

This paper reports a technique for constructing a geometric shape of the surface of contact between the interacting conjugated machine elements using computer technology. A subprogram has been developed in the MATLAB software package.

The comprehensive solution to such problems is a certain scientific challenge and is of great importance when designing the kinematic pairs in mechanical engineering. The main systemic drawback in the construction of complex mechanisms is that the design process does not take into consideration the geometric characteristics of the contact spatial engagement surface in the screw kinematic pairs. As a result, during the manufacture of a kinematic pair, conventionally designed structural elements demonstrate defects that shorten their lifespan. Solving the set task could reduce the time to design tooling, cutting tools, would ensure the required estimation and graphic accuracy, as well as improve the efficiency of the manufacture of parts.

The study of existing procedures for designing screw conjugated surfaces has made it possible to note their unsatisfactory compliance with modern design requirements. Therefore, the manufacture of a kinematic pair that provides for technological accuracy implies the assignment of the curvilinear shapes for a contact spatial engagement surface under the predefined conditions.

The proposed geometric technique for determining the shape of a contact spatial surface of the kinematic pairs of tooling and cutting tools could make it possible to design and manufacture components and mechanisms with the required accuracy

Keywords: kinematic method, geometric shape, cutting tool, conjugated surfaces, contact spatial surface, contact shape, tooling, geometric parameters, screw design, circular design

DEVELOPMENT OF A TECHNIQUE FOR THE GEOMETRICAL MODELING OF CONJUGATED SURFACES WHEN DETERMINING THE GEOMETRICAL PARAMETERS OF AN ENGAGEMENT SURFACE CONTACT IN KINEMATIC PAIRS

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1. Introduction

In the modern stage of building machines with complex structural elements, the methods of descriptive geometry are widely used in design. One of the common methods of geometric object formation is geometric modeling, which makes it possible to obtain the desired geometry of an article and the required conjugation of surfaces at the design stage.

The emergence of new machine-building technologies for machining the articles in the automated industries poses new challenges for manufacturers. In turn, the accuracy of production depends significantly on the accuracy of the design, which led to the need to devise a fundamentally new designing technique, based on the use of the kinematic method.

Therefore, those studies are relevant that address the practical application of the theory for determining a contact surface in conjugated pairs, to improve the efficiency of design

and fabrication operations. In particular, it is interesting to identify the prospects for further possibilities of this theory, combined with a variety of software developed to solve complex industrial tasks. This necessitates undertaking a deeper study into the multifaceted properties of the contact surface determination method as a unique tool for solving present and future technical tasks related to tooling and cutting tools.

2. Literature review and problem statement

The basic provisions and requirements for spatial tooling are stated in work [1]. It is noted that for tooling to operate normally it must have a common tangent plane and a common normal. It is difficult to achieve practical implementation of these requirements during design due to the need to build a large number of axoids. Paper [2] explores the

tasks of forming a discretely-represented engagement curve on special points within the engagement area; however, issues related to the continuity of contact between the conjugated surfaces remain to be fully resolved.

Currently, the kinematic method in combination with computer technology is successfully used in the design of conjugated surfaces, reported in a series of studies. This approach is applied in work [3] but the quasi-statistical design error at high-speed processing becomes the main reason affecting the machining accuracy of articles with complex surfaces, which is a significant factor in the operation of kinematic pairs. An option to overcome the specified problem is proposed in studies [4, 5]. The same approach was employed in work [6]. Reducing, and ideally eliminating, vibration in the machining process avoids the subsequent unstable working conditions. However, these studies are ineffective at the «manual» technique of achieving geometric parameters.

Paper [7] explores those issues that arise in the shape formation of complex conical screw surfaces of machines' work elements. The paper proposes an automated design system that implements a formalized calculation of the tool's trajectory in accordance with the suggested special strategy for screw surface machining. As the raw data, the system needs information about the geometry of the instrument and the screw surface in a parametric form to calculate the tool trajectory. Issues related to the definition of characteristics in the construction of a three-dimensional model of surfaces remained unresolved. A different approach to developing conservative intersections of curves with the boundaries of conjugated surfaces was used in work [8], which is more acceptable. A computer program to determine the curved surface of contacting elements with a fixed point was reviewed and described. Although it is possible to accidentally receive false triggering during the tests on the intersection between normals, the author was able to determine the actual intersection of spatial curves. The method developed and the geometric and computer modeling algorithms could be an option to overcome the emerging difficulties for the formation of a contact surface on CNC machines [9]. Researchers have identified intermediate machine adjustments that ensure milling stability, depending on the conditions and parameters of the process. This approach was applied in works [10, 11]; but the use of a deliberate modification of contact surfaces reduces the sensitivity of machined articles, specifically, worm gears, and leads to errors in the manufacture of their elements. This fact is highlighted in paper [12].

The issues of determining the required parameters of the geometric shape of the contact spatial surface in conjugated kinematic pairs remain unresolved. All this suggests that research into a new technique for the geometrical determination of the shape of a contact surface in toothing involving computer technology is appropriate.

3. The aim and objectives of the study

The aim of this study is to devise a new technique for the geometric modeling of conjugated surfaces, which would make it possible to determine at the design stage the shape of the spatial surface of an engagement contact.

To accomplish the aim, the following tasks have been set:

- to determine the geometric shape of a spatial contact screw surface;
- to investigate the margin of error in the construction of a contact screw surface.

4. Materials and methods to determine the geometric shape of a spatial contact screw surface in the conjugated kinematic pairs

When designing the shapes of conjugated surfaces, we consider geometric modeling as a base for creating computer simulation techniques.

The type of contact surface interacting with two preset conjugated surfaces could predict manufacturing defects at the design stage. We suggest a technique to design the kinematic pairs of the required shape and precision of manufacture using a spatial contact surface. The geometric determination of the shape of the spatial contact surface with a curved transformation includes the circular and screw methods of converting the original curved surface along the curved axis.

5. The essence of the proposed technique for geometric modeling of conjugated surfaces

Let us determine the geometric shape of the contact screw surface in two conjugated curved surfaces. The original curved surface is set by the system of equations (1) and rotated at 30° around its axis (Fig. 1), where

$$0 \leq \omega \leq 7 \text{ mm}, \quad 0 \leq \vartheta \leq \pi, \quad a = 5 \text{ mm}, \quad b = 2 \text{ mm},$$

$$x_0 = -20 \text{ mm}, \quad y_0 = 15 \text{ mm}, \quad z_0 = -10 \text{ mm},$$

$$\begin{cases} x = x_0 + a * \omega * \cos \vartheta, \\ y = y_0 + b * \omega * \sin \vartheta, \\ z = z_0 + 0.5 * \omega^2. \end{cases} \quad (1)$$

To determine the line of intersection of two curved conjugated surfaces, let us assume that curves in space are set by a parametric equation whose solution yields a set of coordinates X, Y, and Z. We shall define the structure of the X, Y, Z coordinates using an example of a curved line set by parametric expression (2).

$$\begin{cases} x = t * \sin t, \\ y = t * \cos t, \\ z = \frac{H}{2\pi} * t, \end{cases} \quad (2)$$

where $0 \leq t \leq 2\pi$.

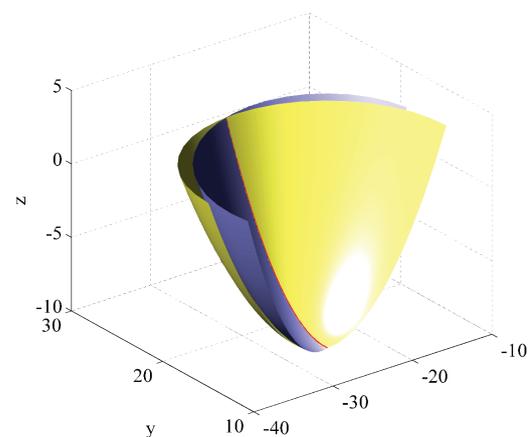


Fig. 1. The original surface and the surface rotated at 30° relative to its axis

Each coordinate X, Y, Z is a one-dimensional array of values; we determine, from the arrays of coordinates of the original and rotated curved surfaces, the coordinates of the surface, which are equal to the coordinates of the original and rotated surfaces.

When rotated around a straight line (the coordinate axis, the surface's rotation axis), the ϑ parameter is constant for any value of ω . The ϑ parameter value is to be determined from the equality of the values of the X coordinates of the original and rotated surfaces.

$$x_0 + a \cdot \omega \cdot \cos \vartheta = (x_0 + a \cdot \omega \cdot \cos \vartheta - x_0) \cos \Omega - (y_0 + b \cdot \omega \cdot \sin \vartheta - y_0) \cdot \sin \Omega + x_0,$$

$$a \cdot \omega \cdot \cos \vartheta = a \cdot \omega \cdot \cos \vartheta \cos \Omega - b \cdot \omega \cdot \sin \vartheta \sin \Omega,$$

$$a \cdot \cos \vartheta = a \cdot \cos \vartheta \cos \Omega - b \cdot \sin \vartheta \sin \Omega,$$

$$1 = \cos \Omega - \frac{b}{a} \operatorname{tg} \vartheta \cdot \sin \Omega,$$

$$\frac{b}{a} \operatorname{tg} \vartheta \cdot \sin \Omega = \cos \Omega - 1,$$

$$\frac{b}{a} \operatorname{tg} \vartheta = \frac{\cos \Omega - 1}{\sin \Omega},$$

$$\operatorname{tg} \vartheta = \frac{a(\cos \Omega - 1)}{b \cdot \sin \Omega}, \quad \vartheta = \arctg \frac{a(\cos \Omega - 1)}{b \cdot \sin \Omega}.$$

Determine the value of the ϑ parameter from the equality of the values of the Y coordinates of the original and rotated surfaces.

$$b \cdot \sin \vartheta = a \cdot \cos \vartheta \sin \Omega + b \cdot \sin \vartheta \cos \Omega,$$

$$1 = \cos \Omega + \frac{a}{b} \operatorname{ctg} \vartheta \cdot \sin \Omega,$$

$$\frac{a}{b} \operatorname{ctg} \vartheta \cdot \sin \Omega = 1 - \cos \Omega,$$

$$\frac{a}{b} \operatorname{ctg} \vartheta = \frac{1 - \cos \Omega}{\sin \Omega},$$

$$\operatorname{ctg} \vartheta = \frac{b(1 - \cos \Omega)}{a \cdot \sin \Omega}, \quad \vartheta = \operatorname{arcc} \operatorname{tg} \frac{b(1 - \cos \Omega)}{a \cdot \sin \Omega}.$$

The resulting parameter ϑ is to be fitted to the equation of the original surface, or the equation of the rotated surface; we shall derive the coordinates of the contact line of these two surfaces.

No. 1. The region is assigned for determining the ϑ parameter – the original surface within $0 \leq \vartheta \leq \pi$, and the contact line built does not match the actual line of intersection of surfaces (Fig. 2).

No. 2. The ϑ parameter of the original surface is within $0 \leq \vartheta \leq \pi$, the line built does not match the actual line of contact of the surfaces, but when the X parameter is mirrored relative to the XZ -plane passing through the surface rotation axis, the contact line coincides with the actual line of intersection of surfaces (Fig. 3).

No. 3. Based on calculation No. 2, but the X value calculation formula is changed (in the formula, sign «+» changed to sign «-»). The contact line coincided with the actual line of intersection of surfaces (Fig. 4).

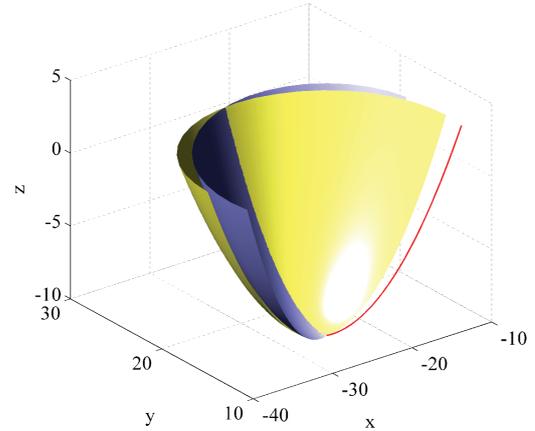


Fig. 2. The contact line does not match the actual line of intersection of surfaces

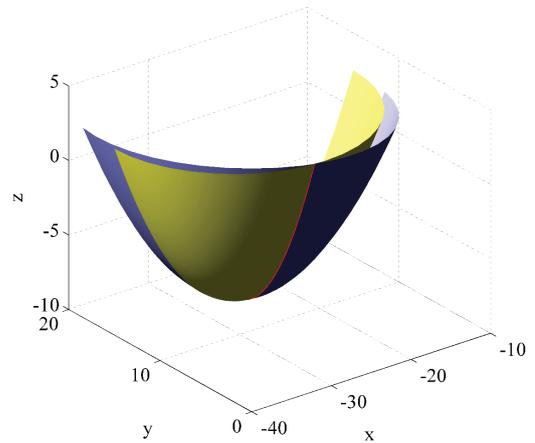


Fig. 3. The contact line matches the actual line of intersection of surfaces

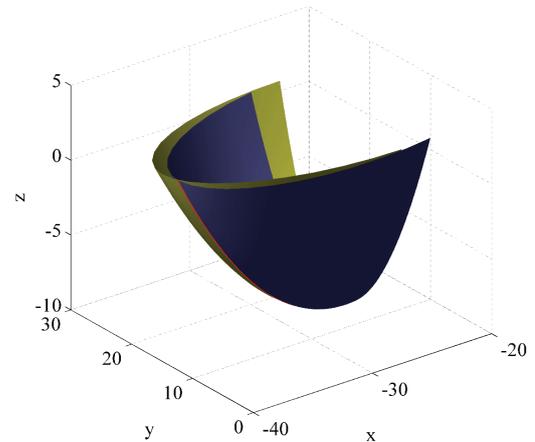


Fig. 4. The contact line coincided with the actual line of intersection of surfaces

Taking into consideration the above results, we shall examine the permissible error in the construction of the geometric shape of the contact engagement surface in the curved conjugated pairs; we shall use the geometric methods of circular and screw transformation. The original surface of the paraboloid is set (Fig. 5) with parametric expression (3).

$$\begin{cases} x = a \cdot \omega \cdot \cos \vartheta, \\ y = b \cdot \omega \cdot \sin \vartheta, \\ z = 0.5 \cdot \omega^2, \end{cases} \quad (3)$$

where $0 \leq \omega \leq 7$ mm, $0 \leq \vartheta \leq \pi$, $a = 5$ mm, $b = 2$ mm.

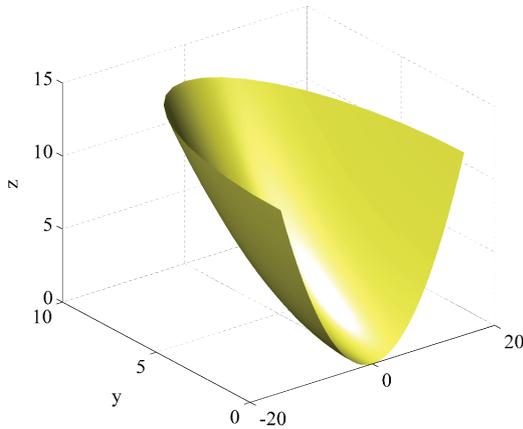


Fig. 5. The original surface of the elliptical paraboloid

Let us consider the rotation of the original surface around the Z-axis at angle Ω set by the general system of equations (4) at a turning angle of 30° :

$$\begin{cases} x' = x \cos \Omega - y \sin \Omega, \\ y' = x \sin \Omega + y \cos \Omega, \\ z' = z. \end{cases} \quad (4)$$

By fitting the parametric values from expression (3) to expression (4), we obtain expression (5) (Fig. 6):

$$\begin{cases} x' = a\omega \cos \vartheta \cos \Omega - b\omega \sin \vartheta \sin \Omega, \\ y' = a\omega \cos \vartheta \sin \Omega + b\omega \sin \vartheta \cos \Omega, \\ z' = 0.5\omega^2, \end{cases} \quad (5)$$

where $0 \leq \omega \leq 7$ mm, $0 \leq \vartheta \leq \pi$, $a = 5$ mm, $b = 2$ mm.

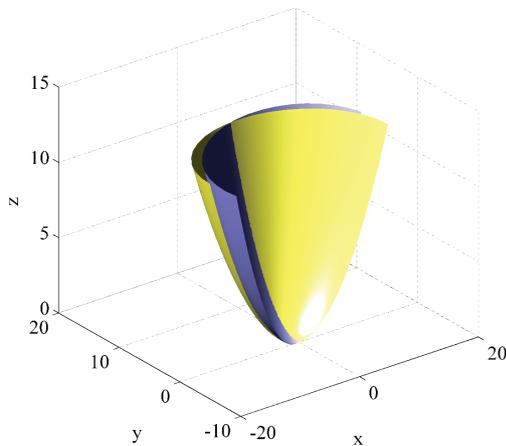


Fig. 6. The original surface and the surface turned at 30° relative to the Z-axis

Assume that we rotate the contact surface around the vertical axis that passes through a certain point $C(x_0, y_0, z_0)$ at angle Ω using the following equation systems (6) to (8):

$$\begin{cases} x' = x - x_0, \\ y' = y - y_0, \end{cases} \quad (6)$$

$$\begin{cases} X' = x' \cos \Omega - y' \sin \Omega, \\ Y' = x' \sin \Omega + y' \cos \Omega. \end{cases} \quad (7)$$

Convert the coordinates (X', Y') to (X, Y) .

$$\begin{cases} x = x' + x_0, \\ y = y' + y_0. \end{cases} \quad (8)$$

Shift the turning point $C(x_0, y_0, z_0)$ relative to the origin of the coordinates and combine the systems of equations (6) to (8) in order to derive a general expression (9) for the rotation around the vertical axis passing through the set point:

$$\begin{cases} x' = (x - x_0) \cos \Omega - (y - y_0) \sin \Omega + x_0, \\ y' = (x - x_0) \sin \Omega + (y - y_0) \cos \Omega + y_0, \\ z' = z. \end{cases} \quad (9)$$

By fitting the parametric values from the system of equations (3) to expression (9) at a turning angle of 30° , we obtain expression (10) (Fig. 7).

$$\begin{cases} x' = (a\omega \cos \vartheta - x_0) \cos \Omega - (b\omega \sin \vartheta - y_0) \sin \Omega + x_0, \\ y' = (a\omega \cos \vartheta - x_0) \sin \Omega + (b\omega \sin \vartheta - y_0) \cos \Omega + y_0, \\ z' = 0.5\omega^2, \end{cases} \quad (10)$$

where $0 \leq u \leq 7$ mm, $0 \leq \vartheta \leq \pi$, $a = 5$ mm, $b = 2$ mm, $C(-10$ mm, -15.0 mm).

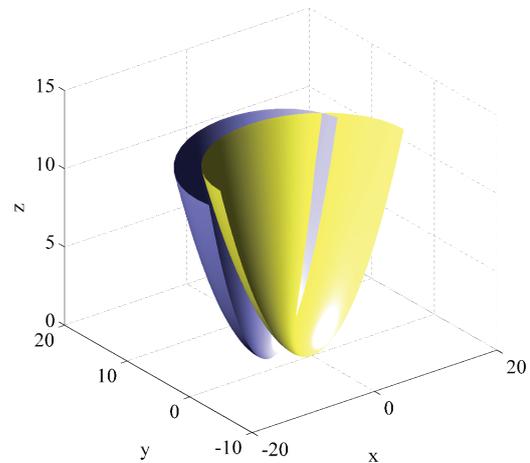


Fig. 7. The original surface and the surface rotated at 30° relative to the assigned point $C(-10, -15.0)$ around the vertical axis

When rotated around the vertical axis, the original surface passes through the contact screw engagement surface. This phenomenon leads to errors in the design of the kinematic pairs.

Consider another case implying the rotation of the curved surface around the curved axis at 30° (Fig. 8). In this case, there are errors in the contact of the conjugated surfaces.

Suppose the curved surface is located in an arbitrary place in the spatial domain and is set by the system of equations (11); rotating around the Z-axis yields expression (12) (Fig. 9).

$$\begin{cases} x = x_0 + a^* \omega^* \cos \vartheta, \\ y = y_0 + b^* \omega^* \sin \vartheta, \\ z = z_0 + 0.5^* \omega^{*2}, \end{cases} \quad (11)$$

$$\begin{cases} x' = (x_0 + a\omega \cos \vartheta) \cos \Omega - (y_0 + b\omega \sin \vartheta) \sin \Omega, \\ y' = (x_0 + a\omega \cos \vartheta) \sin \Omega + (y_0 + b\omega \sin \vartheta) \cos \Omega, \\ z' = z_0 + 0.5\omega^2, \end{cases} \quad (12)$$

where $0 \leq u \leq 5$ mm, $0 \leq \vartheta \leq \pi$, $a = 3$ mm, $b = 2$ mm, $x_0 = -25$ mm, $y_0 = 15$ mm, $z_0 = -10$ mm.

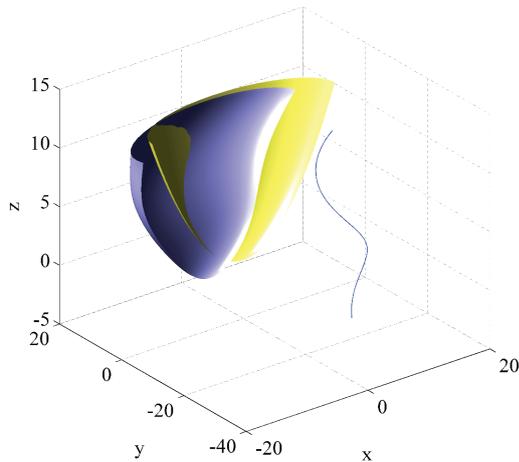


Fig. 8. The original surface and the surface turned at 30° around the curved axis

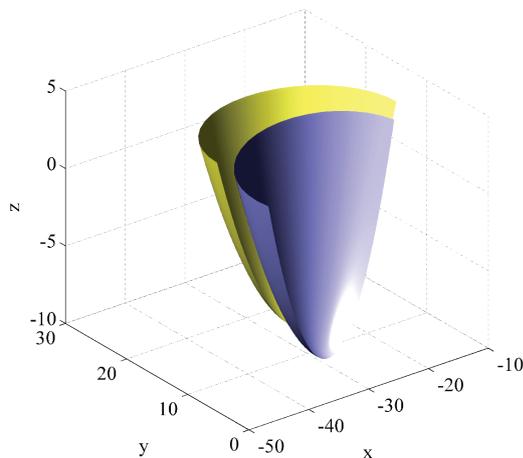


Fig. 9. The original surface and the surface turned at 30° degrees relative to the Z-axis

There is no contact between the surfaces, they are not conjugated. Such kinematic pairs will not work.

When studying and modeling the geometric shape of an engagement surface contact, it was found that the shifted coordinates of the conjugated curved surfaces indicate an uneven change in its geometric parameters. After studying the geometric shape of the spatial contact surface, we revealed heterogeneity in the distribution of the vector beam cross-section field.

6. Discussion of results of studying the shape of a contact screw engagement surface

Our study of the geometric shape of the spatial surface of an engagement contact has shown that the contact surface is determined by two equalities: X and Y . The choice of one of these equalities depends on the region for determining the ϑ parameter of the original surface. By expressing the ϑ parameter through the equality X , we obtain the angle arc tangent within $(-\pi/2; \pi/2)$, and by expressing the ϑ parameter through the Y equality, we derive the angle arc tangent in the interval $(0; \pi)$. Thus, the expression of the ϑ parameter through the equality X defines the region of the intersection line in the vicinity of the x coordinate axis, while the expression of the ϑ parameter through the Y equality – in the vicinity of the y coordinate axis.

The study reported here has made it possible to determine the surface of the contact based on the coordinates of the original surface. At the same time, the required values of the ϑ parameter correspond to the generatrix of the original surface, coinciding or parallel to the coordinate planes XZ or YZ .

The limitations of the geometric technique to determine the contact shape are as follows:

- for each line of intersection, it is necessary to calculate the coordinates according to the formulae specific to them;
- it is required to construct an additional algorithm to determine a formula to use when defining the line of intersection.

The proposed technique could be useful in machine building where it is necessary to determine contact engagement in kinematic pairs for effective geometric design of toothings and cutting tools.

7. Conclusions

1. This paper reports a practical technique for constructing a geometric shape of the contact between conjugated parts providing for the required technical parameters at the design stage. A special feature of the suggested procedure is the possibility to adapt it for use in software packages.

2. The procedure makes it possible to determine the permissible error in the geometric parameters for the shape of contact between conjugated pairs, which is set by the precision quality in the manufacture of toothings and cutting tools.

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