

A method has been proposed to calculate the composite timber-concrete bending elements taking into consideration the non-linear work of a nail joint and the stretched reinforcement in a slab. An acting building code regulates the structure estimation based on the linear-elastic work while the pattern of the joint's deformation under loading demonstrates a pronounced non-linear character. Estimation formulae do not account for the presence of reinforcement in a concrete slab, which leads to the irrational use of the structure's load-bearing properties.

A dependence has been proposed to determine the slip modulus. The determining coefficients are computed based on the rated characteristics given in the acting design standards.

An algorithm for calculating the composite timber-concrete bending structures has been given, taking into consideration the deformation diagram of the joint and reinforcement in the stretched zone of a concrete element.

It has been established that the normal stresses for the considered variants of timber-concrete beams, determined on the basis of the proposed procedure and the linear-elastic model, differ by 1–8 %. At loads corresponding to plastic deformations, those stresses that were estimated in line with the linear-elastic model prove to be understated. At loads exceeding 0.75 kN/m for the beam with a span of 3 m, and 0.5 kN/m for the beam with a span of 5 m, stresses in the stretched region of a concrete slab exceed the concrete stretching strength while the stresses in a timber beam do not reach the ultimate values. In fact, in this case, the structure's load-bearing capacity is underutilized because the stretching effort in the cross-section with a crack is accepted by the reinforcement.

Based on the design features of timber-concrete floors (the thickness of a slab and protective layer), an analysis of the load-bearing capacity considering the reinforcement has been performed. It has been established that the load-bearing capacity of a slab ensures that an estimated bending momentum is tolerated up until the loads that cause the destruction of the timber beam. At the same time, the conditions for the rational operation of compressed concrete and stretched reinforcement are met

**Keywords:** timber-concrete structures, bending elements, nail joint, non-linear work, reinforcement

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# CALCULATION OF THE COMPOSITE TIMBER-REINFORCED-CONCRETE BENDING ELEMENTS CONSIDERING THE NONLINEAR WORK OF THE JOINT

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## 1. Introduction

The use of timber structures in construction is often limited by the size of lumber, the insufficient load-bearing capacity, and rigidity of their elements. To overcome these limitations, as well as to optimize the use of timber, modern construction employs composite structures that rationally use materials with different characteristics. These elements include timber-concrete structures, which typically consist of a concrete slab, reinforced with steel or composite fittings, and a timber beam of a solid or composite cross-section.

Timber-concrete structures are used in buildings of various purposes to make new or strengthen existing floors. The application of timber-concrete floors makes it possible to improve the load-bearing capacity and rigidity compared to

timber floors. This type of flooring is more environmentally friendly due to the reduced volume of reinforced concrete, which, in turn, brings down energy costs during construction, and reduces the carbon footprint of the building as a whole.

An element is typically composed of a concrete slab, reinforced with steel or composite fittings, and a timber beam of a solid or composite cross-section. The effectiveness of such a structure depends on the joint operation of the composite components, which is provided by the mechanical connections between them. The most common are joints of the nail type (bolts, nails, screws, etc.). The special feature of nails is their malleability, which does not make it possible to obtain a rigid connection. The result is the reduced load-bearing capacity and the increased deformability of a structure in general.

In accordance with the current design rules [1, 2], the calculation is performed on the basis of the preconditions for the linear-elastic work of a joint while the deformation pattern of a joint under loading demonstrates a pronounced non-linear character. Estimation formulae do not take into consideration the design features (the presence of reinforcement in a concrete slab), which leads to the irrational use of the load-bearing properties of the structure.

It is a relevant task to improve the methods of designing timber-concrete bending elements by taking into consideration the deformation diagram of the joint and the reinforcement load-bearing capacity as it will make it possible to more accurately determine parameters for the strained-deformed state and to rationally utilize the structure load-bearing capacity.

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## 2. Literature review and problem statement

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Paper [3] reports mathematical models for determining the internal forces and deflections of hybrid timber-concrete structures, derived from the precise solution of the system of balance equations of the beam on malleable links. The deformation dependences, supplemented with the expressions to assess the free oscillations of a single-span hinge-supported composite beam, obtained using the bending theory of beams by Timoshenko, are given in article [4]. The drawback of the cited works is the cumbersomeness of the formulae, as well as the lack of comparison of theoretical results with experimental data.

A simplified procedure for calculating composite beams is reported in [5]. Estimation dependences showed a sufficient convergence with the results of an accurate solution based on [3]. Similar formulae (the so-called  $\gamma$ -method) are given in acting norms [1, 2]. At present, this is the most common technique for calculating composite structures with timber elements in engineering practice. However, the method is based on the linear dependences of the physical-mechanical characteristics of materials and the deformation of the joint; it also has a series of limitations, in particular, it is applicable only to an evenly distributed load of the sinusoidal shape.

The authors of [6] propose a mathematical model to assess the stressed-strained state of a timber-concrete bending element, based on the principle of virtual movements. The study focuses on identifying deflections by solving a system of equations that describe the equilibrium of a composite structure system. However, the proposed analytical expressions do not take into consideration the dependence of the connection deformation on the load magnitude.

Work [7] built a four-tier model for calculating timber-iron concrete beams. Nail is regarded as a beam on an elastic base, which consists of two parts (concrete and timber) and a free area between them (the formwork), according to the procedure reported in [8]. The crack formation in a concrete slab is taken into consideration by reducing the height of the cross-section while the load-bearing capacity of the stretched reinforcement in the cross-section is disregarded. In addition, the authors of [9] note that the presence of formwork also affects the deformity of the joint.

Paper [10] proposes a mathematical dependence to build the complete deformation diagram of the nail connection between the timber and concrete elements. A special feature of the formulae is the relationship between the defining coefficients and the key physical-mechanical characteristics

of the joint (rigidity, strength, and the maximum allowable movement). The downside of the model is the need for experimental research to obtain the required coefficients.

Study [11] reports the results of statistical processing of the available experimental data from various authors on the mechanical characteristics of different types of joints; it analyzes the most common dependences to describe the "load-movement" curve and gives recommendations on their applicability. However, the continued use of the results obtained by the authors is limited to the range of connections considered.

To simulate the work of the connection in the timber-concrete structures, the authors of [12] developed a three-dimensional computer finite-element model, in which nails are modeled by one-dimensional rod finite elements, concrete and timber – by three-dimensional finite elements. The verification of the model by comparison with the experimental data showed a sufficient convergence of the results of the study. However, despite the detailed development, the model does not take into consideration the destruction of materials when timber and concrete reach the limits of strength, strength as the criteria for the failure of the structure.

Study [13] reported the finite-element modeling of a timber-concrete joint. The timber and concrete were simulated by volumetric eight-node finite elements, the nail – by a four-node rod, which made it possible to take into consideration the shape of the nail deformation. The elastic characteristics of timber were set to be orthotropic. The authors note that the theoretical curve of the connection deformation accurately reflects the experimental one up to the maximum allowable deformation magnitude of 15 mm. However, for the case of an analysis of the full-size structure, a given approach is not appropriate due to the laboriousness of the development of the model.

A common drawback of the above works is the complexity of the use in practical calculations and the bulkiness of the finite-element estimation models, as well as the need to experimentally derive the dependences describing the work of the joints. Available models do not take into consideration the presence of reinforcement in the stretched region of a slab with the composite structure.

Therefore, it is necessary to establish dependences to determine the deformation characteristics of the joint and to improve the methods of calculating the composite timber-concrete structures, taking into consideration the design features and patterns of the joint's deformation.

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## 3. The aim and objectives of the study

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The aim of this study is to construct a method to calculate the timber-concrete bending structures taking into consideration the deformation diagram of the joint and reinforcement in the stretched region of a concrete element.

To accomplish the aim, the following tasks have been set:

- to build mathematical dependences to determine the deformation characteristics of the joint and a method to calculate the composite timber-concrete bending elements, taking into consideration the non-linear character of the joint and reinforcement operation in the stretched region of a concrete element;

- to assess the impact of the non-linear operation of the nail joint and reinforcement of a concrete slab on the load-bearing capacity of a timber-concrete bending element.

**4. Mathematical dependences and a method to calculate the timber-concrete bending elements taking into consideration the non-linear operation of the joint and reinforcement**

The dependences given in [5] are widely used to determine the parameters of the stressed-strained state of composite timber-concrete structures. The basic assumptions are:

- for the element under consideration, the Bernoulli hypothesis holds, that is, the cross-sections that are flat and normal to the axis of the element before deformation remain flat and normal to its axis after deformation;
- the deformations are distributed for the height of the cross-section in line with a linear dependence, and there is a gap between the components of the structure caused by a reciprocal shift;
- the components of the structure under load have equal deflections and the axes' curvature;
- the connecting elements are evenly spaced along the length of the structure;
- the structure is fixed, to prevent the loss of stability from the bend plane.

The estimation scheme and the basic parameters of the cross-section are shown in Fig. 1.

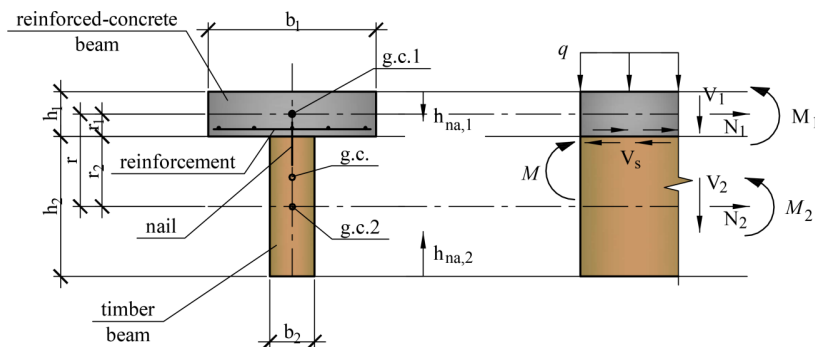


Fig. 1. The estimation scheme and the basic parameters of the cross-section of the timber-concrete bending element:  $h_1, h_2, b_1, b_2$  is the height and width of the concrete slab and the timber beam, respectively;  $r$  is the distance between the centers of gravity of a concrete slab and a timber beam;  $M_1, M_2$  are the bending moments;  $V_1, V_2$  is the transverse force;  $V_s$  is the shear force at the contact boundary, distributed lengthwise;  $N_1, N_2$  is the longitude force in a concrete slab and a timber beam

The basic dependences for determining internal efforts in a composite timber-reinforced concrete bending element take the following form [5]:

$$N_i = \mp \left( 1 - \frac{EI_0}{EI_{eff}} \right) \frac{M}{r}, \tag{1}$$

$$M_i = \frac{E_i I_i}{EI_{eff}} M, \tag{2}$$

$$V_i = \frac{E_i I_i}{EI_{eff}} V + V_s r_i, \tag{3}$$

$$V_s = \left( 1 - \frac{EI_0}{EI_{eff}} \right) \frac{V}{r}, \tag{4}$$

where  $M$  is the estimated bending moment;  $N_i$  is the longitudinal force in an element;  $M_i$  is the bending moment in an element;  $V_i$  is the transverse force in the elements;  $V_s$  is the shear

force at the contact boundary, distributed lengthwise;  $E_i$  is the element's elasticity module;  $I_i$  is the moment of inertia of an element's cross-section;  $EI_{eff}$  is the effective bending stiffness;  $i=1$  – for a concrete element;  $i=2$  – for a timber element.

Based on the condition of the linear distribution of deformations for the height of the cross-section, expressions to determine the normal stresses in the cross-section of a structure's elements can be recorded as follows:

$$\sigma_{max,i} = \mp \frac{E_i}{EI_{eff}} h_{na,i} M, \tag{5}$$

$$\sigma_{min,i} = \mp \frac{E_i}{EI_{eff}} (h_{na,i} - h_i) M, \tag{6}$$

where  $h_{na,i}$  is the distance from the most stressed fiber to the neutral axis of the  $i$ -th element:

$$h_{na,i} = r_i + \left( 1 - \frac{EI_0}{EI_{eff}} \right) \frac{EI_{eff}}{E_i A_i} \frac{1}{r}. \tag{7}$$

The sign «+» corresponds to stretching, «-» – compression. The authors of [3, 5] suggest determining the effective bending stiffness as follows:

$$EI_{eff} = \left[ 1 + \frac{\frac{EI_{\infty} - 1}{EI_0}}{1 + \left( \frac{\mu}{\pi} \right)^2 (\alpha L)^2} \right]^{-1} EI_{\infty}, \tag{8}$$

where  $\alpha L$  is the composite action coefficient;  $\mu$  is the longitudinal bend coefficient that depends on the type of a structure's fixed supports;

$$EI_0 = E_1 I_1 + E_2 I_2,$$

$$EI_{\infty} = EI_0 + EA_p r^2 / EA_0;$$

$$EA_p = (E_1 A_1)(E_2 A_2); EA_0 = E_1 A_1 + E_2 A_2.$$

A composite action coefficient is written as follows:

$$\alpha L = L \sqrt{\frac{Kr^2}{EI_0 \left( 1 - \frac{EI_0}{EI_{\infty}} \right)}}, \tag{9}$$

where  $L$  is the estimated span of the structure;  $K$  is the joint's slip modulus that takes into consideration the shift of the connecting elements when they are exposed to the evenly distributed force along the seam between the reinforced concrete and timber element.

The above approach applies to the arbitrary schemes of leaning and loading, the types of connecting elements, materials, the geometric parameters of a bent structure. However, those dependences do not accurately determine the internal efforts and stresses in a composite timber-reinforced-concrete bending element as the calculation employs a slip modulus based on the linear model of connection operation. In addition, the above formulae do not take into consideration the work of the reinforcement in the stretched region of a slab.

Under actual conditions, the connection's operation has a pronounced non-linear character. For a mathematical notation of the deformation diagram, we shall use the following dependence [11]:

$$V_{s,1}(\delta) = (b + c\delta) \left( 1 - e^{-\frac{a\delta}{b}} \right), \quad (10)$$

where  $V_{s,1}$  is the shear load on a connecting element;  $\delta$  is the displacement of a connecting element due to the shear load;  $a, b, c$  are coefficients.

The characteristic shape and the parameters of a deformation diagram of the nail joint of timber-concrete elements are shown in Fig. 2.

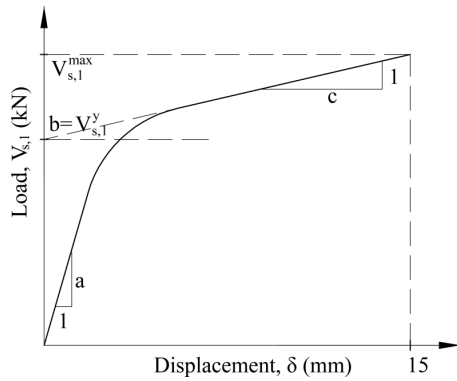


Fig. 2. The characteristic shape and parameters of the nail joint deformation diagram [10]

The dependences for determining the coefficients in the expression to describe the deformation diagram of the nail joint were proposed and experimentally verified in work [14].

The  $a$  coefficient is found as the tangent of the tangent's inclination angle, drawn from the coordinate origin to the deformation diagram; it can be expressed through the theoretical slip modulus  $K_{ser}$  to calculate the connection based on the operational suitability threshold:

$$a = \frac{K_{ser}}{0.65}, \quad (11)$$

where  $K_{ser} = 2\rho_w^{1.5}d / 23$  [1, 2];  $\rho_w$  is the timber density;  $d$  is the nail diameter.

The  $b$  coefficient is equal to the shear force at the limit of connection proportionality:

$$b = F_y, \quad (12)$$

The  $c$  coefficient corresponds to the tangent of the tangent's inclination angle to the deformation diagram at the section corresponding to the fluidity of the connection; it can be recorded as follows:

$$c = \frac{V_{s,1}^{max} - b}{15}, \quad (13)$$

where  $V_{s,1}^{max}$  is the maximally allowable load corresponding to the displacement  $\delta = 15$  mm.

The theoretical value of the maximum load and the load at the limit of fluidity can be determined from the formulae given in [1, 2]:

$$V_{s,1}^y = 1.15 \sqrt{\frac{2\beta}{1+\beta}} \sqrt{2M_y f_{h,w} d}, \quad (14)$$

$$V_{s,1}^{max} = 1.15 \sqrt{\frac{2\beta}{1+\beta}} \sqrt{2M_u f_{h,w} d}, \quad (15)$$

where  $M_y$  is the bending moment of the plastic deformation of the nail;  $M_u$  is the maximum allowable bending moment of the nail;  $f_{h,w}$  is the nail embedment strength into timber;  $\beta$  is the coefficient that takes into consideration friction between the contact surfaces of the connection elements.

The bending moment of plastic deformation and the maximum permissible bending moment of a nail [8]:

$$\frac{f d}{6} \quad (16)$$

$$M_u = \frac{f_u d^3}{6}, \quad (17)$$

where  $f_y, f_u$  is the resistance of a nail's material to the yield point and the ultimate strength, respectively.

A coefficient that takes into consideration friction between the contact surfaces of the connection elements [8]:

$$\beta = \frac{f_{h,w}}{f_{h,c}}, \quad (18)$$

where  $f_{h,w}, f_{h,c}$  is the nail embedment strength into the array of timber and concrete, respectively.

The nail embedment strength into the array of timber and concrete is determined from the following empirical formula [1, 2]:

$$f_{h,i} = 0.082(1 - 0.01d)\rho_i, \quad (19)$$

where  $\rho_i$  is the density of wood ( $\rho_w$ ) and concrete ( $\rho_c$ ).

Since the slip modulus is determined as the tangent of the inclination angle of the deformation diagram chart secant [15], drawn from the coordinate origin through a point on the graph corresponding to the actual load, the following expression is suggested for its mathematical notation:

$$K(\delta) = \frac{1}{\delta} (b + c\delta) \left( 1 - e^{-\frac{a\delta}{b}} \right). \quad (20)$$

A composite timber-concrete bending element is calculated taking into consideration the non-linear character of connection work under a load, using a simple iteration method whose algorithm implies the following. At the beginning of the calculation, a slip modulus is accepted on the basis of the condition for the linear-elastic operation of the joint, equal to theoretical, according to [1, 2]:

$$K = \frac{2}{3} K_{ser}. \quad (21)$$

Using a given value of the slip modulus, formula (9) is used to compute the composite action coefficient  $\alpha L$ ; formula (8) – to calculate the effective bending stiffness  $EI_{eff}$ . Next, one determines the shear force at the contact boundary  $V_s$  whose value is used to calculate the movement of the connection (the reciprocal shift of the structure's elements):

$$\delta = \frac{V_s}{nK}, \tag{22}$$

where  $n$  is the number of connecting elements per unit of structure length.

Based on the derived displacement, formula (20) is used to refine the slip modulus, which is applied to determine the new values of  $\alpha L$ ,  $EI_{eff}$ , and  $\delta$ .

Calculation is performed until the following condition is met:

$$\frac{|\delta_i - \delta_{i-1}|}{\delta_{i-1}} \leq 0.001, \tag{23}$$

where  $\delta_i$  is the displacement at the  $i$ -th (current) iteration;  $\delta_{i-1}$  is the displacement at the preceding iteration.

The resulting values of the slip modulus, composite action coefficient, and the effective bending stiffness are used to determine the internal efforts in a structure's elements.

To take into consideration the work of the reinforcement in the stretched region of a slab, we shall use known provisions from the theory of reinforced concrete [16]. The strength of the elements that operate at bending is determined from the condition that the destruction occurs when stresses in the stretched reinforcement accept the values equal to the ultimate strength, and in the compressed zone of concrete – strength at compression. At the same time, the resistance of concrete to compression  $f_{cd}$  is taken to be evenly distributed throughout the entire height of the compressed zone; the work of concrete in the stretched zone is not taken into consideration, and the stresses in the stretched reinforcement are equal to the ultimate strength of the reinforcement  $f_{yd}$ . The cross-section equilibrium condition is written in the form:

$$\begin{cases} f_{yd}A_s = f_{cd}b_1x, \\ M_1 \leq f_{yd}A_s(d - 0.5x), \end{cases} \tag{24}$$

where  $x$  is the height of a compressed zone of concrete;  $A_s$  is the area of the stretched reinforcement;  $M_1$  is the bending moment in a slab, determined from formula (2);  $d$  is the working height of the cross-section.

At the known sizes and the reinforcement cross-section, the height of the compressed zone of concrete is determined from the following formula:

$$x = \frac{f_{yd}A_s}{f_{cd}b_1}. \tag{25}$$

In order to ensure the rational use of the strength properties of compressed concrete and stretched reinforcement, the following condition must be met:

$$x \leq x_R = \xi_R d, \tag{26}$$

where  $\xi_R$  is the maximum allowable relative height of the compressed zone.

and to take into consideration the work of the reinforcement in the stretched region of a slab. The essence of the experiment was to compare the estimated effective stiffness and the internal stresses based on the proposed method and the linear-elastic model given in [5].

The study used an example of a hinge-supported single-span beam of the composite cross-section, consisting of a concrete slab, cross-section  $b_1 \times h_1 = 500 \times 50$  mm, and a timber beam, cross-section  $b_2 \times h_2 = 50 \times 200$  mm. The slab was made of heavy concrete, strength class C20/25, density 2,500 kg/m<sup>3</sup> [17]. The beam was made of softwood, strength class C24, density 350 kg/m<sup>3</sup> [18]. To connect the beam with the slab, a nail with a diameter of 5 mm is considered, made of steel, strength class 4.8, with a yield point of  $f_y = 320$  MPa and an ultimate strength of  $f_u = 400$  MPa [19]. The nails are arranged in a row along the length of the beam with a step of  $s = 300$  mm. The beam's span is taken equal to  $L = 3, 4$  and  $5$  m. The beam is loaded with an evenly distributed load, lengthwise the beam, of intensity  $q$ .

The characteristics of the timber-reinforced-concrete beam's materials are given in Table 1. A deformation diagram of the nail connection, as well as a chart of the change in the slip modulus depending on the load, which were built using dependences (10) and (20), are shown in Fig. 3.

Table 1

Characteristics of a composite timber-concrete beam's materials

Material	Strength class	Characteristic's title and value			
		Strength, MPa		Elastic modulus, GPa	Density, kg/m <sup>3</sup>
		stretching	compression		
Concrete	C20/25	1	14.5	23	2,500
Reinforcement	Vr-I, Ø3	375		210	7,850
	Vr-I, Ø4	360			
	Vr-I, Ø5	365			
Timber	C24	14	21	11	350

Note: For timber, the characteristics values are specified when the effort is applied along the fibers

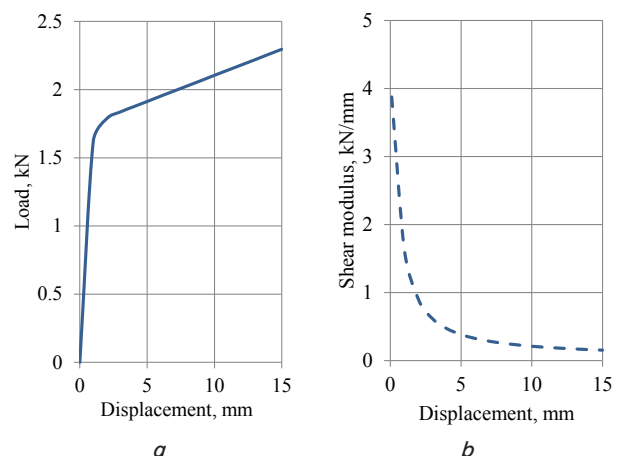


Fig. 3. Nail parameters: a – nail deformation diagram; b – slip modulus

The result of the parametric study is the data, obtained for the variants considered, about the degree of composite action, the magnitude of internal efforts and stresses. Fig. 4 shows the dependence charts of effective stiffness

**5. Assessing the effect of the non-linear work of the connection and reinforcement on the load-bearing capacity of a timber-reinforced-concrete element**

A numerical experiment was performed to determine the effect of the non-linear character of connection operation

on load based on the linear-elastic and non-linear model of connection operation. Tables 2, 3 give the values of normal stresses depending on the load magnitude for beams with spans of 3 and 5 m, determined from the linear-elastic and proposed model.

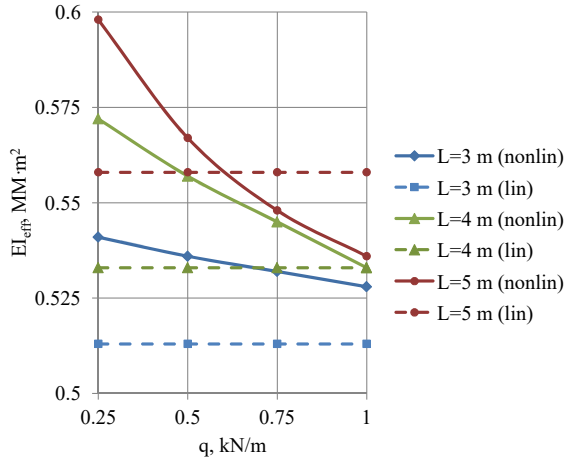


Fig. 4. Dependence chart of effective stiffness on load based on the linear-elastic (lin) and nonlinear model of connection operation (nonlin)

The higher the value of the effective stiffness of a composite component, the higher the degree of its composite action, and the lower the values of internal efforts; hence, the higher the load-bearing capacity. Tables 2, 3 show that at load values of 0.25–0.5 kN/m stresses in the structural elements, based on the linear model, exceed the values derived when considering the non-linear work. The discrepancy in certain values is 1–8 % and decreases with the increase in load. When loads correspond to the plastic deformations of a joint, the stresses, based on the linear-elastic model, are understated.

Our findings suggest that the use of dependences, which take into consideration the non-linear character of the timber-concrete components joints' operation, makes it possible to more accurately determine the parameters of the stressed-strained state. This, in turn, would make it possible to refine the parameters of a structure (for example, to adjust the size of the cross-section depending on the size of the load, as well as the quantity or diameter of the nails).

It should also be noted that at certain load values, the stresses in the stretched region of a concrete slab exceed the strength of the concrete for stretching, while the stresses in a timber beam do not reach the maximum allowable values (Tables 2, 3). In fact, there is a crack in the cross-section of a slab, and the stretching effort is accepted by the reinforcement. The magnitude of the bending moment in a slab for the considered spans is given in Table 4.

The values of normal stresses depending on the load magnitude for a beam with a 3-m span

q, kN/m	Normal stresses in a concrete slab, MPa						Normal stresses in a timber slab, MPa					
	$\sigma_{max,1}$			$\sigma_{min,1}$			$\sigma_{max,2}$			$\sigma_{min,2}$		
	lin	nonlin	%	lin	nonlin	%	lin	nonlin	%	lin	nonlin	%
0.25	-0.32	-0.31	3.9	0.31	0.29	6.9	0.62	0.59	3.5	-0.59	-0.55	7.7
0.5	-0.64	-0.62	3.2	0.62	0.59	6.0	1.23	1.20	2.9	-1.18	-1.11	6.3
0.75	-0.96	-0.93	2.8	0.93	0.89	4.5	1.84	1.80	2.2	-1.77	-1.68	5.4
1	-1.28	-1.25	2.4	1.24	1.19	4.2	2.46	2.41	2.1	-2.37	-2.27	4.4

Table 2

Table 3

The values of normal stresses depending on the load magnitude for a beam with a 5-m span

q, kN/m	Normal stresses in a concrete slab, MPa						Normal stresses in a timber slab, MPa					
	$\sigma_{max,1}$			$\sigma_{min,1}$			$\sigma_{max,2}$			$\sigma_{min,2}$		
	lin	nonlin	%	lin	nonlin	%	lin	nonlin	%	lin	nonlin	%
0.25	-0.87	-0.86	1.2	0.83	0.81	2.2	1.67	1.66	1.0	-1.57	-1.53	2.3
0.5	-1.74	-1.77	-1.6	1.65	1.70	-3.0	3.34	3.39	-1.5	-3.14	-3.24	-3.1
0.75	-2.60	-2.68	-3.0	2.48	2.62	-5.3	5.02	5.15	-2.5	-4.71	-5.00	-5.8
1	-3.47	-3.61	-3.9	3.31	3.54	-6.5	6.69	6.92	-3.3	-6.28	-6.76	-7.1

Table 4  
Bending moment in a slab considering the non-linear operation of the joint

Load q, kN/m	Bending moment in a slab, $M_1$ , kNm, with a span of		
	L=3 m	L=4 m	L=5 m
0.25	0.062	0.105	0.156
0.5	0.126	0.215	0.33
0.75	0.19	0.33	0.514
1	0.255	0.447	0.703

Since in the composite timber-reinforced-concrete floors the thickness of a slab is taken within 50–100 mm, then, due to the design features (providing the necessary thickness of the protective layer), there is a possibility to arrange the reinforcement with-

in the compressed region of concrete. In this regard, we estimated the height of the compressed zone and the load-bearing capacity of the cross-section of a slab measuring  $b_1 \times h_1 = 500 \times 50$  mm, strengthened with the reinforcement  $\varnothing 3, 4$ , and 5, class Vr-I, and a thickness of the concrete protective layer of 15 mm. The calculation results are given in Table 5.

Table 5 shows the variants considered for reinforcing a slab meet the requirements for ensuring the load-bearing capacity of the cross-section of the slab when condition (26) is satisfied. Thus, the calculation of a composite timber-reinforced-concrete element, taking into consideration the reinforcement of the slab, makes it possible to more rationally utilize the load-bearing capacity of the structure.

As shown by the results of the parametric analysis, accounting for the deformation of a nail joint exerts a significant impact on the results of determining the parameters of the joint work and the stressed-strained state. According to the charts in Fig. 3, one can see that for a span of  $L=3$  and 4 m the values of effective stiffness based on the linear-elastic model are lower than those determined considering the non-linear character of connection operation. At the same time, as the workload increases, the degree of composite action decreases. A given feature is clearly visible for a span of  $L=5$ : the chart shows that at a load exceeding 0.6 kN/m, the effective stiffness, taking into consideration the non-linear operation of the connection, is lower than that based on the linear model.

## 6. Discussion of results of studying the composite timber-reinforced-concrete bending elements taking into consideration the non-linear work of the joint

A special feature of the proposed procedure for calculating timber-reinforced-concrete bending elements is the possibility to take into consideration the non-linear character of the deformation of the joint by introducing a function of the slip modulus. The required coefficients are determined on the basis of the rated characteristics, set by the currently acting building code, and do not require additional experimental research.

Depending on the magnitude of the load, the normal stresses, calculated using the proposed procedure and the linear-elastic model, differ, for the options considered, by 1–8 %. At load values of up to 0.5 kN/m, the stresses in the structure elements based on the linear model exceed the values determined considering the non-linear work. The discrepancy in certain values decreases with the increase in load, which is associated with a gradual decrease in the magnitude of the slip modulus. When loads correspond to the plastic deformations of the joint, the stresses based on the linear-elastic model are understated.

At certain load values, the stresses in the stretched region of a concrete slab exceed the strength of the concrete at stretching (Tables 2, 3), while the stresses in a timber beam do not reach the ultimate values. In fact, in the cross-section of the slab, there is a crack, and the stretching force is accepted by the reinforcement. Therefore, when calculating a structure without taking into consideration the work of the reinforcement, the load-bearing capacity is underutilized.

Given the design features of the composite timber-reinforced-concrete structures (the thickness of a slab is 50–100 mm, ensuring the necessary thickness of the protective layer), there is a possibility to arrange reinforcement within the compressed zone of concrete. In this case, the reinforcement in the cross-section would not work on stretching, and, in fact, the physical meaning of the combined work of concrete and reinforcement in the reinforced-concrete bending element is lost. Based on the results of analyzing the cross-section of a slab of the examined structure, it was found that the load-bearing capacity of the slab, taking into consideration the reinforcement, ensures the acceptance of the bending moment (Table 5) up to the loads that cause the destruction of the timber beam. At the same time, the con-

ditions of the rational operation of compressed concrete and stretched reinforcement are met.

The limitation of the proposed procedure is the type of malleable bonds as the dependence (20) and related coefficients (11) to (13) are applicable for round-section steel nails only.

This paper does not consider the calculation of a structure for operational suitability. Analytical dependences need to be developed to determine the deflections, which do not affect the results obtained but are a challenge for follow-up studies. In addition, the next stage of our research is the test of a timber-reinforced-concrete beam, during which it is planned to compare the theoretical and experimental data, to identify the reserves of the load-bearing capacity.

## 7. Conclusions

1. A dependence has been proposed to determine the slip modulus of the nail joint of a timber-concrete structure. The dependence's coefficients are determined on the basis of the rated characteristics, set by the currently acting building code, and do not require additional experimental research. By using known calculation procedures and the derived dependence, an algorithm has been developed to calculate the composite timber-reinforced-concrete bending structures, taking into consideration the deformation diagram of the joint and reinforcement in the stretched zone of the concrete element.

2. During the numerical experiment, it was found that for the variants considered, the normal stresses in the cross-section, calculated using the proposed procedure, are 1.8 % different from those determined using the linear-elastic model. Under the loads corresponding to the plastic deformations of the joint, the stresses that are based on the linear-elastic model are understated compared to the proposed procedure. Given the design features of the timber-reinforced-concrete floors (the thickness of the slab and the necessary protective layer), we analyzed the load-bearing capacity of a slab taking into consideration its reinforcement. It has been established that the load-bearing capacity of the slab, taking into consideration its reinforcement, ensures the acceptance of the estimated bending moment up to the loads that cause the destruction of the timber beam, while meeting the condition for the rational use of the strength properties of compressed concrete and stretched reinforcement.

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