Наявні експериментальні дослідження чутливості характеристик демпфірування до наявності тріщини в елементах конструкції є суперечливими. Деякі дослідження декларують високу чутливість демпфірування, проте інші роблять висновок, що зміна дисипативної здатності конструкції є недостатньою для надійної діагностики тріщини. Ця відмінність може бути пов'язана з впливом багатьох факторів на ефективність демпфірування стосовно виявлення тріщин. Для прогнозування можливої зміни характеристики демпфірування з урахуванням цих факторів була розроблена експериментально-аналітична методика на основі підходів механіки руйнування. Ця методика дозволила виявити умови надійного виявлення крайової тріщини в стержні на двох опорах при поперечних і поздовжніх коливаннях. Було показано, що чутливість характеристики демпфірування до наявності пошкодження є обернено пропорційною рівню демпфірування неушкодженої конструкції. Зміна демпфірування є ефективною для діагностики пошкоджень у відносно жорстких конструкціях. При цьому рівень напруги пошкодженої ділянки повинен бути досить високим, щоб тріщина періодично відкривалася або була постійно відкритою. На основі аналізу результатів досліджень сформульовано умову, яка може допомогти інженерам досить просто визначати ефективність характеристики демпфірування для діагностики тріщини. Характеристика демпфірування є ефективною, якщо відношення розсіяної в тріщині енергії до подвоєної потенційної енергії деформації конструкції перевищує добуток характеристики демпфірування коливань неушкодженої конструкції на відносну похибку її визначення

Ключові слова: логарифмічний декремент коливань, стрижень на двох опорах, крайова тріщина, діагностика, вібраційна діагностика пошкоджень

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ANALYSIS OF CONDITIONS OF EFFECTIVE CRACK DETECTION IN SIMPLY SUPPORTED ROD BASED ON CHANGE OF DAMPING

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1. Introduction

Damping characteristics of structures and their elements are important for accurate prediction of natural and resonant frequencies. In addition, a change of damping can be directly related in some cases to structural damage [1–4].

Thus, a change in the damping ability of a mechanical system can be used for vibration-based diagnostics of damages such as fatigue cracks. At the same time, the effect of a crack on the characteristic of damping structural vibrations depends on a series of factors. These include the level of vibration damping of an undamaged structure, the coefficient of flexibility, the level of tension in the cross-section having a crack, etc. In this regard, the experimental assessment of the effectiveness of damping change to detect damage is extremely time-consuming. The urgency of the problem being solved is determined by the need to develop simple and reliable methods for diagnosing damages such as fatigue cracks. It is proposed to use a change in the characteristic of vibration damping in structural elements as a sign of damage.

2. Literature review and problem statement

The practice of using the change of the characteristics of vibration damping in damage diagnostics is highly controversial. In a series of studies, a significant increase in the characteristics of damping specimen vibrations was observed during the initiation and propagation of cracks. For example, a 63- and 125-time increase in the damping ability of prismatic steel samples when the fatigue crack reached 52 % and 74 % of the cross-section, respectively, was observed in [5]. A 70-fold increase in the damping ability of a cantilever steel specimen when the crack reached 50 % of the cross-section was recorded in [6]. It was demonstrated in [7] that the decrement of vibrations in prismatic steel samples increased up to 30 times with an increase in the fatigue crack to 70 % of the cross-section. When testing a steel rod on two supports with a crack in the middle, and up to 12 times increase in damping characteristic with the crack propagation to 75 % of the cross-section was recorded in [8]. It was noted in [9] that the vibration damping coefficient of cantilever dialuminium specimens increased by 40 % when the crack reached half the cross-section. A significant, up to

80 % increase in damping vibrations of a reinforced concrete beam, a reinforced concrete panel, and a span of a reinforced concrete bridge when multiple cracking of concrete occurred was found in [10–12], respectively. A cantilever specimen with a crack at the base was tested in [13] and a rod on two supports with a crack in the middle was tested in [14]. A 2-fold increase in the characteristic of damping longitudinal vibrations was observed in both cases.

At the same time, in a series of other studies, the change of damping vibrations in some structures was considered insufficient for a reliable diagnosis of the damage that occurred in them. For example, results of vibration tests of a three-span bridge were presented in [15]. They have demonstrated that the occurrence of significant damage changes the damping coefficient by only 10 %. Almost the same result was obtained in [16] in the modal analysis of a cable truss bridge with multiple cracks. In [17], when testing airplane wing caissons, only a 23 % increase in the energy absorption coefficient was observed during the initiation of significant fatigue cracks.

The indicated discrepancy between the data of the above experimental studies [5–17] is explained by the fact that the intensity of the crack-induced change in the damping characteristic depends on a series of factors. For example, it was shown in [15] that the crack parameters (size and location) are one of such factors. Besides, it was found in [7] that the effect of a crack on damping characteristics is largely determined by the damping ability of an undamaged structure (initial level of damping). The stress level in the region with a crack is another factor established in [9, 13] that affects the rate of change in the damping characteristic: a crack located in a weakly stressed region cannot noticeably change the damping characteristic. In addition, experimental studies [18, 19] showed that the intensity of changes in the damping characteristic is different for transverse, longitudinal, and torsional vibrations and strongly depends on the coefficient of flexibility.

Thus, the rate of change in damping characteristics depends on at least five factors. The presence of such a series of factors is a serious problem in assessing the possibility of using the damping characteristics for vibration-based diagnostics of damage. Experimental determination of such conditions is extremely costly and time-consuming. The most effective way to solve this problem consists in the elaboration of an experimental-analytical procedure based on the approaches of fracture mechanics. The change in the damping characteristics caused by a crack that can be predicted with its use.

However, the creation of such a procedure requires knowledge of the analytical relationship between parameters of the crack and the energy that is dissipated in it. There are four approaches that explain energy dissipation in a crack. For example, energy dissipation in a crack was associated with friction between its surfaces [20]. It was assumed in [21, 22] that energy dissipation in the crack is determined by the yield zone in the vicinity of its apex. A mixed mechanism was proposed in [23] for energy dissipation caused by a crack, that is, both friction of the crack surfaces and formation of a plastic zone around its apex. A thermodynamic theory of explaining the mechanism of energy dissipation in a crack was proposed in [13].

The problem we are facing here is that any experimental-analytical method for determining the characteristics of damping vibrations in structures having a crack based on the above hypotheses will be unsuitable for engineering practice because of its complexity. To formulate conditions for assessing the effectiveness of damping change for crack diagnostics suitable for engineering practice is the most effective way to this problem solution.

3. The aim and objectives of the study

The study objective is to determine conditions under which the vibration damping characteristics of structures are effective for detecting damages such as fatigue cracks.

To achieve the objective, the following tasks were set:

– based on the known results of studies in fracture mechanics, develop an experimental-analytical procedure for establishing characteristics of damping vibrations in a rod with a closing crack on two supports;

– to study the effect of crack parameters, coefficient of flexibility, damping level of an undamaged structure, type and mode of vibrations on the intensity of changes in the logarithmic decrement of vibrations.

4. A procedure for determining a characteristic of damping vibrations in a rod with a crack on two supports

In the case of a closing crack, logarithmic decrement of vibrations in the simply supported rod (Fig. 1) can be determined for symmetric vibrations from equation [24]

$$\delta_c(\sigma)=\delta(\sigma)+\frac{\Delta U_c(\sigma_c)}{2U(\sigma)}, \quad (1)$$

where δ is the logarithmic decrement of vibrations in an undamaged rod; U is the potential energy of the rod; ΔU_c is the energy dissipated in the crack; σ is the maximum amplitude of stresses along the rod.

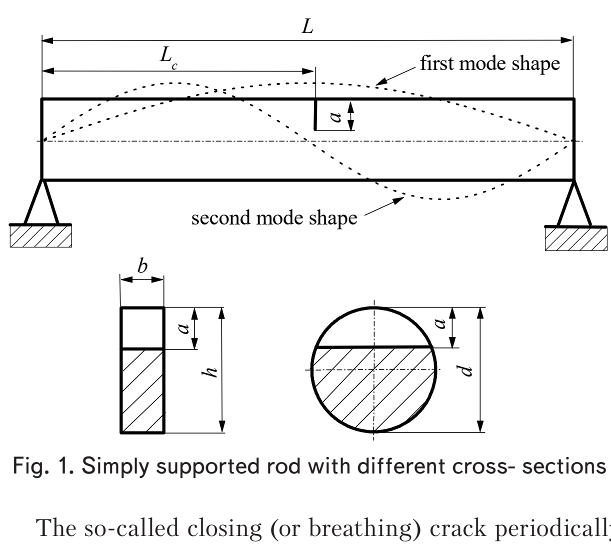

Fig. 1. Simply supported rod with different cross- sections

The so-called closing (or breathing) crack periodically opens and closes during vibrations. The logarithmic decrement of vibrations in the formula (1) is determined in relative units.

Experimental studies [22] have made it possible to relate energy dissipation determined by the plastic zone in the vicinity of the crack apex to a magnitude of the stress intensity factor (SIF) as follows:

$$\Delta\overline{U}_c = 8.634675\times10^{-5}\Delta K_I + +3.87315\times10^{-4}\Delta K_I^2 - 1.29826\times10^{-5}\Delta K_I^3, \quad (2)$$

where $\Delta\overline{U}_c$ is energy dissipated in the crack related to the length of its front; ΔK_I is the SIF range. It should be noted that equation (2) is valid provided that the crack does not propagate.

In a case of symmetric vibrations, the SIF range is determined through the maximum SIF, namely, $\Delta K_I=K_I$. According to [25], for the crack considered, the following is valid for a prismatic rod under transverse vibrations:

$$K_I = \sigma_c\sqrt{\pi a}\begin{pmatrix}1.122-1.40\alpha+7.33\alpha^2-\\-13.08\alpha^3+14.0\alpha^4\end{pmatrix}, \quad (3)$$

where σ_c is the stress amplitude in the damaged section; a is the crack depth; $\alpha=a/h$ is the relative depth of the crack in the prismatic rod; h is the height of the rod's cross-section.

With longitudinal vibrations of a prismatic rod,

$$K_I = \sigma_c\sqrt{\pi a}\begin{pmatrix}1.12-0.231\alpha+10.55\alpha^2-\\-21.72\alpha^3+30.39\alpha^4\end{pmatrix}. \quad (4)$$

With transverse vibrations of a cylindrical rod [26],

$$K_I = \sigma_c\sqrt{\pi a}\left(1.1105-2.6475\gamma+5.6875\gamma^2\right), \quad (5)$$

where $\gamma=a/d$ is the relative depth of the crack in the cylindrical rod; d is the diameter of the rod. In equations (3) to (5), $\alpha=a/h\leq0.6$ and $\gamma=a/d\leq0.6$.

Since stress amplitude in the region of the crack location depends on the rod shape, it is necessary to relate it to the maximum stress amplitude along the beam:

$$\sigma_c = \sigma\sin\left(k_i L_c\right), \quad (6)$$

where L_c is the crack location. The factor $\sin(k_iL_c)$ takes into consideration the influence of vibration mode on the SIF value. For the i-th transverse and longitudinal modes of vibration of the simply supported rod, $k_i=i\pi/L$ (L is the rod length).

Since the SIF is assumed to be constant along the crack front [25], energy dissipated in the crack can be calculated from the formula:

$$\Delta U_c = 2c\Delta\overline{U}_c, \quad (7)$$

where $2c$ is the crack width. In the case of a prismatic rod, $2c=b$ (b is the width of the rod cross-section) and in the case of a cylindrical rod, $2c=2\sqrt{a(d-a)}$.

The potential energy of a prismatic simply supported rod under transverse and longitudinal vibrations and a cylindrical rod under transverse vibrations can be calculated respectively as follows:

$$U(\sigma)=\frac{bhL}{12E}\sigma^2;$$

$$U(\sigma)=\frac{bhL}{4E}\sigma^2;$$

$$U(\sigma)=\frac{\pi d^2L}{64E}\sigma^2. \quad (8)$$

Formulas (8) for determining potential energy of the rod are valid for any vibration mode.

5. Study of the effect of crack parameters, coefficient of flexibility, level of an undamaged structure damping, type, and mode of vibrations on the intensity of change of the logarithmic decrement of vibrations

To study the influence of the flexibility coefficient on change of intensity of the damping characteristic, the rod length was taken L=0.2, 0.4, 1, and 2 m. The damping characteristic of an undamaged rod was taken in the range δ=0.002...0.2. The above range of the logarithmic decrement of vibrations includes almost all materials used in the manufacture of structural elements [27].

The height, width, and diameter of the rod's cross-section were chosen the same: $h=b=d$=0.02 m. Young's shear modulus, density and Poisson's ratio of the rod material were as follows: E=200 GPa and G=80 GPa, ρ=7,800 kg/m^3, ν=0.26, respectively.

The stress amplitude should be as low as possible to prevent crack propagation during testing to detect damage. Experimental study [22] has shown that the condition for non-propagation of a crack is satisfied if stress does not exceed 10 MPa. At the same time, error in determining the logarithmic decrement of vibrations at this stress level was acceptable [19]. Therefore, in calculations of the logarithmic decrement of vibrations of the rod with a crack, it was assumed that σ=10 MPa.

Fig. 2–4 demonstrate the effect of the crack size on the damping characteristic of a prismatic rod with different flexibility factors and low (δ=0.002), average (δ=0.02) and high (δ=0.2) level of the logarithmic decrement of vibrations for an undamaged rod. As can be seen, an increase in the coefficient of the flexibility of the rod significantly suppresses the intensity of changes in the damping characteristics because of the crack propagation.

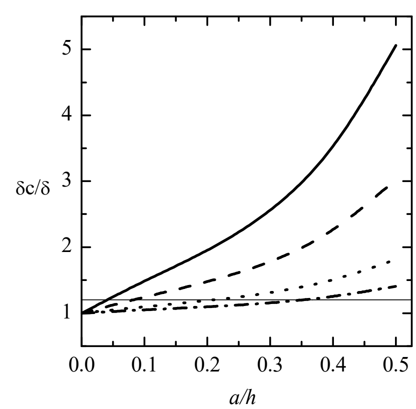

Fig. 2. Dependence of relative change of the decrement of the first mode of transverse vibration of the prismatic rod on the relative depth of the edge crack (δ=0.002; L_c/L=0.5): L/h=10 (solid line); L/h=20 (dash line); L/h=50 (dot line); L/h=100 (dash-and-dot line)

Provided that error in determining the logarithmic decrement of vibrations does not exceed 15 % [19], reliable damage detection is possible if the change in the logarithmic decrement of vibrations exceeds this value. Suppose this change is 20 %. In this case, the relative change in the damping characteristic is δ_c/δ=1.2. This change is shown in Fig. 2–4 with a thin horizontal line. This line was used to estimate the smallest crack size that can be detected based on changes in the logarithmic decrement of vibrations (α_{det}, γ_{det}).

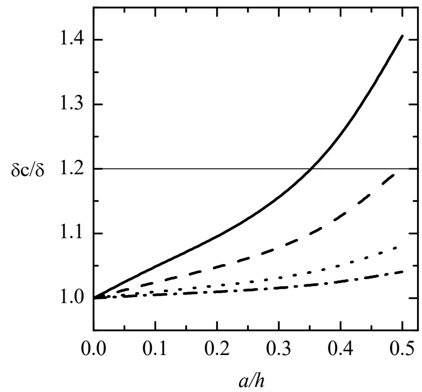

Fig. 3. Dependence of relative change in the decrement of the first mode of transverse vibration of the prismatic rod on the relative depth of the edge crack (δ=0.02; L_c/L=0.5): L/h=10 (solid line); L/h=20 (dotted line); L/h=50 (dash line); L/h=100 (dash-and-dot line)

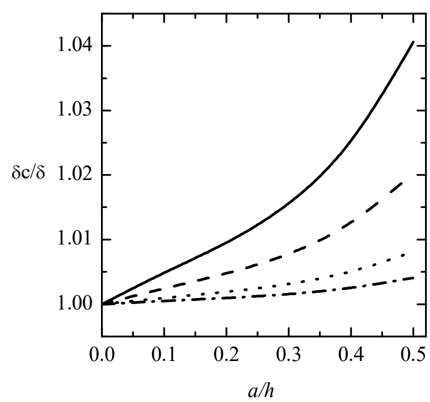

Fig. 4. Dependence of relative change in the decrement of the first form of transverse vibrations of the prismatic rod (δ=0.2; L_c/L=0.5) on relative depth of the edge crack: L/h=10 (solid line); L/h=20 (dotted line); L/h=50 (dotted line); L/h=100 (dash-and-dot line)

As can be seen from Fig. 2, a change in the logarithmic decrement of vibrations can respond to a birth of a rather small crack (α_{det}=0.04) provided that the damping characteristics of the undamaged rod are low (δ=0.002) and the rod is relatively rigid (L/h=10). In the case of δ=0.02, a crack with a size of α_{det}=0.35 can be detected (Fig. 3). At the same time, if the structural material has a high damping ability (δ=0.2), a change in damping can hardly be suitable for detecting cracks of size $\alpha \leq 0.5$ (Fig. 4).

The natural frequency of a rod with a crack on two supports was determined using the analytical procedure and program code developed in [28]. As indicated in [29], reliable diagnostics of damage is possible if a change in the natural frequency of the structure is at least 5 %. Fig. 5 shows that the smallest crack found under the condition f_c/f=0.95 was α_{det}=0.23. Thus, the sensitivity of the relative change in natural frequency to the presence of a crack is much lower than that of the damping characteristics under the condition δ=0.002 but is comparable with the sensitivity of the damping characteristic under the condition δ=0.02. Besides, the higher the beam rigidity, the higher the sensitivity of both damping and natural frequency to the presence of cracks.

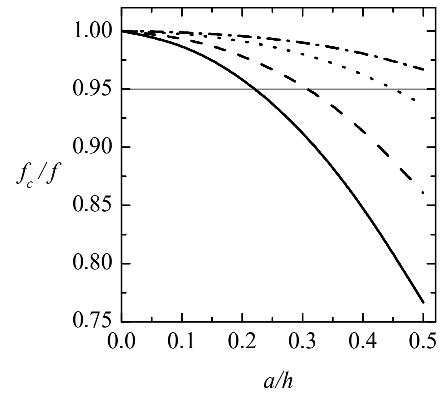

Fig. 5. Dependence of relative change in frequency of the first mode of transverse vibrations of the prismatic rod (δ=0.2; L_c/L=0.5) on the relative depth of the edge crack: L/h=10 (solid line); L/h=20 (dash line); L/h=50 (dot line); L/h=100 (dash-and-dot line)

Fig. 6, 7 show the effect of crack location on the logarithmic decrement of transverse vibrations of the prismatic rod. The most intense change in damping is observed if the crack is located close to a substantially stressed region of the rod. For example, the most stressed region of the rod is in its middle for the first mode of natural vibration and at a quarter of the rod length from the support for the second mode (Fig. 1). Thus, as follows from Fig. 6, 7, change in the logarithmic decrement of vibrations is directly proportional to the level of stress along the rod. As a result, a crack cannot be detected by a change in the damping characteristic if it is located close to supports or weakly stressed rod sections.

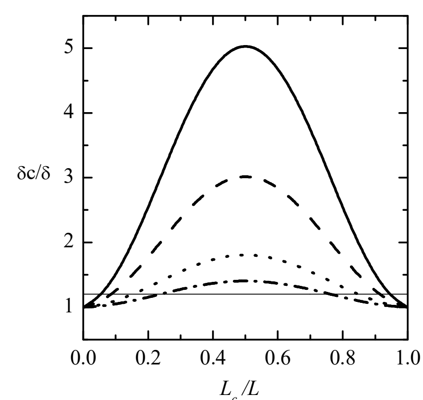

Fig. 6. Dependence of relative change in the decrement of the first mode of transverse vibration of the prismatic rod (δ=0.002; a/h=0.5; L_c/L=0.5): L/h=10 (solid line); L/h=20 (dash line); L/h=50 (dot line); L/h=100 (dash-and-dot line)

A direct comparison of the logarithmic decrement of transverse and longitudinal vibrations demonstrates (Fig. 8) that the characteristic of damping of transverse vibrations is most effective for detecting relatively small cracks in the prismatic rod. At the same time, detection of a relatively large crack in a cylindrical rod (γ>0.4) is more effective when using transverse vibrations.

Generalized data on the sensitivity of vibration-based diagnostics of damage using changes in the logarithmic decrement of vibrations and natural frequency of the first mode (case L_c/L=0.5) are given in Table 1.

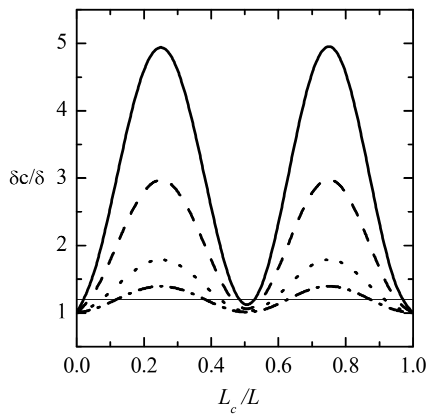

Fig. 7. Dependence of relative change in the decrement of the second form of transverse vibrations of the prismatic rod (δ=0.002; a/h=0.5; L_c/L=0.5): L/h=10 (solid line); L/h=20 (dash line); L/h=50 (dot line); L/h=100 (dash-and-dot line)

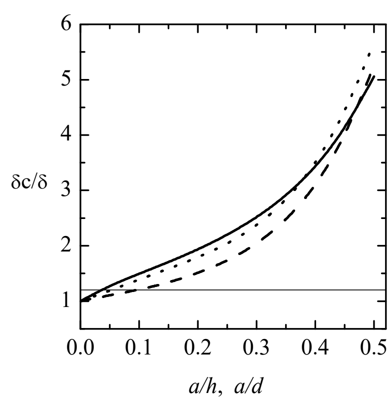

Fig. 8. Dependence of relative change in the logarithmic decrement of the first mode of transverse (solid and dash line) and longitudinal (dash line) vibrations of the prismatic (solid and dash line) and cylindrical (dot line) rod: L/h=10; L/h=10; L/h=0.5; δ=0.002

As can be seen from Table 1, in contrast to the damping characteristic, the relative change in the natural frequency is independent of the damping level of the undamaged structure. This is an obvious advantage of the natural frequency in relation to the damage detection. At the same time, change in the logarithmic decrement of vibrations is more intense than that in frequency. This difference is the greater the lower the logarithmic decrement of vibrations of the undamaged structure. This conclusion is obvious from Table 1 which shows the relative size of the crack determined by the change in the logarithmic decrement of vibrations (δ) and the change in the first natural frequency (f) according to the criteria δ_c/δ=1.2 and f_c/f=0.95, respectively.

Table 1

Crack detection efficiency based on damping (δ) and frequency (f)

Rod cross-sectional shape	Vibration mode	δ	L/h	α_{det}, γ_{det} (δ)	α_{det}, γ_{det} (f)
square	transverse	0.002	10	0.040	0.228
			20	0.082	0.315
			50	0.210	0.457
			100	0.354	>0.5
		0.02	10	0.351	0.228
			20	0.497	0.315
			50	>0.5	0.457
			100	>0.5	>0.5
		0.2	10	>0/5	0.228
			20	>0.5	0.315
			50	>0.5	0.457
			100	>0.5	>0.5
square	longitudinal	0.002	10	0.098	>0.5
			20	0.172	>0.5
			50	0.297	>0.5
			100	0.397	>0.5
		0.02	10	0.397	>0.5
			20	0.492	>0.5
			50...100	>0.5	>0.5
		0.2	10...100	>0.5	>0.5
round	transverse	0.002	10	0.054	0.231
			20	0.104	0.441
			50	0.246	>0.5
			100	0.367	>0.5
		0.02	10	0.367	0.231
			20	0.75	0.441
			50	>0.5	>0.5
		0.2	100	>0.5	>0.5
			10	>0.5	0.231
			20	>0.5	0.441
			50	>0.5	>0.5
			100	>0.5	>0.5

6. Discussion of the results obtained in the study of conditions for effective use of damping characteristics for damage diagnostics

The data presented in Table 1 demonstrate that the change of the logarithmic decrement of vibrations is highly effective for detecting damage in the case when δ=0.002...0.02. Since most metals and alloys have a relatively low ability to dissipate the energy of vibration [27], the vibration-based diagnostics of damage of the structures made of these materials based on damping change seems promising.

An increase in the coefficient of flexibility leads to a decrease in the potential energy of the rod with other conditions being equal. Therefore, according to formula (1), a change in the damping characteristic caused by a crack is directly proportional to the structure rigidity. Thus, a change in damping can be effective for diagnosing damages in relatively rigid structures. In this case, the damaged area must be loaded so that the crack periodically opens during vibration or is constantly open. A permanently closed crack cannot be detected by changing the damping characteristic.

Experimental determination of the effectiveness of the damping characteristics for detecting cracks is a complex task and a laborious process. Based on analysis of the results obtained, a condition was formulated that can help engineers to determine in the simplest way effectiveness of the damping characteristic for crack diagnostics, namely

$$\frac{\Delta U_c}{2U} \geq \kappa\delta. \quad (9)$$

The coefficient κ is determined from the error of experimental determination of logarithmic vibration decrement. For example, if this error does not exceed 20 %, the coefficient κ in relative units should be taken equal to κ=0.2. This value is substituted into equation (9).

Thus, to assess the effectiveness of damping characteristics in order to reliably detect cracks, it is necessary to determine the energy dissipated in the crack, ΔU_c, using equation (2) or (7) (depending on the crack type). Besides, it is necessary to calculate maximum energy of deformation of the structural element, *U,* for a given mode. Data on the initial level of damping of the structural element material for a given vibration mode can be found, for example, in [27].

In the general case, in order to correctly assess the possibility of using the damping characteristic in damage diagnostics, six factors must be taken into consideration. These are size and location of the crack, type of the crack, type of vibration (transverse, longitudinal or torsional), damping characteristic of an undamaged structure (initial level of damping) as well as level of the flexibility coefficient. Therefore, experimental assessment of the effectiveness of the damping characteristic for vibration-based diagnostics of damage taking into consideration all of the above factors is an extremely expensive and time-consuming process.

To solve this problem, a unique experimental and analytical procedure is proposed that takes into consideration the above factors to predict a change in the damping characteristic of the rod on two supports when a crack appears and propagates. Based on the proposed procedure, it was shown that the intensity of change in the damping characteristic is directly proportional to the crack depth and stress level in the section with a crack. In this case, the damping characteristic of longitudinal vibrations changes less intensively than that of transverse vibrations. For example, under the condition of a low initial level of damping (δ=0.002), a crack with relative depth a/h=0.2 located in the most stressed region of the rod increases the damping characteristic by 51 % for longitudinal vibrations and by 94 % for transverse vibrations which is sufficient for reliable crack diagnostics. At the same time, at a high initial level of damping (δ=0.2), the same crack causes an increase in the damping characteristic not exceeding 10 % which does not enable reliable crack detection. In addition, the effect of a crack on the damping characteristic depends on rod compliance. For example, in the case of a low initial level of damping (δ=0.002), a crack with relative depth a/h=0.2 in a rod with relatively high compliance (L/h=100) causes an increase in the damping characteristic by only 1 % and by 94 % in a stiffer rod (L/h=10).

The cause of the revealed regularities consists, firstly, in the physical nature of energy dissipation in a crack and, secondly, in the features of determining the damping characteristics, in particular, logarithmic decrement of vibrations.

Energy dissipation in a crack of normal detachment is determined mainly by the plastic zone at the crack apex. Therefore, energy dissipated in the crack will be greater the deeper the crack and the higher stress in the section with the crack. Thus, as shown in Fig. 6, 7, even a relatively large crack formed in the low-stress region of the structure does not cause a sufficient change of the damping characteristic for a reliable diagnosis.

On the other hand, many damping characteristics including the logarithmic decrement of vibrations are relative values. For their determination, energy dissipated during a vibration cycle belongs to the potential energy of the system (1). A change in the structural element size (in this case, the coefficient of flexibility) changes its potential energy and thus affects relative change in the damping characteristic of a rod with a crack.

The conventional method of vibration-based diagnostics of damage consists of determining changes in natural frequencies of structural vibrations. As shown in Section 5, under certain conditions, a change in the vibration damping characteristic is a much more sensitive indicator of damage than a change in natural frequencies. Given the multifactorial effect of the crack on the damping characteristic of vibrations of a rod with a crack, the problem consists in the determination of these conditions. A solution to this problem was proposed in equation (9) which formulates a rather simple for engineering practice condition for assessing the effectiveness of a change in the characteristic of vibration damping for crack diagnosis.

Determining the damping characteristic of vibrations for actual structures is a more difficult task than that for specimens under laboratory conditions. However, currently, there are well-developed experimental procedures and corresponding equipment for modal analysis of complex structures [30]. They make it possible to determine with high accuracy not only natural frequencies but also modal damping of complex structures with an accuracy sufficient for reliable vibration-based diagnosis of damage.

Further development of this study involves testing of the proposed procedure for vibration-based diagnostics of compressor blades in gas turbine engines at the stage of their repair. These blades are made of low-damping titanium alloys which makes the vibration-based diagnostics of damage, such as fatigue cracks, based on changes in damping characteristics more promising than based on the changes in natural frequencies.

7. Conclusions

1. Based on the approaches of fracture mechanics, a procedure has been elaborated for calculating the characteristic of damping of lateral and longitudinal vibrations of a simply supported rod with an edge crack of mode I. The essence of this procedure consists of the experimental and analytical determination of energy dissipation in a crack of mode I and damping characteristic of vibrations in a rod with a crack taking into consideration the crack parameters, structural features of the rod and properties of its material. Application of this procedure has made it possible to identify conditions under which the change in damping characteristic is a reliable indicator of damage, for example, a fatigue crack.

2. Based on the analysis of the calculation results, it was shown that the sensitivity of the damping characteristic to the presence of damage is inversely proportional to the initial level of damping of the undamaged structure and directly proportional to the structural rigidity and the level of stresses in the damaged section. The damping characteristic is effective if the ratio of the energy dissipated in the crack to the two-fold potential energy of structure deformation exceeds the product of the vibration damping characteristic of the undamaged structure by the relative error in its determination.

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