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У статті виокремлено особливості застосування загальних рівнянь математичної фізики еліптичного типу в задачах моделювання специфічних явищ взаємодії електромагнітних полів з елементами і частинками неоднорідного дисперсного середовища. Такі явища мають місце в установках сепарації органічної та мінеральної сировини або електромагнітної обробки зерна, насіння та ін. Зазначене є актуальним, оскільки звичайний підхід до формулювання математичних моделей у наведених задачах, що базується переважно на диференціальних рівняннях теорії поля у спрощеній формі, не завжди адекватно відображає фізичну сутність згаданих явищ. Тому він обмежує можливості поглибленого дослідження впливу багатьох факторів, які обумовлюють кінцеві результати процесів сепарації та електромагнітної обробки (ЕМО). В роботі запропоновано альтернативний підхід, побудований на використанні інтегральних рівнянь теорії поля, який базується на концепції первинних і вторинних джерел поля і дозволяє значно зменшити порядок системи рівнянь при чисельній реалізації алгоритмів розв'язання задач ЕМО та загальний обсяг потрібних обчислювальних ресурсів. При такому підході виявляються доступними для обрахування локальні параметри поля у взаємодії з окремими частинками та їхнього впливу одне на одне. Зазначений аспект є суттєвим для визначення технологічних характеристик виробничих установок ЕМО. Представлена математична модель, на відміну від поширених спрощених підходів до визначення параметрів поля і пондеромоторних сил, що діють у полі на частинки речовини, адекватно відображає фізичні закономірності розподілу потенціалів і напруженості електричного поля реальних зарядів та індукованих джерел. Завдяки цьому вона наочно відтворює механізм формування головних складових механічних зусиль, що діють на поляризоване тіло з боку електричного поля в цілому, через щільність елементарних зусиль, які справляє поле на поверхневі заряди, що індукуються в діелектричних тілах у зоні дії полів. Така математична модель є універсальним і компактним інструментом аналізу, проектування та оптимізації різноманітних установок і пристроїв, у яких використовується електричне поле та його електромагнітна взаємодія зі середовищем і окремими тілами

Ключові слова: електромагнітна обробка, дисперсні матеріали, математичне моделювання, електричне поле, частинки, речовина, сила

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FEATURES OF MATHEMATICAL MODELING OF ELECTROMAGNETIC PROCESSING OF BULK MATERIALS

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1. Introduction

In recent years, in agricultural production for the purification, separation and processing of cereals, seeds, fruits and other bulk materials, technologies using electric and magnetic fields have spread [1–5]. On a large amount of

experimental data, the effectiveness of the effect of an artificial electromagnetic field for stimulating the vital and growth processes of plants has been proved [4, 5]. Therefore, the use of electrophysical methods of processing biological objects is a good alternative to using it in agricultural production to increase crop yields of chemicals and genetically

modified organisms. Taking into account the environmental friendliness and energy-saving nature of such technologies, their further development and distribution requires full support [4–6]. However, it is possible to admit that to date, the theoretical foundations for the use of these processing methods in the field of agricultural production are not sufficiently developed and, accordingly, the level of engineering is almost at the initial stage of formation.

It is worth noting that the principle of separation (and sorting), which is the basis of this technology, is quite simple and consists in the differentiated force action of electromagnetic fields on parts of a dispersed medium that differ in physical parameters. According to it, it will be updated quite a long time ago in various branches of industrial production technological processes will be applied in which the main operational tool that interacts with the processed object is electric and magnetic fields. In the sectors of the metallurgical industry, they successfully use the technology of magnetic and electric enrichment and separation of minerals and other dispersed raw materials [7], the removal and sorting of non-ferrous scrap from industrial waste [8]. In chemical-technological production by electric fields, plastic granules and powder mixtures are sorted [9, 10]. Magnetic and electric fields are used to process water systems and other process fluids in the power system and industrial water supply systems [11, 12].

In the processes of magnetic and electrical processing of mineral raw materials, mainly static (constant) or low-frequency alternating fields are used. Subsequently, scientific research was developed in the study of the interaction of high-frequency electromagnetic radiation with nanoparticles, matrix-dispersed systems based on them and with plane-inhomogeneous layers (coatings) that separate two homogeneous dielectric half-spaces [13–15]. Now for the processing of grain and other plant materials, feed and waste in animal husbandry, pulsed and high-frequency fields are also used, including microwave [16].

The types of industrial installations used for electromagnetic processing (EMP) of mineral and biological materials are very diverse, but so far the most developed schemes have been developed in the constructive aspect. Among them, it is possible to recognize typical variants of drum (cylindrical type) and belt or conveyor (linear type) installations. For an initial idea of the spatial configuration of the working areas of EMP installations, let's present several schematic images of the mentioned types of structures – Fig. 1–3 [2, 17, 18].

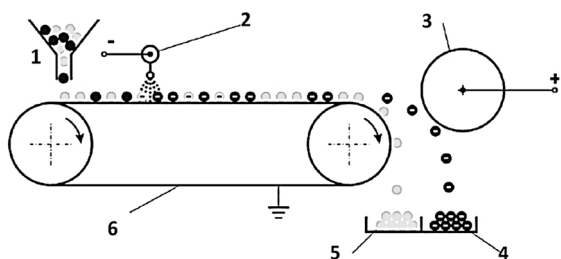


Fig. 1. Scheme of conveyor electrostatic separator [2].

The scheme shows: 1 – feeder; 2 – corona electrode; 3 – static electrode; 4 – charged particles; 5 – uncharged particles; 6 – conveyor belt

The electrostatic separator in Fig. 1 carries out the separation of a mixture of wheat bran useful fractions and low-value waste [2]. In this device, bran flow enters the con-

veyor belt 6 from the feeder 1, and when it passes under the electrode 2, the corona discharge charges light particles – husks, fibers, etc. with a negative charge. When the mixture flow reaches the zone of influence of the electric field of the positive electrode 3 and leaves conveyor belt, the trajectory of charged particles deviates towards this electrode. Thus, charged and uncharged particles fall into different capacities of the receiving hopper – 4 and 5.

The drum electrostatic separator in Fig. 2 is intended to separate the mass of crushed waste particles of various metals or mixtures of dielectric substances, which differ in physical characteristics of conductivity and permittivity [2]. In this embodiment of the separation device, a high-voltage corona discharge channel is used to recharge the particles, and the interaction of the electric field with the particles of matter occurs due to their polarization in this field.

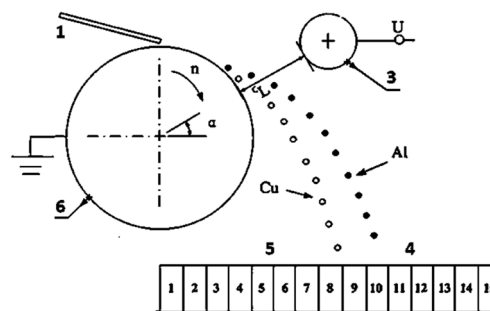


Fig. 2. Scheme of drum electrostatic separator [17]

In particular, Fig. 2 shows various deviation angles of the trajectory of movement of aluminum and copper particles. Designations in Fig. 2 are the same as in Fig. 1: 1 – loading tray; 3 – active positive electrode; 4 and 5 – sections of the receiving hopper; 6 – grounded rotating drum.

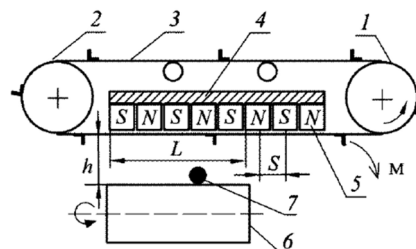


Fig. 3. Suspended iron separator [18]:

1 – drive and 2 – tension drums; 3 – belt; 4 – magnetic circuit; 5 – PM poles; 6 – conveyor; 7 – ferromagnetic particle

In a completely analogous manner, the linear analogue of the considered drum separator – the suspended permanent magnet (PM) separator [18] performs the separation of magnetic particles from non-magnetic ones [18], which is shown in Fig. 3. Magnets 5 are mounted on the magnetic circuit 4. The PM field attracts ferromagnetic particles 7, which deviate from the path of motion of non-magnetic particles and are removed from the conveyor. Thus, in particular, the grain is cleaned from steel chips and other ferromagnetic impurities.

The scheme identical with Fig. 3 is used in an installation for magnetic seed treatment [19].

At the same time, it should be taken into account that the modes of processing bulk materials in EMP installations can differ, in particular, in the nature of the forces of inter-

action of the electric and magnetic fields with particles of a dispersed medium and, accordingly, the specific energy costs and device performance. Factors causing these differences associated not only with the characteristics of the fields (static, variable, pulsed), but also with the electrophysical properties of the particle material, their size, shape, as well as the density and heterogeneity of the dispersed medium. Therefore, in terms of developing a mathematical model and the theoretical foundations of the separation process, it is important to identify local (microscopic) aspects of the effect of the field on small particles against the background of the global (macroscopic) distribution of fields in the working space of EMP installations. Such an integrated approach will provide the model with a universal character and the ability to introduce more reliable practical recommendations and characteristics into the design and optimization techniques of EMP installations.

2. Literature review and problem statement

Despite the difference in the areas of application and the variety of installation designs for performing the above technological processes, the same physical phenomena are the basis of the principle of their action. Therefore, the constructive schemes for the implementation of the working bodies and the construction of the working zone (space) and their interaction with the processed material in devices and apparatuses for electromagnetic processing of various types of dispersed materials have much in common [2–4, 7, 12, 18].

However, in the field of theoretical calculations of work processes and the design of such installations, there are many unexplained issues, and some aspects of the interaction of fields treated with a substance are established generally only empirically. This refers, first of all, to a clear identification of the nature of the inhomogeneities in the distribution of the force parameters of the field, because it is gradients that “pull” polarized and magnetized particles from the total mass. This also concerns the problem of the relationship between the parameters of the filling fields and the behavior of large ensembles of particles in the field.

In particular, in [3], the distribution of electric fields in the working space of the grain separator was determined by modeling on conductive paper without comparison with some calculations. And in joint works [3, 5] almost the entire analysis of electric fields and their interaction with dielectric particles (grains) was performed by the partial capacitance method, in which local details of the field distribution in the vicinity of individual grains do not appear, in principle, respectively, the results of calculations of the force field to a certain extent they are speculative in nature (heuristic).

In [12], the characteristics of a water treatment installation in a DC electric current field were calculated based on the results of experimental studies of current density in individual sections of the pipeline section. The methodology for calculating installations for treating water in fields of natural magnets is also based on recommendations for approximate values of magnetic field induction and size of magnets obtained from experimental studies. No theoretical justification is given in these methods.

Although it was mentioned in [19] that the magnetic system was calculated using the ELCUT finite element method for simulating two-dimensional fields, the essential feature of the grain processing regime – the energy dose of

the magnetic flux – was determined by piecewise linear (by the trapezoidal method) approximated integration induction curve in the gap between the magnets. That is, the theoretical basis for the development of such a technology has not been formed.

In [20], the task was set to determine the trajectories of the motion of plastic particles in the space of an electrically-static separator based on statistics on the distribution of particles by size and charge, obtained on the basis of data from individual studies, and therefore does not constitute a coherent methodology. Although the electric field in the space between the electrodes in the separator is described in this work by the correct equation for the electric potential, the boundary conditions are set along the artificial boundary arbitrarily located relative to the device.

In [22], the graph-analytical method was used to calculate the average value of the force acting on a polarized ball particle, which is located in an electric field by a system of plate infinite double-row electrodes. The distance between the rows of electrodes, on which the distribution and the magnitude of the force acting on the particle in the region between the rows of electrodes, are determined by the selection method.

In [23], the magnetic field in the working gap for calculating the forces acting on particles is determined using a 3D network of magnetic conductivities, which is rather cumbersome, and its parameters are chosen arbitrarily. By iterating the parameters of the network, they acquire a certain certainty, but given that the forces mentioned are calculated through the field gradient and particle volume, the sizes of the latter are limited by the discreteness of the networks. So such a model can't be recognized as adequate in the distribution of forces over an ensemble of particles.

This brief review shows a variety of approaches to the design of EMP installations and methods for calculating electric and magnetic fields, but the authors avoid rigorous field theory problems using either empirical-approximative justifications for design decisions or simplified formulations of differential equations.

This indicates the incompleteness of the development of the theoretical foundations of these processes, for which there are objective reasons, the basis of which, obviously, lies in the structural differences between the electrode and magnetic EMP systems from classical configurations. If traditional designs of electromagnetic devices have closed systems with a small gap and a small volume of the working space, then in EMP installations, on the contrary, the systems are practically open, and the working volume is quite large.

The specified factor, in turn, determines the features of the mathematical interpretation of the description of physical phenomena on which the action of EMP installations is based. It is based on the theory of second-order partial differential equations (PDEs), which is quite complicated for mathematical modeling. But the most significant circumstance, which complicates the problem, is the formulation of the boundary conditions of the boundary-value problem for the PDE in open electrode and magnetic systems, which are characteristic of the described installations. As a rule, in such systems, at least part of the boundary surface is eliminated ad infinitum; therefore, it is necessary to establish artificial boundaries, initially introducing a certain error into the statement of the problem.

So, further studies of this problem remain relevant, as evidenced by the constant increase in the number of new

publications devoted to the calculation of fields and the electromechanical aspects of their interaction with matter in the working area of EMP installations [20–24].

At the same time, mathematical models, which are based on simplified formulations of boundary value problems for EMP processes, are almost impossible to verify with experimental data. At present, there are no means for direct (point) measurement of electric fields in the working area during processing (under voltage). Therefore, the identification of real trajectories of motion of polarized or charged particles to quantify the performance of the installation is possible only indirectly. In fact, the only available way to obtain such information is computer (numerical) modeling based on physically adequate mathematical models. That is, the task is to determine in all details the topography of the fields in the working area and the behavior of individual particles and their ensembles, as well as to establish the relationship of the field parameters with the performance indicators of EMP installations.

In the sources analyzed above, devoted to the issues of calculating EMP installations, as well as other works relating to related fields of application of electromagnetic devices (electric machines and apparatuses), many different methods and techniques for calculating electric and magnetic fields are highlighted. However, due to the variety of types of electromagnetic systems (EMS) in many works, for almost every new version of the system, they make up a separate model and methodology for determining the operational characteristics of electrical devices.

Moreover, in the tasks of designing and optimizing EMP installations, for the calculation of electric and magnetic fluxes, simplified models of the working zone are usually used in the form of electrical equivalent circuits with capacitor chains or similar magnetic circuits [24–26]. Such models are quite simple and convenient for analysis, although, depending on the design of the EMP installations, they may differ in their topology and composition of elements. But the basis of all the options, as a rule, is a whole series of a priori and not always correct assumptions regarding the spatial distribution of the fields and the determination of the capacitance and conductivity of individual EMS elements.

Therefore, in order to increase the reliability of the results of calculating electric and magnetic fields and to reveal the characteristics of the characteristics of EMP installations and similar devices, many natural models are used in which the calculations are based on the correct equations of field theory:

- on second-order partial differential equations of elliptic type (Laplace or Poisson equation) with various boundary conditions, such as Dirichlet or Neumann problems, and mixed boundary value problems;

- on the integral equations of the theory of potentials – volume and surface potentials of a simple and double layer of charges and other scalar and vector sources, as well as their spatial derivatives.

The simplest tasks are when EMS can be represented as a piecewise uniform layered cylinder of a radial or axial assembly (disk type) [27] or its flat development (discrete-homogeneous strip) [28, 29]. In such cases, analytical solutions of differential equations of electro- and magnetostatics for a scalar or vector potential are used, as a rule, by the method of separation of variables in cylindrical or Cartesian coordinates. Such a solution for each of the heterogeneous regions seems to be an infinite Fourier series.

In the simplest case, 2-dimensional Laplace equation in Cartesian coordinates has the form:

- three-dimensional:

$$\Delta U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0;$$

- two-dimensional:

$$\Delta U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = 0, \quad (1)$$

where U – the field potential.

The solution of the formulated problem on the basis of this equation by the variable separation method has the form of a series of products of trigonometric and hyperbolic coordinate functions [30, 31]

$$U(x, y) = \sum_k (A_k \operatorname{sh} k\lambda x + B_k \operatorname{ch} k\lambda x)(C_k \sin k\beta y + D_k \cos k\beta y). \quad (2)$$

The coefficients of this series are determined from the boundary conditions (represented as a series), and convergence depends on these conditions. Let's note that in this method, as a rule, the distribution of the resulting potential or its normal derivative is specified as boundary conditions, whereas only the exciting field is known. In addition, this method is mainly adapted to solving internal problems, while in problems with an open magnetic or electrode system, characteristic of EMP installations, it is necessary to determine the field in the whole space.

Obviously, precisely because of these circumstances, the author [30] noted the inadequacy of the solutions obtained by this method for the natural essence of harmonic functions. Thus, the separation of variables for the Laplace equation does not provide the possibility of solving the problem. On the other hand, the availability of effective computer technology with a powerful arsenal of software, it seemed, would already solve the problem as completely traditional. However, if to limit ourselves to the capabilities of MATLAB, and other similar programs with which the PDE can be solved by finite element methods (FEM), the essence will be the same – the imposition of boundaries and the introduction of boundary conditions of varying degrees of adequacy [32]. In addition to the problem with the adequacy of the boundary conditions, the most important factor characterizing the FEM is the cubic dependence of the number of finite elements (FE) of the partition of the computational domain on its linear size. And the number of solar cells determines another factor – the accuracy and reliability of the solution, and together with them – the order and number of coefficients of the matrix, which approximate the PDE system. If the ratio of the FE size and the computational domain is minimum 1:20, then get 8000 FE, and the number of matrix coefficients (squared) exceeds $6 \cdot 10^7$. Improving the accuracy requires a significant increase in the number of FEs and corresponding increase in computational time in approximately the same proportion, that is, on conventional processors can reach many hours. Thus, according to [33], for a 3D mesh of $3 \cdot 10^6$ size, the counting time was ~ 4000 s, for 2D of $5 \cdot 10^4$ –1400 s, 1500–4000 s, and for some tasks a mesh of $443 \cdot 10^6$ FE was reported.

That is, the use of PDE-FEM-based numerical models, although it provides significant opportunities for optimizing design decisions, however, does not remove the problem of compactness, versatility, and speed of solving the problem itself.

Therefore, none of the known simplified or artificial methods and techniques for solving these problems by EMP does not provide sufficient versatility in the characteristics of the exciting field, as well as the convenience and availability of solutions. As a result, the possibility of an operational analysis of the influence of the design parameters of a device on its engineering efficiency is lost. On the other hand, the above analysis led to the conclusion that a rigorous mathematical approach to the interpretation of the problem in the form of differential equations in the computational plan is burdened with problems in terms of processor capacity, convergence and accuracy. In addition, this approach is not sufficiently informative in terms of physical content, since it does not reveal the induced components of the resulting field, and additional computational operations are required to identify them. Therefore, based on the content of the task under consideration, its integral formulation seems more rational.

So, the analysis of the problems on improving the calculation and design tools for various types of EMP technological systems and their optimization showed that the search for a universal approach and obtaining a compact result remains relevant even in modern computer technologies. From this follows the aim of this research.

3. The aim and objectives of research

The aim of research is to develop a universal compact physically adequate mathematical model of electric and magnetic fields in typical configurations of electromagnetic systems of EMP systems and their interaction with small particles of bulk materials during electromagnetic processing.

To achieve the aim, the following objectives are set:

- determine the conceptual basis of a physically adequate mathematical model for describing physical phenomena on which the principle of EMP installations operation is based, and substantiate the formulations of the corresponding mathematical model for calculating the ponderomotive forces acting on particles in the working zone;

- test the proposed mathematical model in the calculation of the electromechanical interaction of dielectric particles (grains) with an electric field in the working area of the EMP installation and determine the prerequisites for using the proposed model for calculating the working processes of EMP installations of a wider class.

4. The conceptual basis of a physically adequate mathematical model of EMP processes

The grounds for determining the conceptual basis of a physically adequate mathematical interpretation of the description of physical processes in EMP installations turn out to be a detailed examination of the local conditions for the occurrence of the ponderomotive effects of the electric field on small particles (micro- and nanoscale) of the processed bulk material. In [3], it was mentioned in principle that the strength of the interaction of the electric field with

the dielectric ball part is due to the dipole moment, which is formed when it is polarized by the same field due to the appearance of bound surface charges. A similar view on the content of the field interaction with nanoparticles was presented in [13]. It should be recognized that in this approach to the consideration of these phenomena, the actual physical essence is actually reflected.

Obviously, this is precisely why the work [30] gave preference to those mathematical formulations of the problems of calculating electric and magnetic fields, which follow from the conditions of their refraction at the media interfaces, where the concentration of coupled sources of a particular physical nature actually occurs.

Therefore, without going into a detailed comparison of differential and integral devices, the formulation of the field calculation problem, as an advantage of the latter, let's emphasize its physical content – a clear reflection of the relationship of the induced field with the excitation through spatial parameters and electromagnetic characteristics of media and substances that interact with the fields.

5. Construction of a universal physically adequate mathematical model of the spatial structure of electric and magnetic fields in EMP installations

Of the possible interpretations of the integral formulation of the problem, due to the simplicity of the electrical and magnetic systems discussed above, the Greenberg integral method [30] or, in the modern interpretation, the “secondary sources” method [34] will be quite adequate to solve the problem. The most compact form of introducing secondary sources is to use the potential of a simple layer of charges in a scalar form.

In any electrostatic system, field sources are physically real free and bound electric charges, and therefore the whole mathematical apparatus of electrostatics is physically adequate, including the theory of potential, boundary value problems, Green's functions, etc. [35]. As is known, the potential φ_σ of a simple layer of electric charges distributed on which surface S with a density σ creates an electric field whose intensity E_σ is determined by the gradient of this potential:

$$\varphi_\sigma(P) = \frac{1}{4\pi\epsilon_0} \oint_S \frac{\sigma(M)}{r_{PM}} ds_M, \quad (3)$$

$$\vec{E}_\sigma(P) = -\text{grad}_P \frac{1}{4\pi\epsilon_0} \oint_S \frac{\sigma(M)}{r_{PM}} ds_M. \quad (4)$$

In the formulas (3), (4)

$$r_{PM} = \sqrt{(x_P - x_M)^2 + (y_P - y_M)^2 + (z_P - z_M)^2}$$

- the radius vector drawn from the integration point M on the surface S to any observation point P ; the index in the grad operator indicates that the derivatives are taken exactly at the point P ; ϵ_0 – dielectric constant.

From this perspective, the dielectric and conductive elements of the system appear as a combination of secondary sources – bound charges induced on their surfaces distributed with a density σ , which must be determined. To obtain the final solution, it remains only to apply the integral formulas for calculating the potentials and field

strength of the found coupled charges (3), (4) and add them (according to the superposition principle) to the potentials and field strength of free charges (volume, surface or linear) $\varphi^0(P)$ and $\vec{E}^0(P)$.

That is, the resulting field strength at any point in P space should be considered as the sum of the fields of primary and secondary sources:

$$\vec{E} = \vec{E}^0 + \vec{E}_\sigma, \tag{5}$$

where \vec{E}_σ is required by the formula (4), but \vec{E}^0 has a similar form

$$\vec{E}^0(P) = -\text{grad}_P \frac{1}{4\pi\epsilon_0} \int \frac{\rho(K)}{r_{PK}} dV_K, \tag{6}$$

only the integration point K runs through not the surface S , but the volume V , where free charges with bulk density ρ are distributed, although free charges can also be on which other surfaces or thin wires. That is, it is generally said that the field of primary sources (the exciting field) is defined in any way and is known at every point in space, including on the surface S . The universal and most calculable approach to finding the distribution functions of surface charges σ at many boundaries The division of media with different dielectric constants (in a linear formulation) consists in the application of integral relations according to the Greenberg method.

This method is based on the ratio of the normal components of the field strength vector at the media interfaces where they break (jump) due to the difference in the polarization intensity of dielectrics:

$$\epsilon_0(E_{en} - E_{in}) = P_{n2} - P_{n1} = \sigma, \tag{7}$$

where E_{en} and E_{in} – the components of the vector \mathbf{E} from the external and internal sides of the interface.

It follows that

$$\sigma = 2\epsilon_0 \frac{\epsilon_i - \epsilon_e}{\epsilon_i + \epsilon_e} E_{Sn}$$

or

$$\sigma = 2\epsilon_0 \lambda E_{Sn}, \tag{8}$$

where

$$\lambda = \frac{\epsilon_i - \epsilon_e}{\epsilon_i + \epsilon_e},$$

$E_{Sn} = (E_{en} + E_{in})/2$ – the normal component of the field strength, which is formed by the combined action of primary \vec{E}^0 and secondary \vec{E}_σ sources directly at the interface. The field, created by secondary sources, is expressed by the gradient of the potential of a simple layer of the form (3), then with (8) let's obtain the equations for finding the distribution functions σ :

$$\sigma(Q) = 2\epsilon_0 \lambda \left[E_n^0(Q) - \text{grad}_{Qn} \frac{1}{4\pi\epsilon_0} \oint_S \frac{\sigma(M)}{r_{QM}} ds_M \right], \tag{9}$$

which in the classical form of the Fredholm integral equation of the second kind has the form:

$$\sigma(Q) - \frac{\lambda}{2\pi} \oint_S \sigma(M) \frac{\cos(\vec{r}_{QM}, \vec{n}_Q)}{r_{QM}^2} dl_M = 2\epsilon_0 \lambda E_n^0(Q). \tag{10}$$

In two-dimensional (2D) space, the expressions given are modified as follows:

– potential (3) is replaced by a logarithmic potential

$$\varphi_\sigma(P) = \frac{1}{2\pi\epsilon_0} \oint_L \sigma(M) \ln \frac{1}{r_{PM}} dl_M, \tag{11}$$

where L – the contour (line) of the media section;

– equation (9) takes the form:

$$\sigma(Q) = 2\epsilon_0 \lambda \left[E_n^0(Q) + \frac{1}{2\pi\epsilon_0} \oint_L \sigma(M) \frac{\cos(\vec{r}_{QM}, \vec{n}_Q)}{r_{QM}} dl_M \right]. \tag{12}$$

Equations (10) also change accordingly:

$$\sigma(Q) - \frac{\lambda}{\pi} \oint_L \sigma(M) \frac{\cos(\vec{r}_{QM}, \vec{n}_Q)}{r_{QM}} dl_M = 2\epsilon_0 \lambda E_n^0(Q). \tag{13}$$

The general approaches to the methods for solving integral equations (IE) of type (10) and (13), both analytical and numerous, have been sufficiently developed, but their detailed consideration is beyond the scope of this study. Although, of course, these or those of them must necessarily be attached to the totality of tools for mathematical modeling and calculation of various EMIs. Among the common analytical methods, let's note the following, based on the expansion of functions in series, Fourier integral transforms, etc. [36], however, their scope is limited only to simple surfaces and shapes.

The numerical solution of IE with kernels of this type is carried out mainly by replacing the integrals with finite sums with a sufficiently high degree of discretization of the integration surface (contour) and bringing them to systems of linear algebraic equations (SLAEs), which are solved by known methods. It is worth noting, however, that in engineering practice, due to the peculiarities of specific technologies, unusual situations often arise when certain problems arise using conventional methods for solving the aforementioned problems. In particular, depending on the boundary conditions and parameters associated with the characteristics of the medium, the SLAE may turn out to be ill-conditioned, then the problem becomes incorrect [37], and special methods must be applied to solve it. Despite this, it is necessary to process specialized mathematical techniques and computer models adapted to a certain category of problems. It is this situation that the mathematical model is oriented to, it is considered.

In particular, the geometric (spatial) basis for the computational implementation of integral operators in formulas (3), (4), (6), (10) requires special attention, because the geometric characteristics of the modeled objects, as well as the set of coordinates of the collocation points on which the SLAE relies are directly used in mathematical operations of differentiation and integration on curved limiting surfaces. Therefore, to ensure the computational process, first of all, the formation of matrices of SLAE coefficients that approximate IE, the procedure of perfect calculation of the geometric parameters of elements of curved surfaces and contour lines, their curvature, and the normal vectors to them, is of great methodological importance.

One of the variants of such a procedure, quite compact and convenient under the name “Geometric IE platform” [38], was used for calculations in the example considered below.

Within the limited scope of the study, it is difficult to reveal in detail the main characteristic features and capabilities of the presented mathematical model, however, the authors carried out its perfect verification, in particular, on the canonical problem of calculating the circle field in a homogeneous external field, which has an analytical solution. The deviation of the results obtained on the model from the analytical ones appeared only in the 15th sign.

At the same time, in order to test the model's functionality, a demo example of its application to the EMP problems under consideration is presented, implemented in the MATLAB software environment.

6. Testing of the mathematical model in the calculation of ponderomotive forces acting on dielectric particles (grains) in the electric field of the separator

As an approbation of the proposed mathematical model, the calculation of ponderomotive forces acting on dielectric particles, in particular, grains in the electric field of the separator, is performed. The specific forces acting on the side of an inhomogeneous electrostatic field on an elongated dielectric particle (grain) were determined, and the resulting forces and moments that induce this particle to move were found. In this case, the exciting field is formed by two oppositely charged parallel cylindrical electrodes placed horizontally at a distance of 5 diameters (between the centers).

Calculation of the field of these electrodes is a classical problem, the solution of which is given in [39]. Let's use it in the form of the distribution of potential and components of the field strength in the zone where the particle is located (Fig. 4 shows the vector field, Fig. 5 hews the intensity (module) of the field strength characterizing its heterogeneity).

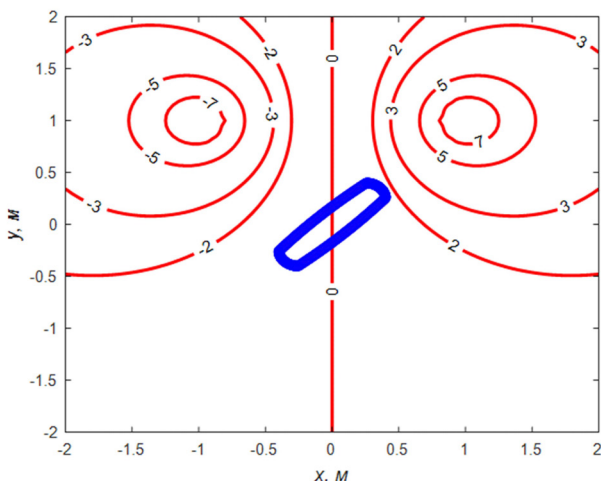


Fig. 4. Sketch of the particle location in the field of charged electrodes

The particle has a length of 3 diameters of the electrode and is three times thinner (Fig. 6). The relative dielectric constant of the particle material ϵ is taken equal to 80 (close to water). All calculations for clarity are carried out in rela-

tive units, but the results are easily translated into physical quantities.

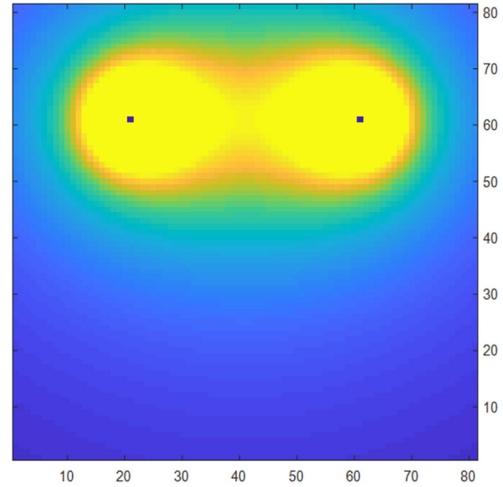


Fig. 5. Visualization of the field strength heterogeneity of the electrodes

Fig. 7 shows the results of the distribution of the density of secondary sources – induced charges in comparison with the primary field – obtained from equation (13).

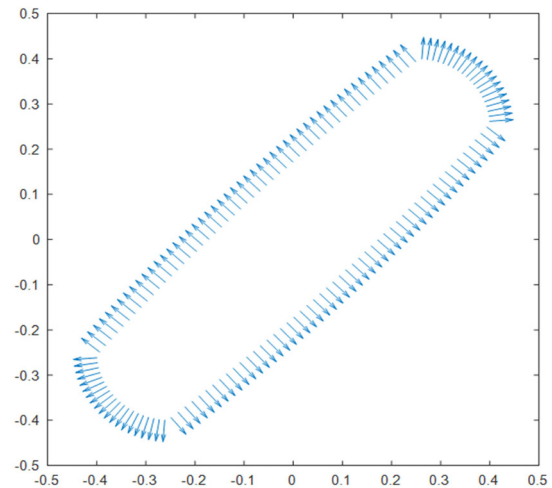


Fig. 6. Particle contour with normal distribution

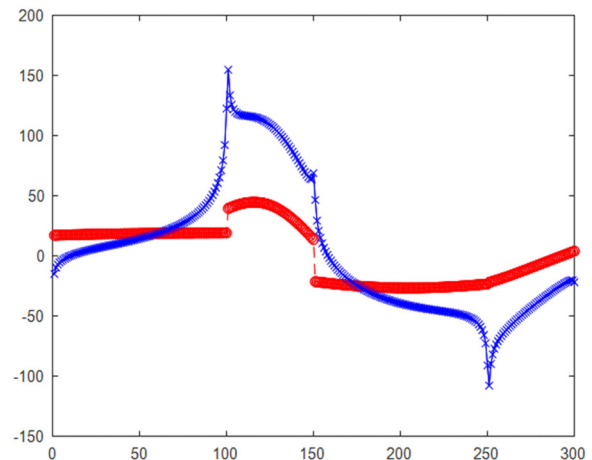


Fig. 7. Distribution of the field of electrodes (red line) and induced charges σ (blue line) along the particle contour

Fig. 8 shows the induced field and lines of equal potential of a polarized particle, and Fig. 9 – equipotentials of the resulting field.

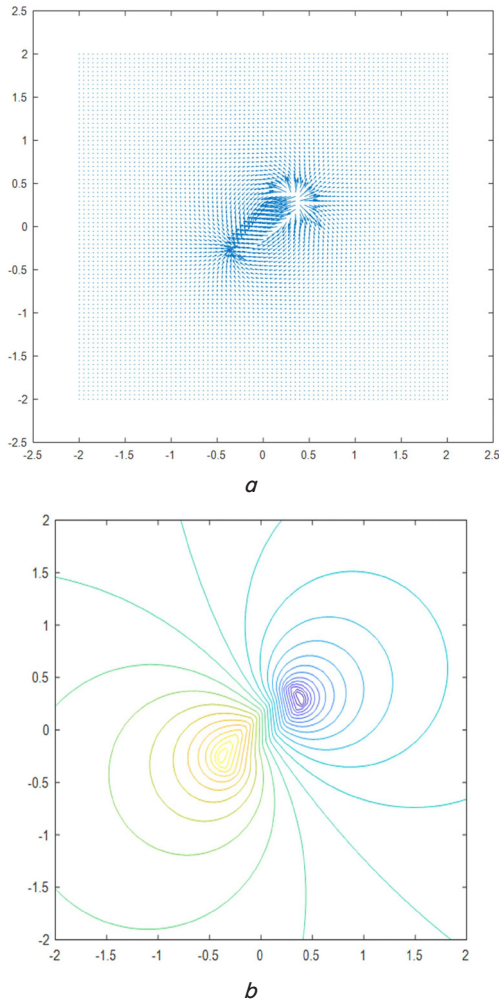


Fig. 8. Induced field: *a* – vector field; *b* – lines of equipotentials of the induced charges of the particle

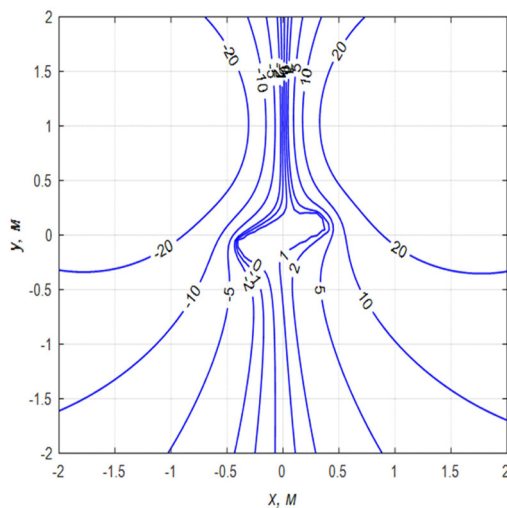


Fig. 9. The lines of equipotentials of the resulting field

From the last diagram, one can imagine the nature of the force action of the field on the introduced particle.

However, the obtained data are sufficient to calculate the specific physical indicators of this force. So, in accordance with the chosen model, the force that the electrostatic field of the electrodes exerts on the dielectric particle introduced into it can be defined as the Coulomb interaction between this field and the charges induced by it, found from equation (13), which are located on the particle's surface [40, 41]. Moreover, each component of the field E_ξ creates a corresponding component of the forces that are distributed over the entire area of the boundary surface with a density:

$$f_\xi(x'_n, y'_n) = \sigma E_\xi(P_n) = \sigma E_\xi(x'_n, y'_n), \tag{14}$$

where the index ξ takes values x, y (or also z in the 3D model), and the point $P_n = P(x'_n, y'_n)$, is the center of the n -th elementary surface area of the particle $\Delta s'_n$. The coordinates of the points P_n are calculated from the geometric data of its contour (Fig. 6). In particular, they can coincide with the collocation points of the aforementioned IE geometric platform.

Carrying out the numerical integration of the force density (14) over the area of the boundary surface, that is, actually summing up each component over all small areas, the components of the main force vector are found that generally act on the particle:

$$F_\xi = \oint f_\xi(x', y') ds \cong \sum_n f_\xi(x'_n, y'_n) \Delta s'_n. \tag{15}$$

Further, according to (14) and (15), the coordinates of the centers of parallel forces are calculated [31] (give full 3D expressions, it is clear that for the 2D version of one component, for example, there will be no z):

$$x'_{C_\xi} = \sum_n x'_n f_\xi(x'_n, y'_n) \Delta s'_n / F_\xi;$$

$$y'_{C_\xi} = \sum_n y'_n f_\xi(x'_n, y'_n) \Delta s'_n / F_\xi;$$

$$z'_{C_\xi} = \sum_n z'_n f_\xi(x'_n, y'_n) \Delta s'_n / F_\xi. \tag{16}$$

Further, for system (15) of forces F_ξ (ξ takes values 1, 2, 3, corresponding to projections on the x, y or z axis) with application points C_ξ , the coordinates of which are defined in (16), let's obtain the principal vector and the principal moment of forces:

$$\bar{F}_M = \sum_{\xi=1}^3 \bar{F}_\xi = \bar{e}_x F_x + \bar{e}_y F_y + \bar{e}_z F_z, \quad \bar{M}_Q = \sum_{\xi=1}^3 \bar{r}_{Q, C_\xi} \times \bar{F}_\xi, \tag{17}$$

where \bar{r}_{Q, C_ξ} – the radius vector drawn from the center of erection of the system of forces Q to the point C_ξ of application of force F_ξ .

So, using the distribution function of the density of secondary sources obtained from Eq. (13), σ , based on (14), let's find the distribution of the force density along the particle contour: the horizontal component $f_x(s)$ (blue line) and the vertical component $f_y(s)$ (red line) shown in Fig. 10. According to the formula (15), the components of the main force vector acting on the particle are obtained: horizontal $F_x=247$ c.u. and vertical $F_y=433$ c.u. According to (16), the coordinates of the centers of parallel forces were found [42], which are the points of application C_ξ of components of the main vector:

$$x'_{c,x} = 1.72; y'_{c,x} = 1.07;$$

$$x'_{c,y} = 0.142; y'_{c,y} = 0.133.$$

Since the centers of application of the components of the main force vector do not coincide, it is necessary to reduce them to a single center Q . Obviously, it is more convenient to choose the point C_0 (0,0) of the center of mass of the particle, then combining the point of application of the components F_x and F_y with the point C_0 , let's obtain by the formula (17) the main moment of forces relative to the axis passing through this point parallel to the z axis.

This moment will have only the z component, which, according to the rules of the vector product, will be:

$$M_Q = M_z = (y'_{c_0} - y'_{c,x})F_x + (x'_{c_0} - x'_{c,y})F_y =$$

$$= (0 - 1,07)(-247) + (0 - 0,133)(-433) = 321 \text{ c.u.},$$

and is clockwise, that is, it tends to unroll a particle along the field lines of force. In this case, the main force vector is $F \approx 500$ c.u. and directed at an angle of 60° to the horizontal; it "draws" a particle towards the highest intensity of the field strength, closer to the electrodes.

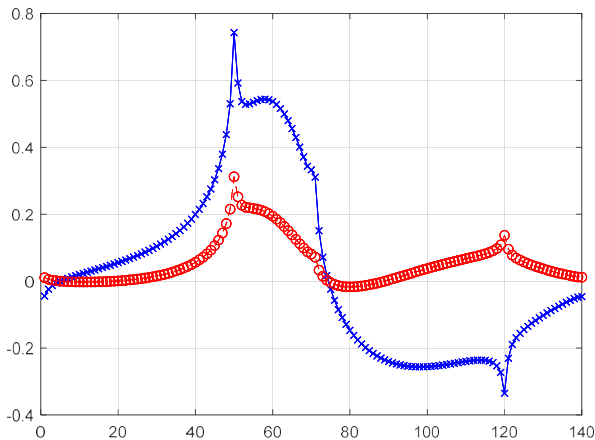


Fig. 10. Distribution of surface forces f_x and f_y along the particle contour (blue and red)

Fig. 11 shows the distribution diagram of surface forces along the contour of the particle surface and shows the direction of the main vector of force and moment acting on the particle by particle, calculated using the above mathematical model.

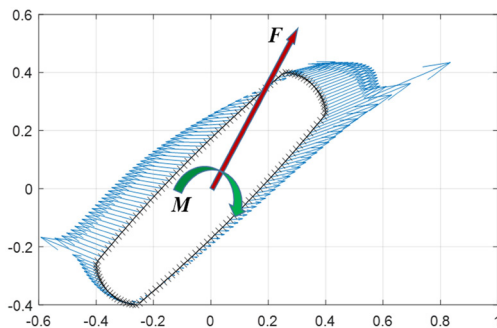


Fig. 11. The plot of surface forces along the contour of the particle

7. Discussion of the research results of mathematical models of the electrostatic field and the interaction with the field of dielectric particles

The given example shows that the presented mathematical model has a significant degree of universality both with respect to the shapes of particles and the shapes of the electrodes, their spatial distribution, the ratio of the geometric proportions of the electrode systems and the microscales of finely dispersed media. The universality of the model is also manifested in relation to the electrophysical characteristics of the processed materials, the material of the electrodes, current conductors and any elements of electrical systems. The indicated properties are laid precisely in the integral form of interpretation of the equations of mathematical physics, represented by a combination of formulas (3)–(13). These equations operate with real physical objects – electric charges, which are located on real surfaces and interact naturally – (14), (15), (17) – at real distances. This is a difference from the common approach to similar problems in the form of differential equations, where the essence of physical phenomena is lost in a series of operations on finite elements and interpolation polynomials or infinite series of harmonics.

At the same time, the model has a compact design of both a mathematical basis and a computational implementation. To solve the demo, the results of which are presented in Fig. 4–11, less than 200 discretization elements of the particle contour were necessary, but this was enough to calculate all the necessary parameters of its interaction with the field, while at least 5000 finite elements and the use of additional algorithms to calculate forces of interaction were necessary to solve this problem by the PDE-FEM method. Thus, thanks to the proposed mathematical model, it is possible to effectively and efficiently obtain physically adequate results when calculating the operating modes of EMP installations and apply them in design techniques.

It should also be added that in fact the same equations (9), (10) that apply to the electric field also express the ratio of the normal components of the vector of the magnetic field strength at the interfaces between media with different magnetic permeabilities, if by the notation σ let's mean the jump of the normal component of the vector magnetization. The fundamental difference between these situations is that there are no "magnetic charges" in nature, while electric charges are quite real. Therefore, the electrostatic analogy can quite justifiably be considered a physically adequate simulation model of magnetic systems in EMP installations, which is suitable for use in the calculations of various magnetic systems and, in this sense, is universal in their topology. The authors will propose the development of the presented mathematical model as applied to the problems of magnetic fields in further studies.

8. Conclusions

1. A distinctive feature of the presented mathematical model is that it adequately reflects the physical laws of the distribution of potentials and electric field strength not only of real charges, but also of induced (secondary) sources,

including local characteristics of the fields around microparticles. At the same time, it clearly reproduces the mechanism of formation of elementary forces that a field produces on surface charges induced in dielectric bodies in the field of action of the fields and the distribution of their density, as well as the main components of mechanical forces and moments acting on the polarized body from the electric field as a whole, that is, its integral action. Due to this, it becomes possible to calculate the forces and energy of interaction between individual particles in the ensemble and determine the resolution of the separator.

2. Testing the model on the canonical problems of electrostatics and testing it on model problems, one of which is given in this paper, shows its full suitability for use as a compact tool for analysis, design and optimization of various electrostatic separator configurations and physical properties of materials installations and devices that use an electric field and its electromechanical interaction with the medium and individual bodies. The presented mathematical model in the form of an electrostatic analogy can be fully applicable to the considered problems of processing materials with magnetic fields.

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