

Важливою складовою ефективного управління проектами підприємства є планування тривалості робіт з метою оптимізації часу реалізації. У роботі створено оптимізаційну економіко-математичну модель визначення тривалості реалізації етапів проекту. Цільовою функцією виступає максимізація ймовірності успішної реалізації проекту та генерації нових організаційних знань на кожному з етапів. Модель передбачає, що сума тривалостей етапів проекту не має перевищувати встановлену тривалість проекту. Модель враховує, що наступний етап може розпочатися після попереднього при ймовірності реалізації завдань та генерації нових знань попереднього на рівні, не менше встановленого. Модель враховує, що з можливих комбінацій тривалостей етапів проекту обирається комбінація з мінімальною сумарною тривалістю проекту та з мінімальними витратами на реалізацію. Модель передбачає застосування елементів комбінаторики для визначення можливих комбінацій тривалості етапів. Також застосовувались знання експертів та метод безпосередньої оцінки для визначення вагових коефіцієнтів етапів проекту. Загальна ймовірність успішної реалізації проекту визначалася як сума ймовірностей успішної реалізації завдань та генерації нових знань на кожному етапі проекту із врахуванням відповідних вагових коефіцієнтів. Практична реалізація моделі здійснювалася для проекту розробки, наповнення та впровадження інформаційної системи та бази даних для управління діяльністю обласного центру фізичного виховання учнівської молоді тривалістю 10 місяців. Проект складається з трьох етапів: проектування, розробка і тестування, впровадження. Встановлено, що оптимальною буде наступна тривалість етапів проекту: 1 етап – 4 місяці, 2 етап – 5 місяців, 3 етап – 1 місяць. При даному розподілі часу ймовірність успішної реалізації проекту становить 0,81, витрати – 5440 USD. Створена модель може бути використана для будь-якого підприємства з метою планування тривалості робіт проекту та його успішної реалізації у встановлений термін

**Ключові слова:** тривалість проекту, оптимізаційна модель, економіко-математична модель, управління знаннями підприємства

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# DEVELOPMENT OF AN ECONOMIC-MATHEMATICAL MODEL TO DETERMINE THE OPTIMAL DURATION OF PROJECT OPERATIONS

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## 1. Introduction

The research into the influence of duration of project works and the generation of new organizational knowledge on the probability of successful project implementation in the set period is rather relevant in the company project management. This is important because the introduction of the subsystem of project knowledge management and the subsystem of knowledge about project management in the system of company knowledge management significantly changes the approaches to company management [1–3]. However, there are objective difficulties that are related to determining the optimal duration of the project works, because the level of completeness of information about the implementation of the current project is insufficient. That is why it is necessary to determine the optimal duration of project works and the generation of new organizational knowledge. This will make it possible to obtain the maximum probability of successful project implementation within the established period and get certain effects from the introduction to production. In particular, the efficiency of a production enterprise can be

increased as a result of obtaining new knowledge during the project implementation and reducing the level of expenditures for the project implementation. The obtained results will enable the improvement of the existing approaches to planning the duration of project works and putting them into actual industrial production.

## 2. Literature review and problem statement

Paper [4] proposed the project on an increase in the knowledge level and enhancement of quality in the medical area that is currently so important and especially relevant and acute. The authors proposed the “Improvement model”, which consists of three stages: setting improvement goals; choosing a balanced set of measures to determine whether the improvement will take place; testing new ideas to change the current process. In addition, it is noted in the research that one should not ignore the knowledge that patients/caregivers can offer to members of the quality improvement group. Thus, a rather acute problem of en-

hancing the quality and level of knowledge of employees was solved, the model was constructed, the stages of model formation were identified. However, this research is of the theoretical character without the practical implementation of the proposed model.

Paper [5] explored the dynamics of NASA action planning in the framework of experimental flight 1 (EFT-1), where the simultaneous change in the percentage of complete and estimated duration for the given project was modeled. In the research, the authors found three attractors: the low-duration attractor, low completion percentage; the attractor, which was approaching full completion, and lasted five months and longer; the long-duration attractor at full completion. The study applied multilevel modeling and established the scenarios of event development, depending on the selected attractor, which makes it possible to facilitate the process of planning further actions. This research once again proves that the process of project duration planning is rather relevant in different spheres of life. The merit of the research is finding three attractors and consideration of three possible event development scenarios. The disadvantage is the lack of consideration of intermediate situations that lie beyond the examined attractors.

The peculiarities of the projects on software development were considered in article [6]. The authors considered project prediction and their implementation in time with the use of information about completed projects both within the company and based on the information about the projects of other companies. This was done to overcome the limitations by the relatively small amount of information about the projects implemented within a company. The authors concluded that the construction of the forecast regarding a current project based on the data on previous projects is not always successful. That is why the study proposed an approach called Dynamic Cross-Company Learning (DCL) to dynamically determine which previous models within or outside a company are currently the most useful for forecasting in this company. The DCL automatically underlines the forecasts given by these models to improve predictive indicators, using information about the previous projects, which is most useful for predicting the situation with a current project. In other words, scientists use the method of analogs, but offer a more thorough approach to the selection of already implemented projects to predict the situation with a current project. The drawback of this approach is the lack of consideration of features of this particular project, as well as consideration of these features in the systemic approach.

The models for forecasting the construction time for both consulting companies and contractors in the Nigerian construction industry were developed in article [7]. The authors developed a multi-variant model, applied to the construction industry of Nigeria. Correlation-regression analysis and quantitative and qualitative impact factors were used. The qualitative factors that appeared as forecasts in derivative models enhanced the accuracy of the models. The disadvantage of this study is the lack of use of the method of expert assessments to take into account the characteristics of each particular project.

It is noted in article [8] that one of the main problems in project management is the use of the management theories, which were developed quite a long time ago, to all types of projects. That is why the forecast, which is made at the be-

ginning of a project, does not correspond to reality at the end of it. The study suggested that a project should be considered as an adaptive complex system, which should take into consideration the interdependence of dynamic components, multiple processes of feedback, nonlinear relationships, etc. The paper proposed project management modeling, applying the PMBOK standard that uses complex networks. The study lacks modeling the process of planning the project works duration.

The effect of the learning curve (or the experience curve) on the project duration is displayed in research [9]. The influence of learning on the project duration was studied using test tasks and actual problems. The duration of the future recurring work is supposed to be shorter due to the learning curve effect if the gap between successive actions is quite small. This effect has been shown to result in the project duration that is by 1–3 % shorter. This “calendar days” calculation led to an integer problem of programming, which was solved by means of MATLAB Parallel Computing Toolbox 2. The disadvantage of this work is the lack of duration distribution between project works to achieve maximum project efficiency.

Paper [10] emphasizes the need to establish a comprehensive definition of the project life cycle and the facilitation of its application for all important projects. This project lifecycle model recognizes that there is always a phase of the project incubation/implementation before the currently existing initial project phase of the majority of project management (PM) standards. In addition, the model recognizes that after an additional project evaluation stage, the project should have an additional stage – the standard project closure stage. It is recommended to consider this complex model of the project life cycle as a standard for important projects. Usually, many practicing doctors and the authorities limit the scope of “project management” by the traditional “start-plan-execution-closure” phases. However, projects start their existence before the traditional starting stage, and their products or results continue to exist and should be evaluated after projects are closed. The authors argue that before and after a phase, projects should be recognized as the ones belonging to the project management area. We believe that it would be expedient to know the vision of the authors as for the percentage division of the project duration into the specified project life cycle phases.

It is stated in paper [11] that PERT (Project Evaluation and Review Technique) became a classic project management tool to assess the project duration when works have an uncertain duration. However, this approach significantly decreases the average duration and overestimates the variation in the duration of actual projects. In the study, the author proposes to use a new modified PERT called M-PERT. The M-PERT is quite accurate in assessing the actual project duration, and also makes it possible to obtain a number of interesting models of network modeling, the number of which was insufficient for the original PERT. These models include probabilistic alternative ways, minimum sets of work, and relationships between activities. The M-PERT makes it possible to do the manual calculation using a recursive merge procedure that reduces the size of a network until the last active operation presents the entire duration of a project. This approach lacks the consideration of the weight factors of each project stage.

In paper [12], the simulation model of the works of the project on production of the screw shaft pipe was developed

based on the use of the method of analysis of the project plan options. This model makes it possible, unlike the existing deterministic approaches, to construct a project plan with a given probability of completion within the specified term. The advantage of this approach is the consideration of probabilities of the project completion in due time, but there is no recommendation for optimal time distribution between project works in order to increase the probability of successful project implementation within the specified period.

In paper [13], a multidimensional system of criteria for forming and evaluating the contents of the project of construction of unique hotel complexes was developed. In addition, a seven-step model for the formation of the content of developer projects of hotel complexes construction was designed. This model includes the following analytical tool-set: a logical-structural sequence of the search for “the best” land plot; a decision support scheme of formation of a variant of a construction site functional; a matrix of formation of the additional value of project beneficiaries.

Despite the significant achievements of scientists in the direction of project management as a whole, and planning the duration of project works in particular, the optimal distribution of the specified duration of the project between its stages requires further studying. This is necessary in order to generate knowledge at each stage and to maximize the probability of successful project implementation taking into consideration the minimization of costs.

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### 3. The aim and objectives of the study

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The aim of this study is to develop an optimization model for establishing the duration of project implementation stages, taking into consideration the weight factors of these stages and the probability of knowledge generation for a different period to obtain the maximal probability of successful project implementation.

To achieve the set goal, the following tasks were solved:

- to set the task and to develop the descriptive model for determining the optimal duration of project phases;
- to construct the mathematical model of the problem of determining the optimal duration of project phases and develop the algorithm of its solution;
- to determine the weight factors of stages and the probability of tasks implementation at each project stage for a set period using the expert survey;
- to form possible combinations of durations of project stages, taking into consideration the constraints of the model;
- to choose the optimal option of the combination of durations of project stages, taking into consideration the constraints and the objective functions of the model.

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### 4. Materials and methods of research to establish the optimal duration of project works and generation of new organizational knowledge

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The proposed model can be implemented at the following input data:

- 1) project duration;
- 2) number of stages, with the help of which a project is implemented;

- 3) weight factors of each project stage;
- 4) probability of tasks implementation and generation of new knowledge at each project stage within the determined time interval;

- 5) costs at each project stage within the specified time unit.

The output data of the model are:

- 1) maximum probability of successful project implementation;
- 2) optimal duration of project stages;
- 3) project cost.

The model involves the use of combinatorics elements to determine the possible combinations of stages' duration. Experts' knowledge and the direct estimation method were used to determine the weight factors of the project stages.

Python programming language was applied for the practical implementation of the model.

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## 5. Results of studying the practical realization of the developed optimization model for determining the duration of project stages

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### 5.1. Problem setting and the development of a descriptive model for determining the optimum duration of project stages

Establishing the optimal duration of project works is one of the most important problems because this makes it possible to increase the probability of successful project implementation. The use of economic and mathematical modeling enables solving this problem.

The task of determining the optimum duration of project works and generation of new organizational knowledge within the framework of this study is solved in the company, which is engaged in software development, is planning to take part in the bidding for the order of the regional center for physical education of school youth (RCPEY). The order is related to the development, content creation, and implementation of the information system and the database for the RCPEY activity management. The customer set the 10-month term of project implementation.

The company is planning three stages of project implementation:

- 1) designing;
- 2) development and testing;
- 3) implementation.

At each of these stages, the project team, which consists of the company employees, plans both to use existing knowledge and to implement the generation of new organizational knowledge relevant to a project. This knowledge can be divided into two groups: knowledge about the project management and knowledge of the subject area, in which the project is implemented.

Table 1 represents new corporate knowledge, which can be acquired in the process of project implementation.

Each of the stages has its own weight in project implementation. During the project implementation, the cost of 1 month at stage 1 is USD 375, at stage 2 – USD 750, and at stage 3 – USD 190.

It is necessary to determine the optimal duration of each of the stages to generate new knowledge, maximizing the probability of successful project implementation, as well as to observe the established project implementation term.

Table 1

Acquired organizational knowledge during the project implementation

Stage No.	Name of stage	Organizational subject knowledge, acquired during the project implementation	Organizational knowledge of the project management, acquired during the project implementation
1	Designing	Knowledge about designing information systems	Knowledge about project duration management, knowledge about project costs management, knowledge about project risks management, knowledge about project management software, knowledge about project communications management, knowledge about project changes management, knowledge about project integration management, knowledge about project team management
2	Development and testing	Knowledge about using the Google Cloud storage and FireBase; knowledge about specific features of using programming language Dart and framework Flutter; knowledge about the database CouchDB using RESTful API;	
3	Implementation	Knowledge about peculiarities of feedback from employees of the regional center of physical education of school youth for their use in future projects of this sphere	

That is, it is necessary to answer the questions:

1) What is the maximum probability of project implementation within the established term, taking into consideration the weight factors and the set constraints?

2) What is the minimum duration of the project implementation to achieve the maximum probability of its implementation?

3) How should the time between the project implementation stages be distributed, taking into consideration the cost minimization?

## 5.2. Mathematical model of the problem on determining the optimal duration of project stages and the algorithm of its solving

The objective function of the optimization model involves maximizing the probability of realization of the whole project taking into consideration weight factors:

$$\sum w_i p_{ij} \rightarrow \max, \quad (1)$$

where  $w_i$  is the weight factors of tasks implementation and knowledge generation at a particular stage, which are established by experts;  $i$  is the project stage number;  $p_{ij}$  is the probability of performing the set tasks at a particular stage, that is, generation of new knowledge within the established period;  $j$  is the variant of the number of days of project tasks implementation and generation of new knowledge at a certain stage.

This model implies constraints, specifically, the sum of the duration of knowledge generation in project stages should not exceed the project duration, and in this case, it is 10 months:

$$\sum t_{ij} \leq T, \quad (2)$$

where  $t_{ij}$  is the duration of project tasks implementation and new knowledge generation of at the determined stage;  $T$  is the total project duration.

Another constraint is that each successive stage can begin if the set tasks were performed at the previous stage with the probability of not less than 0.5:

$$p_{ij} \geq 0.5. \quad (3)$$

Among the selected variants with the same probability of project implementation, it is necessary to choose the one where the total duration is minimal, which will enable saving time and minimizing costs:

$$\sum t_{ij} \rightarrow \min, \quad (4)$$

$$\sum t_{ij} c_i \rightarrow \min, \quad (5)$$

where  $c_i$  is the costs at the  $i$ -th project stage to implement the set tasks and to generate new knowledge within a unit of time.

The proposed optimization model consists of the following stages:

1) Determining the weight factors of each stage with the involvement of experts.

To determine the weight factors, we selected the direct estimation method. Experts assigned points to the indicators points by a certain scale (from 1 to 3). After that, points were added by each indicator and the average ( $C_i$ ) was determined:

$$C_i = \frac{\sum_{i=1}^N C_{ij}}{N}, \quad (6)$$

where  $N$  is the number of experts questioned;  $C_{ij}$  is the number of points for each indicator.

The specified expression is used to calculate weights ( $S_i$ ):

$$S_i = \frac{C_i}{\sum_{i=1}^m C_i}. \quad (7)$$

2) Establishing, by experts, the probability of task implementation and new knowledge generation at each project stage.

3) Determining the total number of combinations  $A_n^k$  of the months of duration of each project stage according to the input data.

The formula of placement combinatorics with the repetition of  $n$  by  $k$ :

$$A_n^k = n^k. \quad (8)$$

4) Search for all combination of months of implementation at each stage of tasks, in which  $T \leq 10$ , that is, consideration of constraint  $\sum t_{ij} \leq T$ .

5) Replacement of  $y$  in combinations  $t_i$  for the corresponding probability, established by experts.

6) Removal of the options, where probabilities are  $\leq 0.5$ , that is, taking into account constraint  $p_{ij} \geq 0.5$ .

7) Search for the product of the corresponding weight factor and corresponding probability  $y$  in combinations ( $w_i p_{ij}$ ) and search for a maximal sum (finding objective function  $\sum w_i p_{ij} \rightarrow \max$ ).

8) Minimization of time  $\sum t_{ij} \rightarrow \min$ , if 2 or more variants of maximized objective function were chosen at the previous stage and minimization of costs  $\sum t_{ij} c_{ij} \rightarrow \min$ .

**5.3. Determining the weight factors of stages and probability of tasks implementation at each project stage within a set period with the use of expert survey**

10 employees of the enterprise were involved as experts. The results of the experts' survey were obtained using formulas (6), (7). They are given in Table 2.

Table 2

Weight factors of project stages, obtained with the use of experts' knowledge

Stage number	Expert's number										Average point	Weights of indicators
	1	2	3	4	5	6	7	8	9	10		
Stage 1	2	1	2	2	2	2	1	2	2	2	1.8	0.3
Stage 2	3	3	3	3	3	3	3	3	3	3	3	0.5
Stage 3	1	2	1	1	1	1	2	1	1	1	1.2	0.2
Total											6	1

The experts established that to implement the project, the weight factor of knowledge generation and execution of set tasks ( $w_i$ ) for stage 1 (designing) is 0.3, for stage 2 (development and testing) – 0.5, and for stage 3 (implementation) – 0.2.

The experts determined the probability of implementation of the established tasks in the interval from 1 to 5 months, which are given in Table 3.

Table 3

Probability of implementation of set tasks and generation of new organizational knowledge within the determined interval of time at various project stages

Stage No.	Probability of project tasks implementation within the period, months ( $t_m$ )				
	1	2	3	4	5
Stage 1	0.2	0.4	0.6	0.8	0.9
Stage 2	0.1	0.2	0.4	0.8	0.9
Stage 3	0.6	0.7	0.9	1	1

As Table 2 indicates, each stage has a different probability of implementation of set tasks and generation of new organizational knowledge within the same time intervals.

**5.4. Forming the possible combinations of the duration of project stages considering the constraints of the model**

In the considered example, according to formula (8),  $n=3, k=5$ , so the number of combinations is calculated:

$$A_n^k = 3^5 = 243. \tag{9}$$

There are 243 possible combinations.

To implement the phase of searching all combinations of months of implementation of set tasks, in which  $T \leq 10$ , at each stage, the Python programming language was used.

```
elements=[]
element=[]
result=[]
```

```
for i in range(1,6):
for j in range(1,6):
for k in range(1,6):
```

```
element.append(list([i, j, k]))
elements.append(element)
element=[]
```

```
for element in elements:
if sum(element[0]) <= 10:
result.append(element[0])
```

```
print(result)
print('Number < 10=', len(result))
[[1, 1, 1], [1, 1, 2], [1, 1, 3], [1, 1, 4], [1, 1, 5], [1, 2, 1], [1, 2, 2],
[1, 2, 3], [1, 2, 4], [1, 2, 5], [1, 3, 1], [1, 3, 2], [1, 3, 3], [1, 3, 4],
[1, 3, 5], [1, 4, 1], [1, 4, 2], [1, 4, 3], [1, 4, 4], [1, 4, 5], [1, 5, 1],
[1, 5, 2], [1, 5, 3], [1, 5, 4], [2, 1, 1], [2, 1, 2], [2, 1, 3], [2, 1, 4],
[2, 1, 5], [2, 2, 1], [2, 2, 2], [2, 2, 3], [2, 2, 4], [2, 2, 5], [2, 3, 1],
[2, 3, 2], [2, 3, 3], [2, 3, 4], [2, 3, 5], [2, 4, 1], [2, 4, 2], [2, 4, 3],
[2, 4, 4], [2, 5, 1], [2, 5, 2], [2, 5, 3], [3, 1, 1], [3, 1, 2], [3, 1, 3],
[3, 1, 4], [3, 1, 5], [3, 2, 1], [3, 2, 2], [3, 2, 3], [3, 2, 4], [3, 2, 5],
[3, 3, 1], [3, 3, 2], [3, 3, 3], [3, 3, 4], [3, 4, 1], [3, 4, 2], [3, 4, 3],
[3, 5, 1], [3, 5, 2], [4, 1, 1], [4, 1, 2], [4, 1, 3], [4, 1, 4], [4, 1, 5],
[4, 2, 1], [4, 2, 2], [4, 2, 3], [4, 2, 4], [4, 3, 1], [4, 3, 2], [4, 3, 3],
[4, 4, 1], [4, 4, 2], [4, 5, 1], [5, 1, 1], [5, 1, 2], [5, 1, 3], [5, 1, 4],
[5, 2, 1], [5, 2, 2], [5, 2, 3], [5, 3, 1], [5, 3, 2], [5, 4, 1]]
Number < 10=90
```

The total number of combinations is 90.

Then replace  $t_i$  in the combinations with the corresponding probability established by the experts. This replacement is carried out according to the data specified in Table 3.

```
In [6]:
ver=[[.2, .4, .6, .8, .9],[.1, .2, .5, .8, .9],[.6, .7, .9, 1, 1]]
In [27]:
print (ver[0][1])
0.4
In [48]:
element, elements=[], []
for z in result:
print(z[0], ' --> ', ver[0][z[0]-1])
print(z[1], ' --> ', ver[1][z[1]-1])
print(z[2], ' --> ', ver[2][z[2]-1])
element.append(ver[0][z[0]-1])
element.append(ver[1][z[1]-1])
element.append(ver[2][z[2]-1])
print(element)
elements.append(element)
element=[]
1 --> 0.2
1 --> 0.1
1 --> 0.6
[0.2, 0.1, 0.6]
1 --> 0.2
1 --> 0.1
2 --> 0.7
[0.2, 0.1, 0.7]
1 --> 0.2
1 --> 0.1
3 --> 0.9
... etc.
```

The following step involves the removal of the variants, where the probabilities are  $\leq 0.5$ , that is, taking into consideration constraint  $p_{ij} \geq 0.5$ .

```
In [59]:
an_elements=[]
for z in elements:
if z[0]>=.5 and z[1]>=.5 and z[2]>=.5:
an_elements.append(z)
print(an_elements)
print(len(an_elements))
[[0.6, 0.5, 0.6], [0.6, 0.5, 0.7], [0.6, 0.5, 0.9], [0.6, 0.5, 1],
[0.6, 0.8, 0.6], [0.6, 0.8, 0.7], [0.6, 0.8, 0.9], [0.6, 0.9, 0.6],
[0.6, 0.9, 0.7], [0.8, 0.5, 0.6], [0.8, 0.5, 0.7], [0.8, 0.5, 0.9],
[0.8, 0.8, 0.6], [0.8, 0.8, 0.7], [0.8, 0.9, 0.6], [0.9, 0.5, 0.6],
[0.9, 0.5, 0.7], [0.9, 0.8, 0.6]]
18
```

Thus, there are 18 combinations left that satisfy this constraint.

At the following stage, we performed the search for the product of the corresponding weight factor and the probability in combinations ( $w_i p_{ij}$ ), the search for the maximal sum.

```
In [67]:
elel=[]
element=[]
for z in an_elements:
element.append(z[0] * .3)
element.append(z[1] * .5)
element.append(z[2] * .2)
elel.append(element)
element=[]

print(elel)

prob=[]
for z in elel:
prob.append(sum(z))
print('-----')
```

```
print(prob)
print(max(prob))
[[0.18, 0.25, 0.12], [0.18, 0.25, 0.13999999999999999],
[0.18, 0.25, 0.18000000000000002], [0.18, 0.25, 0.2],
[0.18, 0.4, 0.12], [0.18, 0.4, 0.13999999999999999],
[0.18, 0.4, 0.18000000000000002], [0.18, 0.45, 0.12],
[0.18, 0.45, 0.13999999999999999], [0.24, 0.25, 0.12],
[0.24, 0.25, 0.13999999999999999],
[0.24, 0.25, 0.18000000000000002], [0.24, 0.4, 0.12],
[0.24, 0.4, 0.13999999999999999], [0.24, 0.45, 0.12],
[0.27, 0.25, 0.12], [0.27, 0.25, 0.13999999999999999],
[0.27, 0.4, 0.12]]
-----
[0.55, 0.57, 0.61, 0.63, 0.70000000000000001,
0.72000000000000001, 0.76000000000000001, 0.75,
0.77, 0.61, 0.63, 0.67, 0.76, 0.78, 0.80999999999999999,
0.64, 0.66, 0.79]
0.80999999999999999
```

Thus, 0.81 is the maximal probability in terms of successful project implementation.

Modeling results are given in Table 4.

Thus, the maximum probability of project implementation under these conditions is 0.81. The combination with probabilities [0.8; 0.9; 0.6] and with durations [4, 5, 1] corresponds to this probability. The probability of project implementation of 0.79 takes second place. The combination with probabilities [0.9; 0.8; 0.6] and with durations: [5, 4, 1] corresponds to this probability. The third place is taken by project implementation probability of 0.78. The combination with probabilities [0.8; 0.8; 0.7] and with durations: [4, 4, 2] corresponds to this probability (Fig. 1).

As Fig. 1 shows, combination No. 15 has the maximal probability of project implementation. Combination No. 18 ranks second, and combination No. 14 ranks third.

Table 4

Probability of project implementation and its rating for different combinations of the duration of project stages, in which probability is more than 0.5

No. by order	Combinations of the duration of project stages	Project duration	Combinations of corresponding probabilities	$w_i p_{ij}$	$\sum w_i p_{ij}$	Rating
1	3, 3, 1	7	0.6. 0.5. 0.6	0.18; 0.25; 0.12	0.55	15
2	3, 3, 2	8	0.6. 0.5. 0.7	0.18; 0.25; 0.14	0.57	14
3	3, 3, 3	9	0.6. 0.5. 0.9	0.18; 0.25; 0.18	0.61	13
4	(3, 3, 3), (3, 3, 4)	9, 10	0.6. 0.5. 1	0.18; 0.25; 0.20	0.63	12
5	3, 4, 1	8	0.6. 0.8. 0.6	0.18; 0.40; 0.12	0.70	8
6	3, 4, 2	9	0.6. 0.8. 0.7	0.18; 0.40; 0.14	0.72	7
7	3, 4, 3	10	0.6. 0.8. 0.9	0.18; 0.40; 0.18	0.76	5
8	3, 5, 1	9	0.6. 0.9. 0.6	0.18; 0.45; 0.12	0.75	6
9	3, 5, 2	10	0.6. 0.9. 0.7	0.18; 0.45; 0.14	0.77	4
10	4, 3, 1	8	0.8. 0.5. 0.6	0.24; 0.25; 0.12	0.61	13
11	4, 3, 2	9	0.8. 0.5. 0.7	0.24; 0.25; 0.14	0.63	12
12	4, 3, 3	10	0.8. 0.5. 0.9	0.24; 0.25; 0.18	0.67	9
13	4, 4, 1	9	0.8. 0.8. 0.6	0.24; 0.40; 0.12	0.76	5
14	4, 4, 2	10	0.8. 0.8. 0.7	0.24; 0.40; 0.14	0.78	3
15	4, 5, 1	10	0.8. 0.9. 0.6	0.24; 0.45; 0.12	0.81	1
16	5, 3, 1	9	0.9. 0.5. 0.6	0.27; 0.25; 0.12	0.64	11
17	5, 3, 2	10	0.9. 0.5. 0.7	0.27; 0.25; 0.14	0.66	10
18	5, 4, 1	10	0.9. 0.8. 0.6	0.27; 0.40; 0.12	0.79	2

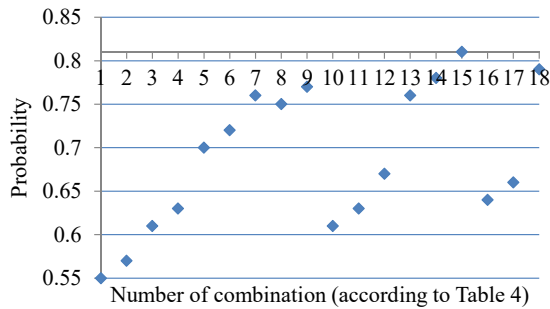


Fig. 1. Total probabilities of successful project implementation for the established combinations of durations of project stages

**5. 5. Selection of the optimal variant of the combination of the duration of project stages taking into consideration the constraints and objective function of the model**

Then we considered minimization of time  $\sum t_{ij} \rightarrow \min$  and costs  $\sum t_{ij}c_{ij} \rightarrow \min$  among the selected variants of combinations of durations of project stages.

In the example under consideration, three selected variants imply the 10-month duration of project implementation (Table 4). If we take into consideration the condition that during the project implementation the cost of 1 month at stage 1 is USD 375, at stage 2 – USD 750, at stage 3 – USD 190, the project cost is USD 1,500+3,750+190=5,440 for combination 1; USD 1,875+3,000+190=5,065 for combination 2; USD 1,500+3,000+380=4,880 for combination 3. That is why from the point of view of maximizing the probability of successful project implementation, it is better to choose the combination with the 4-month duration of the designing stage, 5-month duration of the development and testing stage, and 1-month duration of the implementation stage. With this time distribution between the project stages, the probability of successful project implementation is supposed to be the highest at the level of 0.81. Since the modeling results gave only one combination with the highest probability of successful project implementation and there is no need to choose between several combinations considering minimizing the costs, this combination will be optimal.

If the minimization of costs is a priority for a company, it is possible to choose another ratio of the duration of project stages. However, the combinations with lower costs have a lower probability of successful project implementation.

That is why for a software development company, it is advisable to use the results of the conducted modeling in order to win the bidding and implement a project successfully.

**6. Discussion of results of the study into determining the optimal duration of project stages in the knowledge management system of a company**

The system of company project knowledge management should include the subsystem of management of subject knowledge, project program and portfolio, the subsystem of knowledge about project management, and the combination of these subsystems. A part of this system is planning the duration of project works in order to generate new organizational knowledge for the successful project implementation within the established terms.

In this study, the problem was set and the descriptive model for determining the optimal duration of project stages

was developed. The project duration, the number of stages, at which the project is implemented, the costs at each project stage for the specified unit of time were established. There new corporative knowledge that can be acquired in the process of project implementation was presented (Table 1).

The mathematical model of the problem of determining the optimal duration of project stages was constructed and the algorithm for its solution was developed. The objective function (1), model constraints (2), (3) were determined, and the choice of the optimum variant of the duration of works was considered taking into account the minimization of time and expenses (4), (5). The algorithm implies determining the weight factors using expert estimations (the method of direct estimation) (6), (7), and the elements of combinatorics for finding the possible combinations of the duration of project stages (8).

The weight factors of the stages (Table 2) and the probability of implementation of tasks at each project stage within the established period were determined (Table 3) with the application of an expert survey.

Possible combinations of durations of project stages were formed, taking into consideration the constraints of model (9), Table 4, and Fig. 1.

The best variant of the combination of durations of project stages, taking into consideration the constraints and the objective function of the model, was chosen.

The developed model, unlike the models in papers [5–13], takes into consideration the peculiarities of a particular project, is a combination of the methods of expert evaluation and modeling. The objective function is the probability of successful project implementation. The model takes into consideration the weight factors of each project stage, considers the total probability of project realization in due time as an additive system, enables the optimal distribution of time between the project stages. In addition, the model makes it possible to choose the variant with minimum project duration and its minimum implementation costs. The constraints of this model are the lack of consideration of alternative variants of project implementation.

The drawbacks of the proposed approach include a simplified scheme of project works, as the project, which consists of only three stages, is explored in the model. However, the conceptual scheme for solving the problem of the duration of the project works with an increasing number of stages will not change. In addition, this approach similarly can be applied to determining the duration of works of a particular stage. Since the model was implemented using the Python programming language, only the program code undergoes changes. The number of combinations and arrays of processed information will increase as well. That is why the project that consists of three stages was chosen for the simplified visual representation of the model.

The subsequent research can be directed to the exploration and construction of the model, taking into account the alternative ways of project implementation, which will require taking into consideration the specific features of network planning.

**7. Conclusions**

1. The problem of determining the optimal duration of project works and generation of new organizational knowledge within the framework of this study is solved in the

company that is engaged in software development, plans to take part in the bidding for the order of the regional center for physical education of school youth (RCPESY). The order involves development, creation of the content, and implementation of the information system and database for management of the RCPESY activity. The customer established that the term of project implementation is 10 months. The company plans 3 stages of implementation of this project: designing; development and testing; implementation. At each of these stages the project team, which consists of the company's employees, plans both to use the existing knowledge and to generate new organizational knowledge that corresponds to the project. During the project implementation, the cost of 1 month at stage 1 is USD 375, at stage 2 – USD 750, at stage 3 – USD 190. This stage implies taking into consideration the specific features of the project under consideration.

2. The objective function implies the maximization of the probability of realization of the whole project taking into consideration weight factors. This model implies some constraints, specifically, that the sum of the duration of the knowledge generation at the project stages should not exceed the project duration. Another constraint is that each successive stage can begin if the set tasks are executed at the previous stage with the probability of not less than 0.5. Among the selected variants the same probability of project implementation, it is necessary to choose the one, where the total duration is minimal, which will enable saving time and minimizing costs.

The model algorithm consists of the following steps: determining the weight factors of each stage with the involvement of experts; establishing by the experts of the probability of tasks implementation and generation of new knowledge at each project stage; determining the total number of combinations of months of duration of each of project stages according to the input data; the search for all combinations of months of implementation of tasks at each stage; replacement of  $t_i$  in combinations with the corresponding probability established by the experts; removal of the variants where probabilities are  $\leq 0.5$ ; the search for the product of the corresponding weight coefficient and corresponding probability in the combinations and finding the maximal sum; minimization of time and costs. This algorithm makes

it possible to consider both the maximization of the probability of successful project implementation and the minimization of time for project implementation and costs.

3. Experts determined that to implement the project, the weight coefficient of knowledge generation and execution of set tasks for stage 1 (designing) is 0.3, for stage 2 (development and testing) – 0.5, and for stage 3 (implementation) – 0.2. In addition, the experts established the probability of realization of the set tasks and generation of new organizational knowledge within a certain period (from 1 to 5 months) at different project stages. This stage makes it possible to separate the weight of project stages and to establish the probability of successful implementation of project stages within a certain period.

4. The possible combinations of the duration of project stages were formed the help of the Python programming language, taking into consideration the constraints of the model. The total number of possible combinations is 243. Among them, 90 combinations, in which the sum of the duration of project stages is not more than 10 months, were chosen. 18 combinations, in which the probability of task implementation at each project stage is  $\geq 0.5$ , were chosen among them. This step makes it possible to select all combinations that satisfy the set constraints of the optimization model.

5. We chose the combination of durations of project stages, for which the maximal probability of successful project implementation was established and which satisfies the constraints of the model. The company should choose the combination with the 4-month duration of the stage of designing, the 5-month duration of the stage of development and testing, 1-month duration of the implementation stage. This time distribution between the project stages implies the highest probability of successful project implementation at the level of 0.81. The cost of the project will be USD 5,440. The solution of this problem makes it possible to choose the optimal distribution of time between the project stages and to determine its costs.

The presented approach allows determining the optimal distribution of time between the project stages in order to maximize the probability of successful project implementation, which creates the conditions for its acceptability for various companies.

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**Обґрунтовано вибір проектних рішень систем забезпечення кліматичних умов у приміщеннях на основі використання різних видів відновлюваних джерел енергії. Засоби, якими пропонується вирішення завдань проекту, є використання взаємодії між різними стейкхолдерами, необхідної для ефективного функціонування кількості підсистем у відкритій системі, тобто врахування ефекту синергетики. Доведено, що запровадження комплексної системи кліматизації та енергозабезпечення дозволить об'єднати функції, притаманні розрізним інженерним системам, забезпечити перетворення та перерозподіл енергетичних потоків різних видів, що дозволить мінімізувати експлуатаційні витрати. Побудована ієрархічна структура задачі ухвалення вибору альтернативних проектних рішень систем кліматизації на основі методу Analytic Hierarchy Process, що дозволяє отримати множину оптимальних варіантів застосування відповідного інструментального апарату Data Envelopment Analysis дозволяє побудувати систему оцінювання енергоефективності проектів складних систем кліматизації та енергозабезпечення при використанні різних видів відновлюваних джерел енергії. Побудовано функціонал, призначений для вибору оптимального варіанту проектного рішення системи кліматизації та енергозабезпечення. Запропоновані проектні рішення розглянуто з позиції визначення мінімізації сумарних витрат енергоресурсів та експлуатаційних витрат трьох альфа-стейкхолдерів. Запропонований показник відносної інтегральної енергоефективності дозволяє здійснити оптимальний вибір складних систем із різнорідними входними та вихідними характеристиками**

**Ключові слова:** системи кліматизації, відновлювані джерела енергії, проектні рішення, енергоефективність, ранжування

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## SUBSTANTIATING THE CRITERIA OF CHOOSING PROJECT SOLUTIONS FOR CLIMATE CONTROL SYSTEMS BASED ON RENEWABLE ENERGY SOURCES

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### 1. Introduction

In the field of project management, a portfolio is formed by consistent objective selection from the assigned totality of projects as homogeneous objects. Each of them is charac-

terized by sets of identical inputs and outputs in certain dimensions, division of totality into two classes – leaders and outsiders. The multistage process of evaluating indicators that determine the effectiveness of the relevant managerial decision that was made, consequential dividing homoge-