UDC 536.464+662.612.3
DOI: 10.15587/1729-4061.2020.208440

A THEORETICAL STUDY OF STABILITY OF SOLID FUEL BURNING WITH A TWO-PHASE GASIFICATION AREA

V. Volkov
Doctor of Technical Sciences, Professor
Department of Theoretical Mechanics
Odessa I. I. Mechnikov National University
Dvorianska str., 2, Odessa, Ukraine, 65082
E-mail: viktor@te.net.ua
N. Makoyed
PhD, Associate Professor*
E-mail: okspetr@ukr.net
Yu. Loboda
PhD, Associate Professor*
E-mail: jul.loboda@gmail.com
O. Sokolova
Senior Lecturer*
E-mail: natamakoyed@gmail.com
*Department of Information Technologies and Cybersecurity
Odessa National Academy of Food Technologies
Kanatna str., 112, Odessa, Ukraine, 65039

Theoretically, the combustion stability of solid fuel, which during the combustion process is decomposed according to the "solid phase – liquid phase – gas" scheme, is investigated. The physical and mathematical models for the propagation of small perturbations of combustion are constructed. The medium in all areas of combustion and in combustion products is assumed to be incompressible, and the viscosity of the fuel in the liquid phase is taken into account. Thus, perturbations of hydrodynamic parameters are considered not only in the two-phase gasification zone, but also in the combustion products area and the geometric perturbation of the instantaneous combustion front (flame), distorting the shape of its surface, is also specified. That is the characteristic feature of the presented physical model. The mathematical eigenvalue problem is set and solved. This problem is reduced to an algebraic characteristic equation for a dimensionless complex eigenvalue, which positivity determines the instability. It is proved that in the limiting case of the absence of a liquid phase, absolute instability takes place. At the other limiting case – for perturbations with infinite wavelength – a transition to stability takes place. The latter fact indicates that the presence of a viscous liquid film and changes in the length of the gasification zone under the influence of perturbations have a significant stabilizing effect on solid fuel combustion. In the general case, a sufficient condition for the instability of the roots of the characteristic equation is analytically determined. The physical interpretation of the mathematical results explains the processes of autoturbulization of solid fuel combustion and the possible transition of combustion to deflagration explosion or detonation. The results of the study are in qualitative agreement with experimental data and can additionally be used for theoretical analysis of the stability of the liquid fuel combustion process in the combustion chamber

Keywords: solid fuel combustion, fuel gasification, combustion instability, deflagration explosion, detonation

Received date 09.06.2020
Accepted date 06.07.2020
Published date 31.08.2020

1. Introduction

The problem of hydrodynamic stability of combustion waves is a classical theoretical problem of fluid mechanics, which is still topical [1]. This is primarily due to the practical importance of studies of the stability for flames subjected to small perturbations: it is instability that is the main cause of flame autoturbulization [2, 3] and acceleration [4, 5]. In turn, acceleration of the flame can cause its transition to detonation [6–8] or to deflagration explosion [9]. Thus, the problem of hydrodynamic flame stability is closely related to the general problem of explosion safety [9–11]. A detailed analysis of the evolution of small perturbations [9] for the case of unstable combustion enables us to estimate the time of possible combustion-explosion transition and the length of the so-called detonation induction distance [12]. In addition, studies of flame instability provide great opportunities for describing quasi-laminar and turbulent combustion modes [2, 13]. By the way, the structure of turbulent flame traditionally is of considerable academic interest [14–16].

The relevance of studies of the hydrodynamic flame stability is connected with the emergence of new combustible substances and the development of equipment and technology for fuel burning. For this reason, new models of the stationary combustion process are constantly being created, and the stability of this process is subsequently investigated. In this regard, the study of the stability of solid fuel combustion is of special interest, because, in particular, rocket engines use such kinds of fuel [17].

2. Literature review and problem statement

Initially, studies of combustion stability of liquid and solid fuels were carried out within one-dimensional models [18, 19]. Moreover, the main combustion model of these studies is a model in which solid fuel, as a result of decomposition, passes directly into the gas phase and then instantly burns [19]. However, the multi-dimensionality of the solid-fuel combustion process is quite obvious and experimentally proved [20, 21], which makes it necessary to study the hydrodynamic stability of the combustion front to small multi-dimensional perturbations.

In addition, in many cases, solid fuel during decomposition is not decomposed according to the "solid phase – gas" scheme [19], but according to the "solid phase – liquid phase – gas" scheme [22–24]. The presence of a thin liquid film located on the gasification surface of the fuel solid phase plays, for example, a significant role [24] in the study of so-called erosive combustion [25, 26]. It is the liquid film separating the solid and gas phases that causes the negative erosion effect or the Vilyunov – Dvoryashin effect [24–26]. This effect is that under certain conditions, the burning rate decreases when the burning surface of fuel is blown in a parallel flow. Various attempts to describe and explain the Vilyunov – Dvoryashin effect in more detail [26–28] led to the need of studying the problem of hydrodynamic stability in the zone of solid fuel gasification [24].

The study [24] addresses fuel gasification during the combustion process under the "solid phase – liquid phase – gas" scheme. In this case, it is initially assumed that the length of the liquid phase zone is much less than the length of the gas phase zone. In solving the stability problem, only perturbations with the wave numbers that are inversely proportional to the liquid phase zone extent, i. e., perturbations with the wavelengths of the same order as the thickness of the liquid film are considered. And finally, setting the stability problem in the study [24] proposes that the combustion front is not perturbed: only the interface between the liquid and solid phases of the fuel is perturbed. These assumptions seem quite correct within the narrow problem considered in [24], namely, the analysis of the possibility of the process autoturbulization for the Gusachenko-Zarko mechanism [23, 26] of the occurrence of a negative erosion effect [25] during solid fuel combustion. But in general, the stability study of the solid fuel combustion process with a two-phase gasification area – with liquid phase and gas phase – requires rejection of these assumptions.

In fact, there are no exact data on the thicknesses of the liquid phase zone [24]. It is only known that for some types of solid fuel (for example, containing ammonium perchlorate), the liquid film makes up less than 10 % of the total length of the combustion zone [22–24]. But for the other types of fuel, this ratio may not be fulfilled. However, the thickness of the liquid film should not be neglected, even if it is only 3 μm with a total length of the combustion zone of 40 μm. This is because the liquid film is in a qualitatively different state than the rest of the combustion zone (gas phase).

In addition, in the general case, perturbations of the flame front cannot be neglected also. Even assuming that the perturbations are so weak that they do not affect the burning rate, the shape of the front is distorted. Actually, both the phase boundary and the flame front generate disturbances that interact in the gas phase region of the fuel.

3. The aim and objectives of the study

The aim of the study is a theoretical solution for the problem of hydrodynamic stability of solid fuel combustion. In so doing, flame front perturbations must be taken into account, and the ratio between the lengths of the liquid and gas phase zones in the fuel gasification area is assumed to be arbitrary.

To achieve this aim, it is necessary to accomplish the following objectives:

– to develop physical and mathematical models for the process of solid fuel burning in the presence of small perturbations;

– to set and solve the corresponding eigenvalue problem by obtaining the characteristic equation;

– to analyze the characteristic equation in terms of unstable roots existence;

– to interpret the results physically.

4. Physical and mathematical models of the process of solid fuel burning in the perturbed state

Let us consider a flat flame propagating through a solid fuel at a certain constant velocity. The initial model of the combustion process is a discontinuous scheme of flame, which is represented by a flat front of instantaneous combustion (Fig. 1). The flame front is separated from the solid phase of the fuel by the area with length L, and

$$L = L_1 + L_2, \tag{1}$$

where L_1 and L_2 are the lengths of the zones of liquid and gas phases, correspondingly.

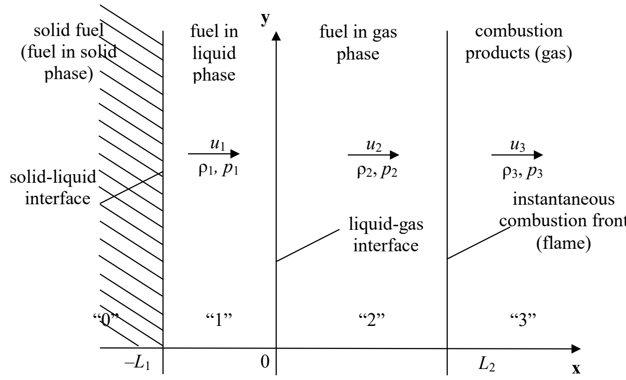

Fig. 1. Stationary solid fuel combustion complex

Moreover, in contrast to (24), no initial assumptions are made regarding the ratio L_1/L_2. In the moving coordinate system connected with the liquid-gas interface, there are the following flow areas (Fig. 1):

– "0": $x \le L_1$ – area of the solid phase of the fuel;

– "1": $-L_1 \le x \le 0$ – area of the liquid phase of the fuel;

– "2": $0 \le x \le L_2$ – area of the gas phase of the fuel;

– "3": $x \ge L_2$ – area of gaseous combustion products.

In this case, $x=0$ is the unperturbed interface between the liquid and gas phases of the fuel, and $x=L_2$ is the unperturbed surface of the flame front (instantaneous combustion front). The problem is considered as two-dimensional in the Oxy plane.

Parameters of the stationary flow in zones "1", "2" and "3" are assumed to be constant and are indicated by the corresponding indices. These parameters satisfy the law of conservation of mass

$$\rho_1 u_1 = \rho_2 u_2 = \rho_3 u_3 \tag{2}$$

and the law of conservation of momentum

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 = p_3 + \rho_3 u_3^2. \tag{3}$$

The medium (liquid, gas) is assumed to be incompressible in all three zones. This assumption is fully justified, since the propagation velocity of combustion is much less than the speed of sound. The fluid in the area "1" is assumed to be viscous, and the medium in the areas "2" and "3" is assumed to be ideal.

In the area "1", the flow of a viscous fluid is described by the system of Navier-Stokes equations and incompressibility equation

$$\begin{cases} \dfrac{\partial u_x}{\partial t}+u_x\dfrac{\partial u_x}{\partial x}+u_y\dfrac{\partial u_x}{\partial y}+\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}=\nu\left(\dfrac{\partial^2 u_x}{\partial x^2}+\dfrac{\partial^2 u_x}{\partial y^2}\right), \\ \dfrac{\partial u_y}{\partial t}+u_x\dfrac{\partial u_y}{\partial x}+u_y\dfrac{\partial u_x}{\partial y}+\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}=\nu\left(\dfrac{\partial^2 u_y}{\partial x^2}+\dfrac{\partial^2 u_y}{\partial y^2}\right), \\ \dfrac{\partial u_x}{\partial x}+\dfrac{\partial u_y}{\partial y}=0, \end{cases} \quad (4)$$

where ρ is the density, p is the pressure, u_x, u_y are the components of the flow velocity along the coordinate axes Ox, Oy, ν is kinematic viscosity.

In the areas "2" and "3", the flow is described by the system of Euler equations and incompressibility equation

$$\begin{cases} \dfrac{\partial u_x}{\partial t}+u_x\dfrac{\partial u_x}{\partial x}+u_y\dfrac{\partial u_x}{\partial y}+\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}=0, \\ \dfrac{\partial u_y}{\partial t}+u_x\dfrac{\partial u_y}{\partial x}+u_y\dfrac{\partial u_x}{\partial y}+\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}=0, \\ \dfrac{\partial u_x}{\partial x}+\dfrac{\partial u_y}{\partial y}=0. \end{cases} \quad (5)$$

Random perturbations result in small displacements $\varepsilon_j(y, t)$ (j=1, 2) of the surfaces x=0 and $x=L_2$ along the Ox axis, i. e., both the liquid-gas interface and the flame front are distorted (Fig. 2).

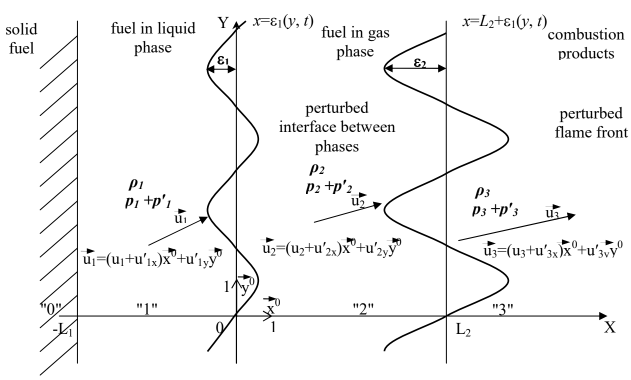

Fig. 2. Combustion complex in perturbed state

Displacements $\varepsilon_j(y, t)$ are written in the form

$$\varepsilon_j(y,t)=A_{0j}h^{-1}\exp(ihy-i\omega t)(j=1,2), \quad (6)$$

where $h=2\pi/\lambda(h>0)$ is the wave number, λ is the perturbation wavelength, ω is some complex (generally speaking) number, i is the imaginary unit, $A_{0j}(j=1,2)$ are indefinite constants.

Thus, the equation of the perturbed liquid-gas interface is represented as

$$x=\varepsilon_1(y,t), \quad (7)$$

and the equation of the perturbed flame front has the form

$$x=L_2+\varepsilon_2(y,t). \quad (8)$$

There are regular medium flows, which are moving along the Ox axis with constant velocities u_j (j=1, 2, 3) and are fully defined by the values of constant parameters p_j, ρ_j (j=1, 2, 3) and ν_1 (in the area of viscous liquid "1"). Let us put small perturbations on this main flow

$$u_x=u_j+u'_{jx}(x,y,t), \quad u_y=u'_{jy}(x,y,t),$$

$$p=p_j+p'_j(x,y,t), \quad (j=1,2,3) \quad (9)$$

and

$$\nu=\nu_1+\nu'_1(x,y,t). \quad (10)$$

It is assumed that $\rho'_j(x,y,t)=0$, i. e. the medium remains incompressible in the perturbed state also.

The quantities u'_{jx}, u'_{jy}, p'_j (j=1, 2, 3) and ν'_1 are assumed to be small first-order quantities together with all their derivatives, i. e. they are of the same order of smallness as the quantities ε_j (j=1, 2).

Substituting expressions (9) and (10) into the system (4) and carrying out the linearization process, i. e., neglecting quantities of the second order of smallness and smaller ones, we get the following system of linear partial differential equations

$$\begin{cases} \dfrac{\partial u'_{1x}}{\partial t}+u_1\dfrac{\partial u'_{1x}}{\partial x}+\dfrac{1}{\rho_1}\dfrac{\partial p'_1}{\partial x}=\nu_1\left(\dfrac{\partial^2 u'_{1x}}{\partial x^2}+\dfrac{\partial^2 u'_{1x}}{\partial y^2}\right), \\ \dfrac{\partial u'_{1y}}{\partial t}+u_1\dfrac{\partial u'_{1y}}{\partial x}+\dfrac{1}{\rho_1}\dfrac{\partial p'_1}{\partial x}=\nu_1\left(\dfrac{\partial^2 u'_{1y}}{\partial x^2}+\dfrac{\partial^2 u'_{1y}}{\partial y^2}\right), \\ \dfrac{\partial u'_{1x}}{\partial x}+\dfrac{\partial u'_{1y}}{\partial y}=0. \end{cases} \quad (11)$$

In turn, the substitution of expressions (9) into the system (5) as a result of linearization leads to the systems

$$\begin{cases} \dfrac{\partial u'_{jx}}{\partial t}+u_j\dfrac{\partial u'_{jx}}{\partial x}+\dfrac{1}{\rho_j}\dfrac{\partial p'_j}{\partial x}=0, \\ \dfrac{\partial u'_{jy}}{\partial t}+u_1\dfrac{\partial u'_{jy}}{\partial x}+\dfrac{1}{\rho_1}\dfrac{\partial p'_j}{\partial x}=0, \quad (\text{j=2, 3}). \\ \dfrac{\partial u'_{jx}}{\partial x}+\dfrac{\partial u'_{jy}}{\partial y}=0, \end{cases} \quad (12)$$

Particular solutions of the system (11) are

$$\frac{u'_{1x}}{u_1}=\begin{bmatrix} -\dfrac{1}{z+1}A_{11}e^{hx}+\dfrac{1}{z-1}A_{12}e^{-hx}+ \\ +A_{13}e^{k_{13}hx}+A_{14}e^{k_{14}hx} \end{bmatrix}e^{ihy-i\omega t}, \quad (13)$$

$$\frac{u'_{1y}}{u_1}=\begin{bmatrix} -\dfrac{i}{z+1}A_{11}e^{hx}-\dfrac{i}{z-1}A_{12}e^{-hx}- \\ -izA_{13}e^{k_{13}hx}-izA_{14}e^{k_{14}hx} \end{bmatrix}e^{ihy-i\omega t}, \quad (14)$$

$$\frac{p_1'}{\rho_1 u_1^2} = \left[A_{11}e^{hx} + A_{12}e^{-hx}\right]e^{ihy-i\omega t}, \tag{15}$$

where

$$k_{11} = 1, \quad k_{12} = -1,$$

$$k_{1l} = \frac{1}{\alpha_1}\left[1 + (-1)^l \sqrt{1 + 2\alpha_1\left(z + \frac{\alpha_1}{2}\right)}\right] (l = 3,4), \tag{16}$$

$$\alpha_1 = \frac{2h\nu_1}{u_1}, \tag{17}$$

$$z = -\frac{i\omega}{hu_1}. \tag{18}$$

Thus, pressure perturbations and perturbations of the velocity vector components in the fuel liquid phase area "1" are represented as a superposition of four types of perturbations corresponding to four indefinite constants A_{1l} (l=1, 2, 3, 4). Perturbation of type "1" (corresponding to A_{11}, k_{11}=1) and perturbation of type "2" (corresponding to A_{12}, k_{12}=1) are acoustic (more precisely, quasi-acoustic) perturbations for which $p_1' \neq 0$.

Perturbation of type "3"

$$\left(A_{13}, k_{13} = \frac{1}{\alpha_1}\left[1 - \sqrt{1 + 2\alpha_1\left(z + \frac{\alpha_1}{2}\right)}\right]\right)$$

corresponds to $p_1' = 0$ and is the vortex perturbation, because for this type of perturbations the vorticity value $\Omega_1 = \frac{\partial u_{1y}'}{\partial x} - \frac{\partial u_{1x}'}{\partial y}$ is different from zero. If the square root in (16) means the principal square root, then the vortex perturbation "3" moves with the flow downstream. In the limit case of ideal liquid $\alpha_1 \to 0$ $(\nu_1 \to 0)$, i. e. with vanishing viscosity, this perturbation turns into a similar perturbation for an ideal liquid (see below).

Perturbation of type "4"

$$\left(A_{14}, k_{14} = \frac{1}{\alpha_1}\left[1 + \sqrt{1 + 2\alpha_1\left(z + \frac{\alpha_1}{2}\right)}\right]\right),$$

like perturbation of type "3", corresponds to the absence of pressure perturbations ($p_1' = 0$) and is the vortex perturbation. If the square root in (17) means the principal square root, then the vortex perturbation "4" diffuses upstream due to viscosity. In the limit case of ideal liquid $\alpha_1 \to 0$ ($\nu_1 \to 0$), this perturbation vanishes, losing its physical meaning, since $k_{14} \to \infty$.

The interface between the solid and liquid phases of the fuel is the unperturbed surface $x=-L_1$, which is modeled by a rigid wall. Suppose that this surface is not a source of vortex perturbations. But since this is the only possible source of vortex perturbations in the fuel liquid phase, the diffusion of the vortex down the stream in the area "1" is absent. Such an assumption seems quite justified, if initially the perturbations are supposed to be so weak that they cannot cause deformation or, at least, vibrations of the surface of the fuel solid phase. In the absence of vortex diffusion down the stream, A_{13}=0 and equations (13) and (14) correspondingly change to

$$\frac{u_{1x}'}{u_1} = \left[-\frac{1}{z+1}A_{11}e^{hx} + \frac{1}{z-1}A_{12}e^{-hx} + A_{14}e^{k_{14}hx}\right]e^{ihy-i\omega t} \tag{19}$$

and

$$\frac{u_{1y}'}{u_1} = \left[-\frac{i}{z+1}A_{11}e^{hx} - \frac{i}{z-1}A_{12}e^{-hx} - izA_{14}e^{k_{14}hx}\right]e^{ihy-i\omega t}. \tag{20}$$

Finally, small perturbations in the viscous liquid phase area "1" are described by the equations (15), (19) and (20).

Particular solutions of two systems (j=2, 3) of equations (12) are

$$\frac{u_{jx}'}{u_j} = \left[\begin{array}{l} -\dfrac{1}{\dfrac{z}{\delta_j}+1}A_{j1}e^{hx} + \\ +\dfrac{1}{\dfrac{z}{\delta_j}-1}A_{j2}e^{-hx} + A_{j3}e^{-\frac{z}{\delta_j}hx} \end{array}\right]e^{ihy-i\omega t}, \tag{21}$$

$$\frac{u_{jy}'}{u_j} = \left[\begin{array}{l} -\dfrac{i}{\dfrac{z}{\delta_j}+1}A_{11}e^{hx} - \\ -\dfrac{i}{\dfrac{z}{\delta_j}-1}A_{12}e^{-hx} - i\dfrac{z}{\delta_j}A_{13}e^{-\frac{z}{\delta_j}hx} \end{array}\right]e^{ihy-i\omega t}, \tag{22}$$

$$\frac{p_j'}{\rho_j u_j^2} = \left[A_{j1}e^{hx} + A_{j2}e^{-hx}\right]e^{ihy-i\omega t}, \tag{23}$$

where

$$\delta_j = \frac{\rho_1}{\rho_j} = \frac{u_j}{u_1} (j = 2,3). \tag{24}$$

Thus, pressure perturbations and perturbations of the velocity vector components in the fuel gas phase area "2" and in the combustion products area "3" are represented as a superposition of three types of perturbations. In each separate area (j=2, 3), these three types of perturbations correspond to three indefinite constants A_{jl} (l=1, 2, 3). Perturbations of the type "1" (corresponding to A_{j1}, k_{j1}=1) and perturbations of the type "2" (corresponding to A_{j2}, k_{j2}=1) are acoustic (more precisely, quasi-acoustic) perturbations, for which $p_j' \neq 0$. It is obvious that these perturbations coincide with the corresponding perturbations for a viscous liquid up to indefinite constants.

Perturbations of the type "3" correspond to indefinite constants A_{j3}(j=2, 3) and are vortex perturbations, for which there is no pressure perturbation $(p_j' = 0)$, and vorticities

$$\Omega_j = \frac{\partial u_{jy}'}{\partial x} - \frac{\partial u_{jx}'}{\partial y}$$

are different from zero. These vortex perturbations move downstream, which is understandable physically, since according to the Helmholtz theorem well known in hydrodynamics, the velocity vortex is transferred together with the flow of the liquid itself in an ideal (inviscid) liquid. Vortex perturbations corresponding to indefinite constants A_{j3}(j=2, 3) are similar to vortex perturbations of the type "3" for a viscous liquid and can be obtained from it by passing to the limit $\alpha_1 \to 0(\nu_1 \to 0)$. Perturbations of the type "4" for a viscous liquid (i. e., diffusing upstream vortex) has no analogue in the case of an ideal liquid precisely because diffusion of the vortex upstream in this case is impossible.

Thus, in the area of gasified solid fuel "2", the expressions for perturbations can finally be represented as

$$\frac{u'_{2x}}{u_2} = \begin{bmatrix} -\frac{1}{\frac{z}{\delta_2}+1} A_{21} e^{hx} + \\ +\frac{1}{\frac{z}{\delta_2}-1} A_{22} e^{-hx} + A_{23} e^{-\frac{z}{\delta_2}hx} \end{bmatrix} e^{ihy-i\omega t}, \quad (25)$$

$$\frac{u'_{2y}}{u_2} = \begin{bmatrix} -\frac{i}{\frac{z}{\delta_2}+1} A_{21} e^{hx} - \\ -\frac{i}{\frac{z}{\delta_2}-1} A_{22} e^{-hx} - i\frac{z}{\delta_2} A_{23} e^{-\frac{z}{\delta_2}hx} \end{bmatrix} e^{ihy-i\omega t}, \quad (26)$$

$$\frac{p'_2}{\rho_2 u_2^2} = \left[A_{21} e^{hx} + A_{22} e^{-hx} \right] e^{ihy-i\omega t}. \quad (27)$$

In the unlimited area of gaseous combustion products "3", the situation with perturbations differs from the situation in the limited area "2". Since the problem of internal stability of solid fuel combustion is considered, the perturbations must satisfy the decay condition (or at least boundedness) under $x\to+\infty$. Obviously, an acoustic perturbation of the type "1" in the area "3" does not satisfy this condition, since it contains the factor e^{hx} ($h>0$). Therefore, in the area "3", $A_{31}=0$ should be set.

Finally, small perturbations in the combustion products area "3" are described by the equations

$$\frac{u'_{3x}}{u_3} = \left[\frac{1}{\frac{z}{\delta_3}-1} A_{32} e^{-hx} + A_{33} e^{-\frac{z}{\delta_3}hx} \right] e^{ihy-i\omega t}, \quad (28)$$

$$\frac{u'_{3y}}{u_3} = \left[\frac{i}{\frac{z}{\delta_3}-1} A_{32} e^{-hx} - i\frac{z}{\delta_3} A_{33} e^{-\frac{z}{\delta_3}hx} \right] e^{ihy-i\omega t}, \quad (29)$$

$$\frac{p'_3}{\rho_3 u_3^2} = A_{32} e^{-hx} e^{ihy-i\omega t}. \quad (30)$$

We also note that the boundedness of all perturbations under $|y|\to+\infty$ is guaranteed by their form: all perturbations contain the factor $\exp(ihy\text{-}i\omega t)$.

The boundary conditions on the solid-liquid interface $x=-L_1$ are the conditions of "adhesion" of the viscous liquid to the surface of the rigid wall. These conditions are

$$u'_{1x}\Big|_{x=-L_1} = 0, \quad u'_{1y}\Big|_{x=-L_1} = 0$$

and can be represented as

$$\frac{u'_{1x}}{u_1}\Bigg|_{x=-L_1} = 0, \quad \frac{u'_{1y}}{u_1}\Bigg|_{x=-L_1} = 0. \quad (31)$$

The boundary conditions on the perturbed liquid-gas interface $x=\varepsilon_1(y, t)$ are the fundamental physical laws of conservation of mass and momentum. In this case, the peculiarity of applying these laws is that on the one side of the discontinuity surface (in the area of the liquid phase "1"), the medium is viscous, and on the other (in the region of already gasified solid fuel "2") it is ideal.

The linearized boundary condition expressing the law of conservation of mass is

$$\rho_1 \left(u'_{1x} - \frac{\partial \varepsilon_1}{\partial t} \right)\Bigg|_{x=0} = \rho_2 \left(u'_{2x} - \frac{\partial \varepsilon_1}{\partial t} \right)\Bigg|_{x=0}. \quad (32)$$

The linearized boundary condition expressing the law of conservation of momentum in the projection onto the Ox axis, is represented as

$$\left(\frac{p'_1}{\rho_1 u_1^2} + 2u'_{1x} - \frac{2\nu_1}{u_1} \frac{\partial u'_{1x}}{\partial x} \right)\Bigg|_{x=0} = \left(\frac{p'_2}{\rho_2 u_2^2} + 2u'_{2x} \right)\Bigg|_{x=0}. \quad (33)$$

The linearized boundary condition, which expresses the law of conservation of momentum in the projection on the Oy axis, has the form

$$\left[u'_{1y} + u_1 \frac{\partial \varepsilon_1}{\partial y} - \frac{\nu_1}{u_1} \left(\frac{\partial u'_{1x}}{\partial y} + \frac{\partial u'_{1y}}{\partial x} \right) \right]\Bigg|_{x=0} =$$

$$= \left(u'_{2y} + u_2 \frac{\partial \varepsilon_1}{\partial y} \right)\Bigg|_{x=0}. \quad (34)$$

An additional boundary condition on the perturbed liquid-gas interface is the condition

$$u'_{1x}\Big|_{x=0} - \frac{\partial \varepsilon_1}{\partial t} = 0. \quad (35)$$

Condition (35) is similar to the Landau condition [29] on the surface of a perturbed front of a laminar flame that is modeled by a discontinuity surface propagating in an ideal incompressible medium. Condition (35) means that the gasification process in the perturbed state proceeds at the same rate as in the unperturbed state, i. e., the perturbations are assumed to be so weak that they cannot affect the evaporation of the liquid.

After simple transformations, the boundary conditions (35) and (33) can be represented as

$$\frac{u'_{1x}}{u_1}\Bigg|_{x=0} - \frac{1}{u_1} \frac{\partial \varepsilon_1}{\partial t} = 0 \quad (36)$$

and

$$\frac{u'_{2x}}{u_2}\Bigg|_{x=0} - \frac{1}{u_2} \frac{\partial \varepsilon_1}{\partial t} = 0. \quad (37)$$

Conditions (33) and (34), respectively, can be transformed to

$$\left(\frac{p'_1}{\rho_1 u_1^2} + \frac{2\nu_1}{u_1} \frac{\partial u'_{1x}}{\partial x} \right)\Bigg|_{x=0} = \frac{\delta_2 p'_2}{\rho_2 u_2^2}\Bigg|_{x=0} \quad (38)$$

and

$$\left[\frac{u'_{1y}}{u_1}-\frac{\nu_1}{u_1^2}\left(\frac{\partial u'_{1x}}{\partial y}+\frac{\partial u'_{1y}}{\partial x}\right)\right]\Bigg|_{x=0}+\frac{\partial \varepsilon_1}{\partial y}=$$

$$=\delta_2\left(\frac{u'_{2y}}{u_2}\Bigg|_{x=0}+\frac{\partial \varepsilon_1}{\partial y}\right). \qquad (39)$$

The boundary conditions on the perturbed surface of the flame front, i. e., the instantaneous combustion front $x=\varepsilon_2(y, t)$ are also the physical laws of conservation of mass and momentum.

The linearized boundary condition expressing the law of conservation of mass at the flame front is

$$\rho_2\left(u'_{2x}-\frac{\partial \varepsilon_2}{\partial t}\right)\Bigg|_{x=L_2}=\rho_3\left(u'_{3x}-\frac{\partial \varepsilon_2}{\partial t}\right)\Bigg|_{x=L_2}. \qquad (40)$$

The linearized boundary condition expressing the law of conservation of momentum to the flame front in the projection onto the Ox axis is represented as

$$\left(\frac{p'_2}{\rho_2 u_2^2}+2u'_{2x}\right)\Bigg|_{x=L_2}=\left(\frac{p'_3}{\rho_3 u_3^2}+2u'_{3x}\right)\Bigg|_{x=L_2}. \qquad (41)$$

The linearized boundary condition expressing the law of conservation of momentum at the flame front in the projection onto the Oy axis is

$$\left(u'_{2y}+u_2\frac{\partial \varepsilon_2}{\partial y}\right)\Bigg|_{x=L_2}=\left(u'_{3y}+u_3\frac{\partial \varepsilon_2}{\partial y}\right)\Bigg|_{x=L_2}. \qquad (42)$$

An additional boundary condition on the perturbed surface of the flame front is the condition

$$u'_{2x}\Big|_{x=0}-\frac{\partial \varepsilon_2}{\partial t}=0. \qquad (43)$$

Condition (43) is absolutely identical to the Landau condition [29], and means the independence of the flame front propagation speed – i. e. combustion rate –of perturbations.

After simple transformations, the boundary conditions (43) and (40) can be changed to

$$\frac{u'_{2x}}{u_1}\Bigg|_{x=L_2}-\frac{1}{u_2}\frac{\partial \varepsilon_2}{\partial t}=0 \qquad (44)$$

and

$$\frac{u'_{3x}}{u_3}\Bigg|_{x=L_2}-\frac{1}{u_3}\frac{\partial \varepsilon_2}{\partial t}=0. \qquad (45)$$

Conditions (41) and (42), correspondingly, can be transformed to

$$\frac{p'_2}{\rho_2 u_2^2}\Bigg|_{x=L_2}=\frac{\delta_3}{\delta_2}\frac{p'_3}{\rho_3 u_3^2}\Bigg|_{x=L_2} \qquad (46)$$

and

$$\frac{u'_{2y}}{u_2}\Bigg|_{x=L_2}+\frac{\partial \varepsilon_2}{\partial y}=\frac{\delta_3}{\delta_2}\left(\frac{u'_{3y}}{u_3}\Bigg|_{x=0}+\frac{\partial \varepsilon_2}{\partial y}\right). \qquad (47)$$

Thus, both physical and mathematical models of the perturbed state of solid fuel combustion are constructed. Explicit expressions are obtained for small perturbations of pressure and velocity components in all three flow areas, as well as expressions for perturbations of the phase interface and flame front. All these perturbations satisfy the boundary conditions at infinity (the boundedness condition under $|y|\to+\infty$ and the decay condition under $x\to+\infty$. Linearized boundary conditions on the perturbed interface between the liquid and solid phases and on the combustion instantaneous front are written.

5. Eigenvalue problem

The expressions for small perturbations (6), (36)–(39) and (44)–(47) contain ten indefinite constants A_{01}, A_{02}, A_{11}, A_{12}, A_{14}, A_{21}, A_{22}, A_{23}, A_{32}, A_{33}. To determine these constants, let us substitute expressions (6), (36)–(39) and (44)–(47) into the boundary conditions (31), (36)–(39), (44)–(47). As a result, we obtain a system of ten linear homogeneous algebraic equations in the above mentioned indefinite constants. This system has nontrivial solutions if and only if its determinant is equal to zero. The fact that the determinant is equal to zero leads to the characteristic equation in the dimensionless eigenvalue

$$z=-\frac{i\omega}{hu_1}.$$

If all the roots of the characteristic equation have negative real parts, then we can talk about the stability of the process to this type of perturbations (conditional stability).

If among the roots of the characteristic equation, there is at least one root with a positive real part, then absolute instability takes place.

The case when all the roots of the characteristic equation have real parts equal to zero is the neutral case. Such a case, as a rule, requires additional studies related to a change in the statement of the stability problem.

The characteristic equation is

$$|a_{rs}|=0, \qquad (48)$$

where $r=1...10$, $s=1...10$.

In the tenth-order determinant on the left side of the equation (48), only the following elements are nonzero:

$$a_{13}=-\frac{e^{-\xi\bar{L}_1}}{z+1},\quad a_{14}=\frac{e^{\xi\bar{L}_1}}{z-1},\quad a_{15}=e^{-k_{14}\xi\bar{L}_1}, \qquad (49)$$

$$a_{23}=\frac{e^{-\xi\bar{L}_1}}{z+1},\quad a_{24}=\frac{e^{\xi\bar{L}_1}}{z-1},\quad a_{25}=ze^{-k_{14}\xi\bar{L}_1}, \qquad (50)$$

$$a_{31}=-z,\quad a_{33}=-\frac{1}{z+1},\quad a_{34}=\frac{1}{z-1},\quad a_{35}=1, \qquad (51)$$

$$a_{41}=-\frac{z}{\delta_2},\quad a_{46}=-\frac{1}{\frac{z}{\delta_2}+1},\quad a_{47}=\frac{1}{\frac{z}{\delta_2}-1},\quad a_{48}=1, \qquad (52)$$

$$a_{52}=-\frac{z}{\delta_2},\quad a_{56}=-\frac{e^{\xi\bar{L}_2}}{\frac{z}{\delta_2}+1},$$

$$a_{57}=\frac{e^{-\xi\tilde{L}_2}}{\frac{z}{\delta_2}-1},\quad a_{58}=e^{-\frac{z}{\delta_2}\xi\tilde{L}_2},\qquad (53)$$

$$a_{62}=-\frac{z}{\delta_3},\quad a_{69}=\frac{e^{-\xi\tilde{L}_2}}{\frac{z}{\delta_3}-1},\quad a_{6,10}=e^{-\frac{z}{\delta_3}\xi\tilde{L}_2},\qquad (54)$$

$$a_{73}=1+\frac{\alpha_1}{z+1},\quad a_{74}=1+\frac{\alpha_1}{z-1},$$

$$a_{75}=\alpha_1 k_{14},\quad a_{76}=-\delta_2,\quad a_{77}=-\delta_2,\qquad (55)$$

$$a_{81}=1-\delta_2,\quad a_{83}=\frac{\alpha_1-1}{z+1},$$

$$a_{84}=-\frac{\alpha_1+1}{z-1},\quad a_{85}=\frac{\alpha_1\left(zk_{14}-1\right)}{2}-z,\qquad (56)$$

$$a_{86}=\frac{\delta_2}{\frac{z}{\delta_2}+1},\quad a_{87}=\frac{\delta_2}{\frac{z}{\delta_2}-1},\ a_{88}=z,\qquad (57)$$

$$a_{96}=\delta_2 e^{\xi\tilde{L}_2},\quad a_{97}=\delta_2 e^{-\xi\tilde{L}_2},\quad a_{99}=-\delta_3 e^{-\xi\tilde{L}_2},\qquad (58)$$

$$a_{10,2}=\delta_2-\delta_3,\quad a_{10,6}=-\frac{\delta_2 e^{\xi\tilde{L}_2}}{\frac{z}{\delta_2}+1},$$

$$a_{10,7}=-\frac{\delta_2 e^{-\xi\tilde{L}_2}}{\frac{z}{\delta_2}-1},\quad a_{10,8}=-ze^{-\frac{z}{\delta_2}\xi\tilde{L}_2},\qquad (59)$$

$$a_{10,9}=\frac{\delta_3 e^{-\xi\tilde{L}_2}}{\frac{z}{\delta_3}-1},\quad a_{10,10}=ze^{-\frac{z}{\delta_3}\xi\tilde{L}_2}.\qquad (60)$$

In addition, the following conventional signs are used

$$\tilde{L}_1=\frac{L_1}{L},\quad \tilde{L}_2=\frac{L_2}{L}\quad \left(\tilde{L}_1+\tilde{L}_2=1\right),\qquad (61)$$

$$\xi=hL=\frac{2\pi L}{\lambda}.\qquad (62)$$

After a series of transformations, the characteristic equation (48) is reduced to the form

$$\left|b_{rs}\right|=0,\qquad (63)$$

where r=1...4, s=1...4.

The elements of the fourth-order determinant on the left side of the equation (63) are:

$$b_{12}=b_{14}=b_{32}=0,\qquad (64)$$

$$b_{11}=z,\ b_{13}=e^{\xi\tilde{L}_1 k_{14}}-e^{\xi\tilde{L}_1}+sh\left(\xi\tilde{L}_1\right)\left(z+1\right),\qquad (65)$$

$$b_{21}=\frac{1}{\delta_2}\left[sh\left(\xi\tilde{L}_2\right)\left(z+\delta_2-1\right)-e^{\xi\tilde{L}_2}z\right],$$

$$b_{12}=\frac{z}{\delta_2},\qquad (66)$$

$$b_{23}=\frac{1}{\delta_2}sh\left(\xi\tilde{L}_2\right)\beta,$$

$$b_{24}=1-e^{\left(\frac{z}{\delta_2}+1\right)\xi\tilde{L}_2}+$$
$$+\left(\frac{1}{\delta_2}-1\right)sh\left(\xi\tilde{L}_2\right)e^{\frac{z}{\delta_2}\xi\tilde{L}_2},\qquad (67)$$

$$b_{31}=-\frac{z}{\delta_2},\quad b_{34}=e^{\frac{z}{\delta_2}\xi\tilde{L}_2}\left(\frac{z}{\delta_2}-2z-\delta_2\right),\qquad (68)$$

$$b_{33}=\left[1+\frac{z}{\delta_2}\left(\alpha_1-1+\delta_2\right)+\alpha_1\right]e^{\frac{z}{\delta_2}\xi\tilde{L}_2}+$$
$$+\left\{\frac{z}{\delta_2}\left[\frac{\alpha_1}{2}\left(zk_{14}-1\right)-z\right]-\alpha_1 k_{14}\right\}+$$
$$+\left[\begin{array}{l}\left(\frac{z}{\delta_2}-z-\alpha_1\right)ch\left(\xi\tilde{L}_1\right)-\\ -\left(\frac{\alpha_1 z}{\delta_2}+1\right)sh\left(\xi\tilde{L}_1\right)\end{array}\right]\left(z+1\right),\qquad (69)$$

$$b_{41}=e^{\xi\tilde{L}_2}\left(-\frac{z}{\delta_2}+\delta_2-1\right),\quad b_{42}=\delta_3-\delta_2-\frac{z^2}{\delta_3},$$

$$b_{43}=\frac{1}{\delta_2}e^{\xi\tilde{L}_2}\left(z+\delta_2\right)\beta,\qquad (70)$$

$$b_{44}=z\left[e^{\left(\frac{z}{\delta_2}-1\right)\xi\tilde{L}_2}-1\right]+\left(\frac{1}{\delta_2}-1\right)e^{\xi\tilde{L}_2}\left(z+\delta_2\right)e^{\frac{z}{\delta_2}\xi\tilde{L}_2},\qquad (71)$$

where

$$\beta=\left(\alpha_1-1\right)e^{\xi\tilde{L}_1}+\left[\frac{\alpha_1}{2}\left(zk_{14}-1\right)-z\right]e^{\xi\tilde{L}_1 k_{14}}+$$
$$+\left[ch\left(\xi\tilde{L}_1\right)-\alpha_1 sh\left(\xi\tilde{L}_1\right)\right]\left(z+1\right).\qquad (72)$$

Thus, a characteristic equation in the dimensionless eigenvalue z is obtained.

6. Analysis of the characteristic equation

Equation (63) can be represented as

$$F\left(z;\xi,\alpha_1,\tilde{L}_1,\tilde{L}_2,\delta_1,\delta_2\right)=0,\qquad (73)$$

where z is the unknown eigenvalue, and, α, $\tilde{L}_1$, $\tilde{L}_2$, δ_1, δ_2 are parameters.

Parameter α_1 – as it follows from equation (17) – characterizes the viscosity of the fuel in the liquid phase. The Reynolds number Re_λ, determined by the perturbation wavelength λ, is represented as

$$Re_\lambda=\frac{\lambda u_1}{\nu_1}.\qquad (74)$$

As follows from (17) and (74), the following relationship takes place

$$\alpha_1=\frac{4\pi}{Re_\lambda},\qquad (75)$$

i. e. α_1 is a quantity inversely proportional to the Reynolds number.

It is known from the combustion theory that the density of incandescent combustion products ρ_3 is always lower than the density of a combustible medium ρ_2 in front of the flame front [2]. On the other hand, the density of the fuel in the liquid phase ρ_1 is obviously higher than the density of the fuel in the gas phase ρ_2. Thus, there is the inequality $\rho_1>\rho_2>\rho_3$, which, taking into account conventional signs (24), takes the form

$$1<\delta_2<\delta_3. \quad (76)$$

The values $\tilde{L}_1$ and $\tilde{L}_2$ are interdependent $\left(\tilde{L}_2=1-\tilde{L}_1\right)$ and in total they express the ratio between the lengths of two zones of the fuel phases – liquid phase and gas phase. For the burning of solid fuel, the inequalities

$$L_1<L_2,\ \tilde{L}_1<\tilde{L}_2 \quad (77)$$

are true.

Following [24], let us consider the limiting case $\tilde{L}_1\to 0$, corresponding to the decomposition of fuel during combustion according to the widespread “solid phase –gas phase” scheme [19]. In so doing $\tilde{L}_2\to 1.$ In this case, the function F on the left side of the characteristic equation (73) can be reduced to the form of a quasipolynomial without a principal term, which is always unstable by the well-known Pontryagin theorem. This result is in agreement with the result of [24], where the characteristic equation is much simpler and can be solved explicitly. But in [24], perturbations of the flame front and perturbations of hydrodynamic parameters in combustion products are not taken into account. Therefore, we can conclude that these perturbations do not impede the development of the instability generated in the gasification zone of the fuel and, possibly, even intensify this process. Furthermore, it is obvious that when $L_1<<L_2$ $\left(\tilde{L}_1\ll\tilde{L}_2\right)$, i. e. in the case of a very thin film, which is the most important from the viewpoint of erosive effect [22–28], the conclusion about the absolute instability of the combustion process remains valid.

Let us consider another limiting case $\xi\to 0(\lambda\to\infty)$, i. e. the case of long-wave perturbations (as compared with the lengths of areas “1” and “2”). Moreover, it follows from (17) that $\alpha_1\to 0$. In this case, the characteristic equation changes to

$$\left(1-\frac{1}{\delta_2}\right)^2 z^2+\frac{1}{\delta_2}\left(2-\frac{1}{\delta_2}\right)z+\left(2-\frac{1}{\delta_2}\right)=0. \quad (78)$$

The roots of the quadratic equation (78) can be found directly, but in this case the Hurwitz criterion of stability of a polynomial with real coefficients can be used. According to this criterion, the quadratic trinomial on the left-hand side of (78) is stable, since the inequality (76) implies that all the coefficients of this trinomial are positive. The reasons for such stabilization at first glance are not entirely clear and are explained below, with a physical interpretation and discussion of the results.

In the general case, the characteristic equation (73) should be solved only numerically, and separately for each set of parameters α_1, $\tilde{L}_1$, $\tilde{L}_2$, δ_1, δ_2. For unstable perturbations, it is possible to find – just as it is done in [30] – the wavelength λ_m with the maximum increment of the amplitude growth. The physical realization of such perturbations should be expected primarily. The quantity λ_m may be adopted as the estimate for the mean dimension of irregularities on the flame front. The space-time structure of the combustion wave also can be calculated at the early stage of autoturbulization, but it is the subject of special research.

Let us calculate the value of the function $F\left(z;\xi,\alpha_1,\tilde{L}_1,\tilde{L}_2,\delta_1,\delta_2\right)$ under $z=0$ in the general case. There is

$$F(0)=\frac{1-\delta_2}{2}\left(e^{2\xi\tilde{L}_2}-1\right)<0. \quad (79)$$

The inequality in (79) holds for any non-zero (i. e., corresponding to multi-dimensional perturbations) values of ξ.

Let us consider the limit of the function $F(z)$ under $z\to+\infty$. If this limit exists, then it belongs to the set of real numbers or is infinite, i. e., it can be considered that

$$F(+\infty)=D(-\infty\le D\le+\infty). \quad (80)$$

It is obvious; that the inequality

$$0\le D\le+\infty \quad (81)$$

is a sufficient condition for the existence of unstable roots of the characteristic equation (73).

To avoid the numerical solution of equation (73), for each specific set of parameters α_1, $\tilde{L}_1$, $\tilde{L}_2$, δ_1, δ_2 the limit $F(z)$ can be calculated under $z\to+\infty$. This limit exists for almost any set of parameters. This makes it possible to apply sufficient condition for instability (81).

Thus, the characteristic equation (73) is analyzed in terms of unstable roots existence. It is proved that in the limiting case $\tilde{L}_1\to 0$, the characteristic equation always has roots with a positive real part. In the other limiting case $\xi\to 0(\lambda\to\infty)$, the roots of the characteristic equation are stable. For the general case of the equation (73), a sufficient condition for instability is obtained.

7. Physical interpretation of the results

Let us consider the obtained mathematical results from the point of view of physics of the solid fuel burning process.

The instability of the roots of the characteristic equation in the limiting case $\tilde{L}_1\to 0$, corresponding to the decomposition of solid fuel during combustion according to the “solid phase – gas” scheme means that the combustion process becomes turbulent and can be accelerated. Such acceleration can lead to the development of explosive processes – deflagration explosion or detonation. The only factor stabilizing the combustion process is the compressibility of the medium [9], which was not taken into account in this problem. But the compressibility of the medium begins to play a significant role only from the moment when the turbulent flame reaches a sufficiently high velocity. If stabilization due to compressibility is not possible at this stage, then the transition of combustion to explosion – deflagration or detonation – is inevitable. Actually, for this reason, only two combustion modes are observed, in which solid fuel passes directly into the gas phase (sublimates) [2, 19, 22]: developed turbulence (developed deflagration [19]) and explosion (usually detonation). In this case, however, the following should be noted.

Firstly, in the present study, the flow area is not limited along the Oy axis, while the real solid-fuel charge has a finite diameter. Therefore, the conclusion about distortion of the flame shape and autoturbulization of the combustion process can only be attributed to solid fuel charges of a sufficiently large diameter. If the diameter of the charge is less than the wavelengths of unstable perturbations, i. e., the charge is quite narrow, then two-dimensional instability does not occur at all, and the question of one-dimensional instability (without distorting the shape of the flame front) must be considered separately. In this case, a vibrational combustion regime may arise [17–19]. Secondly, in the present study, the flow area is not limited along the *Ox* axis. However, each charge of solid fuel has a certain finite extent, so the complete burnout of the charge can occur even before the combustion-explosion transition. This question is related to determining the explosive induction distance and the time of combustion-explosion transition [9].

The physical interpretation of the mathematical results obtained for the limiting case $\tilde{L}_1 \to 0$, corresponding to the absence of a liquid film, naturally extends to the case $\tilde{L}_1 \ll \tilde{L}_2 < 1$, when the liquid film is very narrow. This is in agreement with the results of the study [24], where the case of a narrow liquid film was analyzed in detail from the perspective of the erosive combustion theory for the physical explanation of the negative erosion effect (Vilyunov – Dvoryashin effect).

An interesting fact is the stability of the roots of the characteristic equation in the limiting case $\xi\to 0(\lambda\to\infty)$. The result looks somewhat paradoxical, since the physical reasons for the stabilization of the process are not entirely clear. Viscosity is always a significant stabilizing factor for burning [24, 30]. But in this case, it should not directly influence the development of disturbances. In fact, $\alpha_1\to 0$, and in accordance with (17), α_1 is a quantity that is proportional to the viscosity of the liquid film. However, the fact of taking into account the diffusion of the vortex perturbations upstream in the area "1", which occurs only in a viscous liquid, already represents a viscous effect. In addition, the perturbation of the liquid-gas interface $\varepsilon_1(y, t)$ and the perturbation of the instantaneous combustion front $\varepsilon_2(y, t)$ are perturbations of various amplitudes. Therefore, zone "2" of the gas phase of the fuel has a variable thickness (Fig. 2) in a perturbed state (this factor was not taken into account in [24]). It is known that a change in the thickness of the flame zone under the action of perturbations is a stabilizing factor for the homogeneous gas mixtures combustion [30]. Obviously, this same effect – perhaps just a mechanical one – takes place in this case too. And finally, it should be noted that the passage to the limit $h\to\infty$ $(\lambda\to\infty)$ corresponds not only to long-wave perturbations, but also, with some notes, to one-dimensional perturbations. For such perturbations, there are no geometric distortions of the phase interface and the flame front. If initially to solve the problem of stability of stationary combustion of solid fuel (Fig. 1) with respect to small perturbations proportional to $\exp(-i\omega t)$, then the solution to this problem for an incompressible medium is $\omega=0$. This is a neutral case, which requires a change in the problem statement. The problem should be set taking into account compressibility [31]. Thus, there is no instability of one-dimensional perturbations in an incompressible medium.

The above factors fully explain the stability of the characteristic equation roots in the limit case $\xi\to 0$ $(\lambda\to\infty)$. This fact alone indicates the stabilizing effect of the liquid phase on the combustion process for solid fuel. The latter is consistent both with the results of theoretical studies [24], and with the experimental observations data [22, 23].

The sufficient instability condition (81) contains a quantity D, which, generally speaking, depends on the parameters ξ, α_1 $\tilde{L}_1$, $\tilde{L}_2$, δ_1, δ_2. For each specific case of combustion, the values $\tilde{L}_1$, $\tilde{L}_2$, δ_1, δ_2 as well as the viscosity of the liquid film ν_1, velocity u_1, characterizing the rate of evaporation of the liquid film, and the total length L of the fuel gasification zone (consisting of areas "1" and "2") are completely determined. Therefore, the inequality (81) becomes an inequality for the perturbation wavelength λ. The solution of this inequality allows us to calculate the wavelengths of unstable perturbations, as well as the Reynolds numbers corresponding to them by the formula (74).

Physical interpretation of the results indicates that they are in qualitative agreement with the results of experimental observations and other theoretical studies.

8. Discussion of the results of the study of combustion stability of solid fuels with a two-phase gasification zone

The main results of this study include:

– theoretical confirmation of the well-known conclusion about the instability of combustion of solid fuel, decomposing according to the "solid phase – gas" scheme;

– mathematical proof of the instability of combustion of solid fuel with a two-phase gasification zone for the case when the length of the liquid phase layer is much less than the total length of the gasification zone;

– evidence of the stabilizing effect of the liquid film in the gasification area on the combustion process as a whole;

– a sufficient condition for the instability of solid fuel combustion obtained for the general case of combustion of solid fuel with a two-phase gasification zone.

Confirmation of the conclusion about the instability of solid fuel combustion [19, 24] was obtained under general assumptions. In contrast to [19, 31], in this study perturbations are not assumed to be one-dimensional. In this case, two-dimensional perturbations that are generated by the distorted flame front are taken into account. This factor is not considered in [24]. However, when studying the stability of gas combustion, the perturbation of the flame front (instantaneous combustion front) is taken into account in any statement of the problem [1, 3, 29, 30, 32, 33]. It is proved experimentally [34–36] that combustion acoustic perturbations generated by the flame affect the development of combustion instability.

The case when the length of the liquid phase layer is much less than the length of the gasification zone is considered particularly. For this case, the mathematical proof of the instability of solid fuel combustion was made taking into account variability of the width of the gas phase zone of the fuel. This significant factor was not taken into account in [24] along with the perturbations in the combustion products area.

The stabilizing effect of the liquid film in the gasification area on the solid fuel combustion process is ultimately explained by the liquid viscosity and the variability of the width of the gas phase zone of the fuel. The stabilizing effect of viscosity upon combustion of liquids and gases is known [30]. Obviously, the same effect also occurs for solid fuels with a two-phase gasification zone.

The sufficient condition (81) for the instability of solid fuel combustion in many cases allows proving the instability of the process without solving the characteristic equation (73). This condition is not necessary and does not allow proving the stability of combustion.

This study does not take into account the size of the solid fuel charge. Initially, it is assumed that the charge has an infinite length in all directions. This assumption seems justified for charges of sufficiently large sizes and is typical for problems on combustion stability [1–3, 19, 29]. Taking into account the finite sizes of the charge significantly complicates the problem by setting the boundary conditions on the charge surface.

Concluding the discussion of the obtained results, we can outline the following areas for further research.

First, a detailed numerical analysis of the characteristic equation (73) is possible. But in order for such an analysis to be of practical importance, it is necessary to collect a large amount of confirmed data on the parameters of the stationary combustion complex being studied for stability. These data should contain information on the combustion of different types of solid fuel under various physical conditions. Based on numerical analysis of the characteristic equation (73), it is possible to calculate the space-time structure of the combustion wave at an early stage of autoturbulization – the main ideas of this calculation are indicated in [30, 37].

Secondly, it is of certain interest to take into account the compressibility of the medium in the fuel gas phase area "2" and in the combustion products area "3". The solution to this problem can be of fundamental importance for assessing the possibility of combustion-explosion transition [9]. In addition, the medium in the areas "2" and "3" is much more compressible than in the liquid phase area "1". It can be expected that compressibility will have a certain stabilizing effect on the development of perturbations [9, 37].

Third, the instantaneous combustion front $x=L_2$ (Fig. 1), which models the flame as a discontinuity surface, can also be considered as some area of finite extent [30]. But in this case, viscous effects must be taken into account in all flow areas [30], which from a mathematical point of view significantly complicates the problem and increases the order of the functional determinant on the left side of the characteristic equation.

Fourth, it is possible to consider the stability of the solid fuel combustion process in closed or half-open volumes (combustion chambers) taking into account the corresponding boundary conditions. This should considerably increase the practical significance of the study.

And finally, the solution of the problem of the hydrodynamic stability of solid fuel combustion in the two-phase ("liquid-gas") gasification zone presented in this study can be applied to the problem of stability of liquid fuel combustion in large-diameter chambers [18]. The possibility of such analogy was indicated in [31, 38]. In this case, the rigid wall $x=-L_1$ (Fig. 1) models the end surface of the combustion chamber, and not the surface of the fuel in the solid phase (as in the present study). Then, the area "1" corresponds to the area of liquid fuel. Area "2" corresponds to the gasified state of the fuel. This area may be completely absent if there is no preliminary gasification of liquid fuel during its combustion. Area "3" remains the area of combustion products. In so doing, it makes sense to consider only the limiting case $\tilde{L}_2 \to 0$ (when the combustion of liquid fuel is carried out without its preliminary gasification) and the case $\tilde{L}_1 \gg \tilde{L}_2$.

9. Conclusions

1. In this study, physical and mathematical models of the solid fuel combustion process in the presence of small perturbations are developed. As a scheme of solid fuel decomposition during combustion, the "solid phase – liquid phase – gas" scheme is adopted. A characteristic feature of the simulation is consideration of the hydrodynamic parameters perturbations in the combustion products area and taking into account the geometrical distortion of the flame front.

2. The problem is formulated for finding a dimensionless complex eigenvalue, the positiveness of the real part of which means the instability of the combustion process. The characteristic equation in the indicated eigenvalue is obtained. This equation is a multi-parameter algebraic equation, the left side of which is the tenth-order functional determinant (with the right-hand side equal to zero). The characteristic equation is greatly simplified – the order of the determinant before its direct calculation is reduced to the fourth.

3. The characteristic equation is analyzed in terms of unstable roots existence. It is proved that in the limiting case of the absence of a liquid phase of the fuel, the characteristic equation always has unstable roots; the same conclusion can be made when the thickness of the liquid film is very small in comparison with the length of the entire gasification zone. In another limiting case – for perturbations with an infinite wavelength – a transition to stability is established, which indicates the stabilizing effects of a viscous liquid film and variability of the gasification zone length. A sufficient instability condition is obtained for the general case.

4. The physical interpretation of the results obtained mathematically makes it possible to explain the effects of autoturbulization of the combustion process and possible combustion-explosion transition. An explanation of the Vilyunov-Dvoryashin effect from the standpoint of stability theory is also confirmed. The results obtained are in qualitative agreement with the results of other theoretical studies and experimental data.

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UDC 662.741.3:66.041.49:004.942
DOI: 10.15587/1729-4061.2020.210767

DETERMINATION OF PARAMETERS OF THE CARBON-CONTAINING MATERIALS GASIFICATION PROCESS IN THE ROTARY KILN COOLER DRUM

A. Karvatskii
Doctor of Technical Sciences, Professor, Senior Researcher
Department of Chemical, Polymer and Silicate Engineering*
T. Lazariev
PhD, Leading Researcher
Yuzhnoye Design Office
Kryvorizhska str., 3, Dnipro, Ukraine, 49008
S. Leleka
PhD, Senior Researcher
Scientific Research Center
"Resource-saving Technologies"*
I. Mikulionok
Doctor of Technical Sciences, Professor, Senior Researcher
Department of Chemical, Polymer and Silicate Engineering*
O. Ivanenko
PhD, Associate Professor
Department of Ecology and Plant Polymers Technology*
E-mail: olenka.vasaynovich@gmail.com
*National Technical University of Ukraine
"Igor Sikorsky Kyiv Polytechnic Institute"
Peremohy ave., 37, Kyiv, Ukraine, 03056

An assessment of the feasibility of using the existing equipment of a rotary kiln cooler drum for heat treatment of a carbon-containing filler to produce synthesis gas using production waste in the form of a dust fraction of heat-treated petroleum coke or anthracite is carried out. A mathematical model of the process of gasification of carbon particles is formulated in the continuous-discrete formulation, including thirteen global reactions, of which four are heterogeneous and nine are homogeneous. A numerical model of gasification of a dust fraction of a carbon-containing filler in the rotary kiln cooler drum in the axisymmetric formulation is developed. The convergence of the numerical solution of the gasification problem by the grid step is investigated. It is found that the computational grid, which includes 73,620 cells and 75,202 nodes, leads to an error in determining the main parameters of the model of no more than 1–2 %. Verification of the developed numerical model is performed. It is found that the difference between the molar fractions of CO and H_2, the values of which were obtained by various software products (Fluent, NASA CEA), is in the range of (2.8...5.8) %. Using the developed numerical model of the process of gasification of a carbon-containing filler in the rotary kiln cooler drum, the quantitative composition of the combustible components of the syngas for different initial parameters is determined. It is found that with the ratio O_2/C=(42.7...51.6) %, the predicted quantitative composition of the combustible gases of synthesis gas in molar fractions is CO=(32.8...36.9) %, H_2=(17.1...18.4) % and CH_4=(0.03...0.16) %. The possibility of using the NASA CEA program, intended for operational calculations of equilibrium chemistry, for engineering calculations of the material composition of synthesis gas of industrial furnace equipment, is shown

Keywords: rotary kiln, cooler drum, carbon-containing material, heat treatment, gasification, syngas, numerical simulation

Received date 18.06.2020
Accepted date 17.08.2020
Published date 31.08.2020

1. Introduction

The significant cost of natural gas, as well as the need to comply with stringent environmental safety requirements of industrial enterprises, causes the transition of the energy sector and other industries to an increase in the use of solid fuel with its preliminary gasification. Gasification is known to be a process that involves the use of heat and water vapor to convert carbon-containing materials to syngas, which includes combustible gases such as carbon monoxide, hydrogen and methane.

This, in turn, led to the appearance of a large number of developments of reactor equipment for gasification and research of gasification of solid fuels and biomass, in particular, by numerical methods of computational hydrodynamics [1–13]. These works do not consider the use of existing industrial furnace equipment for solid fuel gasification processes. Thus, the use of synthetic gas obtained from the