Аналітично-числовими методами досліджені усталені вібрації двомасової вібромашини з прямолінійним поступальним рухом платформ і віброзбуджувачем у вигляді кульового, роликового або маятникового автобалансира. Розроблено методику дослідження режимів застрягання вантажів в системах, подібних розглянутої. Методика ґрунтується на ідеї параметричного рішення задачі пошуку частот застрягання вантажів і теорії біфуркацій рухів.

Встановлено, що у двомасової вібромашини дві резонансні частоти обертання ротора і дві відповідні форми коливань платформ. Застосування методики показало, що в разі малих сил опору, у вібромашини:

– п'ять можливих режимів застрягання вантажів, причому перша форма резонансних коливань платформ збуджується на режимах 1 і 2, друга – 3 і 4, а на 5-му режимі частота застрягання вантажів близька до частоти обертання ротора;

– стійкими є непарні режими застрягання (1, 3, 5);

– для збудження резонансних коливань платформ і промислового застосування підходять режими застрягання 1 і 2;

– зі збільшенням швидкості обертання ротора амплітуди коливань платформ, що відповідають певному режиму застрягання, монотонно зростають;

– амплітудою резонансних коливань платформ можна управляти зміною швидкості обертання ротора.

Сили в'язкого опору, що діють на першу платформу, зменшують (аж до повного зникнення) перший діапазон швидкостей обертання ротора, на якому збуджується перша резонансна форма коливань платформ.

Внутрішні сили в'язкого опору, що діють між платформами, зменшують (аж до повного зникнення) другий діапазон швидкостей обертання ротора, на якому збуджується друга форма резонансних коливань платформ.

Сили в'язкого опору, що діють на вантажі при русі щодо автобалансира, зменшують обидва діапазони.

Ключові слова: інерційний віброзбудник, двочастотні вібрації, резонансна вібромашина, автобалансир, двомасова вібромашина, ефект Зомерфельда

STUDYING THE STEADY-STATE VIBRATIONS OF A TWO-MASS VIBRATORY MACHINE EXCITED BY A PASSIVE AUTO-BALANCER

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1. Introduction

Among such vibratory machines as sieves, vibratory tables, vibratory conveyors, vibratory mills, etc., the promising ones are multi-frequency-resonance machines.

Multi-frequency vibratory machines have greater performance [1], resonance vibratory machines are the most energy efficient [2], while the multi-frequency-resonance vibratory machines combine the benefits of both multi-frequency and resonance vibratory machines [3]. Therefore, there is a common issue of designing multi-frequency-resonance vibratory machines [3–20].

The most effective and simple technique to excite resonance two-frequency vibrations is based on the use of a ball-, a roller, or a pendulum auto-balancer as a vibration exciter [10]. This technique is based on the Sommerfeld effect [11]. The feasibility of this technique was tested analytically in studies [12–20]. However, given the significant non-linearity of the problems considered, it was not possible to conduct in-depth research and obtain analytical results for the case of a two-mass vibratory machine.

It is relevant to use the results of papers [17–20] to investigate by analytical-computational methods the steady-state vibrations of a two-mass vibratory machine with a rectilinear translational motion of the platforms.

2. Literature review and problem statement

Two-mass vibratory machines have a series of advantages over single-mass machines. In the two-mass vibratory machines:

– the frequencies of platform oscillations are less dependent on a load mass [4]. frequencies;

– it is possible to excite the anti-resonance fluctuations at which platform oscillations are not transferred to the base [5];

– the resonance modes of motion have large regions of existence and stability [6];

– it is possible to excite the combined (poly-frequency) resonance vibrations of platforms with the natural vibration frequencies of a vibratory machine [7];

– the anti-resonance mode of vibratory machine operation is implemented over a wide parameter range [8], and is less dependent on the mass of a load [9], etc.

It is proposed in [10] to use a ball-, a roller-, or a pendulum auto-balancer to excite two-frequency resonance vibrations in vibratory machines with different kinematic motion of platforms. It is assumed that this technique is applicable for one-, two-, three-mass vibratory machines.

The technique is based on the Sommerfeld effect [11]. The technique employs a special mode of the motion of balls (rollers) [12] or pendulums [13], which occurs under small

forces of resistance to the motion of loads relative to the casing of the auto-balancer. Under this mode, the loads get together, cannot catch up with the rotor, onto which the auto-balancer mounted, and get stuck at one of the resonance frequencies of the vibratory machine. Slow resonance fluctuations of platforms are excited by load jamming. In addition, the casing of an auto-balancer hosts an unbalanced mass. The unbalanced mass rotates in sync with the rotor. That excites the rapid fluctuations of platforms. The parameters of two-frequency vibrations change by changing the rotor rotation velocity, the unbalanced mass, and the total weight of the loads.

The vibrations of rotary machines, which are caused by the Sommerfeld effect, were studied in works [14–16]. The effect of jamming a rotor with an unbalanced mass at the resonance frequency of a platform's vibrations using a dynamic system synchronization method was studied in [14]. The use of an airflow to excite the vibrations of the platform by an impeller hosting an unbalanced mass was studied in [15] by using the energy method. The effect of jamming a pendulum freely mounted on the engine shaft on a platform at the resonance platform vibration frequencies was investigated in [16] by using the motion separation method.

It should be noted that the approximate methods applied in works [14–16] made it possible to establish the very fact of a rotor, an impeller, or a pendulum getting stuck at one of the resonance frequencies of the platform oscillations. At the same time, the laws that govern the platform oscillations were defined in the lowest approximation. Therefore, the above approximate methods and the results obtained cannot be used to study the vibrations of a two-mass machine with a vibration exciter in the form of a passive auto-balancer.

The theoretical justification of the feasibility of the method for exciting two-frequency vibrations by passive auto-balancers was addressed in studies [17–20].

Paper [17] developed the generalized models of single-, two-, and three-mass vibratory machines with a translational motion of the platforms and a vibration exciter in the form of a ball-, roller-, or a pendulum auto-balancer. The differential equations of the motion of vibratory machines have been derived.

Study [18] has analytically established the two-frequency modes of the motion of a two-mass vibratory machine with a rectilinear translational motion of the platforms. At the same time, the motions were not investigated because of the complexity to solve this problem analytically. The difficulties are related to the significant non-linearity of the considered problem.

To investigate the steady-state vibrations of a two-mass vibratory machine, excited by a passive auto-balancer, one can apply the analytical-numerical methods developed in [19, 20] using an example of the single-mass vibratory machines. It was shown in [19] that the various steady-state motions of a single-mass vibratory machine acquire or lose stability only at the bifurcating points. In [20], the task of studying the steady-state modes of the motion of a single-mass vibratory machine was solved parametrically and using computational methods.

3. The aim and objectives of the study

The aim of this study is to analytically-numerically examine the two-frequency motion modes of the vibratory platforms of a two-mass vibratory machine with a rectilinear translational motion of the platforms excited by a passive auto-balancer. This is necessary for the development and design of new two-frequency dual-mass vibratory machines.

To accomplish the aim, the following tasks have been set:
– to devise a methodology for the analytical-numerical analysis of the steady-state vibrations of a two-mass vibratory machine;
– to find, at certain ratios of smallness between the system parameters, different steady-state motions of a vibratory machine and to assess their stability;
– to investigate the influence of external and internal resistance forces on these motion modes.

4. Description of the mechanical-mathematical model of vibratory machine

4. 1. Description of the generalized model of vibratory machine

The generalized model of a two-mass vibratory machine is shown in Fig. 1 [17]. The vibratory machine consists of two platforms of masses M_1 and M_2, forming an angle α with the horizon. Each platform is held by external elastic-viscous supports with a rigidity coefficient k_i and a viscosity coefficient b_i, /i=1, 2/. The platforms are connected via an inner elastic-viscous support with a rigidity coefficient k_{12} and a viscosity coefficient b_{12}.
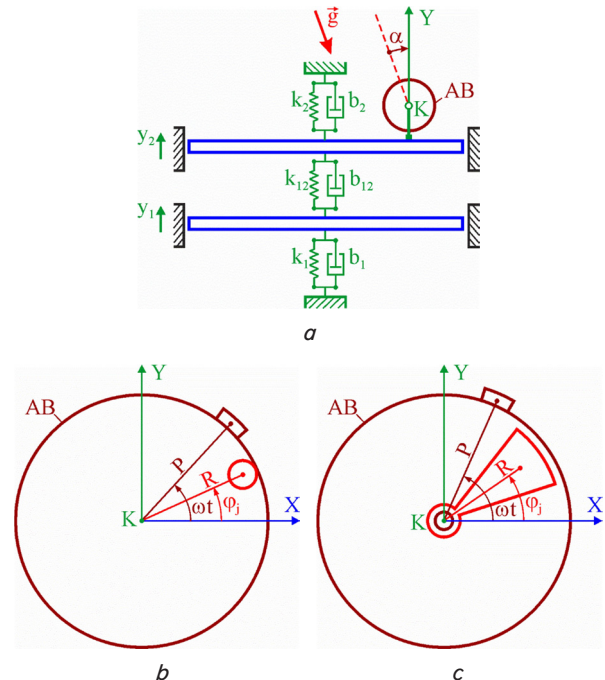

Fig. 1. The generalized model of a two-mass vibratory machine [17]: a — the kinematics of platform motion (the scheme is turned at angle α); b — the kinematics of motion of the unbalanced mass and a ball or a roller; c — the kinematics of the motion of the unbalanced mass and a pendulum

The direction of the platform motion forms angle α with the vertical. The coordinates of the platforms y_1, y_2 are counted from the positions of the static equilibrium of the platforms.

The second platform hosts a passive auto-balancer – ball-type, roller-type (Fig. 1, *b*), or pendulum-type (Fig. 1, *c*).

The casing of the auto-balancer revolves around the shaft, point K, at a constant angular speed ω.

The point unbalanced mass μ is rigidly connected to the casing of the auto-balancer. It is located at distance P from point K. The position of the unbalanced mass relative to the casing is determined by the angle ωt, where t is the time.

The auto balancer is made up of N identical loads. The weight of a single load is m. The load mass center can move along the circle of radius R with the center at point K (Fig. 1 *b, c*). The position of load number j relative to the casing is determined by the angle ϕ_j, $/j = \overline{1,N}/$. The load motion relative to the auto balancer's casing is hindered by the force of viscous resistance, whose module is

$$F_j = b_W v_j^{(r)} = b_W R\,|\phi_j' - \omega|, \quad /j = \overline{1,N}/,$$

where b_W is a viscous resistance force factor,

$$v_j^{(r)} = R\,|\phi_j' - \omega|$$

is the module of the motion speed of the center of the mass of load number j relative to the casing of the auto-balancer with a bar by the value denoting a time-derivative t.

4. 2. Differential equations of the motion of a vibratory machine

The differential equations of vibratory machine motion in a dimensionless form

$$\ddot{v}_1 + 2h_1\dot{v}_1 + n_1^2 v_1 + 2h_{12}(\rho\dot{v}_1 - \dot{v}_2) + n_{12}^2(\rho v_1 - v_2) = 0,$$

$$\ddot{v}_2 + 2h_2\dot{v}_2 + n_2^2 v_2 - 2h_{12}(\rho\dot{v}_1 - \dot{v}_2) - n_{12}^2(\rho v_1 - v_2) + \ddot{s}_y = \delta n^2 \sin n\tau, \tag{1}$$

$$\ddot{\phi}_j + \varepsilon\beta(\dot{\phi}_j - n) + \sigma\cos(\phi_j - \alpha) + \varepsilon\ddot{v}_2\cos\phi_j = 0, \quad /j = \overline{1,N}/. \tag{2}$$

In (1), (2), the following dimensionless quantities are introduced:

– variables and time

$$v_1 = y_1/(\rho\tilde{y}), \quad v_2 = y_2/\tilde{y},$$

$$s_x = \frac{1}{N}\sum_{j=1}^N \cos\phi_j, \quad s_y = \frac{1}{N}\sum_{j=1}^N \sin\phi_j, \quad \tau = \tilde{\omega}t; \tag{3}$$

– parameters

$$n_1^2 = \frac{k_1}{M_1\tilde{\omega}^2}, \quad n_{12}^2 = \frac{k_{12}}{M_{2\Sigma}\tilde{\omega}^2}, \quad n_2^2 = \frac{k_2}{M_{2\Sigma}\tilde{\omega}^2},$$

$$h_1 = \frac{b_1}{2M_1\tilde{\omega}}, \quad h_{12} = \frac{b_{12}}{2M_{2\Sigma}\tilde{\omega}},$$

$$h_2 = \frac{b_2}{2M_{2\Sigma}\tilde{\omega}}, \quad \rho = \frac{M_{2\Sigma}}{M_1},$$

$$n = \frac{\omega}{\tilde{\omega}}, \quad \varepsilon = \frac{Nm}{\kappa M_{2\Sigma}}, \quad \beta = \frac{b_W M_{2\Sigma}}{Nm^2\tilde{\omega}},$$

$$\left(\varepsilon\beta = \frac{b_W}{\kappa m\tilde{\omega}}\right), \quad \delta = \frac{\mu P}{NmR}, \quad \sigma = \frac{g}{\kappa R\tilde{\omega}^2}. \tag{4}$$

In turn, in (3), (4):

$$- M_{2\Sigma} = M_2 + Nm + \mu,$$

$$\tilde{y} = NmR/M_{2\Sigma}, \quad \tilde{\omega} = \sqrt{k_2/M_{2\Sigma}}; \tag{5}$$

– for a ball, a roller, and a pendulum, respectively

$$\kappa = 7/5, \quad \kappa = 3/2, \quad \kappa = 1 + J_C/(mR^2), \tag{6}$$

where J_C is the principal central axial moment of pendulum inertia.

Hereafter, the effect of gravity is not taken into consideration ($\sigma = 0$).

4. 3. The steady-state motion modes of a vibratory machine

The steady-state modes of platform motion are determined at $\varepsilon = 0$ [18]. For actual vibratory machines, the amendment to the law found in [18] does not exceed 2 %.

At steady-state motions

$$\phi_j^{(0)} = \Omega\tau + \psi_j, \quad \Omega, \psi_j - \text{const}, \quad /j = \overline{1,N}/. \tag{7}$$

In this case,

$$s_x = S\cos(\Omega\tau + \gamma_0),$$

$$s_y = S\sin(\Omega\tau + \gamma_0), \tag{8}$$

where

$$S^2 = \frac{1}{N^2}\left[\left(\sum_{j=1}^N \cos\psi_j\right)^2 + \left(\sum_{j=1}^N \sin\psi_j\right)^2\right],$$

$$\text{tg}\,\gamma_0 = \sum_{j=1}^N \sin\psi_j \Big/ \sum_{j=1}^N \cos\psi_j. \tag{9}$$

A two-frequency platform motion mode at zero approximation ($\varepsilon = 0$):

$$v_1(\tau) = D(\Omega, S)\sin(\Omega\tau + \gamma_0) + E(\Omega, S)\cos(\Omega\tau + \gamma_0) + D(n, \delta)\sin(n\tau) + E(n, \delta)\cos(n\tau),$$

$$v_2(\tau) = K(\Omega, S)\sin(\Omega\tau + \gamma_0) + L(\Omega, S)\cos(\Omega\tau + \gamma_0) + K(n, \delta)\sin(n\tau) + L(n, \delta)\cos(n\tau), \tag{10}$$

where

$$D(q, F) = \Delta_1(q, F)/\Delta(q),$$

$$E(q, F) = \Delta_2(q, F)/\Delta(q),$$

$$K(q, F) = \Delta_3(q, F)/\Delta(q),$$

$$L(q, F) = \Delta_4(q, F)/\Delta(q). \tag{11}$$

In turn:

$$\Delta(q) = \left\{ \begin{array}{l} a_{11}(q)a_{33}(q) - \rho\left[a_{13}^2(q) - a_{14}^2(q)\right] - \\ -a_{12}(q)a_{34}(q) \end{array} \right\}^2 +$$

$$+ \left[2\rho a_{13}(q)a_{14}(q) - a_{12}(q)a_{33}(q) - a_{11}(q)a_{34}(q)\right]^2,$$

$$\Delta_1(q,F) = b_3(q,F) \left\langle \begin{array}{l} a_{13}(q) \left\{ \begin{array}{l} \rho\left[a_{13}^2(q) + a_{14}^2(q)\right] - \\ -a_{11}(q)a_{33}(q) + \\ +a_{12}(q)a_{34}(q) \end{array} \right\} - \\ -a_{14}(q) \left[\begin{array}{l} a_{11}(q)a_{34}(q) + \\ +a_{12}(q)a_{33}(q) \end{array} \right] \end{array} \right\rangle,$$

$$\Delta_2(q,F) = b_3(q,F) \left\langle \begin{array}{l} a_{14}(q) \left\{ \begin{array}{l} \rho\left[a_{13}^2(q) + a_{14}^2(q)\right] + \\ +a_{11}(q)a_{33}(q) - \\ -a_{12}(q)a_{34}(q) \end{array} \right\} - \\ -a_{13}(q) \left[\begin{array}{l} a_{11}(q)a_{34}(q) + \\ +a_{12}(q)a_{33}(q) \end{array} \right] \end{array} \right\rangle,$$

$$\Delta_3(q,F) = b_3(q,F) \left\langle \begin{array}{l} \rho \left\{ \begin{array}{l} a_{11}(q)\left[a_{14}^2(q) - a_{13}^2(q)\right] - \\ -2a_{12}(q)a_{13}(q)a_{14}(q) \end{array} \right\} + \\ +a_{33}(q)\left[a_{11}^2(q) + a_{12}^2(q)\right] \end{array} \right\rangle,$$

$$\Delta_4(q,F) = b_3(q,F) \left\langle \begin{array}{l} \rho \left\{ \begin{array}{l} a_{12}(q)\left[a_{13}^2(q) - a_{14}^2(q)\right] - \\ -2a_{11}(q)a_{13}(q)a_{14}(q) \end{array} \right\} + \\ +a_{34}(q)\left[a_{11}^2(q) + a_{12}^2(q)\right] \end{array} \right\rangle. \quad (12)$$

Finally:

$$a_{11}(q) = n_1^2 + \rho n_{12}^2 - q^2, \quad a_{12}(q) = -2q\left(h_1 + \rho h_{12}\right),$$

$$a_{13} = -n_{12}^2, \quad a_{14}(q) = 2qh_{12},$$

$$a_{33}(q) = n_2^2 + n_{12}^2 - q^2, \quad a_{34}(q) = -2q\left(h_2 + h_{12}\right),$$

$$b_3(q,F) = Fq^2. \quad (13)$$

In motion laws (10), the value of the constant parameter Ω that determines the frequency of load jamming is determined from the following equation

$$P(\Omega,n) = 2\beta(n-\Omega)\Delta(\Omega) + \Omega^2\Delta_4(\Omega,S) = 0. \quad (14)$$

Equation (14) is a 9th degree polynomial relative to Ω, which almost defies analytical investigation.

5. Results of studying steady-state vibrations

5. 1. Building a procedure for studying steady-state vibrations and a computational algorithm

The procedure is based on the idea of parametric solution to the problem of finding the frequency of load jamming (14) and a bifurcation theory of motion. The procedure employs the fact that the rotor speed n is linearly included in the

equations of the frequencies of load jamming. Therefore, the specific frequency of load jamming corresponds to one and only one rotor speed. This makes it possible to find all possible modes of load jamming in a parametric form, and bifurcation points at which these modes appear or disappear. The bifurcation theory of motion makes it possible to assess the stability of different jamming modes. Stability can change to instability, and vice versa, only when passing the bifurcation points.

In the absence of resistance forces in the supports (h_1, h_{12}, $h_2=0$)

$$\Delta(q) = \left[\left(n_1^2 + \rho n_{12}^2 - q^2\right)\left(n_2^2 + n_{12}^2 - q^2\right) - \rho n_{12}^4\right]^2. \quad (15)$$

Two different double roots of this equation

$$q_{1/2} = \frac{1}{\sqrt{2}}\sqrt{n_1^2 + n_2^2 + (1+\rho)n_{12}^2 \mp \sqrt{\left[n_1^2 - n_2^2 - (1-\rho)n_{12}^2\right]^2 + 4\rho n_{12}^4}}, \quad (16)$$

determine the system's natural vibration frequencies when the loads are stationary relative to the auto-balancer. These frequencies always exist, and $0 < q_1 < q_2$. They correspond to two shapes of the platform resonance oscillations. The first shape of platform oscillations is dominated by a component at which platforms move in the same direction. The second shape of platform oscillations is dominated by a component at which platforms move in opposite directions.

In the absence of resistance forces in the supports, the term $2\beta(n-\Omega)\Delta(\Omega)$ has five valid positive roots: q_1, q_1, q_2, q_2, n.

For the case of small forces of viscous resistance in the supports, the frequencies of load jamming:

– are close to the natural vibration frequencies or a vibratory machine or the rotor rotation frequency;

– the jamming frequencies, close to the resonance ones, arise and disappear in pairs in the vicinity of each natural frequency.

From (10), the amplitudes of the slow oscillations of platforms are found:

$$A(\Omega,S) = \sqrt{D^2(\Omega,S) + E^2(\Omega,S)},$$

$$B(\Omega,S) = \sqrt{K^2(\Omega,S) + L^2(\Omega,S)}. \quad (17)$$

From (16), such a solution to the equation of the frequency of load jamming is derived in a parametric form

$$n(\Omega) = \Omega\frac{2\beta\Delta(\Omega) - \Omega\Delta_4(\Omega,S)}{2\beta\Delta(\Omega)}, \quad \Omega \in (0,+\infty). \quad (18)$$

In the plane $(\Omega, n(\Omega))$, $\Omega \in (0,+\infty)$, a chart of the function $\Omega(n)$, $n \in (0,+\infty)$ is built. At the points of motions bifurcation, there is the origin or merging of a pair of jam frequencies. At the same time,

$$\frac{dn(\Omega)}{d\Omega} = \frac{1}{2\beta\Delta^2(\Omega)} \times$$

$$\times \left\{ \begin{array}{l} 2\beta\Delta^2(\Omega) - 2\Omega\Delta_4(\Omega,S)\Delta(\Omega) + \\ +\Omega^2 \left[\begin{array}{l} \Delta_4(\Omega,S)\dfrac{d\Delta(\Omega)}{d\Omega} - \\ -\Delta(\Omega)\dfrac{d\Delta_4(\Omega,S)}{d\Omega} \end{array} \right] \end{array} \right\} = 0. \quad (19)$$

A qualitative assessment of the system's performance makes it possible to construct the following computational algorithm for studying the resonance vibrations of a vibratory machine.

1. Equation (19) produced four critical frequencies of load jamming, so that $0 < \Omega_1 < \Omega_2 < \Omega_3 < \Omega_4 < n$.

2. Formula (18) derives four bifurcation angular rotor rotation velocities $n_i = n(\Omega_i)$, $/i = \overline{1,4}/$. For convenience, they are numbered and arranged in ascending order. When passing these velocities, one pair of jamming modes occurs or disappears.

3. For each jamming mode, formula (18) calculates, in a parametric form, the corresponding rotor speeds

$$n_1(\Omega) = n(\Omega), \quad \Omega \in [0, \Omega_1];$$

$$n_2(\Omega) = n(\Omega), \quad \Omega \in [\Omega_1, \Omega_2];$$

$$n_3(\Omega) = n(\Omega), \quad \Omega \in [\Omega_2, \Omega_3];$$

$$n_4(\Omega) = n(\Omega), \quad \Omega \in [\Omega_3, \Omega_4];$$

$$n_5(\Omega) = n(\Omega), \quad \Omega \in [\Omega_4, +\infty). \tag{20}$$

The results of calculations in the plane (n, Ω) are used to build the diagrams of five possible jamming modes

$$\left(n_i(\Omega), \Omega\right), \ /i = \overline{1,5}/.$$

4. In assessing the stability of possible jamming modes, the following rules are applied:

– if there is only one mode of load jamming at a certain rotor speed, it is (globally or locally) asymptotically stable;

– if, at a certain rotor speed, there are three or more load jamming modes, the only the odd modes of jamming are locally asymptotically stable.

5. For each jamming mode, formulae (17) are used to calculate, in a parametric form, the amplitudes of the slow oscillations of the platforms

$$A_1(\Omega, A) = A(\Omega, A), \quad \Omega \in [0, \Omega_1],$$

$$A_2(\Omega, A) = A(\Omega, A), \quad \Omega \in [\Omega_1, \Omega_2],$$

$$A_3(\Omega, A) = A(\Omega, A), \quad \Omega \in [\Omega_2, \Omega_3],$$

$$A_4(\Omega, A) = A(\Omega, A), \quad \Omega \in [\Omega_3, \Omega_4],$$

$$A_5(\Omega, A) = A(\Omega, A), \quad \Omega \in [\Omega_4, +\infty);$$

$$B_1(\Omega, A) = B(\Omega, A), \quad \Omega \in [0, \Omega_1],$$

$$B_2(\Omega, A) = B(\Omega, A), \quad \Omega \in [\Omega_1, \Omega_2],$$

$$B_3(\Omega, A) = B(\Omega, A), \quad \Omega \in [\Omega_2, \Omega_3],$$

$$B_4(\Omega, A) = B(\Omega, A), \quad \Omega \in [\Omega_3, \Omega_4],$$

$$B_5(\Omega, A) = B(\Omega, A), \quad \Omega \in [\Omega_4, +\infty). \tag{21}$$

The results of calculations are applied to build, in the planes (n, A) and (n, B), the diagrams $\left(n_i(\Omega), A_i(\Omega)\right)$ and $\left(n_i(\Omega), B_i(\Omega)\right)$, $/i = \overline{1,5}/$.

5. 2. Search for the steady-state motions of a vibratory machine and the assessment of their stability

All calculations are performed with dimensionless values. The results are also derived in a dimensionless form.

In computational experiments, a vibratory machine is considered without a second external support. This design is most relevant for practice. In this case, the main one is platform 1. Auxiliary platform 2 is attached to it elastically-plastically. Platform 2 is fitted with a vibration exciter in the form of a passive auto-balancer.

Estimated data (dimensionless parameters):

$$n_1 = 1, \quad n_{12} = 2, \quad n_2 = 0, \quad \rho = 0.1, \quad F = 1,$$

$$\beta = 0.3, \quad h_1 = 0.1, \quad h_{12} = 0.1, \quad h_2 = 0, \quad \sigma = 0. \tag{22}$$

Substituting (22) into (16), two natural (resonance) frequencies of system oscillations in the absence of resistance forces are found

$$q_1 = 0.941, \quad q_2 = 2.125.$$

The bifurcation frequencies of load jamming are found as the roots of equation (19):

$$\Omega_1 = 0.9616, \quad \Omega_2 = 1.1606,$$

$$\Omega_3 = 2.1332, \quad \Omega_4 = 3.5256.$$

Substituting (22) into (18), the appropriate bifurcation rotor speeds are derived. Arrange them in ascending order:

$$n_1 = 1.6516, \quad n_2 = 2.1597, \quad n_3 = 6.3684, \quad n_4 = 58.2407.$$

Fig. 2 show the built diagrams of 5 possible modes of load jamming (20).
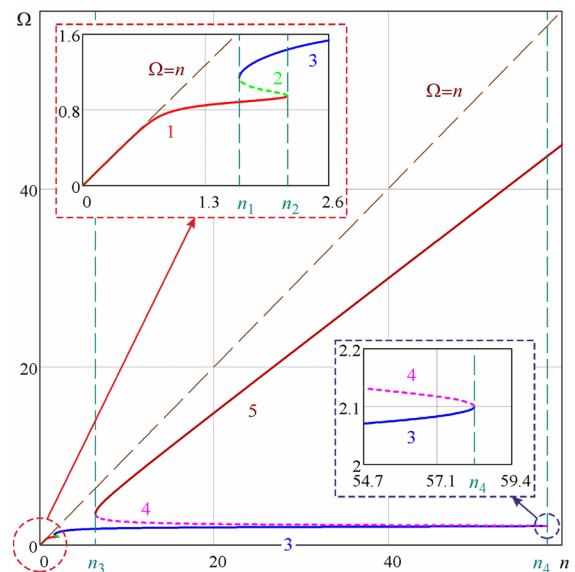

Fig. 2. Diagrams of possible load jamming modes:
i — jamming mode number i, $/i = \overline{1,5}/$

Fig. 2 shows in solid lines the steady-state load jamming modes. It follows from Fig. 2 and rules from point 4 of the algorithm that:

– in the range $(0,n_1)$, the first mode of load jamming is asymptotically stable (globally or locally);

– in the range (n_1,n_2), the first and third modes of load jamming are locally asymptotically stable;

– in the range (n_2,n_3), the third mode of load jamming is asymptotically stable (globally or locally);

– in the range (n_3,n_4), the third and fifth modes of load jamming are locally asymptotically stable;

– in the range $(n_4,+\infty)$, the fifth mode of load jamming is asymptotically stable (globally or locally).

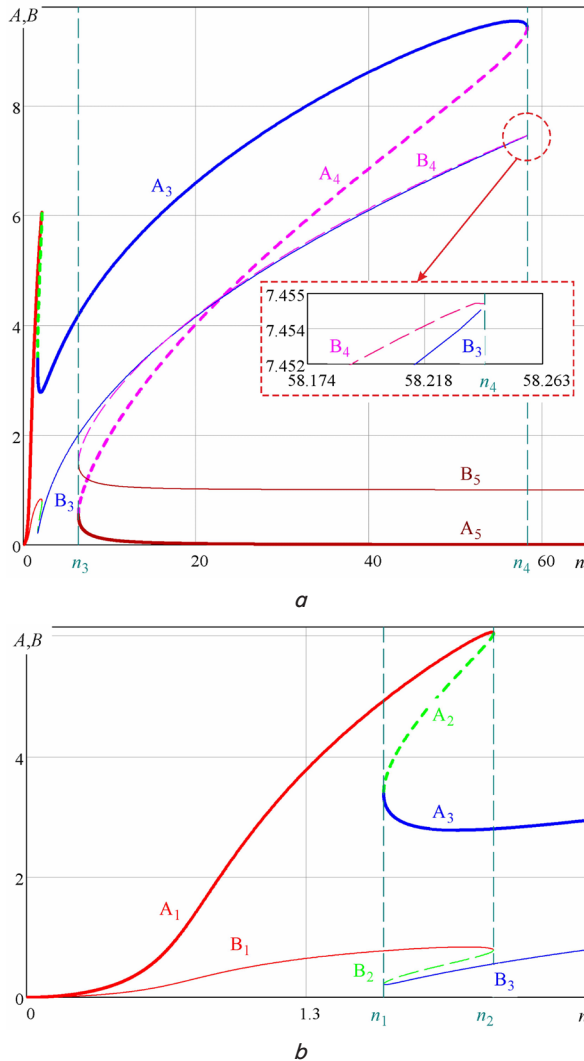Fig. 3, a, b shows the built diagrams of possible amplitudes of the slow oscillations of the platforms.

Fig. 3. Diagrams of the possible amplitudes of the slow oscillations of platforms: a — general view; b — at the low rotor speeds

Fig. 3 shows in solid lines the amplitudes corresponding to the stable jamming modes; in dotted lines – unstable.

Fig. 3 demonstrates that significant fluctuations of the 1st platform can be achieved:

– in the (n_1,n_2) range of rotor speeds when implementing the first jamming mode;

– in the (n_3,n_4) range of rotor speeds when implementing the third jamming mode.

Comparing the single-mass vibratory machine [16] with the considered two-mass machine reveals the following:

– the two-mass vibratory machine has two ranges of angular rotor rotation velocities, at which it is advisable to use a vibratory machine; the single-mass – one;

– the only range of a single-mass vibratory machine roughly corresponds to the first range of the two-mass vibratory machine;

– for the case of a two-mass vibratory machine, the second range (n_3,n_4) is much wider than the first range (n_1,n_2);

– the two-mass vibratory machine, compared to a single-mass one, has twice as many usable load jamming modes, with the second suitable mode having a much larger region of existence and stability.

5. 3. The influence exerted on steady-state motions by the external and internal resistance forces

The estimated data (dimensionless parameters) are taken by default from (22) unless otherwise specified.

When studying the effect of a particular parameter on jamming modes, only this parameter changes.

Fig. 4, a, b shows the dependence of load jamming frequencies on a change in the viscosity factor in support 1.
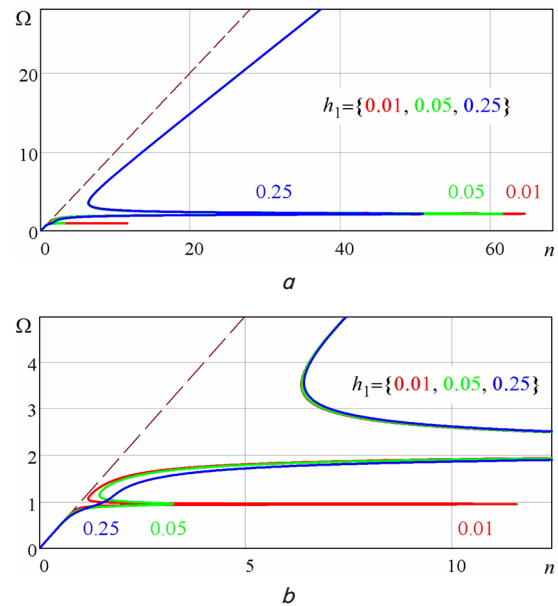

Fig. 4. Dependence of load jamming frequencies on a change in the viscosity factor h_1 in support 1: a — general view; b — in the vicinity of the first jamming frequency

Fig. 4 demonstrates that:

– with an increase in h_1, the first range decreases until the complete disappearance, while the second range almost does not decrease;

– reducing h_1 can significantly increase the first range, up to the intersection with the second one.

Fig. 5, a, b shows the dependence of load jamming frequencies on a change in the viscosity factor in intermediate support 1–2.

Fig. 5 demonstrates that:

– with an increase in h_{12}, the second range decreases until the complete disappearance, while the first range almost does not decrease;

– reducing h_{12} can significantly increase the second range, up to the intersection with the first range.

Fig. 6, a, b shows the dependence of load jamming frequencies on a change in the viscosity force factor acting on the load.
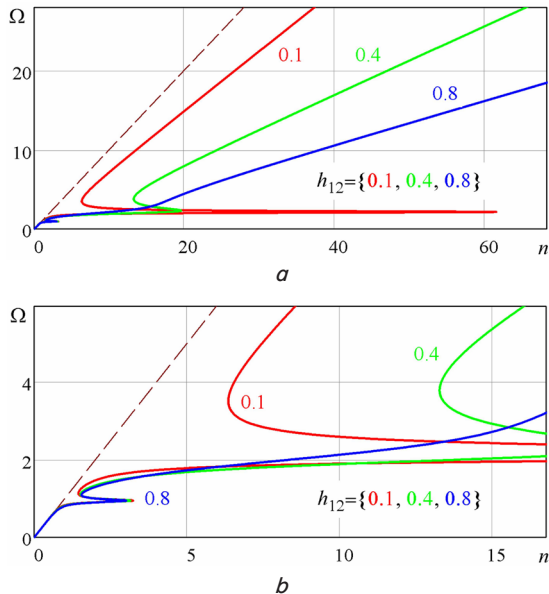
Fig. 5. Dependence of load jamming frequencies on a change in the viscosity factor h_{12} in intermediate support 1—2: a — general view; b — in the vicinity of the first jamming frequency
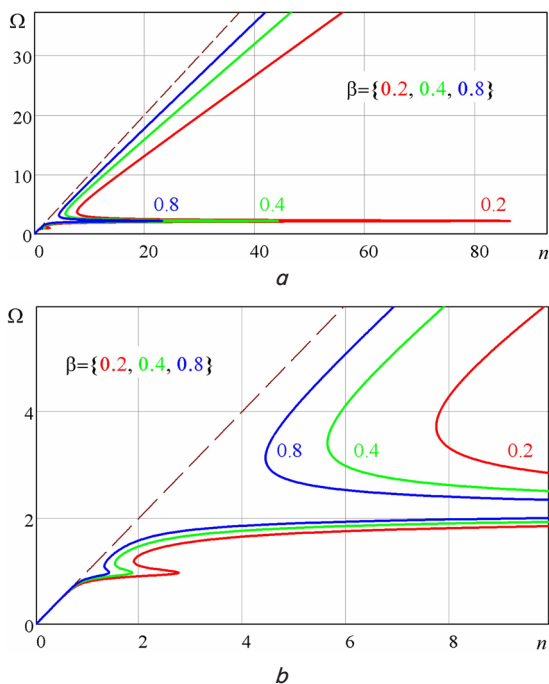

Fig. 6. Dependence of load jamming frequencies on changes in the viscosity force factor β, acting on the load: a — general view; b — in the vicinity of the first jamming frequency

Fig. 6 shows that:

– with an increase in β, both ranges decrease and approach the rotor rotation velocity;

– reducing β can significantly increase both ranges.

6. Discussion of results of studying the two-frequency motion modes of a two-mass vibratory machine

The current study demonstrates the effectiveness of the devised procedure of studying the steady-state vibrations of a

two-mass vibratory machine, excited by a passive auto-balancer. The procedure has made it possible to find all possible modes of load jamming, to investigate them, and to assess stability.

The considered vibratory machine has two resonance rotor rotation frequencies (15) and two corresponding shapes of platform oscillations. The use of the procedure has shown that, for the case of small resistance forces, the vibratory machine:

– has five possible modes of load jamming (Fig. 2), with the first shape of resonance vibrations of platforms being excited under modes 1 and 2, the second shape – 3 and 4, and, under mode 5, the frequency of load jamming is close to the frequency of rotor rotation;

– demonstrates stable jamming modes under the odd (1, 3, 5) load jamming modes;

– shows that the jamming modes 1 and 3 are suitable to excite the resonance oscillations of platforms and for industrial application (Fig. 3);

– exhibits that increasing the rotor speed monotonously increases the amplitudes of platform oscillations corresponding to a certain jamming mode (Fig. 3);

– proves that the amplitude of resonance platform oscillations can be controlled by changing the rotor speed.

The viscous resistance forces acting on the first platform affect the first range of rotor speeds, at which the first resonance shape of platform oscillations is excited (Fig. 4). As the resistance forces increase, the first range decreases until the total elimination.

The internal forces of viscous resistance acting between the platforms affect the second range of rotor speeds, at which the second shape of resonance vibrations of platforms is excited (Fig. 5). As the resistance forces increase, the second range decreases until the total elimination.

The viscous resistance forces acting on the loads when moving relative to the auto-balancer affect both ranges (Fig. 6). As the resistance forces increase, both ranges decrease.

Thus, the two-mass vibratory machine has the following advantages over the single-mass machine:

– a larger number of the resonance modes of platform oscillations;

– a larger range of rotor speeds at which the resonance modes are implemented.

It should be noted that the devised procedure has solved an essentially non-linear problem. The procedure is applicable to solve this class of problems for the cases of single-mass and multi-mass vibratory machines at the different kinematics of platform motions. However, the methodology does not make it possible to obtain the analytical results of research. This needs to be compensated for by a large amount of computations, considering the different ratios of smallness between the system parameters.

In the future, it is planned to investigate the steady-state vibrations of a three-mass vibratory machine using the devised procedure.

7. Conclusions

1. The current study demonstrates the effectiveness of the devised procedure for investigating load jamming modes in systems similar to the one under consideration. The procedure is based on the idea of parametric solution to the problem of finding the load jamming frequencies and a bifurcation theory of motion. The procedure employs the fact that the rotor speed is linearly included in the equations of

load jamming frequencies. Therefore, a specific load jamming frequency corresponds to one and only one rotor speed. This makes it possible to find all possible load jamming modes in a parametric form, the bifurcation points at which these modes appear or disappear. The bifurcation theory of motion makes it possible to assess the stability of different jamming modes.

2. The two-mass vibratory machine has two resonance rotor rotation frequencies and two corresponding shapes of platform oscillations. The use of the procedure has shown that for the case of small resistance forces, the vibratory machine:

– has five possible modes of load jamming, with the first shape of resonance vibrations of platforms being excited under modes 1 and 2, the second shape – 3 and 4, and, under mode 5, the frequency of load jamming is close to the frequency of rotor rotation;

– demonstrates stable jamming modes under the odd (1, 3, 5) load jamming modes;

– shows that the jamming modes 1 and 3 are suitable to excite the resonance oscillations of platforms and for industrial application;

– exhibits that increasing the rotor speed monotonously increases the amplitudes of platform oscillations corresponding to a certain jamming mode;

– proves that the amplitude of resonance platform oscillations can be controlled by changing the rotor rotation velocity.

3. The viscous resistance forces acting on the first platform affect the first range of rotor speeds, at which the first resonance shape of platform oscillations is excited. As the resistance forces increase, the first range decreases until the total elimination.

The internal forces of viscous resistance acting between the platforms affect the second range of rotor speeds, at which the second shape of resonance vibrations of platforms is excited. As the resistance forces increase, the second range decreases until the total elimination.

The viscous resistance forces acting on the loads when moving relative to the auto-balancer affect both ranges. As the resistance forces increase, the two ranges decrease.

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Метою дослідження є обґрунтування умов стійкості відкатних штреків при розробці крутих вугільних пластів.

В процесі моделювання стійкості штреків встановлено, що напружено-деформований стан бічних порід в вуглепородному масиві дільниці покрівлі, який вміщає виробки, залежить від фізико-механічних властивостей покрівлі і ґрунту вугільного пласта, що розробляється, жорсткості охоронних споруд і довжини підтримуваної охоронною спорудою. Зі збільшенням довжини, яка підтримується охоронною спорудою дільниці покрівлі, при мінімальній жорсткості піддатливих опор, збільшується зона плавного прогину бічних порід над відкатним штреком і знижується рівень їх напружено-деформованого стану.

Доведено, що при підтримці гірничих виробок в глибоких шахтах зниження напружено-деформованого стану бічних порід при застосуванні закладки виробленого простору відбувається в результаті ущільнення закладного масиву, на який спираються породи покрівлі, коли значення коефіцієнта ущільнення вихідного матеріалу приймають максимальні значення, що дорівнюють $k_{ущ}=1.5-1.53$. При використанні штучних піддатливих охоронних споруд, що зводяться над штреком, зміна напружено-деформованого стану відбувається в результаті стиснення опор, коли переміщення порід покрівлі і ґрунту обмежуються, а площа контакту бічних порід з засобами охорони збільшується.

При виборі способу охорони відкатних штреків необхідно враховувати параметри охоронних споруд, тому що вплив розмірів одних і тих же опор, при однаковій жорсткості на розподіл напружень в вуглепородному масиві, різноманітний.

Для забезпечення експлуатаційного стану дільничних підготовчих виробок, при розробці крутих вугільних пластів, доцільно застосування піддатливих охоронних споруд, розташованих над відкатним штреком, які обмежують переміщення бічних порід у виробленому просторі

Ключові слова: гірський тиск, очисний вибій, обвалення бічних порід, закладка виробленого простору, піддатливі опори

SUBSTANTIATION OF THE STABILITY OF HAULAGE DRIFTS WITH PROTECTIVE STRUCTURES OF DIFFERENT RIGIDITY

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1. Introduction

The coal industry is the main supplier of high-quality coal for the steel industry and energy. According to experts [1], the possible depth of development of high-quality coking coal, under conditions of steep coal seams, is 1,700 m, with balance reserves reaching 1.14 billion tons.

Despite that, at present, the development of steep high-quality coal seams is characterized by a relatively low level of the technical and economic indicators. In no small