min\limits_{0\leq P_i\leq 1}, \\ P_\Sigma=\prod_{i=1}^{4}P_i \geq P_{pr}, \end{cases} \quad (36)$$

where P_Σ is the probability of compliance of graduates to the regulatory requirements; C_Σ is the conditional cost of EP (creation, development); A_i is the empirical constant.

Solving the system (36) means finding an extremum (minimum) of its first equation when provided the conditions are met relative to P_Σ and P_i. To derive it, it is necessary to satisfy three conditions. These include determining the quantities of empirical constants A_i, the assigned probability of task execution P_Σ, and directly solving the system of equations.

1. Determining the quantities of empirical constants A_i. In the absence of statistical data on their values, it is possible to use those values that were defined by experts as the priorities when comparing them in pairs [14]. The adjacency matrix will consist horizontally and vertically of the A_i results from comparing A_{ik} with A_{ki} based on the rule:

$$A_{ik}=\begin{cases} 10,\ A_{ik}=A_{ki}, \\ 15,\ A_{ik}>A_{ki}, \\ 5,\ A_{ik}<A_{ki}, \end{cases} \rightarrow A_i=\sum_{k=1}^{k=4}A_{ik}.$$

Considering the category of performers at each EP link, it is possible to assert the different conditional values of their functional conformity. The probability P_1 can be assigned with the highest cost and the A_1 coefficient can be provided with some value, for example, 50 arbitrary units. Control measures can be assigned with A_2=40 arbitrary units, practical orientation – A_3=20 arbitrary units, measures to implement the independent work of students A_4=10 arbitrary units, as such that are almost completely formed only by the scientific and pedagogical staff at a specialized department.

2. The desirable values of the probability of functional conformity P_1,..., P_4 and, consequently, P_{pr} are their maximum approximation to 1:

$$P_1 \approx P_2 \approx P_3 \approx P_4 \approx 1 \approx P_{pr}.$$

However, a sufficient level of functional efficiency is to be considered P_{pr}=0.85. This statement is based on predicting the normal law distribution of the probability density of EP fulfilling its functional purpose dependent on a large number of influential factors.

3. We shall solve system (36) and determine specific numerical data for the P_1,..., P_4 distribution using the Lagrange multiplier method; expression (34) at m=4.

Its partial derivatives for variables P_i and the partial extrema are determined from the following equation:

$$\frac{\partial f}{\partial P_i}=\frac{A_i}{(1-P_i)^2}+\frac{\lambda}{P_i}\prod_{i=1}^{4}P=0.$$

It can be recorded in the following form:

$$\frac{A_iP_i}{(1-P_i)^2}=-\lambda\prod_{i=1}^{4}P_i=-\lambda P_{pr}.$$

Since for each i=1, 2, 3, 4 the right-hand side of the previous equation remains constant, one can assert the following:

$$\frac{A_1P_1}{(1-P_1)^2}=\frac{A_iP_i}{(1-P_i)^2},\quad i=2,3,4. \qquad (37)$$

Thus, the application of the Lagrange multiplier method yields the system of four equations, and three of them are the equations of type (37) with four variables:

$$\begin{cases} P_1P_2P_3P_4 \geq 0.85, \\ \dfrac{50P_1}{(1-P_1)^2}=\dfrac{10P_2}{(1-P_2)^2}, \\ \dfrac{50P_1}{(1-P_1)^2}=\dfrac{30P_3}{(1-P_3)^2}, \\ \dfrac{50P_1}{(1-P_1)^2}=\dfrac{40P_4}{(1-P_4)^2}. \end{cases} \qquad (38)$$

Such systems are solved in an iteration. The procedure for iterations is as follows. We find the first approximation of variable P_1 in the following way:

$$P_1^1=\sqrt[4]{P_{pr}}=0.96.$$

The values of the first approximations of functional perfection $P_2^{(1)}$, $P_3^{(1)}$, $P_4^{(1)}$ are calculated from the lower equations of system (38). Validation of the values obtained through the functional conformity equation – the first equation of system (38) – significantly exceeds the limit of 0.85. Carry out two more iterations. The results of the three iterations are given in Table 1.

Table 1

Results of iterative calculations of the functional conformity of EP links

$P_i^{(j)}$	P_1^j	P_2^j	P_3^j	P_4^j	ΠP_i^j
$P_i^{(1)}$	0.96	0.982	0.969	0.964	0.881
$P_i^{(2)}$	0.94	0.974	0.952	0.946	0.824
$P_i^{(3)}$	0.95	0.977	0.961	0.955	0.851

The accuracy of the calculations reached 0.001 and the third iteration is the last.

The result of our calculations is the cost-based rational distribution between the levels of functional conformity of EP links. According to data in Table 1, the highest requirements in terms of functional conformity are put to the quantity P_2 – the probability of a rational organization of self-training of students.

Consequently, the least socially expensive way to improve the EP quality, efficiency at an educational institution implies improving the self-training of students. There are two directions for such work. First, it is the increased role and responsibility of specialized departments for the educational and methodological support of independent work, as well as the management of the corresponding faculty – for the organization and quality of its performance.

Second, it is the improvement of the material and technical support for self-training of students (affordability of the material and technical base of departments for self-mastery of the necessary knowledge by students, as well as creating, by the units at an educational institution, favorable conditions for the implementation of such kind of activity).

There is another conclusion. Inadequate organization of student self-training would block any other improvements in EP.

6. Discussion of results of devising a method for the structural functional-cost modeling of a complex hierarchical system

Thus, the introduction of a polynomial approximation of the dependence of the cost of a complex hierarchical system on its functional suitability under the structural functional-cost modeling makes it possible to relatively simple resolve a series of issues. These include the following:

– a possibility of quantitative evaluation of recommendations concerning further directions for the improvement of the examined system at the parametric level;

– determining the components of the system, the improvement of which is the most rational from the cost point of view on the condition of maintaining the specified level of its functionality;

– putting requirements to the minimum required level of functional perfection of the constituent planned to be included in the system, without changing the assigned level of functional perfection of the system itself.

That can be done, for example, based on the analysis of expression (34) relative to the limiting values for the A_j coefficients [12]. First, in the case of studying a system with the parts identical in terms of cost, at $A_1=A_2=A_3$, the $C_j=C$ equality would hold, and

$$P_1=P_2=P_3=\sqrt[3]{P_{pr}}. \qquad (39)$$

The cost-effectiveness of such a system is matched by the need for a balanced improvement of each of its parts.

A case is possible when $A_1=A_2=A_3=A$, while $C_1\neq C_2\neq C_3$, indicating that there is a sequential combination of similar (equally organized) or identical, but not equally functionally suitable parts. The user (customer) of the system is ready to conditionally pay the same price for each link: all links are equally valuable but have different functional readiness. In this case:

$$\begin{cases} C_\Sigma=\sum\limits_{j=1}^{m}\left(\dfrac{A_jP_j}{1-P_j}\right)=A\sum\limits_{j=1}^{3}\left(\dfrac{P_j}{1-P_j}\right), \\ P_\Sigma=\prod\limits_{j=1}^{m}P_j=P_1P_2P_3\geq P_{pr}. \end{cases}$$

Cost rationalization will correspond to the solution to system (34) at $A_1=A_2=A_3=A$:

$$\begin{cases} P_\Sigma=P_1P_2P_3\geq {}_{pr}, \\ \dfrac{P_1}{1-P_1}=\dfrac{P_2}{1-P_2}, \\ \dfrac{P_1}{1-P_1}=\dfrac{P_3}{1-P_3}. \end{cases}$$

The derived system has a solution at:

$$P_1=P_2=P,\quad P_3=\frac{P_{pr}}{P_1P_2}=\frac{P_{pr}}{P^2}.$$

The task of the researcher is to identify the two components that are most easily directed by their functional perfection to 1 and to determine the minimum permissible value for the third part from the above equation.

The prerequisite for the use of such ratios is that the $P_j>P_{pr}$ inequality should hold because, otherwise, the probability of P_3 would exceed unity, which is physically impossible.

A second extreme case would be the situation where the A_j coefficients differ greatly from each other, such as $A_1>>A_2$, A_3. Before making conclusions, we shall consider the beginning of the second equation in system (34). After regrouping, one can assert the following:

$$A_1P_1 - A_2P_2 = (A_1 - A_2)P_2P_1 \approx A_1P_1P_2. \tag{40}$$

Since $A_1>>A_2$, and the probabilities P_1, $P_2\to 1$, which is the consequence of the need to continually improve the level of functional excellence of the system and its parts, one can assume the following:

$$A_1P_1 >> A_2P_2. \tag{41}$$

Expression (10) is then simplified:

$$A_1P_1 \approx A_1P_1P_2, \tag{42}$$

which is possible only at

$$P_2 \approx 1. \tag{43}$$

If similar conclusions are drawn when satisfying a condition $A_1>>A_3$, the case $A_1>>A_2$, A_3 will correspond to meeting the requirements:

$$P_1 \approx P_{pr};\;\; P_2, P_3 \approx 1. \tag{44}$$

Thus, relations (40) indicate that the relatively cheap parts of the system in terms of its cost rationalization should always have the maximum available level of functional excellence. The relatively expensive parts of the system, which devour the basic funds for the existence of the system, can function in this case only at the level of the specified efficiency.

The experience of performing the tasks of the study allows the following rules of rational structural improvement of a complex system to be formulated.

Rule 1. The rational structure of a complex system (the number of links i and the combinations k between them) should be such as to provide for the predefined level of functional perfection P_{pr}.

In accordance with the structural rationalization, a given rule stresses that the number of links i and combinations k between them should not be redundant. This condition can be written as follows:

$$P = P_{pr} \to i + k = \min.$$

Only in this case, the cost of such a system would be minimal, that is the notion of rationality is aimed at obtaining sufficient benefit from a complex system at a minimum cost. The rule can be termed the rule of a rational structural structure of a complex system.

Rule 2. Any complication of a system (increasing its cost by quantity ΔC) should result in a maximum increase in the functional suitability ΔP.

Considering the introduced designations, one can write it as a condition:

$$C_{\Sigma+1} - C_\Sigma = \Delta C =$$
$$= \min \to P_{\Sigma+1} - P_\Sigma = \Delta P = \max.$$

This is the rule of rational improvement of a complex system. Sticking to this rule would lead to that it becomes necessary to choose, among all possible complications of the system, such that leads to the greatest increase in the level of functional perfection of the system. One can read it as follows: complicating a complex system is advisable only in the case when it is accompanied by an increase in the functional perfection of the entire complex system.

Rule 3. A system is built properly if the decrease ($n\to(n-1)$) or the increase ($n\to(n+1)$) in it by any component (link) leads to, respectively, a reduction in ($-\Delta C$) or a growth in the ($+\Delta C$) system cost:

$$n \to (n+1){:}\Delta C \ge 0,$$

$$n \to (n-1){:}\Delta C \le 0.$$

The rule of the proper structure shows that there are no extra links in a complex system, that is, such links that do not perform activities that are not functionally required for a given system.

The reported results of our research into the method of structural functional-cost modeling of a complex hierarchical system involving the polynomial approximating dependence of the cost of complex systems on the level of their functional perfection confirm the following:

– a method of the functional-cost analysis of a complex system has been constructed, which, in the presence of statistical data, makes it possible to maximally precisely parametrically control the functional suitability of the complex system at its minimum cost. In the absence of such data, it helps more consciously approach the parametric and structural improvement of a complex system;

– a given method can be used to study a complex system at all stages of its existence: development, operation, disposal;

– it can be used to study the weakly formalized and non-formalized complex systems, that is, it makes it possible to translate the qualitative analysis of a complex system into quantitative.

However, the results of our research do not fully cover the peculiarities of using the proposed method for a functional-cost study. This refers to the analysis of both the structural features of the structure of a complex system (parallel, mixed combinations of the system components were not considered in the present paper) and the limited types of mathematical models that we employed. Advancing such directions is the priority in further work on improving the possibilities of the devised method.

However, the limited application of the proposed method should be noted. It is expedient in the presence or establishment of a formal connection between the probability of executing a task and the system parameters, as well as a possibility to justify values for the A_i, B coefficients in the polynomial components. In addition, the use of the built algorithms is aimed not at obtaining specific, physically and mathematically reasonable values and costs and the levels of functional perfection. Rather, such constructs and calculations help understand the bottlenecks, weak links within a complex system, and identify necessary or unnecessary connections among links. To summarize it, in order to more consciously approach the structure of a complex system.

To a large degree, the specified constraints can be removed when individualizing the features of the application of the devised method for complex systems of a certain composition, for example, social, administrative, technical, complex systems. It could be more expedient to categorize them based on the field of application: industrial, educational, public, etc. In addition, to improve the accuracy of predictive conclusions based on the results of applying a given method, one should employ data from the previous studies into similar complex systems.

7. Conclusions

1. We have devised a method for the structural functional-cost modeling of a complex hierarchical system involving the polynomial approximating dependence of the cost of complex systems on the level of their functional perfection, which makes it possible to parametrically control the functional suitability of a complex system at its minimum cost. A given method is adapted to the use at different levels of the *a priori* uncertainty of source data and can be applied at all stages of the existence of a complex system: development, operation, disposal. In addition, it is useful for studying the weakly formalized and non-formalized complex systems, that is, it makes it possible to translate a qualitative analysis of a complex system into quantitative.

2. Initial data for calculations based on the devised method include the choice of the generalized indicator for the structural functional perfection of a system and its components, as well as its cost. Choosing, as the generalized indicator, the probability of fulfilling the functional purpose accounts for the stochastic existence of a complex system and makes it possible to switch to parametric control over the level of functional perfection of the system and to use normative requirements for such indicators. The introduction of a polynomial approximation of the dependence of the cost of a complex system on the generalized indicator of functional perfection reflects the competing relationship between them and makes it possible to carry out the functional improvement of a complex system on the condition of the minimal cost of such an improvement. We have devised the order of structural functional-cost calculations of a complex system with a serial combination of individual parts in order to rationalize its structure. That has allowed us to propose a standard solution to the system of equations for the functional-cost rationalization of a complex system based on the Lagrange multiplier method.

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