Визначено умови настання статичного автобалансування в разі асиметричного ротора на двох ізотропних пружних опорах, що балансується пасивним автобалансиром будь-якого типу. У загальному випадку площина статичної неврівноваженості не збігається з площиною автобалансира.

Застосовано енергетичний метод в припущенні, що маса вантажів автобалансира набагато менше маси ротора.

Встановлено, що статичне балансування ротора автобалансиром будь-якого типу можливе у випадках:

– довгого ротора при обертанні ротора зі швидкостями між першою і другою і над третьою характерними швидкостями;

– сферичного ротора при обертанні ротора зі швидкостями між першою і другою характерними швидкостями;

– короткого ротора на швидкостях, що перевищують деяку характерну швидкість за умови, що автобалансир знаходиться поблизу від центру мас ротора.

Асиметрія ротора збільшує число резонансних швидкостей, але кількість областей настання автобалансування не змінюється.

Неврівноваженість ротора і місце її розташування не впливають на характерні швидкості обертання ротора. Автобалансир в діапазоні швидкостей обертання ротора, що забезпечують автобалансування, прагне максимально зменшити відхилення свого центру від осі обертання ротора. При підході швидкості обертання довгого або сферичного ротора до другої характерної швидкості балансувальної ємності автобалансира перестає вистачати для повного усунення відхилення центру автобалансира від осі обертання ротора.

Отриманий результат узагальнює результати, отримані раніше з застосуванням емпіричного критерію настання автобалансування. Енергетичний метод, на відміну від емпіричного, дозволив оцінити залишкові відхилення поздовжньої осі ротора від осі обертання. Це дозволяє оцінювати запас або розраховувати балансуючу ємність автобалансира.

Тип автобалансира не враховується в таких дослідженнях. Тому отримані результати придатні для автобалансира будь-якого типу, а сам метод придатний для побудови загальної теорії пасивного автобалансування (застосовної для автобалансирів будь-якого типу)

Ключові слова: ротор, ізотропна опора, автобалансир, стаціонарний рух, стійкість руху, рівняння усталеного руху

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INDENTIFYING THE CONDITIONS FOR THE OCCURRENCE OF STATIC SELF-BALANCING FOR AN ASSYMETRIC ROTOR ON TWO ISOTROPIC ELASTIC SUPPORTS

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1. Introduction

Passive automatic balancers are used to balance fast-rotating rotors [1–12]. Under certain conditions, loads in automatic balancers find themselves in the positions in which they balance the rotor.

Constructing a theory of the single-plane automatic balancing of rotors that execute a spatial motion is a relevant scientific task as it describes the operation of many rotor machines with automatic balancers [2–6].

A large number of different types of automatic balancers [1, 4] renders special importance to building a general theory of passive self-balancing suitable for automatic balancers of any type. The empirical [9] and energy [10] methods have been developed for this purpose. The methods make it possible to answer the question on which conditions and over which range of rotation velocities can enable the balancing of a rigid or flexible, fixed in a certain way, rotor using one or more passive automatic balancers of any type.

It is important to find the analytical conditions for establishing static self-balancing for an asymmetric rotor on two isotropic elastic supports for the case when the plane of the automatic balancer does not coincide with the plane of imbalance. This is the case most common in practice. On the other hand, resolving this issue is an important step in building the theory of a single-plane automatic balancing of rotors.

2. Literature review and problem statement

It is a relevant scientific task to construct a theory of the single-plane automatic balancing of rotors that execute a spatial motion. The attempt to build such a theory involved an example of balancing by using a single passive automatic balancer: the impellers of axial fans [2], the drums of washing machines with a horizontal [3] and vertical [4] rotation axis, the drums of extractors, centrifuges, separators [5], CD/DVD discs in the respective drives, etc. Various passive automatic balancers were considered: ring-, ball-, pendulum-type [1], ball-type [2, 3, 5, 6], liquid-type [4].

It should be noted that taking into consideration the type of a rotor machine and the type of automatic balancer significantly complicates the mathematical statement of the problem. The resulting mathematical model is almost impossible to analyze [2–6]. The basic results are derived when using numerical methods for the specific system parameters values. Therefore, the results are of a particular character and defy generalization.

When applying more general approaches, one considers, instead of a specific rotor machine, a certain generalized rotor mounted on two compliant supports. However, the type of automatic balancer is taken into consideration. Thus, the single-plane self-balancing using a two-ball automatic balancer was investigated through numerical modeling for the case of the anisotropy of supports by both the static and dynamic imbalance of the rotor [7]. It was established for specific parameters of the system that the self-balancing is warranted at the speeds of rotor rotation above the resonance. It should be noted that the numerical methods make it possible to model and investigate the dynamics of complex mechanical systems. However, the results are of particular character as they are obtained at the specific parameters of the system for a specific automatic balancer. In addition, the results derived are almost impossible to generalize.

Let us take a closer look at the main analytical results.

The axisymmetric rotor on two isotropic supports was considered in [8]. The plane of the automatic balancer coincides with the plane of imbalance. The automatic balancer is of a two-ball type. The stability of all possible steady motions at which the balls rotate synchronously with the rotor was investigated. The dynamic system synchronization method was used. The resulting analytical conditions for the occurrence of self-balancing are difficult to analyze as they contain the phase angles that determine the positions of the balls relative to the rotor. However, it was found that the conditions for the occurrence of self-balancing significantly depend on the length of the rotor and the distance from the center of the rotor mass to the plane of the automatic balancer.

Let us consider approaches that make it possible to build a general theory of passive automatic balancers (suitable for automatic balancers of any type).

An empirical criterion for the occurrence of self-balancing was proposed in [9]. The criterion examines the rotor's response to the sample elementary imbalances located in the balancing planes. The criterion makes it possible to answer the question on what conditions and what range of rotation velocities can enable the balancing of a rigid or flexible, fixed in a certain way, rotor using one or more passive automatic balancers of any type.

An asymmetric rotor on two isotropic elastic supports was considered in [10]. The asymmetry is caused by the resulting weight of loads in the automatic balancer and the mass of the rotor's imbalance. The plane of the automatic balancer coincides with the plane of imbalance. An automatic balancer can be of any type. The empirical criterion for the occurrence of self-balancing was used. It was found that for the case of a long rotor there are three characteristic rotor rotation velocities, such that the self-balancing is occurred when the rotor rotates at velocities between the first and second and above the third characteristic speeds. For the case of a short rotor, the automatic balancer should be located near the center of the rotor mass. Then the self-balancing would be occurred when exceeding the only characteristic speed. The spherical rotor is balanced in a narrow range of rotor rotation velocities between two characteristic speeds. Characteristic speeds are understood to be speeds that limit the onset of the occurrence of self-balancing. These include both the resonance rotor rotation velocities and some additional speeds located between them.

A general theory of passive self-balancing can also be built using the energy method outlined in [11]. In contrast to the empirical method, the energy method makes it possible to take into consideration the type of automatic balancer, the resulting mass of loads, as well as imbalance. Special features in the application of the energy method to construct a general theory of passive self-balancing are described in [12].

Our review reveals that the analytical conditions for the occurrence of self-balancing were only reported in works [8, 10]. At the same time, the case of an originally asymmetric rotor was not analytically addressed, when the plane of the automatic balancer does not coincide with the plane of imbalance. Note that this is the case most common in practice. In this case, the residual imbalance of the rotor was neither studied nor evaluated.

3. The aim and objectives of the study

The aim of this study is to determine the analytical conditions for the occurrence of static self-balancing for an asymmetric rotor on two isotropic elastic supports and to assess the effect exerted on these conditions by the mismatch between a non-equilibrium plane and the automatic balancer's plane. This would make it possible to find more precise conditions for the occurrence of self-balancing, which is necessary to design automatic balancers for specific rotor machines.

To accomplish the aim, the following tasks have been set:

– to find the conditions for the occurrence of static self-balancing using the modified energy method for the examined rotor system;

– to assess the residual imbalance of the rotor, the residual deviations of the rotor's longitudinal axis from the axis of rotation.

4. A method for determining the conditions for the occurrence of self-balancing

4. 1. Description of the system model

The results of works [11, 12] are used to describe the research methods. Fig. 1 shows the schematic of a rotor on two supports. Fig. 2 illustrates its motion pattern [12]. The rotor is balanced, it rotates at a constant angular velocity ω around the axis passing through the longitudinal axis of the rotor shaft at the undeformed supports. It is rigidly connected to the masses that create imbalance. A passive automatic balancer is mounted onto the rotor to balance the imbalance. The body of the automatic balancer is rigidly connected to the rotor. Therefore, the body relates to the rotor. The unbalanced masses are considered separately from the rotor.

The rotor rests on the isotropic elastic supports whose rigidity ratios are k_1, k_2. The action of gravity is not taken into consideration.

We shall set the rotor into motion using a pair of three axes $OXYZ$ and $P\Xi HZ$. The $P\Xi HZ$ axes are the main central axes of rotor inertia. Under a static equilibrium position of the stationary rotor, these two axis systems are the same (Fig. 1). In the process of motion, the $P\Xi HZ$ axes move in the following way. First, the $P\Xi HZ$ axes move progressively along x, y relative to the $OXYZ$ axes and, as a result, move to the intermediate position $PX_PY_PZ_P$ – Fig. 2, *a*. Next, the $PX_PY_PZ_P$ axes rotate at angles α, β, as shown in Fig. 2, *b*, they then merge with the $P\Xi HZ$ axes. Next, the $P\Xi HZ$ and $OXYZ$ axed rotate around the Z axis at angular velocity ω.

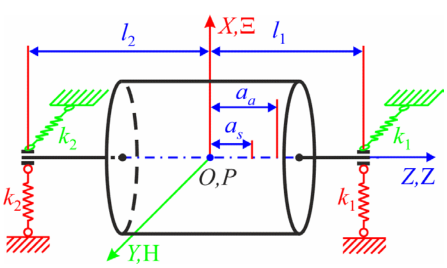
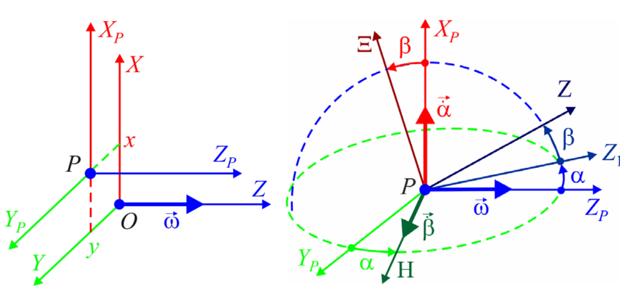


Fig. 1. Schematic of a rotor on two elastic-viscous supports



a *b*

Fig. 2. A rotor motion scheme

Note that at steady motion the system rotates as a rigid whole around the Z axis at constant angular velocity ω.

4. 2. The generalized potential, dissipative function, and the equations of stationary motions

Denote the tensor of rotor inertia through $\mathbf{J}_P^{(r)}$; the imbalance with loads – through $\mathbf{J}_P^{(S)}$. The tensor of the system inertia relative to the axes $P\Xi HZ$ $\mathbf{I}_P = \mathbf{J}_P^{(r)} + \mathbf{J}_P^{(S)}$, hence

$$I_\xi = A + J_\xi,\quad I_\eta = B + J_\eta,\quad I_\zeta = C + J_\zeta,$$

$$I_{\xi\eta} = J_{\xi\eta},\quad I_{\xi\zeta} = J_{\xi\zeta},\quad I_{\eta\zeta} = J_{\eta\zeta}. \tag{1}$$

For a passive automatic balancer, $J_\zeta = \text{const}$ [1]. Thus

$$I_\zeta = C + J_\zeta = \text{const}. \tag{2}$$

Let the system have the center of mass coordinates (G point, not shown in the schematic) ξ_G, η_G, ζ_G. relative to the $P\Xi HZ$ axes.

Note that the centrifugal moments of inertia $I_{\xi\zeta}$, $I_{\eta\zeta}$ and the coordinates of the center of mass ξ_G, η_G are the parameters that characterize the imbalance of the rotor.

Let us assume that the masses of the imbalance and loads are much smaller than the mass of the rotor. Given this, we shall consider the following to be the values of the first order of smallness:

– the coordinates of the center of mass ξ_G, η_G, ζ_G and the components of the inertia tensor $\mathbf{J}_P^{(S)}$ of the imbalance with loads;

– the rotor coordinates α, β, x, y.

With accuracy to the magnitudes of the second order of smallness inclusive, the generalized potential of the system at steady motion:

$$\Pi = -\frac{1}{2}\left\{ \begin{aligned} & v_{11}\alpha^2 + v_{22}\beta^2 + v_{33}\left(x^2 + y^2\right) + \\ & +2k_{14}\left(\alpha y - \beta x\right) + I_\zeta \omega^2 \end{aligned} \right\} + \\ + \left[I_{\eta\zeta}\alpha - I_{\xi\zeta}\beta - M_\Sigma\left(x\xi_G + y\eta_G\right) \right]\omega^2, \tag{3}$$

where

$$v_{11} = (B - C)\omega^2 - k_{33},$$

$$v_{22} = (A - C)\omega^2 - k_{33},$$

$$v_{33} = v_{44} = M_\Sigma \omega^2 - k_{11}; \tag{4}$$

$$k_{11} = k_1 + k_2,$$

$$k_{14} = k_1 l_1 - k_2 l_2,$$

$$k_{33} = k_1 l_1^2 + k_2 l_2^2. \tag{5}$$

Note

$$k_{11}k_{33} - k_{14}^2 = k_1 k_2 \left(l_1 + l_2\right)^2 > 0. \tag{6}$$

It is necessary to investigate the generalized potential for a conditional extremum (3). The conditions are the equations of steady motions, which are to be derived below. At the established steady motions, the generalized potential (3) should have at least a non-isolated local minimum.

5. Results of determining the generalized conditions for the occurrence of static self-balancing

5. 1. The generalized potential at steady motions and the equations of the rotor steady motions

We believe that the rotor is statically unbalanced. The static imbalance is created by a point mass located in a plane located at a distance a_s from point P (Fig. 1). In another plane, which is at distance a_a from point P, there is an automatic balancer (Fig. 1). The centers of mass of the automatic balancer's loads are moving in this plane. The parameters of imbalance $I_{\xi\zeta}$, $I_{\eta\zeta}$, ξ_G, η_G are then dependent on each other, and

$$\xi_G=\left(\xi_s m_s+\xi_a m_a\right)/M_\Sigma,\ \ \eta_G=\left(\eta_s m_s+\eta_a m_a\right)/M_\Sigma,$$
$$\zeta_G=\left(a_s m_s+a_a m_a\right)/M_\Sigma=\text{const},$$
$$I_{\xi\zeta}=a_s\xi_s m_s+a_a\xi_a m_a,\ \ I_{\eta\zeta}=a_s\eta_s m_s+a_a\eta_a m_a. \quad (7)$$

Here: m_S is the mass of the imbalance; ξ_s, η_s are the coordinates of the mass of the imbalance relative to the $P\Xi$HZ; axes; m_a is the mass of loads; ξ_a, η_a are the coordinates of the common center of mass of the loads relative to the $P\Xi$HZ. axes.

A working automatic balancer decreases the deviation from the rotation axis of its center as hard as it can. The coordinates of the automatic balancer's center are $x_a=x+a_a\beta$, $y_a=y-a_a\alpha$. Let us replace the variables

$$x=x_a-a_a\beta,\ \ y=y_a+a_a\alpha. \quad (8)$$

Given (7), (8), the generalized potential (3) takes the form

$$\Pi=-\frac{1}{2}\left\{\begin{array}{l}d_{11}\alpha^2+d_{22}\beta^2+v_{33}\left(x_a^2+y_a^2\right)+\\+2\left(k_{14}+a_a v_{33}\right)\left(\alpha y_a-\beta x_a\right)+I_\zeta\omega^2\end{array}\right\}+$$
$$+\omega^2 m_s\left(a_a-a_s\right)\left(\alpha\eta_s-\beta\xi_s\right)+$$
$$+\omega^2\left[x_a\left(m_a\xi_a+m_s\xi_s\right)+y_a\left(m_a\eta_a+\eta_s\xi_s\right)\right], \quad (9)$$

where

$$d_{11}=v_{33}a_a^2+2a_a k_{14}+v_{11},\ \ d_{22}=v_{33}a_a^2+2a_a k_{14}+v_{22}. \quad (10)$$

The equations of the system's steady motions along the rotor coordinates

$$L_\alpha=\partial\Pi/\partial\alpha=-d_{11}\alpha-\left(k_{14}+a_a v_{33}\right)y_a+$$
$$+m_s\eta_s\left(a_s-a_a\right)\omega^2=0,$$

$$L_\beta=\partial\Pi/\partial\beta=-d_{22}\beta+\left(k_{14}+a_a v_{33}\right)x_a-$$
$$-m_s\xi_s\left(a_s-a_a\right)\omega^2=0,$$

$$L_{x_a}=\partial\Pi/\partial x_a=\left(k_{14}+a_a v_{33}\right)\beta-v_{33}x_a-$$
$$-\left(m_a\xi_a+m_s\xi_s\right)\omega^2=0,$$

$$L_{y_a}=\partial\Pi/\partial y_a=-\left(k_{14}+a_a v_{33}\right)\alpha-$$
$$-v_{33}y_a-\left(m_a\eta_a+m_s\eta_s\right)\omega^2=0. \quad (11)$$

It is necessary to investigate the generalized potential (9) for a conditional extremum. The conditions are four equations of the rotor steady motions (11). The total of unknowns is 6: α, β, x_a, y_a, ξ_a, η_a. Given the conditions (11), 2 independent unknowns remain. Assume these include the deviations of the automatic balancer's center from the axis of rotation x_a, y_a. We make use of the fact that at the main motions $x_a=y_a=0$, provided the automatic balancer's balancing capacity is enough to eliminate the deviation.

5. 2. The transformed generalized potential

The solution to the equation system (11) regarding the generalized coordinates of the rotor α, β, ξ_a, η_a takes the following form

$$\alpha=\left[m_s\eta_s\left(a_s-a_a\right)\omega^2-y_a\left(v_{33}a_a+k_{14}\right)\right]/d_{11},$$

$$\beta=\left[-m_s\xi_s\left(a_s-a_a\right)\omega^2+x_a\left(v_{33}a_a+k_{14}\right)\right]/d_{11},$$

$$\xi_a=-\frac{\left(v_{11}v_{33}-k_{14}^2\right)x_a}{\omega^2 m_a d_{22}}-$$
$$-\frac{m_s\xi_s\left[\left(v_{33}a_a+k_{14}\right)a_s+v_{22}+a_a k_{14}\right]}{m_a d_{22}},$$

$$\eta_a=-\frac{\left(v_{11}v_{33}-k_{14}^2\right)y_a}{\omega^2 m_a d_{11}}-$$
$$-\frac{m_s\eta_s\left[\left(v_{33}a_a+k_{14}\right)a_s+v_{11}+a_a k_{14}\right]}{m_a d_{11}}. \quad (12)$$

The generalized potential (9), upon substitution (12), following the transforms, takes the form

$$\Pi^*=\frac{1}{2}\left(\frac{\left(v_{22}v_{33}-k_{14}^2\right)x_a^2}{d_{22}}+\frac{\left(v_{11}v_{33}-k_{14}^2\right)y_a^2}{d_{11}}-I_\zeta\omega^2\right)+$$
$$+\frac{1}{2}m_s^2\omega^4\left(a_s-a_a\right)^2\left(\frac{\xi_s^2}{d_{22}}+\frac{\eta_s^2}{d_{11}}\right). \quad (13)$$

The function (13) will have a minimum for x_a, y_a if

$$\left(v_{22}v_{33}-k_{14}^2\right)/d_{22}>0,\ \left(v_{11}v_{33}-k_{14}^2\right)/d_{11}>0. \quad (14)$$

Check the first condition in (14). Explicitly

$$f_{22}(x)=v_{22}v_{33}-k_{14}^2=\left[(A-C)x-k_{33}\right]\left(M_\Sigma x-k_{11}\right)-k_{14}^2=$$
$$=M_\Sigma(A-C)x^2-\left[k_{11}(A-C)+k_{33}M_\Sigma\right]x+k_{11}k_{33}-k_{14}^2;$$

$$d_{22}\left(\omega^2\right)=v_{33}a_a^2+2a_a k_{14}+v_{22}=$$
$$=\left(A-C+M_\Sigma a_a^2\right)x-k_{33}-k_{11}a_a^2+2k_{14}a_a, \quad (15)$$

where $x=\omega^2$.

We introduce the discriminant

$$D_1=\left[k_{11}(A-C)+k_{33}M_\Sigma\right]^2-4M_\Sigma(A-C)\left(k_{11}k_{33}-k_{14}^2\right)=$$
$$=\left[k_{11}(A-C)-k_{33}M_\Sigma\right]^2+4k_{14}^2M_\Sigma(A-C). \quad (16)$$

We find, from the first equation in (15), the following squares of the resonance rotor speeds

$$x_{11}=\frac{k_{11}}{2M_\Sigma}+\frac{k_{33}}{2(A-C)}-\frac{\sqrt{D_1}}{2M_\Sigma(A-C)},$$

$$x_{13}=\frac{k_{11}}{2M_\Sigma}+\frac{k_{33}}{2(A-C)}+\frac{\sqrt{D_1}}{2M_\Sigma(A-C)}. \quad (17)$$

We find, from the second equation in (15), the following square of a certain additional rotor speed

$$x_{12}=\frac{k_{33}+k_{11}a_a^2-2k_{14}a_a}{A-C+M_\Sigma a_a^2}=$$
$$=\frac{(k_{11}a_a-k_{14})^2+(k_{11}k_{33}-k_{14}^2)}{k_{11}(A-C+M_\Sigma a_a^2)}. \quad (18)$$

Analyze the roots derived.

5. 3. The case of a long rotor

For the case of a long rotor A≥B>C, the actual rotor systems

$$k_{11}/M_\Sigma<k_{33}/(A-C). \quad (19)$$

Since

$$f_{22}(0)=f_{22}\left(\frac{k_{11}}{M_\Sigma}+\frac{k_{33}}{A-C}\right)=k_{11}k_{33}-k_{14}^2>0,$$

$$f_{22}\left(\frac{k_{11}}{M_\Sigma}\right)=f_{22}\left(\frac{k_{33}}{A-C}\right)=-k_{14}^2<0,$$

then

$$x_{11}<k_{11}/M_\Sigma<k_{33}/(A-C)<x_{13}.$$

Since

$$d_{22}(x_{11})\cdot d_{22}(x_{13})=$$
$$=-\frac{\left\{(M_\Sigma a_a^2-A+C)k_{14}+a_a\left[k_{11}(A-C)-k_{33}M_\Sigma\right]\right\}^2}{M_\Sigma(A-C)}<0,$$

then

$$x_{11}<x_{12}<x_{13}. \quad (20)$$

The first condition in (14) is met over the following range of the angular rotor rotation velocities

$$\omega\in\left(\sqrt{x_{11}},\sqrt{x_{12}}\right)\cup\left(\sqrt{x_{13}},+\infty\right). \quad (21)$$

Similarly, analyze the second condition in (14). We find the following squares of the resonance and additional velocities

$$x_{21}=\frac{k_{11}}{2M_\Sigma}+\frac{k_{33}}{2(B-C)}-\frac{\sqrt{D_2}}{2M_\Sigma(B-C)},$$

$$x_{22}=\frac{k_{33}+k_{11}a_a^2-2k_{14}a_a}{B-C+M_\Sigma a_a^2}=$$
$$=\frac{(k_{11}a_a-k_{14})^2+(k_{11}k_{33}-k_{14}^2)}{k_{11}(B-C+M_\Sigma a_a^2)},$$

$$x_{23}=\frac{k_{11}}{2M_\Sigma}+\frac{k_{33}}{2(B-C)}+\frac{\sqrt{D_2}}{2M_\Sigma(B-C)}, \quad (22)$$

where

$$D_2=\left[k_{11}(B-C)+k_{33}M_\Sigma\right]^2-$$
$$-4M_\Sigma(B-C)(k_{11}k_{33}-k_{14}^2)=$$
$$=\left[k_{11}(B-C)-k_{33}M_\Sigma\right]^2+4k_{14}^2M_\Sigma(B-C) \quad (23)$$

and

$$x_{21}<x_{22}<x_{23}. \quad (24)$$

The second condition in (14) is satisfied over the following range of the angular rotor rotation velocities

$$\omega\in\left(\sqrt{x_{21}},\sqrt{x_{22}}\right)\cup\left(\sqrt{x_{23}},+\infty\right). \quad (25)$$

Conditions (14) are met over the following range of the angular rotor rotation velocities

$$\omega\in(\omega_1,\omega_2)\cup(\omega_3,+\infty), \quad (26)$$

where

$$\omega_1=\min\left(\sqrt{x_{11}},\sqrt{x_{21}}\right),$$
$$\omega_2=\max\left(\sqrt{x_{12}},\sqrt{x_{22}}\right),$$
$$\omega_3=\max\left(\sqrt{x_{13}},\sqrt{x_{23}}\right)$$

are the three characteristic rotor rotation velocities.

If the rotor is symmetrically mounted on the supports, then $k_{14}=0$ and

$$x_{11}=x_{21}=\frac{k_{11}}{M_\Sigma},$$

$$x_{12}=\frac{k_{33}+k_{11}a_a^2}{A-C+M_\Sigma a_a^2},$$

$$x_{22}=\frac{k_{33}+k_{11}a_a^2}{B-C+M_\Sigma a_a^2},$$

$$x_{13}=\frac{k_{33}}{A-C},$$

$$x_{23}=\frac{k_{33}}{B-C}. \quad (27)$$

In this case,

$$x_{11}=x_{21}<x_{12}<x_{22}<x_{13}<x_{23},$$

so the self-balancing is occurred over the following range of angular velocities

$$\omega\in\left(\sqrt{x_{11}},\sqrt{x_{22}}\right)\cup\left(\sqrt{x_{23}},+\infty\right). \quad (28)$$

In this case, we managed to find the boundaries of regions where the self-balancing is occurred (characteristic velocities) in an explicit form.

5. 4. The case of a short rotor

For the case of a short rotor, $C>A\geq B$. We check the first condition in (14). Represent (15) in the following form

$$f_{22}(\omega^2)=\upsilon_{22}\upsilon_{33}-k_{14}^2=-\left[(C-A)\omega^2+k_{33}\right]\times$$
$$\times\left(M_\Sigma\omega^2-k_{11}\right)-k_{14}^2=$$
$$=-M_\Sigma(C-A)\omega^4+$$
$$+\left[k_{11}(C-A)-k_{33}M_\Sigma\right]\omega^2+k_{11}k_{33}-k_{14}^2;$$

$$d_{22}(\omega^2)=\upsilon_{33}a_a^2+2a_ak_{14}+\upsilon_{22}=$$
$$=-\left(C-A-M_\Sigma a_a^2\right)\omega^2-k_{33}-k_{11}a_a^2+2k_{14}a_a. \quad (29)$$

Let the automatic balancer be located at a distance from the center of the rotor mass not exceeding

$$a_a<\sqrt{(C-A)/M_\Sigma}. \quad (30)$$

Then $C-A-M_\Sigma a_a^2>0,\ \ d_{22}(\omega^2)<0$ and the necessary condition for the occurrence of self-balancing is $f_{22}(\omega^2)<0$. We find from (16)

$$D_1=\left[k_{33}M_\Sigma-k_{11}(C-A)\right]^2+$$
$$+4M_\Sigma(C-A)\left(k_{11}k_{33}-k_{14}^2\right)>0. \quad (31)$$

We find from (17)

$$x_{11}=\frac{k_{11}}{2M_\Sigma}-\frac{k_{33}}{2(C-A)}+\frac{\sqrt{D_1}}{2M_\Sigma(C-A)}>0. \quad (32)$$

The necessary condition for the occurrence of self-balancing

$$\omega>\sqrt{x_{11}}. \quad (33)$$

Similarly, we find, from the second condition in (15), the necessary condition for the occurrence of self-balancing

$$\omega>\sqrt{x_{12}},\ \ x_{12}=\frac{k_{11}}{2M_\Sigma}-\frac{k_{33}}{2(C-B)}+\frac{\sqrt{D_2}}{2M_\Sigma(C-B)}>0, \quad (34)$$

where

$$D_2=\left[k_{33}M_\Sigma-k_{11}(C-B)\right]^2+$$
$$+4M_\Sigma(C-B)\left(k_{11}k_{33}-k_{14}^2\right)>0. \quad (35)$$

It follows from (33) and (34) that the self-balancing will be occurred at speeds

$$\omega>\omega_1,\ \omega_1=\max\left\{\sqrt{x_{11}},\sqrt{x_{22}}\right\}, \quad (36)$$

where ω_1 is the only characteristic rotor rotation velocity.

If a rotor is symmetrically mounted on the supports, then $k_{14}=0$ and

$$x_{11}=x_{21}=k_{11}/M_\Sigma. \quad (37)$$

Self-balancing is occurred at speeds

$$\omega>\sqrt{k_{11}/M_\Sigma}. \quad (38)$$

In this case, it was possible to find the boundary of the region where the self-balancing is occurred (the characteristic speed) in an explicit form.

5. 5. The case of a spherical rotor

For the case of a spherical rotor, $C=A=B$. In this case, the first and second conditions in (14) are the same. Represent (15) in the form

$$f_{22}(x)=\upsilon_{22}\upsilon_{33}-k_{14}^2=-k_{33}\left(M_\Sigma x-k_{11}\right)-$$
$$-k_{14}^2=k_{11}k_{33}-k_{14}^2-k_{33}M_\Sigma x;$$

$$d_{22}(x)=\upsilon_{33}a_a^2+2a_ak_{14}+\upsilon_{22}=$$
$$=M_\Sigma a_a^2x-k_{33}-k_{11}a_a^2+2k_{14}a_a, \quad (39)$$

where $x=\omega^2$.

We find from (39)

$$x_2=\frac{k_{33}+k_{11}a_a^2-2k_{14}a_a}{M_\Sigma a_a^2}=$$
$$x_1=\frac{k_{11}k_{33}-k_{14}^2}{k_{33}M_\Sigma},\ \ =\frac{\left(k_{11}a_a-k_{14}\right)^2+\left(k_{11}k_{33}-k_{14}^2\right)}{k_{11}M_\Sigma a_a^2}. \quad (40)$$

We find from (40)

$$x_2-x_1=\left(k_{11}a_a-k_{14}\right)^2/\left(k_{33}M_\Sigma a_a^2\right)>0. \quad (41)$$

Thus, spherical rotors are balanced over a range of velocities

$$\omega\in\left(\sqrt{x_1},\sqrt{x_2}\right). \quad (42)$$

In this case, there are two characteristic rotor rotation velocities and they were found in an explicit form.

By bringing the plane of the automatic balancer to the center of the rotor mass $(a_a\to 0)$, this range can be made (above) as large as possible $\left(\sqrt{x_2}\to+\infty\right)$.

5. 6. Estimating the residual deviation of the rotor's longitudinal axis from the axis of rotation

When the conditions for the occurrence of self-balancing are met, the coordinates of the automatic balancer's center x_a, y_a approach the values that are the smallest by the module. We find from the last two equations (12)

$$x_a=-\frac{\omega^2m_ad_{22}}{\upsilon_{11}\upsilon_{33}-k_{14}^2}\times$$
$$\times\left\{\xi_a+\frac{m_s\xi_s\left[\left(\upsilon_{33}a_a+k_{14}\right)a_s+\upsilon_{22}+a_ak_{14}\right]}{m_ad_{22}}\right\},$$

$$y_a=-\frac{\omega^2m_ad_{11}}{\upsilon_{11}\upsilon_{33}-k_{14}^2}\times$$
$$\times\left\{\eta_a+\frac{m_s\eta_s\left[\left(\upsilon_{33}a_a+k_{14}\right)a_s+\upsilon_{11}+a_ak_{14}\right]}{m_ad_{11}}\right\}. \quad (43)$$

We find from (43) that the complete elimination of the deviation of the automatic balancer's center from the axis of rotation creates the following imbalance

$$\xi_a = -m_s\xi_s\left[\left(v_{33}a_a + k_{14}\right)a_s + v_{22} + a_a k_{14}\right]/m_a d_{22},$$

$$\eta_a = -m_s\eta_s\left[\left(v_{33}a_a + k_{14}\right)a_s + v_{11} + a_a k_{14}\right]/m_a d_{11}. \quad (44)$$

It follows from (44) that when the speed of rotation of a long or spherical rotor approaches the second characteristic speed, the balancing capacity of the automatic balancer ceases to be enough to completely eliminate the deviation of the automatic balancer's center from the rotor's rotation axis as, in this case, $d_{11}, d_{22} \to 0$. Equalities (44) can be used both to assess the reserve and to calculate the balancing capacity of the automatic balancer.

Substituting (44) in the first two equalities in (12), we find the residual angular deviation of the rotor's longitudinal axis from the axis of rotation

$$\alpha = \omega^2 m_s\eta_s\left(a_s - a_a\right)/d_{11}, \quad \beta = -\omega^2 m_s\eta_s\left(a_s - a_a\right)/d_{22}. \quad (45)$$

Note that the empirical criterion for the occurrence of self-balancing does not make it possible to assess the balancing capacity of the automatic balancer, the residual deviations of the rotor's longitudinal axis from the axis of rotation.

6. Discussion of the obtained conditions for the occurrence of static self-balancing

Our study suggests that an asymmetrical rotor that executes spatial motions and is mounted on two isotropic elastic supports can be statically balanced by a single automatic balancer of any type in the following case:

– a long rotor when the rotor rotates at velocities between the first and second and above the third characteristic speeds (26) or (28);

– a spherical rotor when the rotor rotates at velocities between the first and second characteristic speeds (42);

– a short rotor at velocities exceeding a certain characteristic speed provided that the automatic balancer is close to the center of the rotor mass (36) or (38).

As it follows from (17), (18), and (22), the rotor asymmetry increases the number of resonant and additional speeds but the number of regions where the self-balancing is occurred does not change.

This result coincides with the result reported in [10] where an empirical criterion for the occurrence of self-balancing was applied (provided that the planes of the imbalance and the automatic balancer coincide). This confirms the correctness of the results obtained when using the energy and empirical methods. It should be noted that our study has made it possible to arrange the characteristic rotor rotation velocities in ascending order in an explicit form.

It is clear from (13) that the imbalance of the rotor and its location do not affect the characteristic rotor rotation velocities. An automatic balancer in the range of rotor rotation velocities that ensure the self-balancing tends to minimize the deviation of its center from the rotor's rotation axis. It is clear from (44) that when the rotation velocity of a long or spherical rotor approaches the second characteristic speed, the balancing capacity of the automatic balancer ceases to be enough to completely eliminate the deviation of the center of the automatic balancer from the axis of rotor rotation.

The energy method, in contrast to the empirical method, has made it possible to estimate the residual deviation of the rotor's longitudinal axis from the axis of rotation. That allows the estimation of the reserve or the calculation of the automatic balancer's balancing capacity.

The type of automatic balancer is not taken into consideration in such studies. Therefore, the results obtained are suitable for automatic balancers of any type, and the method itself is suitable for building a general theory of passive self-balancing.

The method has flaws inherent in approximate methods. The method produces the approximate boundaries of the regions where the self-balancing is occurred. In addition, the method does not make it possible to study the non-stationary steady motions of the system and transition processes.

In the future, it is planned to use the modernized energy method to investigate the impact of damping in the supports on the conditions for the occurrence of single-plane self-balancing for a rotor on two isotropic supports.

7. Conclusions

1. An asymmetric rotor that executes spatial motion and is mounted on two isotropic elastic supports can be statically balanced by a single automatic balancer of any type in the following cases:

– a long rotor when the rotor rotates at velocities between the first and second and above the third characteristic speed;

– a spherical rotor when the rotor rotates at velocities between the first and second characteristic speeds;

– a short rotor at velocities exceeding a certain characteristic speed provided that the automatic balancer is close to the center of the rotor mass.

The imbalance of the rotor and its location do not affect the characteristic rotor rotation velocities.

2. An automatic balancer in the range of rotor rotation velocities that enable the self-balancing tends to minimize the deviation of its center from the axis of rotor rotation. When the rotation velocity of a long or spherical rotor approaches the second the automatic balancer ceases to be enough to completely eliminate the deviation of the center of the automatic balancer from the axis of rotor rotation.

The energy method, in contrast to the empirical method, makes it possible to assess the residual deviation of the rotor's longitudinal axis from the axis of rotation, to estimate the reserve, or to calculate the balancing capacity of an automatic balancer. In this case, the type of automatic balancer is not taken into consideration. Therefore, the results obtained are suitable for automatic balancers of any type, and the method itself is suitable for building a general theory of passive self-balancing (applicable for automatic balancers of any type).

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