Запропоновано напрямки оптимізації моделі економічного замовлення за умови незначних змін вхідних параметрів методами збурень.

Для досягнення мети отримано аналітичні формули моделі економічного розміру замовлення на основі асимптотичного підходу за умов незначних змін вхідних параметрів. Як змінні параметри системи приймалися: дискретне зростання витрат на виконання замовлення, витрати на виконання замовлення та зберігання запасів, які залежать від «малого параметру», а також періодичні коливання попиту на продукцію.

На основі асимптотичного методу збурень одержано зручну у використанні формулу для визначення економічного розміру замовлення за умови незначного зростання витрат на виконання замовлення. Також визначено відсоткове відхилення «збуреного» розміру замовлення від формули Вільсона. Оцінка чутливості EOQ-моделі виявила, що відносне відхилення «збуреного» обсягу замовлення від оптимального при незначних змінних витратах на його виконання варіюється в межах від 1 % до 15 % в залежності від періоду. Порівняльний аналіз загальних витрат, розрахованих з використанням асимптотичної формули та формули Вільсона, виявив, що урахування зміни обсягів замовлення призводить до зменшення витрат фірми.

Побудована двопараметрична модель оптимального розміру замовлення, яка враховує як незначні зміни витрат на виконання замовлення, так і витрат на зберігання запасів. Одержані двопараметричні асимптотичні формули для визначення оптимального розміру замовлення та загальних витрат, які відповідають «збуреному» розміру замовлення.

Запропонована в роботі асимптотична модель, яка враховує дискретне незначне підвищення витрат на виконання замовлення та періодичний характер коливань попиту на продукцію, має практичне значення. Ця модель може бути використана для оптимізації системи управління логістикою підприємства через наближеність до реалій та зручність у використанні

Ключові слова: модель управління запасами, малий параметр, метод збурень, асимптотичне розвинення, розмір замовлення

OPTIMIZATION OF INVENTORY MANAGEMENT MODELS WITH VARIABLE INPUT PARAMETERS BY PERTURBATION METHODS

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1. Introduction

The current economic situation is characterized by an aggravation of the global crisis phenomena caused by various factors including the pandemic consequences. It affects many countries and leads to the loss of productive capacity and opportunities for the development of both the global and local economies caused by breakage of existing logistic links, disturbance of transport corridors, and fall of consumption and production.

Production potential of a modern enterprise depends on available fixed and circulating assets and other factors. Because of long-term work in conditions of quarantine and economic constraints, circulating assets of enterprises and organizations decreased and fell to a critical minimum in some cases. The companies planning to continue their

activities are forced to look for opportunities to resume financing of circulating assets. In conditions when external borrowings are limited in the number and interest rates, own financial assets of enterprises are the most available sources.

Companies face a problem of optimizing the use of circulating assets which is possible, in particular, by improving their inventory management system and logistics. This necessitates that managers applied new approaches and methods of material flow management. Such methods make it possible to consider the management of material resources as a part of the basic strategic priorities of the company development and application of corresponding modern and effective software. This ensures quick analysis of available material resources coordinating them with the needs of consumers and peculiarities of interaction with suppliers.

There are numerous software products in the market that help improve the quality and efficiency of the logistic process management. Mainly functions of accounting and movement of goods are implemented and used in them but there are almost no effectively implemented functions of calculating purchase volumes and replenishment time. This significantly limits the effectiveness of the proposed software products. Insufficient development of analytical tools, models, and methods of inventory management is one of the causes of the absence of effective implementation of these functions.

Modeling of logistic systems at the present stage of their development requires the use of mathematical optimization tools that have proven their effectiveness, especially in the field of inventory management. The existing models used in the management decision-making process in functional logistics have many limitations, such as invariance of input parameters of the model which significantly narrows possibilities of their use in practice. Divergences of a simplified economic and mathematical model from a real one which describes the company's logistic system may differ. In the case when input parameters of the system undergo minor changes, divergences in simulation results may be economically small in the area of input parameters. In this case, asymptotic methods of "perturbations" can be used to model logistic processes. This makes it possible to obtain an approximate solution of the problem with an acceptable error and in an analytical form convenient for use. Moreover, asymptotic methods make it possible to use the earlier obtained analytical solutions of applied problems to solve similar but more intricate problems by establishing relations between them.

The use of asymptotic methods to solve applied economic problems in the field of logistics will allow the company's management to obtain clear and simple analytical calculation formulas. These formulas will provide an opportunity to optimize the company's overall logistics costs and improve the competitiveness of the companies that will use them.

2. Literature review and problem statement

Of the recently published studies of asymptotic methods, it is necessary to note the study [1] in which asymptotic ideas and methods are presented at a level understood by many readers. The main ideas of asymptotic approaches were stated and peculiarities of their application in various fields investigated. An overview of the perturbation theory methods for solving differential equations describing prob-

lems of applied physics and mechanics was given and the advantages of these methods and fields of their application were discussed in [2].

The study [3] is devoted to the definition of the essence of the perturbation methods which consists in the fact that the problem solution is sought in a form of serial expansion by the small parameter powers. This parameter appears in the model either naturally or is artificially entered for convenience. The method assumes that a corresponding asymptotic sequence which in the simplest and most common variant is taken as the power function of the small parameter ε^n is chosen for the study of an applied problem.

Perturbation methods are used:

– in mechanics of solid structures (for example, solution of the problem of stability of a circular cylindrical shell of variable thickness under axial compression was developed in [4] with the help of asymptotic approach; a variant derivation of the changed Bress-Tymoshenko equations with the asymptotic approach was offered in [5]);

– in differential equations. For example, the perturbation method was applied in [6] to the problem of vibration of a piezoelectric sandwich plate taking into account the effect of shear force. A hybrid method based on a combination of analytical asymptotic WKB approach and numerical Galerkin method was used in [7].

Application of perturbation methods to the problems of mechanics of solid structures and differential equations is explained by the fact that they enable the construction of an approximate analytical solution and estimation of the system sensitivity to the changes in input parameters. These methods have not yet been used in applied economic studies because of their narrow specialization and predominant focusing on the study of the behavior of mechanical objects. However, the scope of these methods can be extended to the problems of optimizing economic processes and making managerial decisions. The analytical formulas obtained through the application of perturbation methods are convenient for enterprise managers and can also expand the scope of the economic models, in particular, the inventory management models.

Problems of inventory management were studied in [8–29]. Features of construction of deterministic single-product and multi-product models of procurement logistics which help managers in the decision-making process were studied in [8, 9]. However, input parameters of these models are considered constant which limits fields of their practical use.

An EOQ model of procurement logistics with the emergence of a shortage caused by the possibility of the presence of defective products in the ordered batch was developed in [10]. This model is limited in terms of accounting minor discrete changes in the order fulfillment costs. A dynamic model of procurement logistics with the emergence of a shortage in demand linearly depending on time was constructed in [11]. This model makes it possible to estimate optimal order quantity, order periodicity, and total costs under this assumption. However, it does not take into account changes in the order fulfillment and inventory storage costs. Further source [12] concerns the study of the behavior of the model of purchasing perishable goods in the presence of inflation and possible delays in payments. The issue of possible payment delays was analyzed in [13] but the models did not take into account possible fluctuations in the demand for the proposed products. A model of inventory management under the condition of the demand

fluctuation and inventory damage caused by suboptimal warehouse location, improper storage conditions, etc. was studied in [14]. In this case, the rate of inventor damage was distributed according to the Weibull function and the inventory maintaining cost was considered a discrete variable. The use of fixed costs for order fulfillment can be considered a disadvantage of this model. The EOQ models constructed in [15] for perishable products take into account possible deterioration of product quality, possible penalties presented in a form of exponential and linear functions. Further study [16] concerned the construction of an integrated inventory management model for one supplier and one customer with a reduction in order costs depending on the time of the order fulfillment but the variability of storage costs was not taken into account. Study [17] was also devoted to the improvement of the single-product model. The study considered peculiarities of the procedures of EOQ-optimization of the supply chain. Such procedures will take into account the feasibility of sharing several vehicles for each order and specifics of payment of the storage costs both in a form of rent and paying only for the occupied warehouse space. However, there are no analytical formulas convenient for application and further analysis by the company's management. Application of the asymptotic approach to determining the single-product order quantity under the condition of variable costs of order fulfillment was given in [18]. However, the cost of inventory storage was calculated taking into account the area occupied by the warehouse for a certain period, and the spatial dimensions of the product unit.

Some researchers consider variations of real situations in the company's logistics management system, namely inflation, sudden rises and falls in demand, etc. and present their models of inventory management in an uncertain environment. For example, a problem of determining economic order quantity (EOQ) for the case when input parameters are probabilistic and optimal probability distribution functions are calculated using a geometric programming model was studied in [19]. A stochastic problem of finding EOQ in a certain time interval was solved in [20]. However, the study was mainly theoretical in nature and did not focus on a practical application.

The study [21] was devoted to the development of an inventory management model that would optimize costs and order quantity taking certain non-deterministic parameters as fuzzy numbers. Demand and related costs are taken in the study [22] as fuzzy variables, and the Jaeger ranking method for fuzzy numbers was used to determine an optimal inventory management policy. However, the obtained models are difficult to apply in practice for want of convenient analytical formulas.

The study [23] has proposed a multi-product inventory management model for a double-level supply chain under the condition of incentive-sensitive demand marketing. In this case, the supplier offers the retailer an opportunity to delay the payment of the purchase price of finished products. Modeling of logistic processes at an enterprise in conditions of ordering a wide range of products from one supplier using asymptotic methods was proposed in [24]. Under the conditions of variable supply costs, a model was obtained that makes it possible to adapt the enterprise's logistic system to the existing business conditions. However, insufficient attention was paid to determining optimal order quantity under conditions of the variability of solely order fulfillment

costs. Further development of the asymptotic approach to multi-item supply was given in [25] which takes into account changes in the storage costs but at a constant demand for the proposed products.

A model of inventory management at the retail level in a system with reverse logistics which allows the company to maximize profits taking into account volumes of supplies and their timing was considered in [26]. However, the proposed model takes into account price variability and not the variability of other input parameters.

Most studies do not take into account transport costs in the analysis of the EOQ or consider the transport costs as a constant part of the order cost which is taken into account in [27] but researchers use an iterative method difficult in application.

A model taking into account limited resources for storage of products, namely allocation of space in refrigeration equipment in retailing is another modification of the EOQ model [28]. This implies the presence of additional costs to maintain a certain storage temperature but the results do not take into account changes in the order fulfillment and inventory storage costs.

Some researchers [29] address the problems of developing analytical tools to determine EOQ but these tools are imperfect and need further study.

The optimization methods analyzed in the known studies can effectively solve the problems of inventory management but they are often incomprehensible to practicing managers because of their high complexity (the use of iterative processes in calculations, complicated mathematical apparatus, etc.). Both researchers and managers find it more convenient to use analytical models that describe the object behavior according to clear formulas and taking into account variability of order fulfillment and inventory storage costs as well as fluctuation of demand for the proposed products.

3. The aim and objectives of the study

The study objective is to optimize the inventory management model under conditions of minor changes in input parameters with the application of perturbation methods. This will expand the scope of this model application in practice.

To achieve this objective, the following tasks were set:
– to obtain an asymptotic formula of the EOQ model with an insignificant discrete increase in the order fulfillment costs;
– to obtain an analytical formula for the EOQ model at variable costs of order fulfillment and inventory storage which depend on the "small parameter";
– to derive an asymptotic formula of the EOQ model under the condition of periodic fluctuation of the demands for products.

4. Studying the problem of determining EOQ by perturbation methods

4. 1. Asymptotic expansion of solution under the condition of discrete growth of the order fulfillment cost

The economic order quantity model (the EOQ model) or Wilson's formula used to estimate the order quantity can be presented as follows [4]:

$$q_{opt} = \sqrt{\frac{2C_0 S}{h}}, \qquad (1)$$

where q_{opt} is the EOQ; C_0 is the cost of the fulfillment of one order, S is the amount of demand in a certain period of time; h is the cost of storage of a product unit for a certain period of time.

The cost of delivery is one of the main components of the order fulfillment costs which is constantly rising because of rising fuel prices. Assuming that the order costs increase by $i\%$ during a certain period of time (for example, each month), then it will reach

$$C_0 \cdot \left(1 + \frac{i\%}{100\%}\right)^n$$

after n periods of time. Taking the ratio $\varepsilon = \frac{i\%}{100\%}$ ($\varepsilon \ll 1$) for a small parameter, the order cost is obtained in this form: $C_0 \cdot (1+\varepsilon)^n$. Since the ε parameter is the small parameter, we can assume that deviation from the initial value of C_0 is insignificant and the condition of constant order fulfillment costs is satisfied.

The "perturbed" EOQ, q^*_{opt}, can be represented as an asymptotic expansion of the artificially introduced small parameter ε:

$$q^*_{opt} = q_0 + q_1 \cdot \varepsilon + q_2 \cdot \varepsilon^2 + \dots . \qquad (2)$$

Formulas for determining the order quantity take the form (3):

$$q_0 + q_1 \cdot \varepsilon + q_2 \cdot \varepsilon^2 + \dots = \sqrt{\frac{2C_0 (1+\varepsilon)^n S}{h}}. \qquad (3)$$

When squaring both parts of equation (3), series expansion in the Taylor row $(1+\varepsilon)^n$ and neglecting the members of order ε^3 and more, we have:

$$\left(q_0 + q_1 \cdot \varepsilon + q_2 \cdot \varepsilon^2 + \dots\right)^2 =$$
$$= \frac{2C_0 S}{h}\left(1 + n \cdot \varepsilon + \frac{n \cdot (n-1)}{2} \cdot \varepsilon^2 + \dots\right), \qquad (4)$$

$$q_0{}^2 + 2q_0 \cdot q_1 \cdot \varepsilon + q_1{}^2 \cdot \varepsilon^2 + 2q_0 \cdot q_2 \cdot \varepsilon^2 + \dots =$$
$$= \frac{2C_0 S}{h}\left(1 + n \cdot \varepsilon + \frac{n \cdot (n-1)}{2} \cdot \varepsilon^2 + \dots\right). \qquad (5)$$

By equating the coefficients at the same powers of the parameter ε, the equation for determining the unknown q_0, q_1, q_2 is obtained:

$$\varepsilon^0 : q_0{}^2 = \frac{2C_0 S}{h}, \qquad (6)$$

$$\varepsilon^1 : q_0 \cdot q_1 = \frac{C_0 S}{h} \cdot n, \qquad (7)$$

$$\varepsilon^2 : q_1{}^2 + 2q_0 \cdot q_2 = \frac{C_0 S}{h} \cdot n \cdot (n-1). \qquad (8)$$

The solution to equations (6) to (8) gives:

$$q_0 = \sqrt{\frac{2C_0 S}{h}}, \quad q_1 = \frac{n}{2} \cdot \sqrt{\frac{2C_0 S}{h}},$$

$$q_2 = \frac{n \cdot (n-2)}{8} \cdot \sqrt{\frac{2C_0 S}{h}}. \qquad (9)$$

To obtain an asymptotic representation of formula (1), the found values (9) are to be substituted into expansion (2). As a result, formula (10) is obtained.

$$q^*_{opt} = \sqrt{\frac{2C_0 S}{h}} \cdot \left(1 + \frac{n}{2} \cdot \varepsilon + \frac{n \cdot (n-2)}{8} \cdot \varepsilon^2\right). \qquad (10)$$

As can be seen from formula (10), the "perturbed" order quantity differs from that obtained from Wilson's formula (1) by a multiplier

$$\left(1 + \frac{n}{2} \cdot \varepsilon + \frac{n \cdot (n-2)}{8} \cdot \varepsilon^2\right).$$

The total company's costs under the condition of an insignificant discrete increase in the cost of order fulfillment are as follows:

$$TC\left(q_{opt}\right) = \frac{C_0 (1+\varepsilon)^n S}{q_{opt}} + \frac{h q_{opt}}{2} =$$
$$= \sqrt{\frac{C_0 S h}{2}} \cdot \left(2 + n \cdot \varepsilon + \frac{n \cdot (n-1)}{2} \cdot \varepsilon^2\right), \qquad (11)$$

$$TC^*\left(q^*_{opt}\right) = \frac{C_0 (1+\varepsilon)^n S}{q^*_{opt}} + \frac{h q^*_{opt}}{2} =$$
$$= \sqrt{\frac{C_0 S h}{2}} \cdot \left(2 + n \cdot \varepsilon + \frac{n \cdot (n-2)}{4} \cdot \varepsilon^2\right). \qquad (12)$$

As can be seen from the obtained formulas, the total costs corresponding to the economic (1) and "perturbed" order quantities (10) at a slight increase in the order fulfillment costs reach a minimum at q which corresponds to (10).

Let us analyze the sensitivity of the obtained model of determining the EOQ to a change of input parameters, namely the order fulfillment costs. Calculate the relative deviation of the optimal batch volume at variable order fulfillment costs by varying the ε parameter. The percentage deviation of the "perturbed" order quantity from the economic one calculated according to formula (10) is presented in Fig. 1.
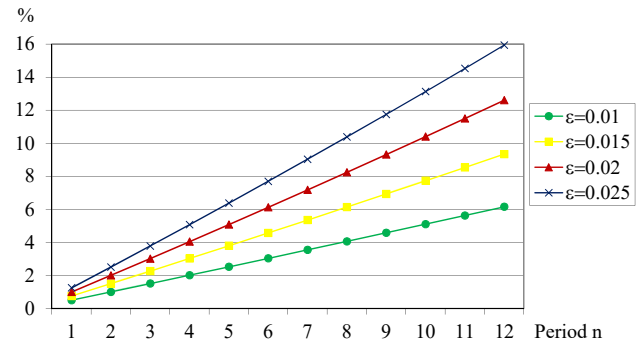

Fig. 1. Deviation (%) of the "perturbed" order quantity from Wilson's formula under the condition of increase in the order fulfillment costs in periods n

Let us consider the cases when there is an increase in the order fulfillment costs by 1 % ($\varepsilon=0.01$), 1.5 % ($\varepsilon=0.015$), 2 % ($\varepsilon=0.02$) and 2.5 % ($\varepsilon=0.025$) in each period. As can be seen in Fig. 1, a gradual increase in the order costs by 1 % ($\varepsilon=0.01$) leads to an increase in the order quantity by 3.03 % and 6.15 % in periods 6 and 12, respectively. If the order cost gradually increases with each period, for example, by 2.5 % ($\varepsilon=0.025$), the value of deviation is 7.69 % in period 6 and 15.94 % in period 12.

Let us test the constructed model for optimization of the inventory management model under the condition of insignificant changes in input parameters on an example of the purchase of coffee and tea by an enterprise working in the HoReCa segment. The initial data and calculation of the model parameters are given in Table 1. The optimal order quantity was calculated from Wilson's formula (1). The perturbed order quantity was determined under the condition of increasing the order fulfillment costs at $\varepsilon=0.025$ using formula (10).

Values of the perturbed order quantity q_{opt}^* at the end of the year, i. e. $n=12$ are given in Table 1. As can be seen, the absolute deviation of the order quantity at the end of the period is 43 packs (15.93 %) for tea and 12 packs (15.58 %) for coffee.

Comparative analysis of total costs calculated from formulas (11), (12) makes it possible to note that the costs corresponding to the perturbed order quantity are less than the costs corresponding to the order quantity calculated by Wilson's formula (1).

Table 1

Initial data and results of the calculation of coffee and tea delivery

Product type	Annual demand S, q_{opt} packs	Storage costs, h, mon. un.	Order fulfillment costs, C_0, mon. un.	Order quantity, packs.		Total costs, mon.un.	
				q_{opt}	q_{opt}^*	TC	TC^*
Tea (10 filter packs)	700	1.0	52	270	313	315.9	312.8
Coffee-beans (1 kg)	240	4.5	56	77	89	407.1	403.2

4. 2. Solving the problem under the condition of variation of the order fulfillment and inventory storage costs

In practice, not only the order fulfillment costs but also the inventory storage costs increase because of rising expenses (for example, rising prices for electricity, utilities, etc.). Let us assume that the storage costs increase with each period of time by j %. Taking the value $\beta=\dfrac{j\%}{100\%}$ ($\beta\ll1$) as the small parameter, dependence of the storage costs is obtained in the form: $h\cdot(1+\beta)^m$.

A change in the delivery costs as one of the main components in the structure of order fulfillment costs and rising utility prices which, accordingly, increase storage costs, often occur in various time periods. Thus, due to various reasons, prices for resources and materials increase and this is reflected in the transport tariffs.

It is advisable to take into account various combinations of values of n and m, ε and β parameters in the EOQ model taking into account the fact that the interval m of changes in storage costs is usually smaller than the interval n of changes in the order fulfillment cost. The β parameter which characterizes an increase in the storage costs may exceed the value

of the parameter ε which characterizes the increasing costs of order fulfillment.

If the change in the storage costs is delayed compared to the change in the order fulfillment costs, the relationship between parameters n and m can be expressed, e. g. by putting $m=\left[\dfrac{n}{4}\right]$, $m=\left[\dfrac{n}{6}\right]$, etc., where $[n]$ means an integer part of the number. Because of small values of the parameters ε and β, assume that deviations from the initial values of C_0 and h are small and the requirements occurring in the model (1) are satisfied.

By representing q_{opt}^* as an asymptotic expansion by two small parameters ε and β and neglecting members of the order of ε^3, β^3, $\varepsilon^2\beta$, $\varepsilon\beta^2$ and above, formula (13) is obtained:

$$q_{opt}^* = q_0 + q_1\cdot\varepsilon + q_2\cdot\beta + $$
$$+q_3\cdot\varepsilon^2 + q_4\cdot\varepsilon\cdot\beta + q_5\cdot\beta^2 + \dots . \qquad (13)$$

When decomposing the functions $(1+\varepsilon)^{\frac{n}{2}}$ and $(1+\beta)^{-\left(\frac{m}{2}\right)}$ into a Taylor series, asymptotic formulas for two parameters ε and β for the "perturbed" order quantity take the following form (14):

$$q_{opt}^* = \sqrt{\frac{2C_0 S}{h}}\cdot\left(\begin{array}{c}1+\dfrac{n}{2}\cdot\varepsilon - \dfrac{m}{2}\beta + \dfrac{n\cdot(n-2)}{8}\cdot\varepsilon^2 - \\ -\dfrac{m\cdot n}{4}\varepsilon\cdot\beta + \dfrac{m\cdot(m+2)}{8}\beta^2\end{array}\right). \qquad (14)$$

Total costs TC at a condition of insignificant increase in the order fulfillment and inventory storage costs for the economic (1) and "perturbed" order quantity (14) are obtained in the form:

$$TC(q_{opt}) = \sqrt{\frac{C_0 S h}{2}}\times\left(\Omega + \frac{1}{2}(n\cdot\varepsilon - m\cdot\beta)^2\right), \qquad (15)$$

$$TC^*(q_{opt}^*) = \sqrt{\frac{C_0 S h}{2}}\times\left(\Omega + \frac{1}{4}(n\cdot\varepsilon - m\cdot\beta)^2\right), \qquad (16)$$

where

$$\Omega = \left(2 + n\cdot\varepsilon + m\cdot\beta - \frac{n}{2}\cdot\varepsilon^2 - \frac{m}{2}\cdot\beta^2 + m\cdot n\cdot\varepsilon\cdot\beta\right).$$

It can be seen that the total costs TC (16) corresponding to the "perturbed" order quantity is less than (15) which corresponds to the economic quantity.

To analyze the sensitivity of the order quantity to the change in the cost of order fulfillment and the cost of inventory storage, the ratio of the "perturbed" order quantity (14) to the optimal one (1) was calculated. Different values of input parameters ε, β and different intervals of cost changes n and m were used.

Percentage deviation of the "perturbed" order quantity from the value of economic order quantity (1) provided that the cost of the order fulfillment is fixed ($\varepsilon=0.0$) is shown in Fig. 2.

As can be seen in Fig. 2, a gradual increase in the storage costs by 5 % ($\beta=0.05$) in periods $n=4$, $n=8$ results in a decrease in the order quantity by 2.41 %, 4.75 %, respectively. If the storage costs change significantly, for example, increases by 20 % ($\beta=0.2$) each quarter (periods $n=4$, $n=8$, and $n=12$), the order quantity decreases by 8.5 %, 16 %, and 22.5 % respectively.
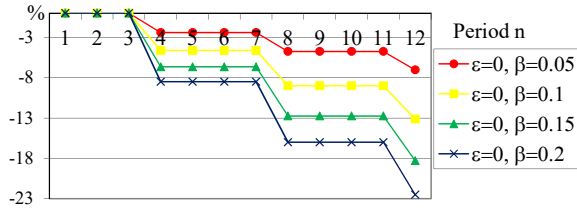
Fig. 2. Deviation (%) of the "perturbed" order quantity from the economic one depending on the period n provided that the costs of the order fulfillment are fixed,

$$\left(\varepsilon = 0, \; m = \left[\frac{n}{4}\right]\right)$$

The ratio of the "perturbed" order quantity (14) to the economic one (1) and the percentage deviation provided that the order fulfillment costs and the of storage costs undergo minor changes are given in Table 2.

Table 2

The ratio of "perturbed" order quantity to the economic one, subject to changes in the order fulfillment costs and the storage costs

Period		ε=0.01, β=0.1		ε=0.01, β=0.2		ε=0.02, β=0.2	
n	m	$\dfrac{q^{*}_{opt}}{q_{opt}}$	%	$\dfrac{q^{*}_{opt}}{q_{opt}}$	%	$\dfrac{q^{*}_{opt}}{q_{opt}}$	%
0	0	1.0	0	0	0	1.0	0
1	0	1.005	+0.5	1.005	+0.5	1.010	+1.0
2	0	1.010	+1.0	1.010	+1.0	1.020	+2.0
3	0	1.015	+1.5	1.015	+1.5	1.030	+3.0
4	1	0.973	−2.7	0.933	−6.7	0.951	−4.9
5	1	0.978	−2.2	0.938	−6.2	0.961	−3.9
6	1	0.983	−1.7	0.942	−5.8	0.970	−3.0
7	1	0.987	−1.3	0.947	−5.3	0.980	−2.0
8	2	0.947	−5.3	0.873	−12.7	0.906	−9.4
9	2	0.951	−4.9	0.877	−12.3	0.915	−8.5
10	2	0.956	−4.4	0.881	−11.9	0.924	−7.6
11	2	0.961	−3.9	0.885	−11.5	0.933	−6.7
12	3	0.921	−7.9	0.819	−18.1	0.865	−13.5

Data in Table 2 show that an increase in storage costs every 4 months leads to a significant reduction in the "perturbed" order quantity compared to the economic one. For example, when n=4 and m=1 at ε=0.01, β=0.1, the order quantity decreases by 2.7 %; when ε=0.01, β=0.2, there is a decrease in the order quantity by 6.7 %. In the next period, n=8 and m=2 at ε=0.01, β=0.1, the order quantity decreases by 5.3 %; and at ε=0.01, β=0.2, the order quantity decreases by 12.7 %. The changes in costs that occur within one specified period (4 months) m=1 and a further increase in the costs of order fulfillment (n varies from 4 to 7) lead to a gradual decrease in deviation from the undisturbed value.

Let us consider, for example, a situation when n=4, and m=1 at a constant percentage increase in the storage costs (β=0.2) and a percentage increase in the order fulfillment costs (from ε=0.01 to ε=0.02). Deviation of the "perturbed" order quantities from those calculated by formula (1) decreases from −6.7 % to −4.9 % and from −12.7 % to −9.4 % at n=8 and m=2.

Table 3 shows the sensitivity of the "perturbed" order quantity to the economic one depending on the rate of growth of the storage costs. The change in costs occurs once

every 3 periods in the first case $\left(m = \left[\frac{n}{3}\right]\right)$, and once every 4 periods in the second case $\left(m = \left[\frac{n}{4}\right]\right)$.

Table 3

Sensitivity of the "perturbed" order quantity to the storage costs

Period		ε=0.01, β=0.1	ε=0.01, β=0.2	Period		ε=0.01, β=0.1	ε=0.01, β=0.2
n	m	$\dfrac{q^{*}_{opt}}{q_{opt}}$	$\dfrac{q^{*}_{opt}}{q_{opt}}$	n	m	$\dfrac{q^{*}_{opt}}{q_{opt}}$	$\dfrac{q^{*}_{opt}}{q_{opt}}$
0	0	1.0	1.0	0	0	1.0	1.0
1	0	1.005	1.005	1	0	1.005	1.005
2	0	1.010	1.010	2	0	1.010	1.010
3	1	0.968	0.929	3	0	1.015	1.015
4	1	0.973	0.933	4	1	0.973	0.933
5	1	0.978	0.938	5	1	0.978	0.938
6	2	0.937	0.864	6	1	0.983	0.942
7	2	0.942	0.868	7	1	0.987	0.947
8	2	0.947	0.873	8	2	0.947	0.873
9	3	0.908	0.807	9	2	0.951	0.877
10	3	0.912	0.811	10	2	0.956	0.881
11	3	0.917	0.815	11	2	0.961	0.885
12	4	0.880	0.758	12	3	0.921	0.819

As can be seen from Table 3, dynamics and interval of increase in the storage tariff rate affects the order quantity. For example, the deviation of the order quantity is −6.3 % at n=6, ε=0.01, and β=0.1 if storage costs change every 3 periods and −1.7 % when storage costs change every 4 periods. At n=6, ε=0.01 and β=0.2, these changes are more significant, namely −13.6 % and −5.8 %, respectively.

Dependence of the ratio of "perturbed" order quantity to the optimal one on parameters ε and β at n=4 is shown in Fig. 3.
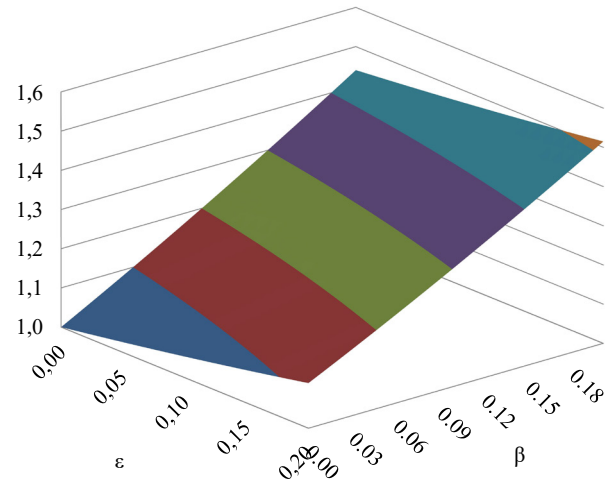

Fig. 3. Dependence of the ratio of "perturbed" order quantity to the optimal one on the parameters ε and β

Fig. 3 shows the nature of deviation of the "perturbed" order quantity from the economic one under different conditions of gradual increase in the order fulfillment and inventory storage costs.

By modeling the nature of changes in the order fulfillment costs and utility tariffs in a short term, managers can

make appropriate adjustments to the organization of the company's procurement by determining the order quantity by means of an asymptotic formula (14).

4. 3. Solution of the EOQ problem using the asymptotic method under the condition of periodic fluctuations in demand for the proposed products

In practice, in addition to the changes in order fulfillment and inventory storage costs, demand S for products may also fluctuate depending on various exogenous and endogenous factors, such as seasonality, etc.

Take the order fulfillment costs as $C_0(1+\varepsilon)^n$. Periodic changes in demand S can be represented as a function $S\left(1-\beta\sin\dfrac{\pi n}{2}\right)$, where $\beta \ll 1$ is the small parameter.

Using the procedure described above, the order quantity q_{opt}^* was defined as the asymptotic expansion of two small parameters ε and β while neglecting the members ε^3, $\varepsilon^2\beta$ and the members of higher orders:

$$q_{opt}^* = (q_0 + \beta \cdot q_1) + (\tilde{q}_0 + \beta \cdot \tilde{q}_1) \cdot \varepsilon + q_2 \cdot \varepsilon^2 + \dots . \qquad (17)$$

The asymptotic formula for the "perturbed" order quantity takes the form:

$$q_{opt}^* = \sqrt{\frac{2C_0 S}{h}} \cdot \left(\begin{array}{c} 1 + \dfrac{n}{2} \cdot \varepsilon - \dfrac{1}{2}\beta\sin\dfrac{\pi n}{2} - \\[2mm] -\dfrac{n}{4}\varepsilon \cdot \beta\sin\dfrac{\pi n}{2} + \dfrac{n \cdot (n-2)}{8} \cdot \varepsilon^2 \end{array} \right). \qquad (18)$$

As can be seen from formula (18), the "perturbed" order quantity differs from Wilson's formula (1) in the multiplier:

$$\left(1 + \frac{n}{2} \cdot \varepsilon - \frac{1}{2}\beta\sin\frac{\pi n}{2} - \frac{n}{4}\varepsilon \cdot \beta\sin\frac{\pi n}{2} + \frac{n \cdot (n-2)}{8} \cdot \varepsilon^2 \right).$$

Depending on the value of the sine function, the value of the order quantity will undergo periodic changes in the direction of increase or decrease relative to the value given by Wilson's formula (1).

Fig. 4 shows the deviation of the "perturbed" order quantity from the economic one under the condition of a gradual increase in the order fulfillment costs and insignificant fluctuations in demand (while fixing the amplitude of demand fluctuation $\beta=0.02$).
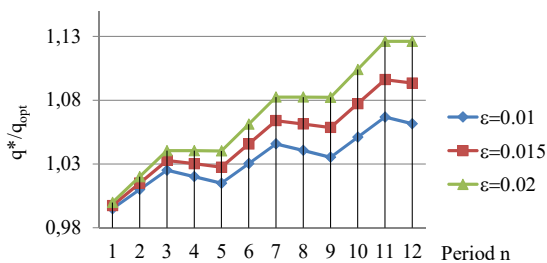

Fig. 4. Sensitivity of deviation of the perturbed order quantity from the optimal one (q^*/q_{opt}) to a gradual increase in the order fulfillment costs depending on the period n at insignificant demand fluctuations

As can be seen in Fig. 4, the increase in the order fulfillment cost and insignificant demand fluctuations cause fluctuations in the order quantity. Moreover, the larger the value of the perturbation parameter ε, the more significant is

a deviation from the economic size of the order calculated by formula (1). For example, the deviation of the order quantity from the optimal one is +6.4 % at $\varepsilon=0.015$ in the 7th period and this deviation is as high as +8.24 % at $\varepsilon=0.02$.

Fig. 5 shows sensitivity of the economic order quantity to the amplitude of demand fluctuations at fixed rates of growth of the order fulfillment costs ($\varepsilon=0.01$ was recorded in Fig. 5).
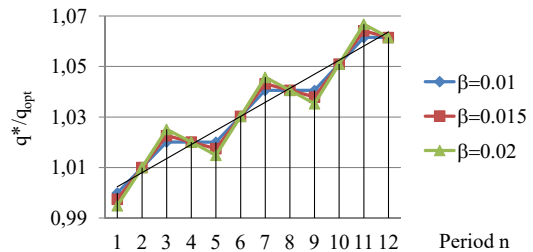

Fig. 5. Sensitivity of the order quantity (q^*/q_{opt}) to the amplitude of demand fluctuations depending on the period n at fixed rates of growth of the order fulfillment costs

Fig. 5 shows that provided the parameter ε is fixed, an increase in the amplitude of the demand fluctuations leads to an increase in the order quantity in odd periods $n=3, 7, 11,\dots$. On the contrary, in odd periods $n=5, 9,\dots$, it decreases depending on the form of the function chosen for the demand approximation. Therefore, the company's management must take these fluctuations into account when deciding on an order.

Let us consider, for example, period 3 when the order fulfillment costs increase by 1 % ($\varepsilon=0.01$) and the amplitude of demand fluctuations is 1.5 % ($\beta=0.015$). The "perturbed" order quantity increases by 2.26 % compared to the economic order quantity and the deviation will be +6.4 % in the 11th period under these conditions. However, if the order is fulfilled in the 5th period, the deviation will decrease.

Fig. 6 shows the sensitivity of deviation of the order quantity to the gradual increase in the order fulfillment costs at insignificant demand fluctuations. To build the graph, it is necessary to fix the amplitude of demand fluctuations (the value of parameter $\beta=0.015$) and change ε in the range from 0 to 0.02 (the rate of change of the order fulfillment costs does not exceed 2 %).
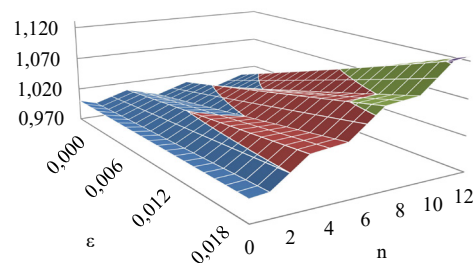

Fig. 6. Sensitivity of deviation of the order quantity to the gradual increase in the order fulfillment costs (ε) in periods n at insignificant demand fluctuation amplitude

Fig. 7 shows the dependence of deviation of the order quantity on the amplitude of demand fluctuations at an insignificant increase in the order fulfillment costs. To build a graph, fix $\varepsilon=0.01$ (growth rates for the order fulfillment are 1 %) and change β, i. e. the amplitude of demand fluctuations from 0 to 0.2.
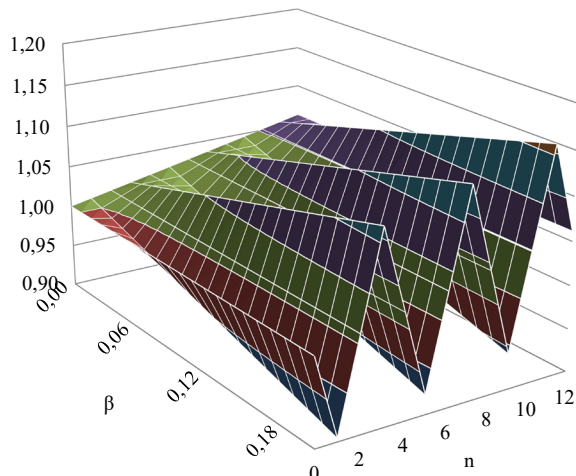
Fig. 7. Sensitivity of deviation of the order quantity to the amplitude of demand fluctuations (β) at a slight increase in the order fulfillment costs in periods *n*

Thus, Fig. 6, 7 make it possible to visually assess the nature of the dependence of deviation of the "perturbed" order quantity from the optimal one calculated from Wilson's formula.

5. Discussion of the results of optimization of the inventory management model using the perturbation methods

The proposed asymptotic approach to the development of the inventory management model using perturbation methods makes it possible to find a solution to the problem in a small range of the input parameter variation. This significantly expands the field of application of the EOQ model. A small parameter in the problem of finding the optimal order size can be understood as the percentage rate of growth of the storage and order fulfillment costs, the amplitude of demand fluctuations, and other factors.

In this study, the range from 0 to 0.025 (that is, 0–2.5 %) was adopted as the range of variation of the small parameter ε which characterizes rates of growth of the order fulfillment costs. The range from 0 to 0.2 (that is, 0–20 %) was chosen as the range of variation of the parameter β which determines the rate of growth of the storage costs. Such values of parameters are determined by the specificity of each of them. The parameter β takes values larger than the parameter ε due to a significant increase in utility costs (water, electricity, heating, etc.). However, since members of the order of smallness greater than ε^2 and β^2 were neglected during the model construction, the calculation error is insignificant from the economic point of view.

The derived formulas of the EOQ model contain parameters *n* and *m* which describe the dependence of the inventory storage and order fulfillment costs. They characterize the intervals of changes in the inventory storage costs (*m*) and changes in the order fulfillment costs (*n*). In practice, the change in the storage costs is delayed compared to the change in the order fulfillment costs, so the relationship between parameters *n* and *m* can be expressed using a mathematical function of the integer part of the number $y=[x]$.

Analytical solution of the problem of determining the EOQ under the condition of a discrete increase in the order fulfillment costs was obtained in the form of a one-parameter formula (10). According to it, formula (12) was derived for calculating the total costs of the order fulfillment and the inventory storage. The application of formula (10) instead of Wilson's formula (1) reduces the company's overall costs (12). Evaluation of the sensitivity of the EOQ model has shown that the relative deviation of the optimal order quantity (Fig. 1) at insignificant changes of the order fulfillment costs varied from 1 % to 15 % depending on the period.

An asymptotic solution of the problem of determining the EOQ under the condition of variation of the order fulfillment and inventory storage costs was obtained using the perturbation method in the form of a two-parameter formula (14) which reduces total costs (16) of the company in this case. The study of the sensitivity of the order quantity to the changes in the order fulfillment and the inventory storage costs (Fig. 2, 3) has found that it depends on the periods of change in the input parameters and the percentage change in the corresponding costs. Calculation of deviation of the order quantity according to formula (14) relative to Wilson's formula (1) (Tables 2, 3) at different values of input parameters has shown a multidirectional dynamics. For example, the deviation can range from +3 % to −16 % for respective periods.

The solution of the EOQ problem under the condition of growing costs of order fulfillment and periodic fluctuations in demand for the proposed products was obtained in the form of formula (18). The nature of the dependence of deviation of the order quantity on the increase in the order fulfillment costs at minor demand fluctuations is shown in Fig. 6. Changes in the optimal order quantity from −1 % to +12 % occur with an increase in the order fulfillment costs. Dependence of deviation of the perturbed order quantity on the amplitude of the demand fluctuations with increasing costs of the order fulfillment (Fig. 7) is multidirectional. The nature of changes depends on the period. For example, the minimum order quantities were in periods 1, 5, 9 and maximum order quantities were in periods 3, 7, 11. The order quantity varied from −10 % to +15 %.

However, the use of selected forms of functional dependence of the order fulfillment and the inventory storage costs as well as demand for the company's products.is a limitation inherent in this study.

The obtained one-parameter and two-parameter solutions of the EOQ model are of practical significance as analytical asymptotic formulas are convenient for analysis and use by company managers. They can be used in further studies provided that the nature of cost variation changes.

The total costs corresponding to the order quantity obtained from the asymptotic formulas in this paper did not differ significantly from those that correspond to the order quantity of the classical EOQ model. However, cost reduction can be significant in the scale of general enterprise procurement.

In contrast to [4–7], the proposed method of "perturbations" applied to the model of inventory management is the development of analytical tools for procurement management and inventory logistics. In particular, the proposed asymptotic formulas make it possible to model the inventory management system of the enterprise under the condition of variable order fulfillment and inventory storage costs, as well as take into account fluctuations in demand for goods and services offered by the company in the market. The available user-friendly model that takes into account chang-

es in demand and costs makes it possible to introduce timely adjustments to the procurement process of the enterprise, minimize overall costs and improve the company's competitiveness in the market.

The prospects for further studies are connected with the adaptation of the method of "perturbations" to other logistics models in which the small parameters having economic significance can be identified. In addition, it is advisable to develop analytical asymptotic models for managing logistic processes in the case of variable input parameters of the system. In particular, the method of "perturbations" can be applied to the construction of a model of multi-item deliveries under the condition of variable system parameters.

6. Conclusions

1. Based on the method of perturbations, an asymptotic formula was derived to determine economic order quantity on conditions that there is a small discrete increase in order fulfillment costs. This formula contains a small parameter that characterizes variation of the order fulfillment costs depending on periods. It is easy to use and it enables obtaining refined values of the order quantities and total costs which allows the company's management to optimize logistic processes. Deviation of the disturbed order quantities was in the range from 1 % to 15 % depending on the period. Comparative analysis of total costs calculated using the Wilson formula and the asymptotic formula has made it possible to state that taking into account changes in order quantities corresponding to the perturbed order quantity leads to a decrease in total logistic expenditures of the company.

2. A two-parameter inventory management model was constructed. It takes into account both minor changes in order fulfillment and inventory storage costs. Asymptotic formulas with two small parameters were derived to determine the optimal order quantity and total costs corresponding to the order quantity determined by Wilson's formula (1) and the perturbed quantity. The study of the nature of deviation of the "perturbed" order quantity from the economic one under different conditions of gradual increase in the order fulfillment and the inventory storage costs has shown that it is mostly linear due to the small input parameters of the model. Deviation of the disturbed order quantities was in a range from +3 % to −16 % for the corresponding periods.

3. A two-parameter model was obtained taking into account the discrete increase in the order fulfillment costs and the periodic nature of fluctuation of demand for the proposed products under the condition of minor expenditure changes. It contains two small parameters characterizing the percentage change in the order fulfillment costs and the amplitude of fluctuations in the demand for products. The sensitivity of deviation of the perturbed order quantity from the optimal one at a gradual increase in the order fulfillment costs depending on the period with minor demand fluctuations was from −2 % to +13 %. Percentage deviation of the order quantity from the optimal one depending on the amplitude of demand fluctuations and the period n at a fixed rate of growth of the order fulfillment costs was from −1 % to +6.5 %. According to the study results, the rate of growth of the order fulfillment costs has a greater impact on the optimal order size than the amplitude of fluctuations in the demand for products which are small parameters of the constructed model. This model is of practical importance for the company's management because, in addition to changes in the order fulfillment and the inventory storage costs, demand for the company's products may also fluctuate because of the changes in various internal and external factors.

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