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To develop the methods for predicting deformations on floodplain areas in the zone of influence of bridge crossings, a mathematical model of a suspended flow with grass vegetation was developed. The problem of calculating the hydrodynamic fields of velocities and pressure in artificially compressed flows refers to the theory of shallow water since the vertical size (flow depth) is substantially smaller than the horizontal dimensions, such as length and width. In accordance with this, the proposed model is based on the equation of distribution of velocity structure and the depth of a floodplain flow in approximation to two-dimensional dependences taking into consideration force factors. Force factors determine resistance at flowing around vegetation in floodplain areas and resistance of washout of fine-grained soil.

To obtain an unambiguous solution of the considered problem, boundary and initial conditions were added to the presented closed system of original equations. These conditions make it possible to determine the level of a free surface of flow and the zone of influence of a bridge crossing at different stages of the estimated flood. Based on finite-difference analogs of transfer equations, the distribution of velocities and depths in estimated sections was calculated. By iteration, the longitudinal velocity in a flood flow with vegetation elements was determined. The results of the calculation of washout on floodplain areas of a sub-bridge watercourse of the lowland river Siversky Donets were obtained. The depth of a flood flow after a washout was determined based on the ratios of actual and flood-free velocities. When compared with the initial bottom marks, the washout of the larger floodplain is 0.96 m, that of the smaller floodplain – 1.28 m.

The proposed scientifically substantiated solution for ensuring optimum interaction of floodplain flows with bridge crossings makes a certain contribution to improving the reliability of their operation due to the quality of design works and the corresponding reduction of construction and operating costs

Keywords: zone of bridge influence, bridge crossing, floodplain vegetation, suspended flow, deformation on floodplains, floodplain flow, turbulence models

PREDICTING DEFORMATIONS IN THE AREA OF IMPACT EXERTED BY A BRIDGE CROSSING BASED ON THE PROPOSED MATHEMATICAL MODEL OF A FLOODPLAIN FLOW

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1. Introduction

The relevance of research is proved by the analysis of destructions of bridge crossings, which suffer most from natural disasters – high floods. Deformation of a watercourse, floodplain areas, washouts of intermediate supports and props can reach such critical dimensions, which become the main cause of emergencies on bridges. The main requirement

for a bridge crossing as one of the most important components of the road transport industry is the best service of transportation. A bridge crossing includes complex and expensive structures, the costs of construction and operation of which depends on the choice of a crossing location, the correct purpose of structures, their general dimensions.

In turn, bridge crossings belong to such a category of structures that violate the natural forms of a watercourse and create additional vortex by their external contours. Encompassing not only a riverbed but also the floodplain parts, the structures cause the changes in the velocity field, at which the transporting ability of flow is distributed very unevenly. After blocking the parts of floodplain approaches by width using the banks, water will start arriving at the bridge and an increased amount of water will pass through a compressed cross-section of a river.

The territory of water resources is re-distributed on floodplain areas, and there arises a problem of runoff regulation, forecasting the throughput capacity and deformation development. The factors causing washout development in floodplain areas are fundamentally different from the relevant processes in watercourses. Unlike a watercourse, floodplains have a significant vegetation cover. Vegetation acts according to the principle of speed-absorber and causes additional turbulence in the lower zone. Velocities on a floodplain stretch, and therefore potential energy of flow, will be significantly lower than in the watercourse due to less discharge.

The geomorphometry of floodplain areas is characterized by the existence of significant vegetation, alluvium from previous floods and freshets, and causes genetic dissimilarity with watercourses.

A floodplain is represented by plant soil at the top of it with a high probability of the existence of sandy and loam soils in the lower zone. A floodplain flow, like a watercourse flow, is heterogeneous, saturated with suspension that due to suppression of flow velocity will settle on the bottom. During freshets, velocity increases and plant soil, as a rule, does not have enough mechanical properties to resist destructive processes. At the same time, a considerable duration of freshets causes leveling the marks of watercourse bottom and the floodplain bottom.

As a result of this interaction, one can observe the narrowing of a watercourse and the formation of new floodplains or watercourse expansion through the floodplain washout. The washout processes are dangerous for bridge crossings.

Proper choice of the size of bridge crossing structures, including the width of an opening as a dominant characteristic, depends on the conditions of their operation, prediction of the possible water inflow to the bridge, and inevitable watercourse and floodplain deformations.

Existing methods for calculation of washouts in the zone of influence of a bridge crossing do not fully take into account the processes that occur in floodplain areas. Washouts on two characteristic parts of a bridge opening occur for various reasons. The fact that the characteristic differences of action of floodplain and watercourse flows results in distortion of results and incorrect assessment of a situation. The watercourse flow that floods a floodplain during the flood period is saturated with the solid phase particles and is thus heterogeneous. Modern methods do not take into consideration the interaction of non-uniform flow with a vegetation cover, which significantly influences the flow formation, as

well as washout processes inside it. This problem is quite significant because the relief of floodplain areas is favorable for the location of a bridge crossing.

2. Literature review and problem statement

Bridge crossings are the most common transport facilities on public roads. Their significance, as noted in paper [1], for technical and functional state and cost estimates is essential in comparison with most road elements. The actual state of such transport facilities is very difficult to assess. This is due to many factors. The simultaneous influence of such processes as the unsustainable movement of water medium and river structures; the existence of significant areas with turns; re-formation of watercourse and floodplain shape in the process of deformation leads to the development of general and local washouts in the area of influence of bridge crossings. In order to study the devastating effects of natural floods as consequences of climate change, paper [2] represents laboratory research into a physical model – the area of river flow with a bridge crossing. Based on the conducted studies, it was concluded on the significant influence of a bridge crossing due to blocking the floodplain flow, corresponding re-formation and an increase in velocities, increased intensity of sediment transfer in the watercourse, at the same time, deformation processes on floodplains were not considered. This is due to the problem of modeling of such flows under laboratory conditions.

Research into the water mode, determining the concentration of suspended alluvium under conditions of extreme climatic phenomena, which significantly affected the hydrological state of a river system are presented in article [3]. But this does not make it possible to disclose fully the mechanism of deformation both in watercourses and in floodplains and, accordingly, to conduct a more objective quantitative assessment of the degree of influence of various factors on this process.

The actual water flow velocity on a floodplain is less than that of a watercourse and, accordingly, of a washout. Soil particles are generally stationary. A washout in this area will begin only if the flow velocity exceeds the flood-free velocity for sediment particles. The features of the relief of floodplains and transition of river flow to a floodplain, coastal washouts, conditions of formation of a floodplain flow are analyzed in paper [4].

Large floodplains have multiple and complex relief formations, the surface level of which changes in different scales. During a flood, floodplain flows are formed and wetland areas are created in lowlands. Paper [4] analyzes the zones of the interaction of a river and a floodplain flow, peculiarities of changes of a floodplain relief due to erosion and alluvium settling in certain areas. But the problem of an impact of hydro-technical facilities or buildings on a change of relief formations of floodplains, especially during a flood or a freshet, was not explored.

The floodplain surface was formed in previous periods. Taking this into consideration, researchers are interested in the throughput capacity of overgrown sites of floodplains. The study of changes in morphological processes, the configuration of a winding watercourse under conditions of overgrown floodplains during flooding based on the two-dimensional model is presented in paper [5]. It should be noted that the proposed solutions are mostly empirical in nature, and

the generalized term that takes into consideration hydraulic resistance of the floodplain surface is determined from the equations of flow motions.

The creation of a digital model of a river taking into account the topographic and morphometric changes enabled the authors of paper [6] to get an artificial landscape of a river system. The proposed morphodynamical and stochastic approaches are based on the generalization of natural quantitative patterns of the landscape changes, but do not make it possible to determine the magnitudes of washout deformations of the watercourse and the floodplain parts of the bridge port.

According to the above [7] results of laboratory research with a movable vegetation layer, deformation development rate, the formation of a floodplain flow and of the relief of floodplains themselves are caused by vegetation distribution, growth and origin. The lack of vegetation contributes to the flow formation with the smallest winding, river vegetation slows down the deformation development on floodplains and promotes watercourse deepening. The vegetation of heterogeneous origin contributes to alluvium settling on one floodplain and to the deformation development on the other one, increases the watercourse winding, and creates conditions for the development of new floodplains and isles.

The study of roughness in floodplain areas, depending on the type and parameters of vegetation, was conducted in paper [8]. A comparison of air and water conditions showed that roughness decreases significantly in flooded areas, due to their hydraulic conditions, vegetation removal and deformation development on floodplains.

The influence of floodplain vegetation on the river shape in the plan and the transportation capacity of a floodplain flow were explored in paper [9] with the help of a 2D morphodynamical model with sub-models for flow resistance and vegetation distribution. The components of resistance to flow are divided into soil resistance and vegetation resistance. The impact of the type, configuration, and density of floodplain vegetation on the transportation and settling of river suspension, development of deformations on floodplains was described in article [10].

No analytical approximation for determining the magnitudes of both general, and local washouts on floodplains, taking into consideration the distribution of velocities within the vegetation layer, namely, in the zone of influence of bridge crossings, was found in the modern scientific literature. This is due to the complexity of modeling of deformation processes on floodplains and the lack of an unambiguous solution to basic equations of hydrodynamics.

3. The aim and objectives of the study

The aim of this study is to develop the method for prediction of deformations on floodplains in the zone of influence of a bridge crossing based on the proposed mathematical model of suspended flows, taking into consideration the existence of vegetation elements.

To achieve the aim of the research, it is necessary to solve the following tasks:

– to analyze and determine the force factors that take into consideration the resistance to the washout of fine-grained soil and resistance during flowing around the vegetation in floodplain areas;

– to develop a mathematical model for a suspended floodplain flow with herbal vegetation, observing consistency with pressure field;

– to formulate initial and boundary conditions to implement the proposed mathematical model of a floodplain flow with vegetation elements;

– to carry out the calculation of deformations in floodplain areas of a plain river in the zone of influence of a bridge crossing by the proposed mathematical model.

4. Mathematical model of a suspended flow with herbal vegetation to predict the distribution of depths and velocities on a floodplain

4. 1. Source equations

The basis for the development of a mathematical model of the motion of a floodplain flow in the zone of influence of a bridge crossing is the equation of actual liquid dynamics in "stresses", or, which is the same thing, the Navier-Stocks equation. To obtain the equations of averaged turbulent motion, Reynolds offered the method of replacement of actual velocities V_i and pressure p with averaged $\overline{V_i}$, $\overline{p}$ and pulse V_i', p_i' [11–14]. These equations differ from the Navier-Stocks equation by the existence of an additional tensor of turbulent or Reynolds stresses $\overline{V_i'' V_j''}$. For practical purposes and the operation convenience, it is appropriate to represent the Reynolds equation in the scalar form through projections onto the axis of Cartesian coordinates, taking into consideration the above, neglecting viscous stresses $\dfrac{\partial \tau_{ij}}{\partial x_j}$:

$$\frac{\partial \overline{V_i}}{\partial t} + \overline{V_j} \frac{\partial \overline{V_i}}{\partial x_j} = -\frac{1}{\overline{\rho_{LS}}} \frac{\partial p}{\partial x_i} - \frac{\partial \overline{V_i' V_j'}}{\partial x_j} - g_i - \sum \overline{f_i}, \qquad (1)$$

$$\frac{\partial \overline{V_j}}{\partial x_j} = 0, \qquad (2)$$

where x_i (i=1, 2, 3) is the coordinates; $\overline{\rho_{LS}}$ is the water density, kg/m^3; g is the free fall acceleration, m/s^2; $\sum f_i$ is the sum of mass forces.

In the expanded form equation (1) and (2) are written down as follows:

$$\frac{\partial \overline{V_1}}{\partial t} + \frac{\partial \overline{V_1^2}}{\partial x_1} + \frac{\partial \overline{V_1 V_2}}{\partial x_2} + \frac{\partial \overline{V_1 V_3}}{\partial x_3} =$$
$$= -\frac{1}{\overline{\rho_{LS}}} \frac{\partial p}{\partial x_1} - \frac{\partial \overline{V_1'^2}}{\partial x_1} - \frac{\partial \overline{V_1' V_2'}}{\partial x_2} - \frac{\partial \overline{V_1' V_3'}}{\partial x_3} - \overline{f_{P1}}, \qquad (3)$$

$$\frac{\partial \overline{V_2}}{\partial t} + \frac{\partial \overline{V_2 V_1}}{\partial x_1} + \frac{\partial \overline{V_2^2}}{\partial x_2} + \frac{\partial \overline{V_2 V_3}}{\partial x_3} =$$
$$= -\frac{1}{\overline{\rho_{LS}}} \frac{\partial p}{\partial x_2} - \frac{\partial \overline{V_2' V_1'}}{\partial x_1} - \frac{\partial \overline{V_2'^2}}{\partial x_2} - \frac{\partial \overline{V_2' V_3'}}{\partial x_3} - \overline{f_{P2}}, \qquad (4)$$

$$\frac{\partial \overline{V_3}}{\partial t} + \frac{\partial \overline{V_3 V_1}}{\partial x_1} + \frac{\partial \overline{V_3 V_2}}{\partial x_2} + \frac{\partial \overline{V_3^2}}{\partial x_3} =$$
$$= -\frac{1}{\overline{\rho_{LS}}} \frac{\partial p}{\partial x_3} - \frac{\partial \overline{V_3' V_1'}}{\partial x_1} - \frac{\partial \overline{V_3' V_2'}}{\partial x_2} - \frac{\partial \overline{V_3'^2}}{\partial x_3} - g - \overline{f_{Ch}} - \overline{f_{P3}}, \qquad (5)$$

$$\frac{\partial \overline{V_1}}{\partial x_1}+\frac{\partial \overline{V_2}}{\partial x_2}+\frac{\partial \overline{V_3}}{\partial x_3}=0, \tag{6}$$

where f_{Ch} is the cohesion force or the force of resistance to soil grain break-off; f_{Pi} is the force of vegetation resistance; g is the projection of the volumetric force onto axis x_3.

The presented system of turbulent motion equations, describing the hydrodynamic field of velocities, pressure in runoffs, is not closed, therefore its integration and numeric implementation of discrete analogs are associated with great difficulties. To solve the applied problems of hydrodynamics, one uses mathematical models of currents, which take into consideration only the basic, determining factors, and as a rule, neglect the action of secondary ones.

4. 2. Determining the force factors of resistance to soil washout and vegetation flow-over in floodplain areas

According to the results of a fairly large number of experimental studies [12], the properties of the bottom of a suspended flow depend on the clogging process. The river suspended flow when influencing the bottom, clogs it over time. Clogging turns the non-bound bottom into the bound bottom as a result of filling the pores between the grains of sand and stones with colloid-like alluvium. As a result of clogging, the irregularities of the bottom are aligned and smoothed, and cohesion of bottom soil grains increases.

Cohesion force is one of the factors, on which depends an increase in resistance to washout of fine-grained soil (with the diameter of less than 0.1–0.15 mm), settled usually on floodplain areas of plain rivers. According to the research conducted in [12], the force of resistance to a particle set break-off can be determined from the following dependence:

$$\overline{F}_{Ch}=\frac{k_{Ct}C_{ch}\cdot \overline{S}g}{C_{sh1}D}, \tag{7}$$

where k_{Ct} is the boundary of fatigue of tearing bound soils, according to experiments [12]; C_{ch} is the soil cohesion in the state of complete water saturation, kg/m²; $\overline{S}$ is the distribution of suspended sediments along the vertical or flow mutability, a formula for determining it is presented below; C_{sh1} is the first coefficient of the sediment particles shape, it is equal to the ratio of a particle volume to the diameter of an equidimensional layer in third degree; D is the diameter of alluvium particles, m.

The surface of the floodplain part is mainly covered with vegetation, different in density and dimensions, which affects the throughput capacity of a floodplain. The strength of resistance to vegetation, according to [13], is determined from the following dependence:

$$\overline{f}_{Pi}=\frac{C_X s_p}{2\overline{V}}\left(\overline{V^2 V_i}+\overline{V_j V_j' V_i'}\right), \tag{8}$$

where

$$V=\sqrt{\sum_{i=1\div 3}\overline{V_i^2}}$$

is the actual velocity; C_X is the resistance factor that is determined by experimental studies, for the case of resistance to vegetation, according to [13]; s_p is the general middle area, distributed around the volume of the layer, determined from the following dependence:

$$s_p=\frac{D_{st}\Delta z_{st.}}{\Delta x_1 \Delta x_2 \Delta x_3}, \tag{9}$$

where D_{st} is the diameter of the plant stem that is flown around, $D_{st}=7\cdot 10^{-3}$ m; Δz_{st} is the part of the plant stem that is flown around, by height, m.

The problems of calculating hydrodynamic fields of velocities and pressures in broken and artificially compressed flows relate to the problems of the shallow water theory since the vertical size (flow depth) is substantially smaller than the horizontal dimensions, such as length and width. This makes it possible to consider the distribution of velocities that are medium in depth in a two-dimensional statement.

4. 3. Development of a mathematical model of a floodplain flow in compliance with consistency with pressure field

We will present the equation of distribution of the velocity structure and the depth of a floodplain flow in two-dimensional approximation. These equations will be obtained from general three-dimensional equations of hydrodynamics (3)–(6) by the integration of the latter vertically from bottom mark z_o to free surface H, that is, along the flow depth $h=H-z_0$.

Using the method and the designations shown in articles [11, 18], the depth-averaged value of any variable of the function – the characteristics of flow $\Phi(x_1, x_2, x_3, t)$ will be designated by broken brackets:

$$\langle \Phi \rangle = \frac{1}{h}\int_{z_0}^{H}\Phi dx_3. \tag{10}$$

When integrating over time and depth the summands containing derivatives of horizontal coordinates, we used the properties of integrals that depend on the parameters [11].

Integrating equations (5), we obtained the law of vertical distribution of averaged hydrostatic pressure that was used when integrating equations (3) and (4).

To do this, we will be restricted to the case of steady motion. Taking into account that we consider the problem of the shallow water theory, that is, $h \ll L$, components, which have the order of derivative magnitudes $\langle \overline{V}_i \rangle$ and their squares and express turbulent stresses, can be neglected. The condition that $\overline{V'_{3H}}=0$. is met on the flow surface. At $\overline{V}\approx \overline{V_1}$ or $\frac{\overline{V_1}}{\overline{V}}\approx 1$, pressure distribution in a flow will be written down in the form of:

$$\overline{p}=\overline{p}_H+\overline{\rho_{LS}}g(H-x_3)-\overline{\rho_{LS}V_3^{2''}}-$$
$$-\frac{C_X s_p}{2}\overline{\rho_{LS}}\langle \overline{V_1'V_3'}\rangle(H-x_3)-\frac{k_{Ct}C_{Ch}\cdot \langle \overline{S}\rangle g}{C_{sh1}D}(H-x_3). \tag{11}$$

The vertical distribution of suspended sediment and flow turbidity are determined, as a rule, from the Routh formula [11]

$$\overline{S}(x_3)=\overline{S}(z)\left(\frac{h-x_3}{x_3}\frac{z}{h-z}\right)^{\frac{W_z}{\kappa U_\tau}},$$

where W_z is the hydraulic velocity, κ is the Karman parameter $\kappa=0.435$, U_τ is the dynamic velocity $U_\tau=\sqrt{ghI_{x_1}}$, I_{x_1} is the inclination of the flow free surface.

To make calculations from this formula, it is necessary to know the bottom content of alluvium $\overline{S}(z)$. However, the magnitude $\overline{S}(z)$ must be added from the rationally constructed theory of motion of drawn alluvium, but such a theory does not exist. A certain step in the elimination of the above drawback of the existing theories of bottom sediments is made in the procedure explored in paper [2], containing the dependences for determining specific discharge of suspended sediments. Such dependence

$$q_{sus} = \int\limits_{\Delta}^{h} \overline{VS} \, dx_3$$

can be represented as $q_{sus} = \langle \overline{VS} \rangle h$, then the dependence for determining average turbidity $\langle \overline{S} \rangle$ will take the following form:

$$\langle \overline{S} \rangle = FS_0, \tag{12}$$

where S_0 is the countdown concentration of bottom sediments determined using the procedure [2]; F is the factor that takes into account the parameter of alluvium suspension, determined using the procedure [2].

Accepting that $\overline{p}_H = \mathrm{const}$, that is, excluding baroclinic currents from consideration, having substituted value $\overline{p}$ from equation (11) and $\overline{f}_{P1}$, $\overline{f}_{P2}$ from (12) in formulas (3) and (4) in the longitudinal and transverse directions, obtain the equations:

$$\frac{\partial \overline{V_1}}{\partial t} + \frac{\partial \overline{V_1^2}}{\partial x_1} + \frac{\partial \overline{V_1 V_2}}{\partial x_2} + \frac{\partial \overline{V_1 V_3}}{\partial x_3} =$$

$$= -g\frac{\partial h}{\partial x_1} - \frac{C_X s_p}{2}\frac{\partial}{\partial x_1}h\langle\overline{V_1'V_3'}\rangle - \frac{k_{Ct}C_{Ch}\cdot g}{C_{sh1}D\rho_{LS}}\frac{\partial}{\partial x_1}h\langle\overline{S}\rangle -$$

$$- \frac{\partial}{\partial x_1}\left(\overline{V_1^{2\prime}} - \overline{V_3^{2\prime}}\right) - \frac{\partial \overline{V_1'V_2'}}{\partial x_2} - \frac{\partial \overline{V_1'V_3'}}{\partial x_3} -$$

$$- \frac{C_X s_p}{2}\left(\overline{V_1^2} + \overline{V_1^{2\prime}} + \frac{\overline{V_2}}{\overline{V_1}}\overline{V_2'V_1'} + \frac{\overline{V_3}}{\overline{V_1}}\overline{V_3'V_1'}\right). \tag{13}$$

$$\frac{\partial \overline{V_2}}{\partial t} + \frac{\partial \overline{V_2 V_1}}{\partial x_1} + \frac{\partial \overline{V_2^2}}{\partial x_2} + \frac{\partial \overline{V_2 V_3}}{\partial x_3} =$$

$$= -g\frac{\partial h}{\partial x_2} - \frac{C_X s_p}{2}\frac{\partial}{\partial x_2}h\langle\overline{V_1'V_3'}\rangle - \frac{k_{Ct}C_{Ch}\cdot g}{C_{sh1}D\rho_{LS}}\frac{\partial}{\partial x_2}h\langle\overline{S}\rangle -$$

$$- \frac{\partial \overline{V_2'V_1'}}{\partial x_1} - \frac{\partial}{\partial x_2}\left(\overline{V_2^{2\prime}} - \overline{V_3^{2\prime}}\right) - \frac{\partial \overline{V_2'V_3'}}{\partial x_3} -$$

$$- \frac{C_X s_p}{2}\left(\overline{V_1 V_2} + \overline{V_1'V_2'} + \frac{\overline{V_2}}{\overline{V_1}}\overline{V_2^{2\prime}} + \frac{\overline{V_3}}{\overline{V_1}}\overline{V_3'V_2'}\right). \tag{14}$$

Multiply equations (13) and (14) by dx_3, integrate over depth and evaluate the order of summands, using the method and assessment given in research [11].

Before integrating nonlinear summands of equations of the $\frac{\partial \overline{V_i^2}}{\partial x_i}\left(\frac{\partial}{\partial x_i}\overline{V_i V_j}; i, j = 1, 2\right)$, form, which express inertial forces, caused by convective acceleration, it is necessary to take into consideration that in an actual flow, there is an uneven vertical distribution of velocities, and as a result of integration over the flow depth, the coefficients, with regard to this non-uniformity, appear in summands. To obtain the values

and physical content of the specified coefficients, represent local velocity V_i in the following form:

$$\overline{V_i} = U_i + u_i, \tag{15}$$

where $u_i = \overline{V_i} - U_i \neq 0$ is the deviation of local velocity $\overline{V_i}$ from the depth-averaged U_i.

In addition, magnitude u_i must meet the condition:

$$\langle u_i \rangle = \frac{1}{h}\int\limits_{z_0}^{H} u_i \, dx_3 = 0. \tag{16}$$

Then, according to [11], the coefficient that takes into account non-uniformity of the vertical distribution of velocities and accepts the values that are larger than unity, that is, $\alpha_h > 1$ is equal to

$$\alpha_h = 1 + \frac{\int\limits_{z_0}^{H} u_i^2 \, dx_3}{U_i^2 h}.$$

Integration of summands that contain derivatives of turbulent stresses by vertical coordinate x_3, will give the following:

$$\int\limits_{z_0}^{H} \frac{\partial \overline{V_i'V_3'}}{\partial x_3}\, dx_3 = \int\limits_{z_0}^{H} d\left(\overline{V_i'V_3'}\right) = \left(\overline{V_i'V_3'}\right)_H - \left(\overline{V_i'V_3'}\right)_{z_0}. \tag{17}$$

That is, the difference of tangents of turbulent stresses that act on a free surface

$$\tau_{iH} = \left(\overline{V_i'V_3'}\right)_H$$

and on the bottom of a flow

$$\tau_{iz_0} = \left(\overline{V_i'V_3'}\right)_{z_0}.$$

After integrating equations of motion (13) and (14), represent two-dimensional equations. In this case, we will neglect the summands of a higher order of smallness in comparison with basic summands $\frac{V^2 h}{L}$ and $\frac{\tau h}{L}$. The order of summands of the force of resistance to vegetation

$$\frac{C_x s_p h}{2}\frac{\partial\langle\overline{V_1''V_3'}\rangle}{\partial x_1} \approx \frac{h\tau}{L}$$

and friction force on the bottom and free surface τ_{iH}, τ_{iz_0} $(i = 1, 2)$ depend on specific conditions of the problem and in the general case can be co-dimensional with the order of basic summands mentioned above. That is why these summands will also exist in final two-dimensional equations of motion. Accept that:

$$N_p = \frac{C_x s_p}{2},$$

$$N_{Ch} = \frac{k_{Ct}C_{Ch}\cdot g}{C_{sh1}D\rho_{LS}}. \tag{18}$$

Two-dimensional equations of motions after dividing by h and taking into account (18) will be written down in the following form:

– equation of the amount of motion (towards axis X_1) –

$$\frac{\partial U_1}{\partial t}+\alpha_h\left[\frac{\partial U_1^2}{\partial x_1}+\frac{\partial U_1 U_2}{\partial x_2}\right]=$$

$$=-\left(g+N_P\left\langle\overline{V_1'V_3'}\right\rangle-N_{Ch}\left\langle\overline{S}\right\rangle\right)\frac{\partial h}{\partial x_1}-$$

$$-h\left(N_P\frac{\partial\left\langle\overline{V_1'V_3'}\right\rangle}{\partial x_1}+N_{Ch}\frac{\partial\left\langle\overline{S}\right\rangle}{\partial x_1}\right)-$$

$$-\frac{\partial}{\partial x_1}\left(\left\langle\overline{V_1^{2\prime}}\right\rangle-\left\langle\overline{V_3^{2\prime}}\right\rangle\right)-\frac{\partial\left\langle\overline{V_1'V_2'}\right\rangle}{\partial x_2}-$$

$$-\frac{1}{h}\left[\left(\overline{V_1'V_3'}\right)_H-\left(\overline{V_1'V_3'}\right)_{z_0}\right]-$$

$$-N_P\left[\alpha_h U_1^2+\left\langle\overline{V_1^{2\prime}}\right\rangle+\frac{U_2}{U_1}\left\langle\overline{V_2'V_1'}\right\rangle\right], \tag{19}$$

– equation of the amount of motion (towards axis X_2) –

$$\frac{\partial U_2}{\partial t}+\alpha_h\left[\frac{\partial U_1 U_2}{\partial x_1}+\frac{\partial U_2^2}{\partial x_2}\right]=$$

$$=-\left(g+N_P\left\langle\overline{V_1'V_3'}\right\rangle+N_{Ch}\left\langle\overline{S}\right\rangle\right)\frac{\partial h}{\partial x_2}-$$

$$-h\left(N_P\frac{\partial\left\langle\overline{V_1'V_3'}\right\rangle}{\partial x_2}+N_{Ch}\frac{\partial\left\langle\overline{S}\right\rangle}{\partial x_2}\right)-$$

$$-\frac{\partial\left\langle\overline{V_2'V_1'}\right\rangle}{\partial x_1}-\frac{\partial}{\partial x_2}\left(\left\langle\overline{V_2^{2\prime}}\right\rangle-\left\langle\overline{V_3^{2\prime}}\right\rangle\right)-$$

$$-\frac{1}{h}\left[\left(\overline{V_2'V_3'}\right)_H-\left(\overline{V_2'V_3'}\right)_{z_0}\right]-$$

$$-N_P\left[\alpha_h U_1 U_2+\left\langle\overline{V_1'V_2'}\right\rangle+\frac{U_2}{U_1}\left\langle\overline{V_2^{2\prime}}\right\rangle\right], \tag{20}$$

– equation of continuity –

$$\frac{\partial H}{\partial t}+\frac{\partial U_1 h}{\partial x_1}+\frac{\partial U_2 h}{\partial x_2}=0. \tag{21}$$

The calculation of steady two-dimensional currents on floodplains with herbal vegetation is carried out in order to predict the distribution of depth-averaged velocities and depths over flow width. The equation of distribution of depth-averaged velocities and depths can be obtained from dynamic two-dimensional equations (19)–(21), if we accept that $U_1=U$, $U_2\approx 0$ in the latter, and use the stationarity condition $\frac{\partial}{\partial t}=0$. As a result of averaging turbulent charac-teristics over flow depth, in two-dimensional equations (19), (20) there are summands $\frac{\partial\left\langle\overline{V_i'V_j'}\right\rangle}{\partial x_i}$ and $\frac{\partial}{\partial x_i}\left(\left\langle\overline{V_i^{2\prime}}\right\rangle-\left\langle\overline{V_3^{2\prime}}\right\rangle\right)$, where ($i=1, 2$), which express the contribution of tangent and normal stresses to the dynamic balance of the pulse of forces in two-dimensional currents.

Unlike equations of three-dimensional turbulent currents, in two-dimensional equations the influence of normal turbulent stresses on the formation of two-dimensional currents is expressed only by the difference of depth-averaged normal stresses in vertical and horizontal directions. Paper [11] presented an analysis of estimation of values of difference of dispersion of vertical pulse component of

velocity $\overline{V_3^{2\prime}}$ and dispersions of longitudinal and transverse components of velocity $\overline{V_1^{2\prime}}$ and $\overline{V_2^{2\prime}}$. This makes it possible to substantiate the assumption about neglecting normal turbulent stresses when considering the problems within two-dimensional idealization of actual currents.

The existence of turbulent stresses in equations leads to the necessity of approximation with the help of certain ratios and models of turbulence of members of turbulent transportation.

In some cases, only an approximated description of turbulence is sufficient. Thus, in problems of large water masses, the value of turbulent viscosity is accepted as constant [11]. More complex models in such problems do not justify themselves because of significant uncertainty in assigning the boundary conditions and errors in numerical solutions.

To calculate two-dimensional currents, it is necessary to express the magnitudes of tangent and normal turbulent stresses, acting between the jets of two-dimensional currents, through velocity U. The values $\left\langle\overline{V_i'V_j'}\right\rangle$, $\left\langle\overline{V_i^{2\prime}}\right\rangle$ will be determined, as accepted in papers [11], based on the Boussinesq hypothesis, and taking turbulent viscosity v_t as proportional to velocity U, according to [11]:

$$\nu_t=2\Lambda U, \tag{22}$$

where Λ is the coefficient of turbulent exchange,

$$\Lambda=\frac{h}{2MC};$$

C is the Shezi factor,

$$C=\frac{Q_{f-p}}{B_{f-p}h_{av}^{3/2}I_{x_1}^{1/2}};$$

M is the function of the Shezi factor, at $10\leq C\leq 60$ $M=0.7C+6$, at $C>60$ – $M=48$; Q_{f-p} is the discharge of floodplain flow; B_{f-p} is the width of floodplain flow; h_{av} is the average depth of a floodplain flow.

Then, based on the hypothesis of Boussinesq, taking into consideration formula (22) for a two-dimensional problem:

$$-\left\langle\overline{V_1'V_2'}\right\rangle=\Lambda\frac{\partial U^2}{\partial x_2},$$

$$-\left\langle\overline{V_1'V_3'}\right\rangle=\Lambda\frac{U^2}{h},$$

$$-\left\langle\overline{V_1^{2\prime}}\right\rangle=2\Lambda\frac{\partial U^2}{\partial x_1}. \tag{23}$$

To calculate tangent stresses on the bottom, the quadratic law of friction [11] was accepted, taking into account $U_1=U$, $U_2\approx 0$:

$$\left(\overline{V_1'V_3'}\right)_{z_0}=\frac{C_f\overline{U^2}}{K_\Phi},$$

$$\left(\overline{V_2'V_3'}\right)_{z_0}=0. \tag{24}$$

where C_f is the empirical friction factor,

$$C_f = \frac{n^2 g}{h^{1/3}};$$

n is the roughness factor.

For watercourses of arbitrary cross-section, it is necessary to introduce the correction coefficient K_{sh}, taking into account the watercourse shape, to the expression of calculating the local value of the bottom friction force. The relevance of the introduction of K_{sh} coefficient is shown in papers [11, 18]. The value of K_{sh} coefficient is determined from the following dependence:

$$K_{sh} = \frac{C^2}{x_2 V_{av}^2} \int_0^{x_2} \frac{U^2}{C_h^2} dx_2, \qquad (25)$$

where V_{av} is the average velocity,

$$V_{av} = Q_{f-p}/B_{f-p},$$

C_h is the Shezi factor by height:

$$C_h = \frac{1}{n} h_{av}^{1/6}. \qquad (26)$$

To predict the distribution of depths and velocities, taking into consideration the dependence for turbulent stresses (23), after the transformations, represent steady equations of two-dimensional currents on floodplains with herbal vegetation:
– of longitudinal equilibrium of a flow –

$$[\alpha_h - N_P 2\Lambda] 2U \frac{\partial U}{\partial x_1} =$$
$$= -\left(g - N_P \Lambda \frac{U^2}{h} + N_{Ch} \langle \overline{S} \rangle \right) \frac{\partial h}{\partial x_1} -$$
$$- h \left(-N_P \frac{\partial}{\partial x_1} \left(\Lambda \frac{U^2}{h} \right) + N_{Ch} \frac{\partial \langle \overline{S} \rangle}{\partial x_1} \right) +$$
$$+ \frac{\partial}{\partial x_2} \left(\Lambda \frac{\partial U^2}{\partial x_2} \right) - \left[\frac{C_f}{h K_{sh}} + N_P \alpha_h \right] U^2, \qquad (27)$$

– of transverse equilibrium of a flow –

$$\left(g - N_P \Lambda \frac{U^2}{h} + N_{Ch} \langle \overline{S} \rangle \right) \frac{\partial h}{\partial x_2} =$$
$$= -h \left(-N_P \frac{\partial}{\partial x_2} \left(\Lambda \frac{U^2}{h} \right) + N_{Ch} \frac{\partial \langle \overline{S} \rangle}{\partial x_2} \right) +$$
$$+ \frac{\partial}{\partial x_1} \left(\Lambda \frac{\partial U^2}{\partial x_2} \right) + N_P \Lambda \frac{\partial U^2}{\partial x_2}, \qquad (28)$$

– of continuity equation –

$$\frac{\partial U h}{\partial x_1} = 0. \qquad (29)$$

To obtain an unambiguous solution to the considered problem, specifically, to determine the flow depth on a floodplain after a washout, it is necessary to add boundary and initial conditions to the presented closed system of original equations.

4. 4. Initial and boundary conditions for the implementation of the proposed mathematical model of a floodplain flow

At the boundaries of the calculation region, in the nodes, adjacent to solid surfaces – dams of a bridge crossing, the dependence for velocity is used as boundary conditions:

$$U_{i,j} = \frac{U_{\tau_{i,j}}}{\kappa} \ln \left((x_2)_c^+ E \right), \qquad (30)$$

where E is the roughness coefficient [14]; $U_{\tau_{i,j}}$ is the dynamic velocity on floodplains; κ is the Carman parameter; $(x_2)_c^+$ is the dimensionless distance from the wall [14], determined from the following dependence:

$$(x_2)_c^+ = \frac{\Delta x_2 U_{\tau_{i,j}}}{\nu}. \qquad (31)$$

The sticking condition, when the value of flow velocity is accepted as equal to zero, is true directly on the most solid surfaces (dam walls, bridge support), as well as for gentle banks.

The maximum mark of free surface H, both for the case of gentle banks and for the case of vertical walls (steep banks, bridge poles, dams), is determined by means of linear extrapolation [11] according to the values of the marks of the free surface of the internal points of the calculation region:

$$h(x_1, x_2) =$$
$$= 2h(x_1, x_2 - 1) - h(x_1, x_2 - 2). \qquad (32)$$

All calculations are carried out accordingly for each branch of flood or freshet for a certain day. Any of the above values $\mathbf{\Phi}_i$ are calculated by the longitudinal coordinate for the entire calculation section (a series of values), $\mathbf{\Phi}_{ij}$ are calculated by two coordinates x_1, x_2 as a numerical matrix.

The marks of flow free surface on all calculated cross-sections are determined in a one-dimensional statement based on the equation of steady uneven motion in open watercourses, based on the solution, presented in paper [2]. The proposed approach to solving the problem is based on the method by Y. V. Abramov using the properties of the central jet of a river flow in the artificial compression area. The finite-difference analog of the differential equation, which describes a change in the flow depth, takes the following form:

$$h_i - h_{i-1} = J(l_i - l_{i-1}) -$$
$$- JR \left[\left(1 - \frac{l_i}{R} \right)^{-1} - \left(1 - \frac{l_{i-1}}{R} \right)^{-1} \right] \pm$$
$$\pm \frac{J h_r^{4/3} m^2}{2g} \left[\left(1 - \frac{l_i}{R} \right)^{-2} - \left(1 - \frac{l_{i-1}}{R} \right)^{-2} \right], \qquad (33)$$

where J is the longitudinal inclination of the bottom; m is the inverse magnitude to the coefficient of watercourse roughness n, $m = \frac{1}{n}$; R is the parameter of the central jet; h_r is the watercourse depth, m; l_i is the longitudinal coordinate of calculation section, m; g is the free fall acceleration, m/s². Initial conditions: $h=h_0$ at $l=0$; $h=h_i$ at $l=l_i$.

4. 5. The sequence of calculation of deformations on floodplains of a plain river in the area of influence of a bridge crossing

The proposed system of equations (27) to (29) in conjunction with boundary conditions (30) to (33) is solved in the following sequence:

1. Source data for a floodplain flow, if necessary, their calculation, constants, boundary conditions, are entered. The data are shown in Table 1. In order to reduce the source information volume, we introduce interpolation polynomials, which make it possible to shorten it by 2–3 times.

2. The parameters of the calculation region, determined by the area of influence of a bridge crossing, are determined. Substantiation of the boundaries of compression and spreading zones, plotting a curve of free surface on bridge crossings, the transformation of riverbed discharge at bridge crossings are performed using the calculation methods proposed in research [15].

The values of the curve of free surface of a river flow in the area of influence of a bridge crossing are calculated according to the equation (33). The calculations are performed from the bottom up, starting with the section of complete spreading, where all hydraulic flow characteristics are known, because they have natural values, the third summand of the equation (33) is taken with the "+" sign. In this case, $l_{i-1}=0$ and l_i varies from 0 to l_{z_s} in the spreading zone. The magnitude of the parameter of the central jet in the spreading zone is $R=R_{z\_s}$. The magnitude of the sub-bridge support corresponds to value $l_i=l_{z\_s}$.

Having reached the bridge section, in order to continue calculation in the compression zone, it is necessary to replace the sign before the third summand with "–", give the value of the resulting depth under a bridge to magnitude h_{i-1}, to accept the distance $l_{i-1}=0$ and to change the magnitude of the central jet parameter in the compression area $R=R_{z\_c}$. Value l_i varies from 0 to l_{z_c} of the magnitude that is the distance from a bridge to the beginning of a compression zone, where full support under a bridge is formed.

Based on equation (33), calculate the inclination of a river flow free surface:

$$I_{x_1} = \frac{h_i - h_{i-1}}{l_i - l_{i-1}} =$$

$$= J - \frac{JR}{l_i - l_{i-1}}\left[\left(1-\frac{l_i}{R}\right)^{-1} - \left(1-\frac{l_{i-1}}{R}\right)^{-1}\right] \pm$$

$$\pm \frac{J h_r^{4/3} n^{-2}}{2g(l_i - l_{i-1})}\left[\left(1-\frac{l_i}{R}\right)^{-2} - \left(1-\frac{l_{i-1}}{R}\right)^{-2}\right]. \quad (34)$$

3. After obtaining parameters of the calculation zone in the first approximation, the calculation based on the specified source data is carried out: two-dimensional matrices of the marks of the bottom and free surface of a river flow. These matrices describe the configuration of the bottom and the free surface in complete accordance with the real conditions. The distribution of velocities and depths in estimated sections in floodplain areas is calculated by finite-difference analogs of equations (27), (28).

Using iterations, longitudinal velocity in a steady floodplain flow with vegetation elements is determined.

On floodplain flows, it is possible to accept $\frac{\partial h}{\partial x_1}=0$ and carry out calculations according to simplified equations:

$$U_{i+1,j} = U_{i,j} - \frac{\Delta x_1}{[\alpha_h - N_P 2\Lambda]2U_{i,j}} \times$$

$$\times \left[h_{i,j}\left[\begin{array}{c} -\frac{N_P}{\Delta x_1}\left[\left(\Lambda\frac{U^2}{h}\right)_{i+1,j} - \left(\Lambda\frac{U^2}{h}\right)_{i,j}\right] + \\ +N_{Ch}\frac{\langle S\rangle_{i+1,j} - \langle S\rangle_{i,j}}{\Delta x_1} \end{array}\right] - \right.$$
$$\left. -\frac{1}{\Delta x_2^2}\left[\begin{array}{c}\Lambda_{i,j}\left(U^2\right)_{i,j+1} - \left(\Lambda_{i,j}+\Lambda_{i,j-1}\right)\times \\ \times\left(U^2\right)_{i,j} + \Lambda_{i,j-1}\left(U^2\right)_{i,j-1}\end{array}\right] + \right.$$
$$\left. +\left[\frac{C_f}{h_{i,j}K_{sh}} + \alpha_h N_P\right]\left(U^2\right)_{i,j} \right].$$

The depth distribution in transverse cross-sections of a floodplain flow is determined:

$$h_{i,j+1} = h_{i,j} - \frac{\Delta x_2}{g - N_P\left(\Lambda\frac{U^2}{h}\right)_{i,j} + N_{Ch}\langle S\rangle_{i,j}} \times$$

$$\times \left[h_{i,j}\left(\begin{array}{c} -\frac{N_P}{\Delta x_2}\left[\left(\Lambda\frac{U^2}{h}\right)_{i,j+1} - \left(\Lambda\frac{U^2}{h}\right)_{i,j}\right] + \\ +N_{Ch}\frac{\langle S\rangle_{i,j+1} - \langle S\rangle_{i,j}}{\Delta x_2} \end{array}\right) - \right.$$
$$\left. -\frac{1}{2\Delta x_1 \Delta x_2}\left[\begin{array}{c}\Lambda_{i+1,j}\left(\left(U^2\right)_{i+1,j+1} - \left(U^2\right)_{i+1,j-1}\right) - \\ -\Lambda_{i,j}\left(\left(U^2\right)_{i,j+1} - \left(U^2\right)_{i,j-1}\right)\end{array}\right] - \right.$$
$$\left. -N_P\Lambda_{i,j}\frac{\left(U^2\right)_{i,j+1} - \left(U^2\right)_{i,j}}{\Delta x_2} \right]. \quad (35)$$

Boundary conditions near the jet-directed dams are calculated from dependence (32).

Depths after washout are determined from the following ratio:

$$h_{i,j}^{n+1} = \frac{h_{i,j}^n U_{i,j}}{U_{fl-fi,j}}, \quad (36)$$

where $U_{fl-fi,j}$ is the flood-free soil velocity.

The continuity condition is verified according to (29). During verification of the continuity condition, specific flow discharge on verticals $q_{f-pij} = U_{i,j}\cdot h_{ij}$ is calculated. After that, complete discharge is calculated by summing specific discharges for each transverse cross-section and multiplying by the width of the calculation band

$$Q_{f-pi} = \sum_j q_{f-pij}\cdot \Delta B,$$

where $\Delta B = x_{2i} - x_{21}$. Coefficients

$$k_i = \frac{Q_{f-pi}}{Q_{f-p0}},$$

where Q_{f-p0} is the discharge in the initial section, are calculated. Velocities

$$U_{i,j}^{II} = \frac{U_{i,j}^{I}}{k_i},$$

where I, II are the numbers of corresponding approximations, are adjusted. In calculations, velocities are adjusted twice. It is necessary to verify the continuity condition, calculating specific discharges

$$q_{f-pij} = U_{i,j} \cdot h_{ij}.$$

Velocities

$$U_{i,j}^{II} = \frac{U_{i,j}^{I}}{k_i}$$

are calculated again. Specific discharges are calculated by corrected velocities, the coefficients that must be equal to unity are found again. If necessary, it is possible to increase the number of iteration cycles for verification of arrays of magnitudes. The final verification is performed by the discrete analog of equation (29):

$$(U \cdot h)_{i,j} = \frac{(U \cdot h)_{i+1,j}}{\Delta x_1}. \tag{37}$$

The longitudinal inclination of the water surface of a floodplain is re-calculated:

$$I_{x_1} = \frac{h_{i+1,j} - h_{i,j}}{\Delta x_1}. \tag{38}$$

Then we recalculate in the following approximation the magnitudes of discharge of floodplain flows in the compression and spreading area; coefficient, which takes into consideration uneven vertical distribution of velocity; coefficient of turbulent exchange; coefficient, which takes into consideration resistance of vegetation elements on the floodplain; the averaged turbidity over vertical $\langle \overline{S} \rangle$. Longitudinal velocity in a steady floodplain flow with vegetation elements and depth distribution in transverse cross-sections of a floodplain flow are determined by iterations until the continuity condition is met. The matrix of marks of the surface of floodplain bottom relief on the calculation region Z_{f-p} and the matrix of depths of floodplain flow h_{f-p} are updated after each step of high waters.

5. Results of calculation of washouts on floodplains in the area of influence of a bridge crossing

Calculation of general and maximal washout was carried out on the floodplain sections of the sub-bridge watercourse on the River Seversky Donets near the village of Brusivka on motor road T-05-14, km 83+575, Donetsk oblast (Ukraine). The floodplain is mainly 2-sided, alternating on the banks. The soils are silt-sandy and clay, peat on watermarked areas. Every year during spring flood and rain freshets, the floodplain is flooded to the depth from 0.5 m to 3.3 m for 1–2 weeks, the water remains in lowlands for 1–3 months. The designed motor road bridge crosses the river Siversky Donets at the angle of 26° and has a length of 297 m.

5. 1. Calculation of the position of a curve of the free surface of river flow in the area of influence of a bridge crossing

To carry out the calculation, we collect the source data, which are shown in Tables 1, 2. Then the parameters of the zone of influence of a bridge crossing were determined: the length of the compression zone and of the spreading zone.

Table 1

Source data of calculation

No by order	Parameter	Designation	Measurement unit	Notes (in what form it is assigned)
1	Longitudinal inclination of the bottom	J	–	0.00018
2	Average mark of watercourse bottom	Z_r	M	143.7
3	Average mark of floodplain bottom	Z_{f-p}	M	146.64
4	Free fall acceleration	g	m/s²	9.81
5	Width of river spreading	B_{riv}	m	466.3
6	Average width of watercourse	B_r	m	58.11
7	Average width of a larger floodplain	b_{f-pb}	m	212.26
8	Average width of a smaller floodplain	b_{f-ps}	m	149.08
9	Coefficient of roughness on floodplains	n	–	0.067
10	Width of a bridge crossing port	B_{br}	m	267

Table 2

Source parameters of calculation region

Day	Total discharge of river Q_{riv}, m³	Water-course discharge Q_r, m³	Level of river surface, H, m	Water-course depth, h_r, m	Floodplain depth h_{f-p}, m
1.2	448	207.65	147.24	3.54	0.6
4	506	215	147.5	3.8	0.86
8.4	864	293.32	147.98	4.28	1.34
13.6	1193	368.08	148.27	4.57	1.63
18.8	1658	465.03	148.62	4.92	1.98
24.6	1992	534.73	148.83	5.13	2.19

Fig. 1, 2 show in first approximation the depth in the area of influence of a bridge crossing and inclination of the free surface of river flow. At steps 10 and 11, it is possible to see clearly a change in depth and inclination in the bridge crossing section.

Table 3

Calculated parameters of the estimation region

| Day	Natural specific discharge of floodplains, $q_{f\text{-}p}$, m	Compression coefficient β_{br}	Length of influence of bridge crossing R_{z_br}, m	Froude number F_r	Value of complete support, ξ	Length of compression zone l_{z_c}, m	Length of spreading zone l_{z_s}, m	Parameters of central jet in spreading zone R_{z_s}, m	Parameters of central jet in compression zone R_{z_c}, m
1.2	0.589	1.355	1,194.8	0.01061	0.794	245.7	949.1	3,621.6	937.5
4	0.713	1.391	1,349.2	0.0095	0.823	239.5	1,109.8	3,950.5	852.4
8.4	1.398	1.476	1,673.6	0.01578	0.7232	464.4	1,209.3	3,748.1	1,439.3
13.6	2.021	1.51	1,842.2	0.0224	0.613	713.7	1,128.6	3,341.3	2,112.9
18.8	2.923	1.542	2,035.0	0.0314	0.464	1,090.3	944.7	2,687.9	3,102.1
24.6	3.57	1.556	2,144.3	0.0379	0.355	1,382.0	762.3	2,133.2	3,867.5
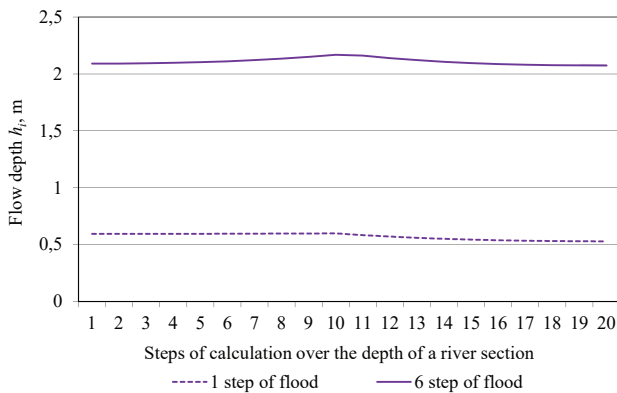
Fig. 1. Calculation of flow depths h_i (1−10 steps − compression zone, 11−20 steps − flow spreading zone)
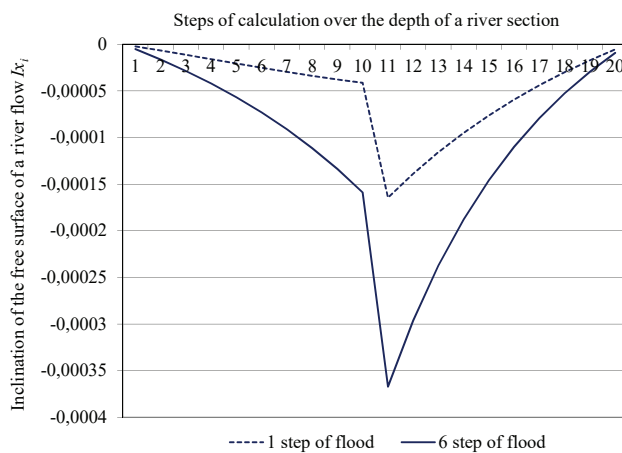

Fig. 2. Calculation of inclination of the free surface of a river flow Ix_i (1−10 steps − compression zone, 11−20 steps − flow spreading zone)

The obtained initial conditions over the length of the estimated section (Fig. 1, 2) in conjunction with boundary conditions (30) to (33) make it possible to solve the proposed system of equations (27) to (29).

5. 2. Calculation of deformations on floodplains in the zone of influence of a bridge crossing

Based on the determined initial conditions (Fig. 1, 2) and initial marks of the surface and river flow depths (Fig. 3, 4), the calculation was carried out on the section of the river

Siversky Donets of the length of 220 m, of the width of 540 m. The solution was obtained based on the source data given in Table 4.

We determined the hydrodynamic parameters of floodplain flows from average velocity (Fig. 5), to meet continuity conditions, to calculation of depths in transverse cross-sections (Fig. 6) and marks of the bottom area after the flood at the calculation steps (Fig. 7).
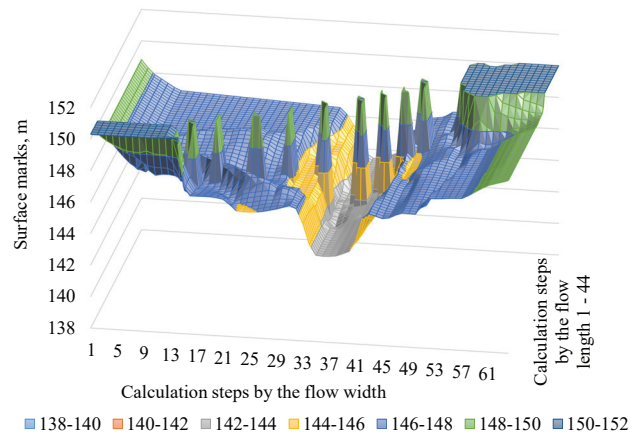

Fig. 3. Initial marks of surface $Z_{f\text{-}p}$ of relief of watercourse flow bottom and floodplains on the calculation region before a flood

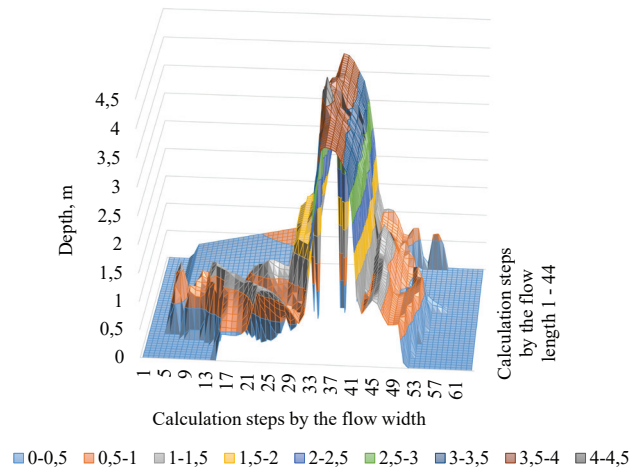

Fig. 4. Initial depth of river flow on the calculation region

Source data for calculation of the distribution of longitudinal velocities and depths of the floodplain flow

| No. of entry	Parameter	Designation	Measurement unit	Notes (in what form it is assigned)
1	Coefficient of kinematic viscosity of water (depending on water temperature)	ν	m^2/s	$115 \cdot 10^{-8}$
2	Density of river flow water	ρ_{LS}	kg/m^3	998
3	Carman parameter	κ	–	0.435
4	Average or medial diameter of soil fraction	d_m	mm	0.41
5	Density of bottom soil	ρ_s	kg/m^3	2,620
6	Hydraulic velocity	W_z	m/s	0.185 (determined through calculation)
7	Cohesion of soil in the state of complete water saturation	C_{Ch}	kg/m^2	0.19
8	Function of Shezi factor	M	–	at $10 \le C \le 60$ $M=0,7C+6$, at $C>60 - M=48$
9	Coefficient to determine cohesion force	k_{Ch}	–	0.035
10	First coefficient of a form of particle of alluvium	$C_{\Phi 1}$	–	$\pi/6$
11	Coefficient of resistance to vegetation	C_X	–	0.4
12	Height of vegetation	$h_{veg.}$	m	1.5
13	Diameter of the plant stem, that is flown around	$D_{st.}$	m	$7 \cdot 10^{-3}$
14	Part of the plant stem that is flown around vertically	$\Delta z_{st.}$	M	if $h_{veg}>h_{f-p}$, then $\Delta z_{st.}=h_{f-p}$, if $h_{veg}<h_{f-p}$, then $\Delta z_{st.}=h_{veg.}$
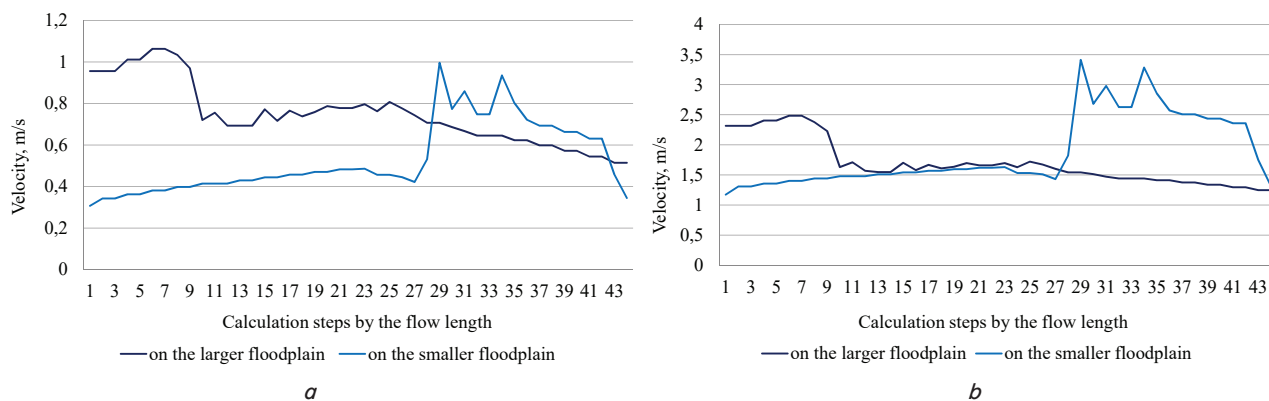
Fig. 5. Calculation of average velocity of floodplain flow $V_{f-pb(s)av}$:
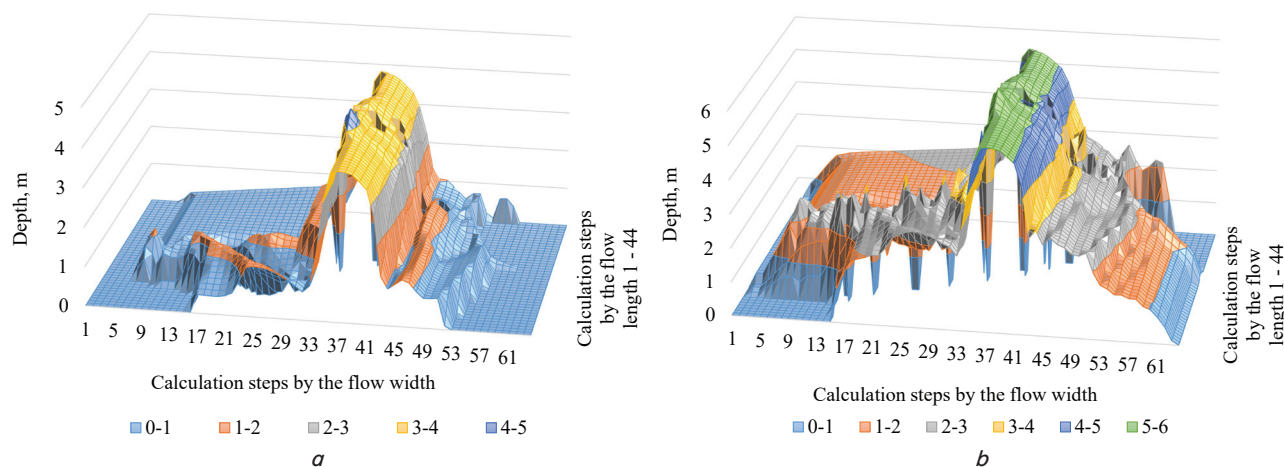a — after step 1 of the estimated flood; b — after step 6 of the estimated flood

Fig. 6. Calculation of depth in transverse cross-sections of floodplain flow $h_{i, j}$:
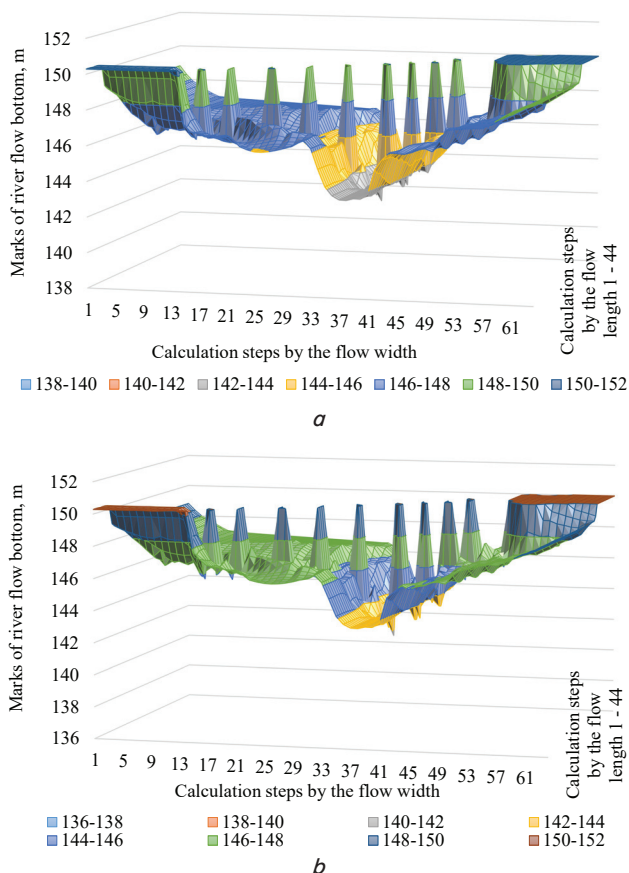a — after step 1 of the estimated flood; b — after step 6 of the estimated flood

Fig. 7. Marks of the surface of floodplain bottom relief:
a — after step 1 of the estimated flood;
b — after step 6 of the estimated flood

The washout of the watercourse of the river Seversky Donets stops only at the recession of a spring flood after the 6th calculation step by the source hydrograph. According to the calculations carried out using this method, when compared with the initial marks, the washout of the larger floodplain is h_p=0.96 m, the washout of the smaller floodplain is h_p=1.28 m.

6. Discussion of results of calculation of deformations on floodplains in the area of bridge crossing influence

The mathematical model of the floodplain flow is based on general equations for the transfer of a non-homogeneous flow and continuity equation (3) to (6). The model form of equations was obtained based on the assumption of the smallness of particles and smallness of flow acceleration in comparison with gravity acceleration. Transfer equations (27), (28) take into account the law of distribution of averaged vertical hydrostatic pressure (11), which gives the following advantages: a direct dependence between pressure and velocity, the existence of flow depth. The developed equations of a floodplain flow take into consideration in their composition the distribution of suspended alluvium due to average turbidity, the cohesion force of fine-grained soils and the force of resistance to vegetation (18).

To solve specific applied tasks and obtain, consequently, an unambiguous solution, the initial and boundary conditions throughout the entire numerical region (30) and (32) to (34) were substantiated and stated. The depth of a flood-

plain flow after a washout is determined based on the ratios of actual and flood-free velocities (36).

The proposed approach makes it possible to determine the parameters of the zone of influence of a bridge crossing in a two-dimensional statement (Fig. 1, 2), and to meet continuity conditions, to calculate average velocity on each floodplain. Determined initial and boundary conditions are the basis for the realization of finite-difference analogs of transfer equations (34), (35). In accordance with this, the data arrays of depths in transverse cross-sections of a flow (Fig. 6) and the marks of the surface of floodplain relief (Fig. 7) were obtained, which makes it possible to provide an accurate forecast of the development of deformation processes.

The developed mathematical model makes it possible to take into consideration the peculiarities of floodplain areas with regard to vegetation and alluvium distribution, it enables determining the actual distribution of velocity structure in the layer of plants of heterogeneous flow and deformations on floodplains.

The proposed approaches [7–10] to studying the processes of floodplain flow formation correspond to their natural state, rather than the artificially compressed state. The presented results of the numerical experiment allow studying the development of deformations on floodplains, in particular, in the zone of influence of a bridge crossing taking into consideration peculiarities of relief and interaction with the watercourse flow on actual objects of transport facilities.

The next step in the authors' research is to determine the parameters of maximum washout, taking into consideration in the model equation the resistance force at flowing around the bridge poles. This will make it possible to provide a complete forecast of deformation development on floodplains in the area of influence of a bridge crossing when designing road facilities and structures on plain and foothill sections of rivers.

7. Conclusions

1. In order to take into consideration the peculiarities of the formation of floodplain flows, we used an approach, according to which the vertical distribution of velocity fields is determined concerning distributed force acting in the vegetation layer. A significant influence of resistance of break-off of fine-grained soil grains on dynamics of s floodplain flow was taken into consideration. Appropriate dependences were obtained for the constituent elements of resistance to vegetation and soil cohesion.

2. The geomorphometry of floodplain sections is characterized by the existence of considerable vegetation, alluvium from previous floods and freshets and causes genetic dissimilarity with watercourses. The flowing over vegetation by a flow in floodplain areas of bridge crossings creates specific conditions for the transfer of both convective, and turbulent characteristics of a current. To take into consideration these conditions, the two-dimensional mathematical model of the suspended flow with grass vegetation for forecasting the distribution of depths and velocities on a floodplain was proposed.

The proposed mathematical model makes it possible to obtain the distribution of depth-averaged velocities in compliance with the pressure field in a two-dimensional statement. This greatly simplifies its implementation and meets

the conditions of problems of the shallow water theory for artificially compressed open flows.

3. The formulated and calculated initial and boundary conditions for the river Siversky Donets provide an opportunity to obtain parameters of the zone of influence of a bridge crossing and depth distribution in the first approximation and a change of a free surface inclination. The consideration of characteristic features of floodplain flow motion makes it possible to carry out the numerical implementation of the proposed mathematical model by iterative methods of calculation of hydrodynamic characteristics on the determined length of a river flow in compliance with continuity conditions. At the beginning of a spring flood, after the first, according to a source hydrographer, estimated step in first approximation, the average depth of a river flow varies in the range from 0.598 to 0.529 m. During flood recession, after the 6th step, it ranged from 2.16 m to 2.09 m.

4. The proposed method for calculation of washouts on floodplains is based on specific features of open artificially compressed flows in the area of influence of a bridge crossing. This method makes it possible to take into consideration

a significant influence of resistance from vegetation elements and resistance of soil grains break-off during a washout on the dynamics of a floodplain flow. The depth of the washout of the floodplain area was determined based on the ratio of actual and flood-free velocities.

Calculation after each step of the estimated flood was performed based on clarified source data, which describe the bottom and free surface configuration in full accordance with actual conditions. The determined hydrodynamic indicators of floodplain flows of the river Siversky Donets during flood recession vary in quite a wide range. The maximum average velocity on a larger floodplain is 2.68 m/s, at a smaller floodplain – 3.41 m/s. The maximum depth of water flow in transverse cross-sections, both on the larger, and the smaller floodplains can reach up to 3 m, and the difference of maximum values of washouts of the bottom region is 0.32 m. A higher value of the washout magnitude after performing the calculations on the river Siversky Donets was determined on a smaller floodplain that has a lower throughput capacity and more compression, which meets actual conditions.

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