Розроблені математична модель (ММ) опису динаміки елемента ГЗ МТС, в якості приклада якої розглядається підводна буксирувана система (ПБС), та ММ МТС з ГЗ.

ММ динаміки елемента ГЗ дає можливість враховувати:

1) рух судна-носія (СН);

2) особливості конструкції ГЗ, які впливають на функціональні характеристики МТС;

3) рух підводного апарату (ПА);

4) вплив перешкод на шляху руху ПА та ГЗ;

5) значні переміщення ГЗ в складі МТС.

Математична модель МТС з ГЗ дозволяє вирішувати наступні завдання:

1) визначати зміну форми ГЗ та сил її натяжіння в процесі маневрування СН та ПА з урахуванням морських хвиль, вітрових навантажень на СН, глибини моря та її зміни у визначеній акваторії, маси та пружних якостей ГЗ;

2) визначати відносне положення СН та ПА в процесі їх маневрування;

3) визначати максимальні навантаження на ГЗ, необхідні для оцінки його міцності в процесі маневрування СН і ПА.

Аналіз проектних завдань при створенні морських прив'язних систем (МПС), як різновиду МТС, показує, що значну теоретичну складність та наукоємність отримують розрахунки ГЗ МПС. Запропонована методика удосконалення проектування МТС з ГЗ, заснована на ММ опису динаміки ГЗ МТС (а також МТС з ГЗ), дає можливість досліджувати різноманітні режими експлуатації практично всіх класів МПС. За її допомогою виникає можливість удосконалювати існуючі методи розрахунків і проектування МПС з ГЗ, довести їх до рівня інженерного додатку

Ключові слова: гнучкий зв'язок, морська технічна система, математична модель опису динаміки гнучкого зв'язку, удосконалення проектування МТС з ГЗ

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CONSTRUCTION OF A MATHEMATICAL MODEL TO DESCRIBE THE DYNAMICS OF MARINE TECHNICAL SYSTEMS WITH ELASTIC LINKS IN ORDER TO IMPROVE THE PROCESS OF THEIR DESIGN

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1. Introduction

Given the changes in the earth's ecology and climate, the requirements for the technology of studying and developing the oceans and its shelf zone have increased. The study and development of the oceans and other water resources needs new technical means to investigate the ocean, which requires the development of new methods of calculating and designing marine technical systems (MTS). A characteristic feature of modern MTS is the presence of elastic links – chains, ropes, cables, cable-hose systems, etc., which are an integral part of them. Elastic links (ELs) are used in a wide range of modes of operation (different depths, currents, extreme modes when towing, multi-link technical systems, their mutual influence, etc.). The elastic link in the water, which is a spatial curvilinear object, retains its shape until any forces begin to affect it. In the operation of MTS, the efforts of tugboats, currents, underwater vehicles (UV) and emergency modes of operation could serve as an example of such impacts. In this case, one should take into consideration not only the technical conditions but also the mechanical loads on EL, which arise due to the influence of the wind, sea disturbance, sea currents, and maneuvers of boats.

An example of MTS is the class of marine tethered systems (MTdS), which in turn includes underwater self-propelled tethered systems and underwater towed systems (UTS) [1, 2]. When designing and operating MTdS with self-propelled underwater vehicles (SUV), the stationary problems in which objects occupy an unchanged position in space with constant flow characteristics and perceive external loads that are constant in time, are quasi-static. The state of the system is described by static equations, although the forces acting on it are hydrodynamic by nature.

A dynamic problem for the considered system is associated with the hydrodynamic one. Experience of designing and operating systems with SUV for shelf tasks shows that the equations of motion of the elements within such systems employ simplified expressions for the external forces, attached fluid masses, hydrodynamic forces of fluid flow. The process dynamics are taken into consideration by the inertial terms of the motion equations, the dynamic component of the speed of system elements in the liquid.

This makes it relevant to improve the theory and methods of designing MTdSs with EL in order to refine existing calculation procedures.

In many cases, changes in the EL characteristics need to be taken into consideration quickly, in real time. In this case, it is important to take into account the change in the shape of the EL cross-section by depth caused by a change in hydrostatic pressure:

1) when used in still liquid;

2) in the steady movement, when the attached masses of water could be neglected; 3) under dynamic modes of EL operation.

Currently, when designing MTS with EL, it is necessary to use new MMs to describe the dynamics of MTS with EL, which would make it possible to address the design issues with the maximum consideration of their operational properties.

The models of EL dynamics require additional research for specific schemes (new problems on EL mechanics) and specific perturbations. The existing calculations of EL mainly employ their MM for ideal round sections.

In this case, it is necessary to take into consideration a change in the EL natural characteristics during operation (a change in their characteristics over time over a long period of operation):

– during hydrostatic compression, the EL changes its shape from the ideal circle;

– there is a significant stretching of the EL lengthwise, including the internal wires in cables, as a result of which the hydrodynamic resistance of the EL changes;

– there is uneven aging and wear of shells and braiding of EL cables made of different materials;

– the curvy rigidity of the EL changes depending on the depth of the dive;

– the emergence of transverse EL vibration in the flow of water is possible, which significantly affects its reliability.

Today, it is a relevant task to promptly calculate efforts in the EL for the problems related to research design and use in the automatic control systems of MTdSs in real time.

2. Literature review and problem statement

Paper [3] considers the hydrodynamic model of the towed system. The towed cable model in the cited paper is based on the Ablow and Schechter method. The paper proposes a way to create a towed system. The basic equation is solved using the fourth-order Runge-Kutta numerical method for a stable cable. The results of the calculations are close to measured. A given model of the description of UTS EL does not make it possible to investigate the modes of maneuvering of the system, leading to the vibration of poorly flowed-around EL in the stream.

Article [4] considers a method of motion control for underwater towed system (UTS) with movable wings. The UTS is affected by the non-linearity and uncertainty of the position of the flexible towing cable, hydrodynamic forces, parametric fluctuations, and external disturbances. In the cited article, the cable is approximated by the method of concentrated masses, so that the number of segments of cables determines the order of the system. Direct consideration of the non-linear dynamics is one of the main features of the cited article. However, the effect of hydrodynamic compression on the cable during its spatial movement is not considered.

Work [5] developed a method of mathematical modeling of the EL dynamics based on the automatic control over the axial movement of its elements. The regulator of distances between the elements of the EL as a component of the mathematical model was synthesized. A method of modeling EL with a changeable length was proposed. The effectiveness of the developed method was shown in comparison with the method of concentrated masses and elastic links in the modeling of non-stretchable ELs, but the change in the EL hydrodynamic characteristics under the influence of hydrostatic pressure was not taken into consideration.

Study [6] examines the process of modeling and speed control of underwater wheeled tethered towed vehicles. The simulation was carried out in a two-coordinated system. The hydrodynamic UV model and the hydraulic mechanism of the EL model were combined, the dynamic characteristics of the original towed system were studied using computer simulations. A feature of the flexible towed system is the combination of the non-linearity of temporal load regulation, environmental restrictions. Real speed control is based on a developed and implemented theory using a high-speed sliding mode. However, the simulation of the dynamics was carried out in a two-dimensional coordinate system, which is currently insufficient to describe the dynamics of UTS at large movements.

Paper [7] examined the resistance of an underwater vehicle without EL, but the issues related to the description of the EL dynamics remained unresolved.

Article [8] considers a model of flexible segments, adopted for dynamic calculations. In this model, the cable is divided into a certain number of flexible segments and is described by the non-linear equations at moments of the uniform segment movement. In a given example, the dynamics modeling was also performed in a two-coordinated system, which is currently insufficient to describe the ~~dynamics~~ of the spatial motion of UTS.

Study [9] considers the application of a method of dynamic optimization of the trajectory of a free-floating cable in the water depth. The model was considered in a three-dimensional system of coordinates with the splitting of EL into the interrelated elements. The cable is modeled as a chain of rigid rods connected to each other by hinges with two degrees of freedom, which could describe the bend of the cable in two planes (three coordinates). The cable is considered to be very flexible, but not able to lengthen. A given model provides an opportunity to obtain the motion trajectory of a vessel and a cable, but without taking into consideration the change in the hydrodynamic characteristics of the cable, which is not enough to fully study the dynamics of EL.

Paper [10] considers the application of a dynamic optimization method for a drill column fixed at one end on the seabed, similar to the method proposed in [9]. A given model takes into consideration the loads from the stretching and rotation of the drill column. However, the specified model is unsuitable for use in cable lines calculations.

There are many tools to describe the MTS EL, mainly in the statement of a flat problem. However, the solution to the task of improving the design of MTS with EL is the need for comprehensive use of existing and developed methods, taking into consideration the full amount of information available about MTS with EL.

Currently, there is no single theory of the design of MTS with EL – surface, underwater (towed and tethered), stationary and drifting, which would take into consideration all the significant factors of the operation of EL and could reliably allow their design. The reason for this may be objective difficulties associated with significant financial and time costs, which makes the relevant research quite labor-consuming.

Analysis of project tasks in the construction of MTdS with EL shows that significant theoretical complexity and science-intensiveness are required by the calculations of MTS EL, the strength and reliability of their elements.

In this regard, there are tasks to build a MM for the operation of MTS under quasi-stationary modes of operation, a MM to describe the MTS dynamics, development of software for computer simulation of quasi-stationary and dynamic modes of MTS operation.

3. The aim and objectives of the study

The aim of this study is to improve the design of marine technical systems with elastic links based on a refined mathematical model of their dynamics, describing the phenomenon of the longitudinal rigidity and change in the shape of the EL during operation.

The construction of such models would make it possible to describe the functioning of almost all types of MTS with EL (surface, underwater, stationary, and drifting) operating under actual conditions. From the point of design, the use of such mathematical models would make it possible to abandon the organization and carrying out complex field experiments, some of which are almost impossible to perform under natural conditions. The construction of such models would serve as the basis for the development of computer research and design technologies of MTS EL and MTS with EL.

To accomplish the aim, the following tasks have been set:

– to build an MM to describe the MTS EL dynamics using an underwater towed system (UTS) as an example, which, in contrast to existing models, would make it possible to take into consideration the EL as part of MTdS, and the features of the EL design;

– to build an MM to describe the dynamics of MTS with EL, which would determine a change in the shape of the EL and the forces of its tension in the process of maneuvering the CV and UV taking into consideration sea disturbances and underwater currents; the relative position of the CV and UV in the process of maneuvering them;

– to improve the design methodology of MTS with EL, based on the use of the MM of MTS EL and MTS with EL.

4. Mathematical model to describe the dynamics of an elastic link in the marine technical system

The work of EL within MTS and UTS, specifically in the course of operation, is characterized by the following conditions:

1) the ropes are exposed to air temperature changes (from –40 °C °C to 50 °C) of sea water (from –2 °C to 30 °C);

2) the ropes are repeatedly curved on blocks with a diameter of D_b=200–500 mm and larger;

3) the blocks have the diameter of the groove D_g, reaching up to three or more diameters of the rope, therefore, the groove could be considered almost flat;

4) the ropes on the drums of towing winches are reeled in 15 to 20 layers;

5) the ropes are crushed on the drums of the winch because of multi-layered winding, which directly affects the integrity of the rope design;

6) when pulling the rope through the blocks, there are significant angles of deviation (up to 30–45°);

7) the ropes are operated under harsh conditions of corrosive fatigue as they are exposed to variable environments: air – sea water;

8) the ropes experience dynamic loads at sea waves and especially when hitting the ground and underwater obstacles;

9) the ropes are subjected to significant mechanical wear due to friction against the blocks and the seabed.

As marine practice shows, in the course of long-term operation of MTdS and UTS:

– the ropes receive a large residual lengthening (up to 15 to 20 m or longer per 1,000 m of length);

– the uneven laying of ropes on the drums of winches and pins leads to the snatching of individual wires and whole strands from the body of the rope;

– there is a squeezing of the organic core along the entire length of the rope;

– there is a destruction of the drum flanges;

– there is an intense wear of rope wires due to mechanical rubbing on blocks and rolls, in the pecking slips and on bollards. This leads to a break of the rope, especially when hooked by an underwater obstacle, and eventually to the loss of towed carriers and other expensive equipment;

– CR experiences the pulling of electrical wires, which causes changes in their resistance and, when water enters the break, to the short circuit and failure.

In this regard, mathematical models describing the dynamics of MTS with EL should take into consideration the nature of design tasks as fully as possible, taking into consideration such operational factors as the characteristics of sea waves, the structural features of the EL, new approaches to determining the reserves of strength of the EL and MTS elements. This would make it possible to comprehensively account for, from a single theoretical position of the EL hydrodynamics, all the operational characteristics of MTS with EL.

In general, the MTdS consists of a carrier vessel (CV), an underwater towed system (UV) and the EL that connects them (Fig. 1) [2]. The right $0xyz$ coordinate system is used. The $0x$ and $0y$ axes lie in the sea surface plane, and the $0z$ axis coincides with the direction of the vector of gravity q.

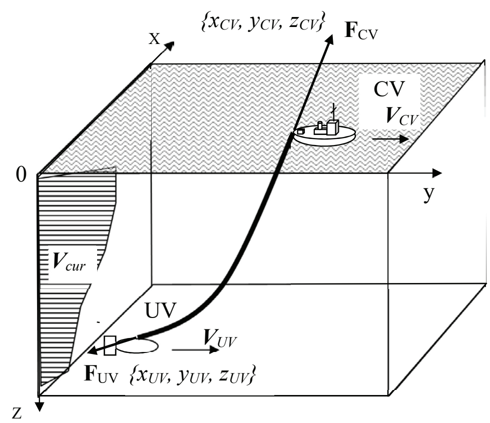

Fig. 1. General scheme of MTS using UTS as an example

The root end of the EL is fixed on CV and has the coordinates x_{CV}, y_{CV}, z_{CV}. The running end of the EL is fixed on UV and has the coordinates x_{UV}, y_{UV}, z_{UV}. The speed of the CV is set by the vector $\boldsymbol{V}_{CV}$, and the speed of the UV is set by the vector $\boldsymbol{V}_{UV}$. The speed of the sea current $\boldsymbol{V}_{cur}$ is stationary, it has an arbitrary diagram.

EL could be represented as a combination of individual small elements of length ds interconnected by elastic links (Fig. 2).

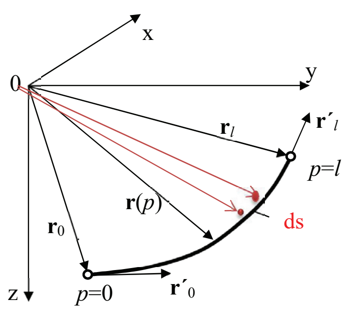

Fig. 2. Axial line of the EL element

The full length of EL is defined as the sum of the elementary lengths ds, the number of which is $p=0, ..., l$. The small element of the EL of length ds is affected by stretching forces, which are created as a result of the forces applied to the root $\mathbf{F}_{CV}$ and the running $\mathbf{F}_{UV}$ ends of the EL from the side of CV and UV.

In the process of movement of the CV and UV, under the influence of forces of the hydrodynamic nature and the thrust force of the carrier vessel, the spatial form of the EL and, as a result, the distance between the CV and UV, change. The gravity forces q of the EL are characterized by the linear density of the EL itself.

If the EL retains an unchanged d_0 diameter along the length, then in the mathematical model it is set by a numerical value, and if d_0 depends on the s coordinate along the length of the EL (for example, under the influence of hydrostatic compression), then it is set by a function.

The radius-vector of EL $\mathbf{r}(p)$ is related to the vector of coordinates $\mathbf{e}$ via dependence

$$\vec{\mathbf{r}}(p)=\mathbf{S}(p)\cdot\vec{\mathbf{e}}, \tag{1}$$

where $\vec{\mathbf{e}}$ is the vector of the generalized coordinates;

$$\vec{\mathbf{e}}=\left\{\vec{\mathbf{r}}_0^{0T}\ \vec{\mathbf{r}}_0^{1T}\ \vec{\mathbf{r}}_l^{0T}\ \vec{\mathbf{r}}_l^{1T}\right\}^T;$$

$$\vec{\mathbf{r}}_u^k=\partial\vec{\mathbf{r}}^k/\partial p^k\Big|_{p=u}=\begin{Bmatrix}x_u^k\\ y_u^k\\ z_u^k\end{Bmatrix};$$

– the radius vectors (k=0) and tangent vectors (k=1) to the EL axial line at its end points (p=0 and p=l) when u takes values 0 (p=0) or l (p=l); $\mathbf{S}(p)=\left\| s_1\mathbf{I}\ \ s_2\mathbf{I}\ \ s_3\mathbf{I}\ \ s_4\mathbf{I}\ \ 0\ \ 0\right\|$ is the matrix of EL shape functions; $\mathbf{I}$ is a unity matrix (3×3) [11].

As the functions of the shape of the EL, the Hermite functions are used, which make it possible to approximate the shape of the EL element for the magnitude and derivative of the EL radius-vector

$$x(p)=s_1(p)\cdot x_0+s_2(p)\cdot x_0'+s_3(p)\cdot x_l+s_4(p)\cdot x_l', \tag{2}$$

$$y(p)=s_1(p)\cdot y_0+s_2(p)\cdot y_0'+s_3(p)\cdot y_l+s_4(p)\cdot y_l', \tag{3}$$

$$z(p)=s_1(p)\cdot z_0+s_2(p)\cdot z_0'+s_3(p)\cdot z_l+s_4(p)\cdot z_l'. \tag{4}$$

The EL element motion equations could be obtained using the Lagrange equations of second kind

$$\frac{d}{dt}\frac{\partial T}{\partial\dot{\mathbf{e}}}-\frac{\partial T}{\partial\mathbf{e}}+\frac{\partial U}{\partial\mathbf{e}}=\frac{\delta W}{\delta\mathbf{e}}, \tag{5}$$

where T is the kinetic energy of the EL element, J;

$$T=\frac{1}{2}\int_0^l\rho_l\cdot\dot{\vec{\mathbf{r}}}^T\cdot\dot{\vec{\mathbf{r}}}\cdot\mathrm{d}p; \tag{6}$$

ρ_l is the linear density of the EL material, kg/m; U is the potential energy of the deformation of the EL element, J; δW is the virtual work of active forces, J;

$$\delta W=\int_0^l\delta\vec{\mathbf{r}}^T\cdot\rho_l\cdot\vec{\mathbf{g}}\cdot\mathrm{d}p+\int_0^l\delta\vec{\mathbf{r}}^T\cdot\vec{F}\cdot\mathrm{d}p; \tag{7}$$

$\mathbf{g}$ is the vector of acceleration of gravity, m/s^2; F is the vector of active (external) forces operating on the EL, N.

The use of the vector of generalized coordinates in the absolute system of coordinates does not require the recalculation of the coordinates of the EL nodes between it and the auxiliary coordinate systems. In this case, there is no need to take into consideration the attached fluid masses to the EL once determining the hydrodynamic resistance forces of the EL employs the resistance ratios defined empirically. Using the absolute coordinates method has significant advantages over the methods used earlier.

After substitution of the time-derived vector radius $\dot{\vec{r}}=S\cdot\dot{\vec{e}}$ (1) and its variation $\delta\vec{r}=S\cdot\delta\vec{e}$ in equation (5), the motion equations in the matrix form take the form

$$\mathbf{M}\cdot\ddot{\vec{\mathbf{e}}}+\vec{\mathbf{Q}}^e=\vec{\mathbf{Q}}^g+\vec{\mathbf{Q}}^F, \tag{8}$$

which includes the constant mass matrix (M) and the column of the generalized forces of gravity ($\boldsymbol{Q}^g$), as well as the vector of external active forces ($\boldsymbol{Q}^{\mathrm{F}}$) and reactions of elastic deformation of the ($\boldsymbol{Q}^{\mathrm{e}}$). Equation (8) is written without taking into consideration the rigidity of bending and torsion, which would be taken into consideration in the further improvement of the mathematical model of EL as part of MTS.

The matrix of masses is determined from formula

$$\mathbf{M}=\frac{\partial^2 T}{\partial\dot{\vec{\mathbf{e}}}\partial\dot{\vec{\mathbf{e}}}^T}=\rho_l\int_0^l S^T S\cdot \mathrm{d}p=\text{const}, \tag{9}$$

and the column of the generalized forces of gravity

$$\vec{\mathbf{Q}}^g=\frac{\partial W^g}{\partial\vec{\mathbf{e}}}=\int_0^l \rho_l\cdot S^T\cdot\vec{\mathbf{g}}\cdot \mathrm{d}p=\text{const}. \tag{10}$$

The potential energy of the internal forces of the elastic deformations of the EL element is determined from formula

$$U=\frac{1}{2}\int_0^l E\cdot A\cdot\varepsilon^2\cdot \mathrm{d}p, \tag{11}$$

where E is the Jung module of the EL material, Pa; A is the area of the cross-section of the EL, m^2; ε is the longitudinal deformation of the EL axial line with longitudinal rigidity E·A

$$\varepsilon=\sqrt{\vec{\mathbf{r}}'^T\vec{\mathbf{r}}'}-1\approx\frac{1}{2}\left(r'^T r'-1\right)=$$
$$=\frac{1}{2}\left[\left(\frac{\partial x}{\partial p}\right)^2+\left(\frac{\partial y}{\partial p}\right)^2+\left(\frac{\partial z}{\partial p}\right)^2-1\right]. \tag{12}$$

The longitudinal forces could be defined from formula

$$Q_i^\varepsilon=\frac{\partial\Pi^\varepsilon}{\partial e_i}=EA\int_0^l\varepsilon\frac{\partial\varepsilon}{\partial e_i}\mathrm{d}p=$$
$$=\sum_{k=1}^{4}E\cdot A\cdot\overline{\varepsilon}\cdot\overline{\mathbf{S}}_{ik}^{11}\cdot e_k=\sum_{k=1}^{4}K_{ik}^\varepsilon\cdot e_k, \tag{13}$$

where K_{ik}^ε is the matrix of the EL longitudinal rigidity;

The elements of the mass matrix could be determined from formula (9) by replacing a variable of integration

$$M_{ij}=\rho_l\int_0^l s_i^T s_j\cdot \mathrm{d}p=$$
$$=\rho_l\cdot l\int_0^l s_i^T s_j\cdot \mathrm{d}\frac{p}{l}=\rho_l\cdot l\int_0^1 s_i^T s_j\cdot \mathrm{d}\xi,\quad i,j\in[1;4]. \tag{14}$$

Similarly, the remaining elements of the M matrix are calculated.

To determine K_{ik}^ε, the matrix of the longitudinal rigidity of the EL elements, it is necessary to calculate the components of its elements. The elements of the matrix $\overline{S}_{ik}^{11}$ are symmetrical relative to the main diagonal.

$$Q_i^\varepsilon=\sum_{k=1}^{4}E\cdot A\cdot\overline{\varepsilon}\cdot\overline{\mathbf{S}}_{ik}^{11}\cdot e_k=\sum_{k=1}^{4}K_{ik}^\varepsilon\cdot e_k=K^\varepsilon\cdot\vec{\mathbf{e}}\ ,$$
$$K^\varepsilon=K_0^\varepsilon\cdot\overline{\varepsilon}. \tag{15}$$

By substituting the functions of Hermit (2) to (5) into the formula for the matrix element M_{22}, we would obtain

$$\vec{\mathbf{Q}}_1^g=\rho_l\cdot\vec{\mathbf{g}}\cdot\int_0^l s_1\cdot \mathrm{d}p, \tag{16}$$

$$\vec{\mathbf{Q}}_2^g=\rho_l\cdot\vec{\mathbf{g}}\cdot\int_0^l s_2\cdot \mathrm{d}p, \tag{17}$$

$$\vec{\mathbf{Q}}_3^g=\rho_l\cdot\vec{\mathbf{g}}\cdot\int_0^l s_3\cdot \mathrm{d}p, \tag{18}$$

$$\vec{\mathbf{Q}}_4^g=\rho_l\cdot\vec{\mathbf{g}}\cdot\int_0^l s_4\cdot \mathrm{d}p, \tag{19}$$

The generalized mathematical model of the dynamics of the EL element is defined from a system of equations:

$$\ddot{x}_1=f_{1X}\left(x_1,y_1,z_1,\frac{\partial x_1}{\partial t},\frac{\partial y_1}{\partial t},\frac{\partial z_1}{\partial t}\right), \tag{20}$$

$$\ddot{y}_1=f_{1Y}\left(x_1,y_1,z_1,\frac{\partial x_1}{\partial t},\frac{\partial y_1}{\partial t},\frac{\partial z_1}{\partial t}\right), \tag{21}$$

$$\ddot{z}_1=f_{1Z}\left(x_1,y_1,z_1,\frac{\partial x_1}{\partial t},\frac{\partial y_1}{\partial t},\frac{\partial z_1}{\partial t}\right), \tag{22}$$

$$\ddot{x}_2=f_{1X}\left(x_1,y_1,z_1,\frac{\partial x_1}{\partial t},\frac{\partial y_1}{\partial t},\frac{\partial z_1}{\partial t}\right)+$$
$$+f_{2X}\left(x_1,y_1,z_1,x_2,y_2,z_2,\frac{\partial x_2}{\partial t},\frac{\partial y_2}{\partial t},\frac{\partial z_2}{\partial t}\right), \tag{23}$$

$$\ddot{y}_2=f_{1Y}\left(x_1,y_1,z_1,\frac{\partial x_1}{\partial t},\frac{\partial y_1}{\partial t},\frac{\partial z_1}{\partial t}\right)+$$
$$+f_{2Y}\left(x_1,y_1,z_1,x_2,y_2,z_2,\frac{\partial x_2}{\partial t},\frac{\partial y_2}{\partial t},\frac{\partial z_2}{\partial t}\right), \tag{24}$$

$$\ddot{z}_2=f_{1Z}\left(x_1,y_1,z_1,\frac{\partial x_1}{\partial t},\frac{\partial y_1}{\partial t},\frac{\partial z_1}{\partial t}\right)+$$
$$+f_{2Z}\left(x_1,y_1,z_1,x_2,y_2,z_2,\frac{\partial x_2}{\partial t},\frac{\partial y_2}{\partial t},\frac{\partial z_2}{\partial t}\right), \tag{25}$$

The system of equations (20) to (25) depends on the parameters that characterize the properties of EL and the external influences on it. The right-hand sides of dependences (20) to (25) are the functions from the desired variables and their derivatives for time.

5. Mathematical model of the dynamics of a marine tethered system with an elastic link

The following parameters are needed to perform mathematical modeling of the MTdS dynamics:

1) the amplitude, length, and speed of harmonic waves on the surface of the sea;

2) the coordinates of the velocity vector of CV depending on time;

3) the coordinates of the sea current velocity vector;

4) the EL maximum length;

5) the coefficient (modulus) of elasticity of the EL for stretching at each of its points;

6) the permissible forces (or stresses) of the EL stretching at each of its points;

7) the coordinates and masses of loads fastened to the EL;

8) the coordinates and buoyancy of floats and buoys fixed to the EL;

9) the hydrodynamic drag coefficients of the EL;

10) the linear density of the EL;

11) the mass, buoyancy, hydrodynamic resistance ratios and the UV velocity vector in relation to stationary water depending on time;

12) the coordinates of obstacles in water;

13) the size and mass of obstacles;

14) the coordinates of the speed of the obstacles;

15) the depth of the sea.

The characteristics of the MTdS, which are determined by a mathematical model:

1) the forces operating at the root and running end of the EL;

2) the EL length;

3) the relative stretching of the EL;

4) the UV coordinates in relation to CV;

5) the shape of the EL, depending on the time;

6) the distribution of forces and stresses due to stretching the EL along its length;

7) to determine the quasi-static position of the MTdS and the shape of the EL, if it is possible under the specified parameters of the MTdS and EL, by using the developed software under a "solution-setting" mode when, in the process of solving the problem, the desired functions are no longer dependent on time.

The mathematical model of the dynamics of a marine tethered system (MTdS) includes not only the EL equations but also the equations of the dynamics of CV and the towed UV, whose movement determines the boundary conditions in the EL nodes with numbers i=0 and i=N. We assume that an EL node with number i=0 is fixed on the UV and the EL node with number i=N is fixed on UV.

The dynamics of the CV and the towed UV differ only in parameters, so the equations of their dynamics take the same form. We believe that CV and UV are the absolutely rigid rods, whose position in space is determined by the coordinates of their center of mass (x_i, y_i, z_i), the course angle ($\varphi_{k\,i}$), the differential angle ($\varphi_{d\,i}$) and the roll angle ($\varphi_{kr\,i}$). The length of the rods (L_i) is equal to the length of the CV or UV, and the position of the center of mass is determined by the distance to it from the stern (L_{ki}). The position and orientation of these variables and parameters are shown in Fig. 3.

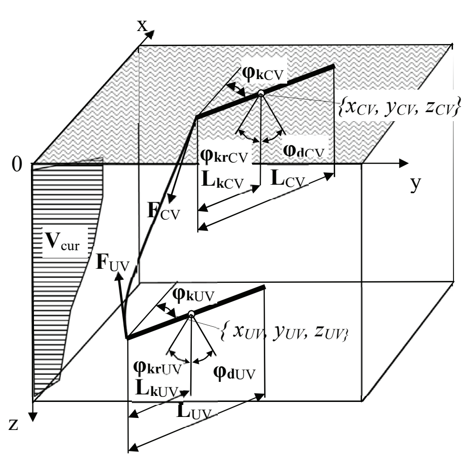

Fig. 3. Scheme of the CV and UV geometrical parameters

The system of equations of the dynamics of the CV or UV consists of six equations [13].

$$M_{ci}\cdot\ddot{x}_i=F_{cxi}, \tag{28}$$

$$M_{ci}\cdot\ddot{y}_i=F_{cyi}, \tag{29}$$

$$M_{ci}\cdot\ddot{z}_i=F_{czi}, \tag{30}$$

$$J_{ki}\cdot\ddot{\phi}_{ki}=M_{zi}, \tag{31}$$

$$J_{d\,i}\cdot\ddot{\phi}_{d\,i}=M_{di}, \tag{32}$$

$$J_{kr\,i}\cdot\ddot{\phi}_{kr\,i}=M_{kri}, \tag{33}$$

where M_{ci} is the mass of the vessel; J_{ki}, J_{di}, J_{kri} are the moments of inertia of the vessel with respect to the axes of turning, the differential, and the roll; F_{cxi}, F_{cyi}, F_{czi}, M_{zi}, M_{di}, $M_{kr\,i}$ are the external forces and torques acting on the vessel.

The system of equations of two related elements of the EL could be solved using numerical methods. Taking into consideration the derived matrices, the equation takes the form [13]

$$\begin{gathered}\mathbf{M}_1\cdot\ddot{\vec{\mathbf{e}}}_{i-1}+\mathbf{M}_2\cdot\ddot{\vec{\mathbf{e}}}_i+\mathbf{M}_3\cdot\ddot{\vec{\mathbf{e}}}_{i+1}+\\+\mathbf{K}_1\cdot\vec{\mathbf{e}}_{i-1}+\mathbf{K}_2\cdot\vec{\mathbf{e}}_i+\mathbf{K}_3\cdot\vec{\mathbf{e}}_{i+1}=\vec{\mathbf{Q}}_i,\end{gathered} \tag{34}$$

The second-order time derivative of the vector of generalized coordinates must be replaced with its discrete analog.

$$\ddot{\vec{\mathbf{e}}}\approx\frac{\dfrac{\vec{\mathbf{e}}^{n+1}-\vec{\mathbf{e}}^{n}}{\Delta t_{n+1}}-\dfrac{\vec{\mathbf{e}}^{n}-\vec{\mathbf{e}}^{n-1}}{\Delta t_{n}}}{\dfrac{1}{2}\left(\Delta t_n+\Delta t_{n+1}\right)}, \tag{35}$$

where n is the number of the calculated step over time t;

$$\Delta t_n=t_n-t_{n-1}; \tag{36}$$

$$\Delta t_{n+1}=t_{n+1}-t_n. \tag{37}$$

As a result of transform (35), we would obtain

$$\ddot{\vec{\mathbf{e}}}\approx\frac{2}{\left(\Delta t_n+\Delta t_{n+1}\right)}\left(\frac{\vec{\mathbf{e}}^{n+1}-\vec{\mathbf{e}}^{n}}{\Delta t_{n+1}}-\frac{\vec{\mathbf{e}}^{n}-\vec{\mathbf{e}}^{n-1}}{\Delta t_{n}}\right), \tag{38}$$

By introducing equation coefficients (38)

$$C_{t0}=\frac{2}{(\Delta t_n+\Delta t_{n+1})\cdot\Delta t_n}, \tag{39}$$

$$C_{t1}=\frac{2}{\Delta t_{n+1}\cdot\Delta t_n}, \tag{40}$$

$$C_{t2}=\frac{2}{(\Delta t_n+\Delta t_{n+1})\cdot\Delta t_{n+1}}, \tag{41}$$

expression (38) transforms to the form

$$\ddot{\vec{e}}\approx C_{t2}\vec{e}^{n+1}-C_{t1}\vec{e}^{n}+C_{t0}\vec{e}^{n-1}. \tag{42}$$

We transform equation (34) considering (42)

$$\begin{gathered}\mathbf{M}_1\cdot\left(C_{t2}\vec{\mathbf{e}}^{n+1}-C_{t1}\vec{\mathbf{e}}^{n}+C_{t0}\vec{\mathbf{e}}^{n-1}\right)_{i-1}+\\+\mathbf{M}_2\cdot\left(C_{t2}\vec{\mathbf{e}}^{n+1}-C_{t1}\vec{\mathbf{e}}^{n}+C_{t0}\vec{\mathbf{e}}^{n-1}\right)_{i}+\\+\mathbf{M}_3\cdot\left(C_{t2}\vec{\mathbf{e}}^{n+1}-C_{t1}\vec{\mathbf{e}}^{n}+C_{t0}\vec{\mathbf{e}}^{n-1}\right)_{i+1}+\\+\mathbf{K}_1\cdot\vec{\mathbf{e}}^{\mathbf{n}}_{i-1}+\mathbf{K}_2\cdot\vec{\mathbf{e}}^{n}_{i}+\mathbf{K}_3\cdot\vec{\mathbf{e}}^{n}_{i+1}=\vec{\mathbf{Q}}^{n}_{i}.\end{gathered} \tag{43}$$

The matrix algebraic equation (43) is recorded in an implicit form using three temporal layers. We use a two-step iterative method to solve it. In the first step, let us assume that the M_1 and M_3 matrices are zero, then equation (43) takes an explicit form

$$\mathbf{M}_2 \cdot \left(C_{t2}\tilde{\vec{\mathbf{e}}}^{n+1} - C_{t1}\vec{\mathbf{e}}^{n} + C_{t0}\vec{\mathbf{e}}^{n-1}\right)_i + \\ +\mathbf{K}_1 \cdot \vec{\mathbf{e}}_{i-1}^{n} + \mathbf{K}_2 \cdot \vec{\mathbf{e}}_{i}^{n} + \mathbf{K}_3 \cdot \vec{\mathbf{e}}_{i+1}^{n} = \vec{\mathbf{Q}}_i^n, \quad (44)$$

in which the "tilde" sign marks the approximated value of the vector of generalized coordinates on the n+1 layer by time. The solution to equation (44) could be obtained from the inverse matrix method

$$\mathbf{M}_2^{-1} \cdot \mathbf{M}_2 \cdot \left(C_{t2}\tilde{\vec{\mathbf{e}}}^{n+1} - C_{t1}\vec{\mathbf{e}}^{n} + C_{t0}\vec{\mathbf{e}}^{n-1}\right)_i + \\ +\mathbf{M}_2^{-1} \cdot \mathbf{K} \cdot \vec{\mathbf{e}}_{i-1}^{n} + \mathbf{M}_2^{-1} \cdot \mathbf{K}_2 \cdot \vec{\mathbf{e}}_{i}^{n} + \mathbf{M}_2^{-1} \cdot \mathbf{K}_3 \cdot \vec{\mathbf{e}}_{i+1}^{n} = \mathbf{M}_2^{-1} \cdot \vec{\mathbf{Q}}_i^n. \quad (45)$$

Adjustment (refinement and regularization) of the generalized coordinate vector is performed using it in equation (43), which is transformed according to the method of Abarbanel and Zvas [14] to the form

$$\mathbf{M}_1 \cdot \left(C_{t2}\tilde{\vec{\mathbf{e}}}^{n+1} - C_{t1}\vec{\mathbf{e}}^{n} + C_{t0}\vec{\mathbf{e}}^{n-1}\right)_{i-1} + \\ +\mathbf{M}_2 \cdot \left(C_{t2}\vec{\mathbf{e}}^{n+1} - C_{t1}\vec{\mathbf{e}}^{n} + C_{t0}\vec{\mathbf{e}}^{n-1}\right)_{i} + \\ +\mathbf{M}_3 \cdot \left(C_{t2}\vec{\mathbf{e}}^{n+1} - C_{t1}\vec{\mathbf{e}}^{n} + C_{t0}\vec{\mathbf{e}}^{n-1}\right)_{i+1} + \\ +\mathbf{K}_1 \cdot \tilde{\vec{\mathbf{e}}}_{i-1}^{n+1} + \mathbf{K}_2 \cdot \tilde{\vec{\mathbf{e}}}_{i}^{n+1} + \mathbf{K}_3 \cdot \tilde{\vec{\mathbf{e}}}_{i+1}^{n+1} = \vec{\mathbf{Q}}_i^n. \quad (46)$$

Equation (46) is also solved by the inverse matrix method

$$\mathbf{M}_2^{-1} \cdot \mathbf{M}_1 \cdot \left(C_{t2}\tilde{\vec{\mathbf{e}}}^{n+1} - C_{t1}\vec{\mathbf{e}}^{n} + C_{t0}\vec{\mathbf{e}}^{n-1}\right)_{i-1} + \\ +\mathbf{M}_2^{-1} \cdot \mathbf{M}_2 \cdot \left(C_{t2}\vec{\mathbf{e}}^{n+1} - C_{t1}\vec{\mathbf{e}}^{n} + C_{t0}\vec{\mathbf{e}}^{n-1}\right)_{i} + \\ +\mathbf{M}_2^{-1} \cdot \mathbf{M}_3 \cdot \left(C_{t2}\tilde{\vec{\mathbf{e}}}^{n+1} - C_{t1}\vec{\mathbf{e}}^{n} + C_{t0}\vec{\mathbf{e}}^{n-1}\right)_{i+1} + \\ +\mathbf{M}_2^{-1} \cdot \mathbf{K}_1 \cdot \tilde{\vec{\mathbf{e}}}_{i-1}^{n+1} + \mathbf{M}_2^{-1} \cdot \mathbf{K}_2 \cdot \tilde{\vec{\mathbf{e}}}_{i}^{n+1} + \mathbf{M}_2^{-1} \cdot \mathbf{K}_3 \cdot \tilde{\vec{\mathbf{e}}}_{i+1}^{n+1} = \mathbf{M}_2^{-1} \cdot \vec{\mathbf{Q}}_i^n. \quad (47)$$

Consistent application of formulae (45) and (47) makes it possible to determine the vector of generalized coordinates at all nodes of the EL on the n+1 layer over time.

The vectors of the generalized coordinates at the EL boundary nodes ($i=0$ and $i=N$) on the n+1 layer of time could be determined from equations (39) to (44). The generalized coordinates at the EL boundary nodes (i=1 and $i=N-1$) in the n+1 layer over time are determined from formulae (45) and (47).

The Δt value must be set by using the principle of cause-and-effect relation between the wave processes of EL deformation. This principle is taken into consideration if the Courant-Friedrichs-Levy condition is met, which determines the stability of the iterative computational process by formulae (45) and (47). According to the condition by Courant-Friedrichs-Levy [13]

$$\Delta t \le C_k \cdot l / C_T, \quad (48)$$

where C_k is the computational stability reserve ratio ($C_k \le 1$); C_T is the speed of longitudinal waves of EL stretching, m/s, determined from formula

$$C_T = \sqrt{\frac{T}{\rho_l}}. \quad (49)$$

The resulting formulae (45) and (47) to solve the equation system (34) make it possible to construct an algorithm for modeling the EL dynamics under the influence of external influences on it.

The algorithm consists of the following steps:

1) the introduction of the main parameters for EL, CV, UV;

2) the introduction of the main functions that determine the sea waves, the kinematics of obstacles, the speed of the sea current and the external forces operating on the EL;

3) setting the initial and boundary conditions for EL, CV and UV;

4) sampling of the EL and the calculation of the desired functions at the nodes of the grid, in accordance with the specified initial and boundary conditions;

5) determining a step in the time change Δt in line with condition (48) and the moment of time on the n+1 layer $t^{k+1}=t^k+\Delta t$;

6) checking the conditions to end the calculation $t^{n+1} \le t_m$. If the condition is met, then we move on to step 7 of a given algorithm to calculate the desired functions on the n+1 layer of time, otherwise we consider the calculation process complete and move on to step 12;

7) calculating the change in the coordinates of the EL nodes at the predictor stage;

8) adjusting the change in the coordinates of the EL nodes at the regularization stage from formula (49);

9) calculating the MTdS characteristics on the n+1 time layer from formulae (initial and boundary conditions, simulation results);

10) recording the results of the calculation of MTdS characteristics on the n+1 time layer to the assigned files;

11) transition to step 5;

12) end of calculations.

The flow chart of the constructed algorithm is shown in Fig. 4 [13].

The resulting algorithm for modeling the EL dynamics makes it possible to proceed in the future to the development of software that describes the MTdS dynamics.

The MM of the EL element dynamics makes it possible to take into consideration the large movements of the EL as part of MTdS and take the following into account:

1) the movement of the carrier vessel (CV), which is determined by the following factors: the sea waves; the kinematic characteristics of the CV movement; the sea current speed;

2) the features of the EL design, which affect the functional characteristics of the MTdS, are determined by the length and change in the length of the EL in the course of the CV movement, the elastic and strength characteristics of the EL, the positive or negative buoyancy of the EL, as well as loads, floats and buoys related to the EL. It should also be taken into consideration: the forces of hydrodynamic resistance of the EL in the course of its movement in the water, as well as the forces operating on the root and running end of the EL;

3) the UV movement, which is determined by the mass and buoyancy of UV, the relative location of the UV with respect to CV, and the kinetic characteristics of its move-

ment, by the forces of hydrodynamic resistance of the UV in the course of its movement in the water;

4) the effect of obstacles along the path of the UV and EL, which is determined by the following factors: the location of obstacles in the water; the size of obstacles; the kinetic characteristics of the movement of obstacles.

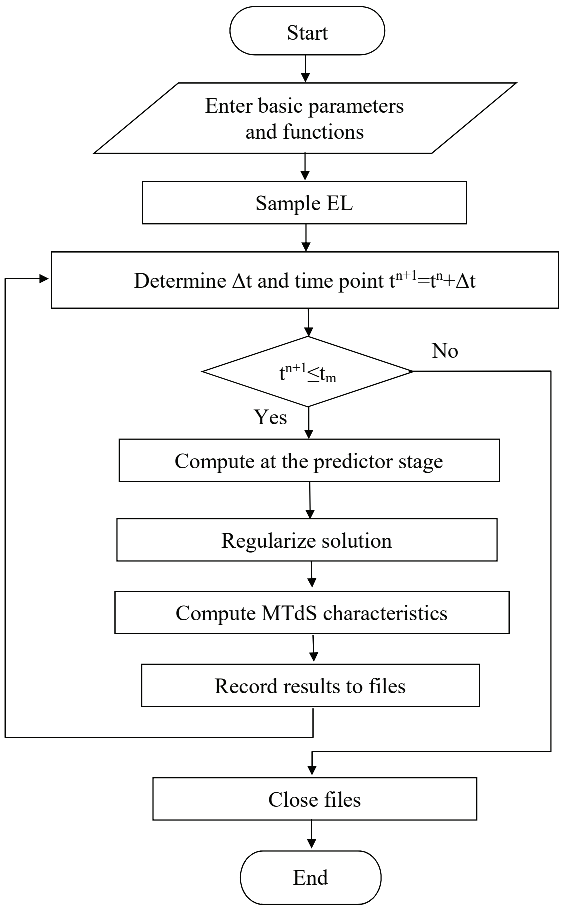

Fig. 4. Flow chart of the MTdS dynamics simulation algorithm

The MM of two related elements of the MTdS EL makes it possible to construct an algorithm for calculating the EL dynamics at its large movements and makes it possible to solve the following problems, which were not taken into consideration in the existing MMs:

1) to determine a change in the shape of the EL and the forces of its tension in the process of maneuvering the CV and UV taking into consideration the sea waves and underwater currents. It is also possible to take into consideration the wind loads on the CV, the depth of the sea and its changes in the assigned water area, the mass and elastic properties of the EL;

2) to determine the relative position of the CV and UV in the process of their maneuvering;

3) to identify the resonant modes of EL stretching and maximum loads on EL to assess its strength during the maneuvering of the CV and UV;

4) to determine the efforts of stretching the EL;

5) to define a system of equations describing the dynamics of the EL element as a result of external forces and stretching reactions.

The developed mathematical model is aimed, first of all, to solve the non-stationary problems on the dynamics of tethered systems. Therefore, the stationary towing modes are determined as a result of the completion of mathematical modeling of the non-stationary (transitional) system process from the initial position (EL is suspended on the CV vertically in the direction of the Z axis) to its steady state. To simplify the modeling of the steady state of the tethered system, we used the principle of "reversal of exposure" when the elements of the system were considered stationary while they are flown over by water at the speed that is equal to the speed of CV movement and opposite in direction.

We tested the mathematical model of the tethered system and the calculation algorithm using the stationary towing mode of three types of tethered systems as examples.

Tethered system 1 [15]. The tow vessel (TV) is connected to a towed carrier (TC) by a cable-tug with a diameter of 30 mm, a length of 400 m, the rigidity of the cable-tug is $6.75 \cdot 10^7$ N, the linear weight is 1.43 kg/m. The TC hosts a stabilizing section of an additional rope of zero buoyancy, which generates effort (F_{TC}) 1 kN at a towing speed of 3 m/s. The TC is a circular cylinder with a diameter of 0.8 m, 2 m long, weighing 1,030.44 kg, which ensures its zero buoyancy. The TC resistance ratios $C_x=C_z=0.77$; cable-tug $C_n=1.35$, $C_\tau=0.04$ [15].

The cable-tug line was calculated in the above work [15], without taking into consideration the hydrodynamic resistance of the cable-tug. Their lines were also calculated based on the developed model without taking into consideration the hydrodynamic resistance of the cable-tug and taking it into account. The distribution of the tension force along the cable tug in the direction of the S coordinate was also derived.

The study shows that the results of modeling are consistent with the calculations given in [15] without taking into consideration the hydrodynamic resistance of the cable-tug. Accounting for the resistance of the cable-tug in mathematical modeling leads to a decrease in the depth of its immersion, which corresponds to physical laws.

Tethered system 2 [16]. In a given example, the TC is used with its own engine, the thrust of which accepted values of 50, 100, 150, and 250 H in the direction of TV movement at a speed of 0.8 m/s at a deepening angle of 30° in relation to the X axis. The diameter of the cable is 12 mm, and its length is 100 m [16].

The cable-tether line in the above-mentioned work was calculated, as well as the lines calculated according to the developed model. The distribution of a tension force along the cable-tether in the direction of the S coordinate was derived. The modeling results are consistent with the calculations given in work [16] both for the cable-tether profile and the magnitude of the tether tension force near TC (S=100 m).

Tethered system 3 [17]. A given example uses a TC with its own engine whose thrust (F_x) is in the direction of the TV movement at a speed of 0.5 m/s and whose deepening force is (F_z). The diameter of the cable-tether is 12 mm, and its length is 100 m. For calculation, data from work [17] were used.

We compared the cable-tether line calculated in work [17] and the lines calculated according to the developed model. We also derived the distribution of a tension force along the cable-tether in the direction of the S coordinate as a result of mathematical modeling and the module of the tension force of the cable-tether near TC (S=100 m) from work [17]. The results of modeling are consistent with the calculations given in [17] both for the cable-tether profile and the amount of the tether tension force near TC (S=100 m).

6. Refining the design methodology of a marine technical system with an elastic link

Improving the designing of MTS with EL is based on improving the existing methods of their design by using the constructed MMs that describe the dynamics of MTS EL and MTS with EL using UTS as an example.

It is proposed to use a comprehensive approach in the design of MTS with EL taking into consideration operational factors, which implies:

1) taking into consideration the minimum radius of the bend and the number of layers of the EL winding on the winch drum, which affects the size of the drum and the entire winch depending on the required length of the EL;

2) taking into consideration the dynamic impact loads on the EL, which requires modeling the operating modes of the MTdS EL to determine the strength of the EL and the nodes of fixing the running and root end of the EL tug line;

3) taking into consideration the forces of hydrodynamic nature, which emerge at the EL transverse vibration in the flow of water and the design features of the EL, allowing its elimination;

4) ensuring the assigned buoyancy of the EL;

5) ensuring the predefined radius of the bend of the EL on the drum.

The most important design element of any TS at present is the need to develop a concept. The concept of improving the design of MTS with EL is to construct a comprehensive model of improving the design of MTS with EL (mathematical and computer), making it possible to take into consideration the internal and external factors that influence the MTS EL operation.

The main features of the concept of improving the design of MTS with EL are:

– applying classical design theory techniques, taking into consideration the features (specificity) of MTS design depending on the type of MTdS;

– the construction of an MM for the dynamics of MTdS and MTdS with EL so that it reflects their characteristic physical processes during the construction of a specific MTdS with EL;

– the development of software to describe the dynamics of MTdS EL and MTdS with EL;

– the optimization of the elements and characteristics of an object as a system at all levels of its hierarchy and at all stages of design as reliable information accumulates (an example of this hierarchical level is to supplement the object with the main elements of the system (equipment);

– synthesis of classical methods and research design using the constructed MM, refining the results of optimization of the elements and characteristics of an object, obtained based on developed mathematical and computer models of MTS;

– organizing the interaction between the designer, customer, developers of the basic samples of the equipment, and the feedback of the stage results to the initial technical tasks on MTS with EL and assembly equipment. Argued and agreed assignment adjustments, that is, the use of the sequential approach method;

– improving the stability of optimal solutions by exploring the technical capabilities of MTS with EL in emergency and ensuring the overall safety of MTS with EL.

A brief list of the concept's most important objectives:

– take into consideration the specific operating conditions of the EL and refine the problem of external forces based on experience and model experiment, scientific research;

– analyze the sea wave parameters and various aspects of operation;

– explore the technological processes of the use of EL under marine conditions;

– assess emergencies and take steps to prevent them;

– reasonably select the strength criteria and their assessment;

– assess the reliability of the EL and MTS elements;

– improve the design methodology, taking into consideration the purpose of the object (EL), the results of previous scientific research, existing advances in science and technology and design experience.

The concept of improving the design of MTS with EL could be constructed in the form of an algorithm, making it possible to take into consideration the internal and external environmental factors that influence the operation of the MTS EL, their parameters.

7. Discussion of results of studying the dynamics of the elastic link in a marine technical system and the marine technical system with an elastic link

The advantage of this study compared to similar ones is that the MM of two related elements of the MTS EL allows one to:

1) determine a change in the EL shape and the forces of its tension, mass and elastic properties of the EL during operation (maneuvering of the CV and UV, etc.). This takes into consideration the sea waves, underwater currents, wind loads on the CV, the depths of the sea and its changes in the assigned water area;

2) identify resonance modes of EL stretching and maximum loads on EL to assess its strength during the maneuvering of the CV and UV;

3) determine the efforts of stretching the EL.

The resulting algorithm for modeling the dynamics of the EL makes it possible to move in the future to the development of software that describes the MTdS dynamics.

The mathematical model of the dynamics of the marine tethered system includes not only the EL equations but also the equations of the dynamics of CV and the towed UV, the movement of which determines the boundary conditions at the EL nodes with numbers i=0 and i=N.

The drawback of this study is that a given model does not take into consideration the stretching of the EL, its bending and torsion. Alternative solutions may be to expand this model and introduce new parameters to the software describing the dynamics of the MTS EL, for example, taking into account the stretching of the EL, its bend and torsion. In the future, the MM for a MTS with

EL would be expanded by introducing new equations that account for the stretching of the EL, bending and torsion.

The limitation of this study is that the Δt value should be set using the principle of cause-and-effect relation between the wave processes of EL deformation. This principle is taken into consideration if the Courant-Friedrichs-Levy condition is met, which determines the stability of the iterative computational process in line with formulae (45) and (47).

The number of EL elements determines the error of modeling and the time it takes to complete the calculations. The more elements the EL has, the lower the error of calculation, but the longer its duration. The software that we implemented based on the constructed MM could employ no more than 100 elements of the EL. Approximately, the length of one element should not exceed 20 meters. The final number of EL elements could be selected by comparing the solutions to the problem derived for a different number of EL elements.

In addition, the expansion and improvement of a given MM would require adjustments to the algorithm for modeling the dynamics of MTdS.

The mathematical model for MTS with EL makes it possible to solve the following tasks, which were disregarded in the existing MMs:

1) to determine a change in the shape of the EL and the forces of its tension in the process of maneuvering the CV and UV taking into consideration the sea wave and underwater currents. It is also possible to take into consideration wind loads on the CV, the depth of the sea and its changes in the assigned water area, the mass and elastic properties of the EL;

2) to determine the relative position of the CV and UV during their operation;

3) to identify the resonance modes of EL stretching and maximum loads on EL to assess its strength during the maneuvering of the CV and UV.

Owing to the features of the proposed solutions using the developed MMs, it is possible to conduct a model experiment without resorting to a physical experiment. At the same time, it is possible to model the different operational modes of the designed MTS with EL, up to extreme ones and emergency.

To calculate the MTdS for strength, in this case, one could use data on loads derived from the model experiment.

We tested the mathematical model of the tethered system and the calculation algorithm based on the examples of a stationary towing mode of three types of tethered systems. The results of our modeling are consistent with the calculations given in [12, 13, 15, 16].

The proposed procedure for designing MTdS with EL, based on the MM that describes the dynamics of the MTdS EL (and MTdS with EL), makes it possible to investigate the different modes of operation of almost all classes of MTdS, to obtain the values of forces operating on the EL and the working bodies of the MTdS. At the same time, it is possible to assess the real destructive efforts in the EL and MTdS and further develop recommendations for predicting possible loads in the design of their elements, to improve the existing methods of calculating and designing MTdSs with EL and to bring them to the level of an engineering application.

8. Conclusions

1. Analysis of existing models of the dynamics of the MTdS EL has revealed that most models of the EL element in the MTdS consider the dynamics of the EL at relatively small movements and bends, which shows the relevance of the development of a given mathematical model of the dynamics of the EL element to explore the real operating conditions (large movements of the EL within MTdS, etc.). We have derived equations for the dynamics of the MTdS EL element, which make it possible to describe significant magnitudes of its movements.

2. The developed mathematical model of the EL dynamics and the algorithm of the mathematical model would make it possible for a designer of MTdS, which includes the EL, to better and promptly design almost all classes of MTdS. It would also solve the problems on describing the dynamics of the MTdS EL under extreme and emergency conditions.

3. We have refined the methodology for designing the MTS with EL, based on the concept of improving the design of MTS with EL. The use of the MM for the MTS EL and MTS with EL makes it possible to bring the design of MTS elements to engineering methods of calculation. Using a comprehensive approach in the design of MTS with EL implies the so-called "single button" approach, whereby when it is pressed the entire MTS is designed. At the same time, it is possible to design not only the individual elements of MTS – EL, TVM, CV, etc., but the entire MTS as a single system in general, taking into consideration the influence of MTS elements on each other. Using such a comprehensive system to improve the design of MTS with EL would reduce the time spent in the early stages of MTS design.

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