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Two-phase flow with gas-liquid component is commonly applied in industries, specifically in the refinery process of liquid products. Oil products with bubbles contents are undesirable in a production process. This paper describes an investigation of a process mechanism regarding the bubble breakup of the two-phase injection into quiescent water. The analytical model was developed based on the force mechanism of water flow at the bubble interface. The inertia force of water flow continually pushes the bubble while the drag force resists it. The bubble gets shapes change that affects the hydrodynamic flow around the bubble. Vortices with high energy density impact and make the stress interface over its strength so that the interface gets tear. The experiment was carried out by observing in the middle part of the injected flow. It was found that the forming process of bubble breakup can be explained as the following steps:

1) sweep model is a bubble pushed by the inertial force of water flow. The viscous force of water shears the surface of the bubble. The effect of both forces, the bubble changes its shape. Then trailing vortex starts to appear in near bubble tail. The second flow of water is in around of the bubble to strengthen the vortex energy density that causes fragments to detach from the parent bubble;

2) stretching model, the apparent bubble has high momentum force infiltrated in stagnant water depth and bubble ends are stretched out by the inertial force of the bubble and viscous force of water. The bubble surface has experienced stretching and tearing become splitting away. Based on the finding, the breakup process is highly dependent on the momentum of water flow, which triggers the secondary flow as the initial process of vortex flow, and it causes the tear of the bubble surface due to angular momentum

Keywords: inertial and viscous force, angular momentum, quiescent water, energy density, bubble interface

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## 1. Introduction

A liquid product from the two-phase (liquid-gas) separation process is not desired to contain bubbles. Separation of gas bubbles in the process of the refinery of liquid products is not all separated because there are split bubbles of various sizes. This split bubble is caused by the presence of vortices which affect the bubbles. These vortices arise due to flow patterns that are influenced by arbitrary bubble shapes. The separation of the bubbles is based on the buoyancy force of the bubbles. For small bubbles they are difficult to be since small bubbles have small buoyancy force compared to inertial flow force.

Water flow injection velocity could generate flow pattern around the bubble. This flow pattern affects the bubble shape. Bubble shape contour and flow pattern will trigger to generate vortices around the bubble. Flow pattern and bubble shape in certain conditions change vortices size to be small ones. The small vortex has a high energy density which has a high tearing power to the bubble interface. The UDC 628.54 DOI: 10.15587/1729-4061.2020.208333

# ANGULAR MOMENTUM TEARING MECHANISM INVESTIGATION THROUGH INTERMOLECULAR AT THE BUBBLE INTERFACE

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> novelty of this study is the finding of the interface tearing started from weaken intermolecular induce force. That is intermolecular induce force between the Hydrogen atom of water (H<sub>2</sub>O) and Nitrogen (N<sub>2</sub>) molecules. The vortex twists the water molecule then the induce force is cut off. The interface is torn after that the bubble gets split up to be several parts, thereby increases the surface tension of the smaller bubble. This finding is very important for practical breakup analysis as well as for bubble liquid separation in oil processing equipment.

#### 2. Literature review and problem statement

Injection of two-phase flow is a process in industries. When a bubble is injected in quiescent water, the bubble experiences breakup for several times after exit from a horizontal straight nozzle. This breakup mechanism is the research observation as the effect of distortion between the bubble and fluid surround or pressure difference between

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them. The bubble breakup is affected by both the unstable surface and wake vortices in a stagnant liquid [1]. The bubble is penetrated by fluctuating vortices that cause tearing of the bubble surface. The number of bubbles breakup depends on the number of vortices which have higher energy than the energy of the bubble surface. Breakup of a bubble is caused by hydrodynamic forces. These forces try to shear the bubble surface and then the surface is resisted by surface tension forces [1–3].

A bubble is influenced by the liquid flow field. The bubble experiences stretching at the middle part of the bubble so its surface tension increases. The bubble can be imaged as an elongated dumbbell which will collapse [1]. On the other hand, an expanded bubble volume causes decreasing of gas pressure; Young-Laplace equilibrium gets disturbances. Hence the gas pressure is reduced up to lower than liquid pressure. This condition causes penetration from surrounding pressure to that the value of curve coefficient will decrease. Then the daughter bubbles separated from a mother bubble. At this time of separation all the bubbles do not move up because of the absence of gravity [4].

A submerged nozzle discharges gas by downward jets into a liquid, where penetration length is an important parameter that is used in equipment systems and devices design [5]. The flow rate of gas-injection increases bubble size and decreases liquid velocity also comparatively not correlation with a diameter size of gas injection and the composition of gas [6].

Bubble breakup is related to hydrodynamics, the mechanisms of the breakup can happen in the laminar and turbulent flow [1]. Viscous shear will stretch the bubble surface that can elongate the bubble and cause the bubble breakup in laminar flow. The bubble breakup is resulted by surface instabilities and wake vortices. Variations of bubble shape are a consequence of hydrodynamic pressure around the bubble. The bubble breakup is significantly affected by the pressure [7]. The bubble surface is acted by dynamic pressure forces which determine the part of curvature form and establishes the bubble shape as well as the motion [8]. The eddy kinetic energy is to deform the droplet surface as seen at surface energy increase properly, as a theoretical model [9].

The breakup rate of rising bubbles significantly increases in water. Observing them a significant effect of surface tension was found, while the viscosity effect was neglected in the bubble Reynolds number value from 60 to 3000, which has been studied [10]. Bubble breakup processes for the bubbly jet in stagnant water and visual observations of the rising bubbles have been investigated [11].

The depth of the downward penetration was gas jet injection into water has been experiment done [12]. The nozzle was submerged slightly to the underwater level to prevent air entrainment that indicated the penetration depth was proportional to nozzle diameter, gas velocity, and ratio of gas to liquid density. Penetration length of the void generally is influenced by the gas velocity of injection, the injected gas properties, diameter of discharging pipe, and liquid in the container [5]. The size of the bubble is influenced along the injected early steps [6]. The increasing bubble size is the impact of flow rate increase of gas-injection and velocity decrease of liquid.

Bubbles injected into turbulent flowing liquid in the pipe line have investigated [1]. The bubble break-up mechanisms result from hydrodynamics. Breakup processes of droplets in turbulent distributions are multifarious. It is stated that more than one breakup mechanism may be present in turbulent distribution. A droplet is available in the turbulent field and also it gets both viscous and inertial forces [9]. The effect of viscosity on bubble elongation was tried to correlate with the maximum bubble size. Gas bubbles deformation subjected to simple shear flow, which effect of shear fields in processes is very little [13]. Before the bubbles are swept off, they can get the maximum diameter into the flowing liquid [14]. Bubbles surface conditions and liquid flow rate affect bubbles diameter.

Developed volume of bubbles has an effect that the inside pressure of the ones becomes lower than before. A change of bubbles volume is due to an increase or decrease of the ambient liquid pressure [15]. The pressure difference is available outside the interface as an effect of hydrodynamic flow. The momentum interaction between the liquid and the bubble correlates between the surface tension and the hydrodynamic pressure [16, 17]. In this experiment, the effect of heat is not considered because it is work done at a constant temperature of 31.4 °C.

Turbulent phenomena are represented in the bubble swarm linkage and separation models [18]. A bubble breakup is affected by the bubble-induced turbulence [19]. The size of collided turbulence is an effective contribution to the bubble breakup process based on the change of surface energy [20]. The energy exchange mechanism is taken into analysis between vortex and surface configuration.

The papers [21, 22] present the results of research about the operating pressure that describes the rate of injection velocity. This velocity will generate flow pattern in the penetrated medium which indicates significant effects on the hydrodynamics of bubble surround. The turbulent velocity that occurs will change the gas-liquid medium and cause bubbles to break up. It is shown that the effect of pressure is closely related to the bubble breakup behaviors. Small-size bubbles tend to follow liquid streams that are difficult to separate. But there were unresolved issues related to the effect of pressure on the hydrodynamics of a bubble surround has been extensively studied by experiment, the underlying mechanism is not well understood. The pressure operation is one of many factors, for example, fluid characteristics and properties; operation conditions (flow capacities, pressure and operation temperature of fluid) that influence bubble breakup. This approach was used in [23], however, there was an attempt to construct a theoretical model for breaking gas bubbles up in the turbulent flow. The theory model is formulated as a calculation problem in which the change in density is likely to be reflected from the distribution of gas bubbles in the turbulent isotropic flow of incompressible liquid. Bubble breakup in turbulent flow occurs when the intensity of turbulent velocity fluctuations exceeds several threshold values. The condition of turbulent flow intensity and bubble size supports a bubble break up. Many studies are limited in revealing the mechanism of bubble splitting.

To overcome the bubble breakup, an artificial technique is needed for removing or reducing the vortex generation. By giving additional flow in the direction of injection flow against the secondary flow at the locus where the vortex tends to develop could eliminate or reduce the presence of vortex. The other technique is to take advantage of the downward pull of the gravitational effect. All this suggests that it is advisable to conduct a study on the bubble breakup mechanism to find ways to overcome bubble breakup in the separation process. On the other hand, the work can contribute to improving the next design of the device.

This research work is needed to do for determining the distance between the nozzle and the bubble start to break up. The distance could indicate what the value of optimum operational injection velocity when applied in the separation process. It was an important thing to minimize small or even micro bubbles trap in the liquid and sticky on the container. Gas jets are examined in the downward direction to a liquid and affected some factors that are gas density, liquid viscosity, nozzle diameter for jet penetration, surface tension of liquid, and gas jet velocity [24]. Penetration air bubbles are to predict the depth into the liquid pool [25]. The momentum of the initial jet can also determine the depth.

Bubble breakup is in turbulent flow inside a Plexiglas pipe [1]. On the other side, this work was to study bubble breakup in stagnant water by a bubble flow injection. Hydrodynamic condition in stagnant liquid there is two states, these are surface instability and wake vortex while in this work there are three steps:

1) bubble volume development means height and length increase;

2) bubble stretching means height decrease and length proceed increase;

3) bubble collapse means separated volume from the parent bubble to be daughters and a new mother bubble.

Mechanisms of the breakup and distortion experiment of liquid drops are sprayed in atmosphere with air jet at room temperature [26]. Meanwhile, this work injected two-phase flow into stagnant water it can be assumed in the same analogy with the sprayed liquid drops. The detachment of bubbles from the nozzle is influenced by the surface tension of the surrounding fluid. The growth of bubbles increases the surface tension. Stretching of the bubble surface starts from the initial growth [27].

The effect of an elongated bubble is formed by solid packing around it [28]. In this experiment, the elongated bubble was influenced by hydrodynamic distortion of water. A bubble breakup through a T-junction channel divided to become two steps: pinch-off and squeezing stage [29]. This paper proposed a bubble collapse in two models such as sweep and stretching models. The angular momentum of hydrodynamic water flow takes a role in the mechanism of the models. The angular momentum has a high shearing power to tear the bubble surface.

Two-phase flow injected in quiescence water, the bubble experiences deformation. Inertial water flow continually pushes the bubble while hydrodynamic flow patterns contribute to the bubble change [30, 31]. The flat bubble surfaces mean the flow patterns over them are streamlined. On the other hand, the curve bubble surfaces are predicted the flows are vortex. The vortex will be small if it is pressed between the bubble surface and streamline flow in static pressure conditions. The small size vortex has high energy density which has high tearing power to the bubble surfaces.

The bubble experiences jet flow after the terminal velocity point, where the bubble still has a high inertial flow force. The effect of the force, rise second flow over the bubble then that gets shearing force at the bubble surface. The impact of two forces is in the opposite direction and then the bubble gets stretched. The stretched effect makes bubble surface stress increase. If any added stress on the surface may be the bubble easy to split up.

The small size vortex with high energy density is easy to tear the surface. So where the bubble part experiences breakup, the part can be identified as vortex place. The vortex will affect the water molecules layer at the bubble interface. The interface structure consists of water molecules outside the bubble and oxygen and nitrogen molecules inside it. The water molecules are induced by oxygen and nitrogen molecules in the bubble. Because hydrogen bonds in the water molecule are stronger than inducing of oxygen or nitrogen molecules there is no hydrogen electron displacement. Then the bubble and water make equilibrium force as the interface. The effect of the impact, water molecules turn and disturb the equilibrium. The inducing of nitrogen atoms to hydrogen atoms break at first than oxygen, because induced nitrogen atom is weaker than oxygen. Interface tearing starts from induced nitrogen to hydrogen atom in water molecules.

The bubble changing process is studied based on the image of the experimental observations. Conversely, the characteristics of this process are analyzed based on predictions from available knowledge. Then this is matched with the phenomenon of bubble changes due to the complex flow pattern that is difficult to predict and analyze. The size of the vortex roughly follows the bubble surface curvature at the location before the bubble breakup. At least the main causes can be identified.

#### 3. The aim and objectives of the study

The aim of the study is an investigation of the breakup mechanism of the bubble separation process in the cyclone separator which is not desired in the bubble separation. For obtaining the breakup phenomena, an investigation was done through simple injection in the water pool.

To achieve this aim, the following objectives are accomplished:

– investigate the effects of inertial water flow to the bubble shape since water flow inertia due to injection tends to continuously push bubbles to change its shape. The condition of the bubble and its surroundings can be identified based on the Young-Laplace balance;

- predict the bubble shape changes related to the vortices generation. This prediction is important since the flow of water around the bubble tends to follow the surface contours of the bubble. When there is a region with lower pressure due to moving the bubble, the flow of water will fill into that region. Because of the different velocity and direction of movement of water molecules, vortices arise;

 inspect the bubble stretching due to the interaction between the bubble inertial force and hydrodynamic force due to secondary flow;

– identify the cause of the bubbles body part get breakup since the presence of the vortex induces pressure gradient around the bubble surface that tends to press the surface of the bubble towards the pinch off;

- study the tearing mechanism of the bubble interface. This is important because the changes in the shape of objects need energy. The energy obtained from around the bubbles is hydrodynamic energy. The requirement for a bubble to break up is larger hydrodynamic energy than the surface energy of the bubble. The highest specific energy in flow patterns is the small size vortex. Angular momentum from the vortex impact on the bubble surface causes the interface to tear. Starting from conventional equipment to improve equipment that is more advanced, lightweight, practical, and inexpensive in operation, but there are still obstacles that must be corrected from the results obtained. To examine this, research is needed from a simplified operation side to look for causes and effects. For the next step, it is hoped that there will be new innovations towards improving the results based on information from the results of this research.

#### 4. Research work methodology

# 4.1. Theoretical model

The magnitude of momentum flow injection is followed by proportionality to the second flow velocity that restricts the daughter bubble to become low its velocity as illustrated in Fig. 1.

#### Fig. 1. Velocity diagram at the time of the bubble breakup

A sphere bubble is an ideal shape where around the bubble ble there is uniform pressure (Fig. 2, a). When two-phase (air-water) flow is injected in quiescent water, the bubble will deform from sphere to ellipsoid disk shape in the initial part of the injected flow. Then the bubble is in terminal velocity, the bubble nose becomes near flat surface that means the surface curve factor value is close to one [32] or outside pressure of the bubble nose approaches to its inside (Fig. 2, b). Furthermore, the inertial force of the flow in front of the bubble nose induces it, after that ellipsoid disk change to be bullet shape (Fig. 2, c).



Fig. 2. Change of bubble shape in the initial part of injection flow: a – sphere shape of a bubble because uniform pressure surround; b – ellipsoid disk shape is formed by the inertial force and drag force toward balance at the terminal velocity point; c – bullet shape is the effect of induced water in front of the bubble nose

Change from sphere to ellipsoid shape in Cartesian coordinate, the surface at the *x* axis region experiences decreasing of the value of the surface curve factor,  $\kappa$  from point *A* to *A*', and at the y axis region increasing of the surface curve factor from point *B* to *B*' happens (Fig. 2, *b*). After the terminal velocity point, ellipsoid shape changes the bullet shape, where surface energy increases from point *A*' to *A*''. On the other hand, point *B*' is flattening cause of ef-

fect second flow over this point and also point B" becomes low surface energy (Fig. 2, *c*). The effect of this, drag area decreases and then bubble velocity increases again. Under this condition, the bubble breakup mechanism is started. In the middle part of the injected flow, the bubble breakup process has found two models that are sweep and stretching mechanism models.

# 4.1.1. Sweep mechanism model

The moving bubble experiences deceleration because of the restriction effect of the second flow. Meanwhile, the bubble tail is pushed by the main flow that gets deformation as a hooky bubble (Fig. 3, *a*). Hydrodynamic flow patterns are always following the profile form that is crossed. When a bubble starts to jet, the bubble surround happen a second flow. Because of the bubble contours form, the trailing vortex will raise on it (Fig. 3, b). Bubble travel causes rising of stream velocities of the second flow that will sweep the trailing vortex. The vortex is continuing pressured by water static pressure and sweeping of the second flow. Then vortex size becomes small (Fig. 3, c). Assume that linear momentum, mV is constant and then changes to angular momentum, L as illustrated in Fig. 4. If vortex radius reduces from  $r_{v1}$  to  $r_{v2}$ then infinitesimal water velocity,  $V_1$  goes up to  $V_2$  or increasing angular velocity,  $\omega$ . Vortex energy density,  $e_D$  dominates to tear fragment from the parent bubble (Fig. 3, d).



Fig. 3. Schematic of sweep model: a - deformed bubble isthe effect of the inertial flow stream; b - rising of trailing vortex follows the bubble profiles; c - vortex becomes smaller and stretches between the fragment and parent bubble; d - breakup bubble because of tearing by small vortex

The small vortex size has a higher vortex energy density than the big one (Fig. 4, *a*). Angular momentum (1) impact and tear at the bubble interface as Fig. 4, *b*. The momentum depended on vortex radius,  $r_v$  and a velocity difference,  $V=V_2 - V_1$  of infinitesimal mass, m within time difference,  $\Delta t$ . The vortex energy density of water flow (2) is given on the bubble surface. This is equal to vortex stress,  $\sigma_v$  that impact on the interface (3). If allowable surface stress of the bubble is less than vortex stress, the bubble breaks up.

Fig. 4 may be explained with three equations below,

$$L = m \cdot V \cdot r_v = m \cdot r_v^2 \cdot \omega = I\omega; \tag{1}$$

$$e_{D} = \frac{E}{\forall_{w}} = \frac{\frac{1}{2}I\omega^{2}}{\forall_{w}} = \frac{\frac{1}{2}L\omega}{\forall_{w}};$$
(2)

$$\sigma_v = \frac{m\frac{V_2^2}{r_v}}{A_s} = \frac{L \cdot \omega}{A_s \cdot r_v},\tag{3}$$

where  $E = \frac{1}{2}I\omega^2$  is infinitesimal water flow kinetic energy, *I* is rotation mass,  $\forall_w$  is infinitesimal water volume, and

 $A_s$  is bubble surface area unit.



Fig. 4. Change of a small vortex: a - vortex radius decreases from radius  $r_{v1}$  to  $r_{v2}$  and velocity  $V_1 < V_2$ , b - angular momentum impact at bubble interface

Inducing of water flow inertia affected to elongate the parent bubble nose and then the second flow streams swept the hooky bubble tail to different position, so the fragment twisted and then detached from the parent bubble. The kinetic energy of the second flow changed the bubble to be mother and daughter bubble forming energy and new second flow kinetic energy (Fig. 3, d).

#### 4.1.2. Stretching mechanism model

This model is a bubble flow with its inertial force and viscous force restricting it. The bubble gets stretched then splits up (Fig. 5).

Hydrodynamic flow around the bubble affects water pressure on the bubble surface. The second flow moves away from the bubble, water pressure on the surface becomes low or the bubble expands (Fig. 5, a). Water streams shear the bubble surface and the bubble moves against. Hydrodynamic pressure always presses the surface (Fig. 5, b). When bubble volume increases and its pressure decreases it makes the bubble surface unstable. This means the bubble surface is easy to deform. The bubble deforms when it is distorted by the second flow (Fig. 5, c). When the bubble is in deep water, the static pressure or vortex plays to press it. Bubble ends are stretched by the viscous force of the second flow and inertial force of bubble flow and then the bubble splits up (Fig. 5, d).

When the second flow flows over the bubble it gets viscous force significantly because of the rounded contour of the bubble surface. On the other hand, the bubble surface is pressed by the second flow pressure,  $P_V$  (Fig. 6, *a*). So, the convex shape of the surface becomes flat. The resultant force,  $F_R$  (combination of inertial force of the bubble,  $F_{ib}$  and viscous force,  $F_V$ ) and inertial force of water make the bubble to be canoe shape. In addition, the second flow pressure is distributed to any directions at the surface as surface stress (Fig. 6, *b*). Then, the bubble is in the Young-Laplace balance (4). After that, the bubble surface is stretched by the inertial force of the bubble and the second flow coincides with the vortex. The surface stress,  $\sigma$  increases to be a new surface stress,  $\sigma$ '. Surface curvature (Fig. 6, *c*) is influenced by the bubble outside pressure, which is characterized by the surface curve factor,  $\kappa$ . For convex surface at point *A*,  $\kappa>1$ , flat surface at point *B*,  $\kappa=1$  and concave surface at point *C*,  $\kappa<1$ . If the bubble is slightly concave it is at first start to collapse.



Fig. 5. Schematic of stretching model: a - a bubble expands with moving effect of the second flow; b - expanded bubble is deformed by the inertial force of the bubble and hydrodynamic pressure; c - stretched bubble effect of the viscous force of the second flow and water inertial force in front of the bubble nose; d - the bubble splitting process because of external force stress larger than allowable surface stress

Second flow pressure pushed the bubble surface from point of A to B and then continued by vortex from point of B to C. Adding pressure of the second flow to the surface, curve factor decreased from one value at point B to less than one ( $\kappa$ '). Moreover, gas pressure,  $P_G$  is slightly increased to be a new gas pressure,  $P'_G$  and than the bubble towards a new balance as of (5). If the left side is less than the right side, this means the collapse process of the bubble is happening.

$$P_G = P_L + \kappa \sigma, \tag{4}$$

$$P_G' = P_L + \kappa' \sigma' + P_V, \tag{5}$$

where at first the bubble in balance as eq. 4 then to be a new balance as eq. 5 because vortex presses on it. The effect of vortex pressure on the bubble surface, the specific surface energy,  $\kappa\sigma$  becomes  $\kappa'\sigma'$ .



Fig. 6. Bubble stretching mechanism: a – flow across the bubble is the effect of the bubble flow; b – pressure force balance is to follow outside pressure condition;
 c – stretching bubble cause of the second flow restriction

force against the bubble inertial flow force

#### 4.2. Experimental setup

The test bed used for the experiment is a water bath with sizes, length 100 cm, height 20 cm, and depth 20 cm. The syringe is connected with the nozzle line where the outlet of it is placed on the centre of  $20 \times 20$  cm plane as in Fig. 7. The syringe and the nozzle line are filled with clean water. A single bubble is made by inserting manner of a lengthen pipette with a tiny flexible tube into the nozzle line. The end of the tiny tube is placed around the middle nozzle line. The relative size of the bubble can be determined by crushing rubber of the pipette. Now a bubble is available inside the nozzle line, after that it is injected in the water bath by the impact on the push-rod of the syringe.



Fig. 7. Sketch of two-phase flow injection test bed

The image of the bubble flow in the quiescent water will be captured by Fuji Film Finepix HS 55 Exr digital high-speed camera with lighting aid of the lamp and the infrared LED (Fig. 8).



Fig. 8. Horizontal bubble injection flow images

This research work is focused on the bubbles breakup mechanism process. The image (Fig. 8) of the bubbles is used as investigation.

# 5. Results and angular momentum tearing mechanism investigation

The bubble breakup mechanism was investigated based on images captured (Fig. 8). Because of arbitrary bubbles shape, logical approaching of its shape or representative size of the main object observations was used in the investigation.

A bubble is pushed by inertial water flow from the bubble tail to become an arbitrary bubble shape as a hooky bubble (Fig. 9). This shape is affected by hydrodynamic flow pattern while expanding and it increases the bubble drag (Fig. 10). The water stream of the second flow induces the bubble surface that makes its surface unstable.



Fig. 9. Hooky bubble is formed by the inertial injection force and hydrodynamic flow force surround



Fig. 10. Expanded bubble is the effect of decreasing pressure surround that is influenced by the hydrodynamic flow pattern

Meanwhile, the parent bubble velocity is slightly increasing so its drag gets low which is affected by the second flow distortion. Fragment bubble velocity is slightly decreasing because it crashes with the second flow velocity. Parent and fragment have different velocity magnitudes, which means the bubble experiences stretching. If the bubble gets stretched, its drag becomes low so that the velocity increases. Furthermore, the parent bubble velocity decreases, and then increasing is continued again. It can be identified as maximum drag at the moment. It is shown in the figure of the parent bubble curve (Fig. 11). Furthermore, increasing of the parent velocity affects strengthening of the second velocity. The shear force on the bubble surface is becoming different velocity between the fragment and parent bubble as illustrated in Fig. 11. This is as stretching phenomena on the bubble.

Water inertial flow of injection determines the bubble shape as Fig. 2. Hydrodynamic flow pattern is determined by the shape. Increasing of the second flow velocity affects vortices rising on the bubble surround. Vortex energy density continually increases up to the fragment detach from the parent bubble that is contributed by the second flow. Specific hydrodynamic energy for the bubble detachment is the difference of energy between the parent and fragment (Fig. 12).



Fig. 11. Velocity of the fragment and parent bubble is in the breakup process



Fig. 12. Specific energy in the bubble collapse process

Reducing of the bubble mass is detached to split of fragment become mother and daughter bubble. In addition, the mother bubble drag gets decreased or increased in its velocity that is indicated in the sharpened shape of the bubble nose. The more sharpened shape shows the inertial force of the bubble,  $F_{ib}$  and hydrodynamic force,  $F_{hyd}$  increase as the function of inertial bubble velocity,  $V_{ib}$  and hydrodynamic velocity,  $V_{hyd}$  respectively. Bubble stretching happens when the inertial force of the bubble is opposite to the shear force or hydrodynamic force on the bubble surface (Fig. 13). Any time the mother bubble velocity increases in stagnant water, it will strengthen the wake vortices on the bubble tail surround. The energy density of these wake vortices breaks the mother bubble up to be several small bubbles as daughter bubbles (Fig. 14).



Fig. 13. Stretched bubble by inertial and hydrodynamic force



Fig. 14. Breakage bubble is the effect of vortices surround

Along bubble moving, the bubble experiences a change of its volume (3D) that is represented by an area change in 2D. It is assumed that bubble sides only applied the same value of water height static pressure. Meanwhile, on the lower and upper of the bubble it is different height of water deep. Because the bubble is small, the pressure on it around is assumed as uniform. Moreover, the internal bubble pressure gets decreased as the effect of hydrodynamic and inertial force. Fig. 10 illustrates expanding of the bubble volume that slightly decreases the inside pressure of the bubble. Expanded bubble volume can be identified by the change of bubble length from  $\ell_1$  to  $\ell_2$  ( $\ell_1 < \ell_2$ ) and bubble height from  $h_1$ to  $h_2$  ( $h_1 \le h_2$ ). There are two possibilities of bubble collapse,  $P_G > P_L + \kappa \sigma$  means bubble growth (gas pressure increases or liquid pressure decreases) and  $P_G < P_L + \kappa \sigma$  means bubble penetration (gas pressure decreases or liquid pressure increases). Eq. (4) becomes

$$\sigma = \frac{P_G - P_L}{\kappa} = \frac{P_G - P_L}{2}R,\tag{6}$$

where  $\kappa = 2/R$ ;  $0 < \kappa < 1$  interface starts to tear or unstable condition where the bubble may be collapsed or back to the previous shape, it depends on surrounding condition. Shape change of the bubble causes interface stress,  $\sigma$  as the effect of the bubble radius, *R*. If the stress is more than allowed interface stress, the bubble gets collapsed.

Naturally, the bubble follows the Young-Laplace balance. Change of the surface is the effect of different pressure between the bubble and surrounding pressure [15]. For the moment, pull on the bubble surface is the effect of the initial liquid slug injection [5]. Both of these work conditions continuously and tend to increase up to the bubble breakup. Change of conditions usually follows a new balance that affects the change of the bubble shape as illustrated in Fig. 5.



Fig. 15. Steps of bubble break up: bubble length and height have an increased trend, the opposite trend indicates stretched bubble, and the same down trend is the bubble breakup

Fig. 15, the size of bubble length and bubble height is getting increased, it means that the bubble is in stretching. The bubble has the inertial force and the second flow force shears the bubble surface in the opposite direction. Meanwhile, bubble length increases and bubble height decreases, the bubble experiences yield or detachment process. Air molecules in the bubble make the estrangement process. Finally, bubble length and bubble height are the same in decreasing of size, the bubble has broken up. Fragment and parent bubbles do not influence each other.

#### 6. Discussion of experimental results

Decreasing of bubble pressure and eddy effect may change the curve factor value. Based on the Young-Laplace principle, when liquid pressure,  $P_L$  becomes higher than gas pressure,  $P_G$  so that the bubble is penetrated by the liquid pressure. The bubble breakup is caused by a significant effect of pressure [7]. Besides that, wake available on the part of the bubble curve will trigger the bubble collapse.

The bubble shape is influenced by fluid hydrodynamics. Penetration of eddy flow changes the bubble surface because flow energy density is higher than the surface energy density. Moreover, the bubble breaks up to deform be a new balance as daughters and a mother bubble.

Injected bubble in the deep place of water tends to shorten the vortex radii. The vortex is pressured by the static pressure of water. Then the vortex energy density increases towards bubble break up. Initially, the deformed bubble is subjected to a stagnation pressure on the bubble surface and then it is stretched by the inertial and hydrodynamic force on both its ends (Fig. 13). The effect of an elongated bubble, the bubble volume increases and gas pressure inside one decreases, and then the Young-Laplace balance is disturbed. Surface tension increases, the bubble surface becomes unstable  $(0 < \kappa < 1)$  as illustrated in Fig. 16. Curve factor changes from  $\kappa > 1$  (convex shape) to become  $\kappa < 1$  (concave shape) value because of hydrodynamic flow [32]. The bubble surface experiences hydrodynamic stress,  $\sigma_h$ . Available vortex close to the bubble adds stretching on the surface as vortex stress,  $\sigma_v$ . The bubble attempts to toward the Young-Laplace balance by collapsing itself. Part of the bubble tail is not stable because of hydrodynamic effect and mainly wake effect. The sum of them is as applied stress on the bubble surface,  $\sigma_a$ which has total value stress,  $S_{tot}$ . The bubble collapsing process resembles what had been investigated experimentally in two steps, these are pinch-off step and squeezing step [29].





When ambient pressure on a bubble is lower than the inner pressure which effect of bubble dynamic behaviour, the bubble expands towards the ambient [33]. Because of the arbitrary bubble shape, the expanded bubble is predicted by measuring the shape with two dimensions as shown in Fig. 10. The bubble height and length increase and then combine both of them as area change. This indicates the change of volume by neglecting the bubble depth dimension. The expanded bubble volume is the evidence of pressure decrease. The bubble length elongates starting from the terminal velocity point to the fragment detaches from the parent bubble. Moving of each bubble surface by its velocity and probably deform cause a liquid act effect [34]. Deformation is in induced mechanic to a locked air volume can produce single bubbles [35]. In this case, a section area or a diameter equivalent of the mother bubble gets low and consequently the bubble drag decreases and the bubble velocity increases. After that, water stream changes wakes around the parent bubble tail that causes the fractured bubble tail part. The wakes contact with the fragments, consequently they tear the parts of the parent bubble tail into several sub volumes (or fragments) illustrated in Fig. 14. The stretch of the particle before breakup and then interaction with turbulent eddies causes the particle break up [36]. Turbulence is the main cause of bubble breakup [9].

Bubble breakup happens continuously up to wake energy density is equal to the bubbles surface energy or lack of bubble breakup energy. In other words, small bubbles have higher shape energy than surround them.

The bubble breakup is studied molecularly that is separate between molecules bond than atoms bond. Atoms bond is stronger than molecules bonds. A bubble consists of Oxygen and Nitrogen molecules, which have a nonpolar molecule. Meanwhile, water around the bubble has a polar molecule. Oxygen and Nitrogen molecules induce Hydrogen atoms and form interface. Water and a bubble make a force balance molecularly at the interface in Fig. 17.



Fig. 17. Water flow turns dipole force,  $F_{d1}$  of water molecules at the interface

The dipole force of the water molecule at the interface,  $F_{d1}$  and dipole force of the water molecule in the water flow,  $F_{d2}$  form the resultant force, R. If there is molecule movement of water flow, this dipole force of the water molecule at the interface is getting add water flow force,  $F_{d2h}$  become  $F_{d2}$  plus  $F_{d2h}$ . Then, the resultant force, R turns to become a new resultant force, R'. This new resultant force will tear the interface and the bubble breaks up if the stress is over the allowed interface stress.

Water flow has kinetic energy which is transferred to water molecules at the interface [37, 38]. The flow will turn the water molecule that is followed by dipole force turning [39]. A polar substance is more easily polarized than a nonpolar substance [40]. The turning of water molecules makes its Hydrogen atom to Oxygen and Nitrogen atoms of the bubble vibrate [41]. Then the interface of the bubble is stretch. This interface may tear if the induce force is less

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than the hydrodynamic force. The induce force, F between molecules can be determined by the Coulomb equation (7).

$$F = k \frac{(q^{+})(q^{-})}{r^{2}},$$
(7)

where k is a proportionality constant,  $q^+$  is positive charge,  $\bar{q}$  is negative charge, and r is the separation distance.

Atom configuration of Oxygen and Nitrogen to Hydrogen (Fig. 18) shows the distance of the orbital electron charge that Nitrogen-Hydrogen  $(r_{\text{N-H}})$  is longer than Oxygen-Hydrogen  $(r_{\text{O-H}})$ . Based on Equation (7), the induce force of Nitrogen is smaller than of Oxygen. Hydrogen charges are always induced by Oxygen and Nitrogen molecules [41]. While an individual hydrogen bond at a water molecule is stronger than between an individual bond of Nitrogen and Hydrogen. Because of water moving, Hydrogen to Nitrogen bond experiences stretching and may then start to tear.



Fig. 18. Distance of Hydrogen charge to Oxygen and Nitrogen charge

Hydrogen atoms in the water are induced by Oxygen and Nitrogen molecules of the bubble. The dipole force vector of water at the interface is turned by the dipole force vector of water flow (Fig. 17). If weaken water flow turns the dipole force vector of water at the interface, the bubble only deforms its shape. In contrast, the angular momentum of strong water flow crashes the molecule of water at the interface and then turns its dipole force vector. The angular momentum makes the interface tear. Forming a new moment dipole is the result of the dipole force turning that can tear the interface [42]. Vortex is very strong to turn the dipole force of water at the interface (Fig. 19). Because the induced force between hydrogen and oxygen is stronger than hydrogen and nitrogen, the bubble tear starts from the bonds of the Hydrogen atom of water molecules to the Nitrogen atom of Nitrogen molecules [37].

After tear of the interface, the bubble is easy to break up because molecule bonds between Oxygen and Nitrogen in the bubble are very weak. Furthermore, the bubble breaks up into several parts.

This research is still looking for the root of the problems and has not been adjusted to the real application. Further research is needed to try horizontal injection towards the curved wall as like GLCC and continued by downward incline injection towards the curved wall. Possible restrictions to results where fluid viscosity is different and may be not uniform that results will be different from those studied. But at least there are problems that can be identified.



## Fig. 19. Vortex turns the dipole force that can create a dipole moment between Nitrogen atoms and water molecules at the interface

This research can be developed step by step to reduce the complex hydrodynamic flow in the device with an effort to overcome any problems, those are found towards a perfect device design. This research is a part of a number of problems and it is far from exactly perfect. Difficulties available in the development of this research are looking for ways to overcome problems and adjust to the conditions of available devices.

#### 7. Conclusions

1. The inertial flow of water is an important role to deform a bubble which affects its flow pattern.

2. A sharp bubble flow causes the increase of the second velocity of water then creates vortex following bubble profiles. The vortex with a higher energy density due to strengthening by a static water pressure triggers to break the bubble up.

3. The fragment of the bubble detaches from the parent bubble, the water stream around the mother bubble distorts the bubble tail, and the bubble inertial force pushes the bubble nose toward. The bubble gets stretched. Then the middle part of the bubble gets increased stress. For the bubble toward in balance condition, the bubble splits itself.

4. Wake vortices behind the bubble tail affect the bubble surfaces. Static pressure of water supports to reduce vortices size or increases the energy density of vortices. Because vortices stress is larger than the interface stress value, the bubble tail cannot stand to angular momentum.

5. Hydrogen bonds to Nitrogen are weaker than to Oxygen. When the weak bonds are applied any forces, the bonds become stress and then fracture. The interface starts to tear at the weak bonds, moreover the bubble is broken up.

# References

- Walter, J. F., Blanch, H. W. (1986). Bubble break-up in gas liquid bioreactors: Break-up in turbulent flows. The Chemical Engineering Journal, 32 (1), B7–B17. doi: https://doi.org/10.1016/0300-9467(86)85011-0
- Hinze, J. O. (1955). Fundamentals of the hydrodynamic mechanism of splitting in dispersion processes. AIChE Journal, 1 (3), 289–295. doi: https://doi.org/10.1002/aic.690010303
- Chen, Z., Ata, S., Jameson, G. J. (2015). Break-up of bubble clusters in turbulent flow Theory. Powder Technology, 279, 228–232. doi: https://doi.org/10.1016/j.powtec.2015.04.016
- Tomiyama, A., Kataoka, I., Zun, I., Sakaguchi, T. (1998). Drag Coefficients of Single Bubbles under Normal and Micro Gravity Conditions. JSME International Journal Series B, 41 (2), 472–479. doi: https://doi.org/10.1299/jsmeb.41.472

- Rassame, S., Hibiki, T., Ishii, M. (2016). Void penetration length from air injection through a downward large diameter submerged pipe in water pool. Annals of Nuclear Energy, 94, 832–840. doi: https://doi.org/10.1016/j.anucene.2016.04.046
- Bai, H., Thomas, B. G. (2001). Bubble formation during horizontal gas injection into downward-flowing liquid. Metallurgical and Materials Transactions B, 32 (6), 1143–1159. doi: https://doi.org/10.1007/s11663-001-0102-y
- Xing, C., Wang, T., Guo, K., Wang, J. (2014). A unified theoretical model for breakup of bubbles and droplets in turbulent flows. AIChE Journal, 61 (4), 1391–1403. doi: https://doi.org/10.1002/aic.14709
- Bari, S. D., Robinson, A. J. (2013). Experimental study of gas injected bubble growth from submerged orifices. Experimental Thermal and Fluid Science, 44, 124–137. doi: https://doi.org/10.1016/j.expthermflusci.2012.06.005
- Han, L., Luo, H., Liu, Y. (2011). A theoretical model for droplet breakup in turbulent dispersions. Chemical Engineering Science, 66 (4), 766–776. doi: https://doi.org/10.1016/j.ces.2010.11.041
- Wichterle, K., Wichterlová, J., Kulhánková, L. (2005). Breakup of Bubbles Rising in Liquids of Low and Moderate Viscosity. Chemical Engineering Communications, 192 (5), 550–556. doi: https://doi.org/10.1080/00986440590495034
- Lima Neto, I. E., Zhu, D. Z., Rajaratnam, N. (2008). Bubbly jets in stagnant water. International Journal of Multiphase Flow, 34 (12), 1130–1141. doi: https://doi.org/10.1016/j.ijmultiphaseflow.2008.06.005
- 12. Zhang, C., Sa, R., Zhou, D., Jiang, H. (2017). Effects of gas velocity and break size on steam penetration depth using gas jet into water similarity experiments. Progress in Nuclear Energy, 98, 38–44. doi: https://doi.org/10.1016/j.pnucene.2017.02.006
- 13. Canedo, E. L., Favelukis, M., Tadmor, Z., Talmon, Y. (1993). An experimental study of bubble deformation in viscous liquids in simple shear flow. AIChE Journal, 39 (4), 553–559. doi: https://doi.org/10.1002/aic.690390403
- Al-Hayes, R. A. M., Winterton, R. H. S. (1981). Bubble diameter on detachment in flowing liquids. International Journal of Heat and Mass Transfer, 24 (2), 223–230. doi: https://doi.org/10.1016/0017-9310(81)90030-2
- Yang, B., Prosperetti, A., Takagi, S. (2003). The transient rise of a bubble subject to shape or volume changes. Physics of Fluids, 15 (9), 2640–2648. doi: https://doi.org/10.1063/1.1592800
- Liu, L., Yan, H., Zhao, G. (2015). Experimental studies on the shape and motion of air bubbles in viscous liquids. Experimental Thermal and Fluid Science, 62, 109–121. doi: https://doi.org/10.1016/j.expthermflusci.2014.11.018
- 17. Lee, H. S., Merte, H. (1996). Spherical vapor bubble growth in uniformly superheated liquids. International Journal of Heat and Mass Transfer, 39 (12), 2427–2447. doi: https://doi.org/10.1016/0017-9310(95)00342-8
- Nguyen, V. T., Song, C.-H., Bae, B.-U., Euh, D.-J. (2013). Modeling of bubble coalescence and break-up considering turbulent suppression phenomena in bubbly two-phase flow. International Journal of Multiphase Flow, 54, 31–42. doi: https://doi.org/10.1016/ j.ijmultiphaseflow.2013.03.001
- Shi, W., Yang, X., Sommerfeld, M., Yang, J., Cai, X., Li, G., Zong, Y. (2019). Modelling of mass transfer for gas-liquid two-phase flow in bubble column reactor with a bubble breakage model considering bubble-induced turbulence. Chemical Engineering Journal, 371, 470–485. doi: https://doi.org/10.1016/j.cej.2019.04.047
- 20. Chen, Y., Ding, J., Weng, P., Lu, Z., Li, X. (2019). A theoretical model describing bubble deformability and its effect on binary breakup in turbulent dispersions. European Journal of Mechanics B/Fluids, 75, 352–360. doi: https://doi.org/10.1016/j.euromechflu.2018.09.004
- Zhang, H., Yang, G., Sayyar, A., Wang, T. (2020). An improved bubble breakup model in turbulent flow. Chemical Engineering Journal, 386, 121484. doi: https://doi.org/10.1016/j.cej.2019.04.064
- Hreiz, R., Lainé, R., Wu, J., Lemaitre, C., Gentric, C., Fünfschilling, D. (2014). On the effect of the nozzle design on the performances of gas–liquid cylindrical cyclone separators. International Journal of Multiphase Flow, 58, 15–26. doi: https://doi.org/10.1016/ j.ijmultiphaseflow.2013.08.006
- 23. Sosinovich, V. A., Tsyganov, V. A., Kolovandin, B. A., Puris, B. I., Gertsovich, V. A. (1995). Model of gas bubble breakup in a turbulent liquid flow. Journal of Engineering Physics and Thermophysics, 68 (2), 164–175. doi: https://doi.org/10.1007/bf00862856
- Emami, A., Briens, C. (2008). Study of downward gas jets into a liquid. AIChE Journal, 54 (9), 2269-2280. doi: https://doi.org/10.1002/aic.11524
- Clanet, C., Lasheras, J. C. (1997). Depth of penetration of bubbles entrained by a plunging water jet. Physics of Fluids, 9 (7), 1864–1866. doi: https://doi.org/10.1063/1.869336
- 26. Liu, Z., Reitz, R. D. (1997). An analysis of the distortion and breakup mechanisms of high speed liquid drops. International Journal of Multiphase Flow, 23 (4), 631–650. doi: https://doi.org/10.1016/s0301-9322(96)00086-9
- Das, S., Weerasiri, L. D., Yang, W. (2017). Influence of surface tension on bubble nucleation, formation and onset of sliding. Colloids and Surfaces A: Physicochemical and Engineering Aspects, 516, 23–31. doi: https://doi.org/10.1016/j.colsurfa.2016.12.010
- Jo, D., Revankar, S. T. (2011). Investigation of bubble breakup and coalescence in a packed-bed reactor Part 2: Development of a new bubble breakup and coalescence model. International Journal of Multiphase Flow, 37 (9), 1003–1012. doi: https:// doi.org/10.1016/j.ijmultiphaseflow.2011.06.015
- Wang, X., Zhu, C., Wu, Y., Fu, T., Ma, Y. (2015). Dynamics of bubble breakup with partly obstruction in a microfluidic T-junction. Chemical Engineering Science, 132, 128–138. doi: https://doi.org/10.1016/j.ces.2015.04.038

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- 30. Mortuza, S. M., Gent, S. P., Kommareddy, A., Anderson, G. A. (2012). Investigation of Bubble and Fluid Flow Patterns Within a Column Photobioreactor. Journal of Fuel Cell Science and Technology, 9 (3). doi: https://doi.org/10.1115/1.4006052
- Kajero, O. T., Abdulkadir, M., Abdulkareem, L., Azzopardi, B. J. (2018). Experimental study of viscous effects on flow pattern and bubble behavior in small diameter bubble column. Physics of Fluids, 30 (9), 093101. doi: https://doi.org/10.1063/1.5045160
- 32. Tomita, Y., Robinson, P. B., Tong, R. P., Blake, J. R. (2002). Growth and collapse of cavitation bubbles near a curved rigid boundary. Journal of Fluid Mechanics, 466, 259–283. doi: https://doi.org/10.1017/s0022112002001209
- Han, R., Wang, S., Yao, X. (2016). Dynamics of an air bubble induced by an adjacent oscillating bubble. Engineering Analysis with Boundary Elements, 62, 65–77. doi: https://doi.org/10.1016/j.enganabound.2015.09.009
- Ellingsen, K., Risso, F. (2001). On the rise of an ellipsoidal bubble in water: oscillatory paths and liquid-induced velocity. Journal of Fluid Mechanics, 440, 235–268. doi: https://doi.org/10.1017/s0022112001004761
- Chu, P., Waters, K. E., Finch, J. A. (2016). Break-up in formation of small bubbles: Break-up in a confined volume. Colloids and Surfaces A: Physicochemical and Engineering Aspects, 503, 88–93. doi: https://doi.org/10.1016/j.colsurfa.2016.05.037
- 36. Rodríguez-Rodríguez, J., Gordillo, J. M., Martínez-Bazán, C. (2006). Breakup time and morphology of drops and bubbles in a high-Reynolds-number flow. Journal of Fluid Mechanics, 548, 69–86. doi: https://doi.org/10.1017/s002211200500741x
- Ratajczak, H., Orville-Thomas, W. J. (1973). Charge-transfer properties of hydrogen bonds. III. Charge-transfer theory and the relation between the energy and the enhancement of dipole moment of hydrogen-bonded complexes. The Journal of Chemical Physics, 58 (3), 911–919. doi: https://doi.org/10.1063/1.1679344
- Zivkov, E., Yarusevych, S., Porfiri, M., Peterson, S. D. (2015). Numerical investigation of the interaction of a vortex dipole with a deformable plate. Journal of Fluids and Structures, 58, 203–215. doi: https://doi.org/10.1016/j.jfluidstructs.2015.08.009
- Egger, D. A., Zojer, E. (2013). Anticorrelation between the Evolution of Molecular Dipole Moments and Induced Work Function Modifications. The Journal of Physical Chemistry Letters, 4 (20), 3521–3526. doi: https://doi.org/10.1021/jz401721r
- Joshi, S., Kumari, S., Bhattacharjee, R., Sarmah, A., Sakhuja, R., Pant, D. D. (2015). Experimental and theoretical study: Determination of dipole moment of synthesized coumarin–triazole derivatives and application as turn off fluorescence sensor: High sensitivity for iron(III) ions. Sensors and Actuators B: Chemical, 220, 1266–1278. doi: https://doi.org/10.1016/j.snb.2015.07.053
- Starikov, V. I., Petrova, T. M., Solodov, A. M., Solodov, A. A., Deichuli, V. M. (2019). Study of the H2O dipole moment and polarisability vibrational dependence by the analysis of rovibrational line shifts. Spectrochimica Acta Part A: Molecular and Biomolecular Spectroscopy, 210, 275–280. doi: https://doi.org/10.1016/j.saa.2018.11.032
- Ngafwan, N., Wardana, I. N. G., Wijayanti, W., Siswanto, E. (2018). The role of NaOH and papaya latex bio-activator during production of carbon nanoparticle from rice husks. Advances in Natural Sciences: Nanoscience and Nanotechnology, 9 (4), 045011. doi: https://doi.org/10.1088/2043-6254/aaf3af