UDC 539.3
DOI: 10.15587/1729-4061.2020.209228

STABILITY AND RATIONAL DESIGN OF THE «BARREL-OGIVE» TYPE STRENGTHENED SHELL STRUCTURES UNDER COMBINED LOADING

This paper reports a study into the stability of a shell structure of the barrel-ogive type, supported by the discretely arranged intermediate frames, under the joint action of the uniform external pressure and axial compressive efforts.

A case of the sinusoidal approximation of the meridian of the middle surface of shell compartments has been considered.

Governing differential equations have been built to study the stability of a compound shell structure taking into consideration the curvature radii of the "barrel" and "ogive" compartments under the joint action of axial compression and uniform external pressure. A finite difference method has been used to integrate the fourth-order governing equations with variable coefficients. It is shown that an increase in the meridian curvature parameter exceeding 4 % leads, in some cases that involve the loading by axial forces, to an increase in the critical external pressure by 1.5–2 times.

The effect of stabilizing the growth of critical pressure with an increase in the rigidity of the frames is illustrated for the different values of the meridian curvature and the number of supporting elements. A given effect makes it possible to draw conclusions about the possibility of determining the rational rigidity characteristics of the structure.

The effect of increasing critical pressure in the presence of a compressive force in the shells of the positive Gauss curvature, which is the result of internal stretching efforts in the circumference direction, has been investigated. In this case, a generatrix deviation from the ideal shape leads to an increase in wavenumbers in the circumferential direction while the stability is lost, which indicates an increase in the critical pressure. A further increase in the axial compression of the structure leads to the emergence of annular compressive efforts, which is a consequence of the reduction in the critical stresses of external pressure

Keywords: shell, barrel-ogive structure, external pressure, axial compression, intermediate frames

V. Gristchak
Doctor of Technical Sciences, Professor, Head of Department
Department of Applied Mathematics and Mechanics*
E-mail: grk@znu.edu.ua

D. Hryshchak
PhD, Leading Researcher
Central Scientific Research Institute of Armament and Military Equipment of the Armed Forces of Ukraine
Povitroflotsky ave., 28, Kyiv, Ukraine, 03049
E-mail: gristchak@gmail.com

N. Dyachenko
PhD, Associate Professor
Department of Fundamental Mathematics*
E-mail: dyachenkonata69@gmail.com

P. Degtiarenko
Head of Complex
Yuzhnoye Design Office
Kryvorizhska str., 3, Dnipro, Ukraine, 49008
E-mail: d-ko@i.ua

*Zaporizhzhia National University
Zhukovskoho str., 66, Zaporizhzhia, Ukraine, 69600

Received date 20.05.2020
Accepted date 21.07.2020
Published date 27.08.2020

Copyright © 2020, V. Gristchak, D. Hryshchak, N. Dyachenko, P. Degtiarenko
This is an open access article under the CC BY license
(http://creativecommons.org/licenses/by/4.0)

1. Introduction

Compound shell structures are the force elements of construction structures, aircraft, and other systems of new equipment. During operation, shells may be subjected to external pressure and axial compression, which leads to a loss of stability of the original shape. The need to improve performance of, for example, rocket and space equipment (RSE), as well as the reduced material consumption in structures, leads to the search for better shapes and the rational reinforcement by stiffeners, in particular, by a transverse force set (frames).

This work analyzes the geometric shapes of the median surface of shell structures, which, in some cases of loading, can lead to an increase in the stability of the compound system under a combined external force impact. From this point of view, convex rotation shells are of interest, specifically, compound shell structures of the “barrel-ogive” type.

One of the requirements for engineering practice is to reduce the material consumption of a structure while maintaining the stability of its original shape. This work focuses on solving the task of the equilibrium of a compound shell structure with respect to the local and overall buckling modes. In this case, we analyze the effect exerted by the character of change in the curvature of the meridian of compartments’ components on the rational rigidity characteristics of the supporting frames.

The relevance of this scientific issue is predetermined by the need to develop a method to study compound shell structures and analyze their stability under combined loading. The study results could be used to rationalize the RSE design.

2. Literature review and problem statement

A significant number of theoretical and experimental studies address the issue of the stability of thin shells, specifically rotation shells of the ideal geometric shape. Actual shell structures have imperfections in a geometric shape, which, in some cases, significantly affects their performance. In this case, there are the unresolved issues related to the stability of the components of geometrically imperfect shell structures, especially those supported by stiffeners.

In this body of research, one should note work [1], in which the analysis is limited to the effect of the axisymmetric initial imperfections on the stability of shells of the cylindrical shape under axial compression. The influence of the supporting annular stiffeners on the carrying capacity of cylindrical shells is studied in [2]. In this case, the researchers tackle certain types of external loading of the studied structures. Work [3] reports a numerical analysis into the performance of the geometrically imperfect composite cylindrical shells under combined loading by external efforts based on a non-linear method of finite elements. However, there are unresolved issues related to the influence exerted by supporting elements on the carrying capacity of the specified types of structures.

A study of the stability and rational design of the combined "cone-cylinder" structure, taking into consideration its performance patterns when it is supported by the annular stiffeners, is reported in work [4], which could serve a basis to analyze the equal stability of the components of shell elements.

Worth noting is review [5] that reports the results of studies on the stability of smooth and supported shells of different geometric shapes and compound structures, including experimental research. It focuses on the possibility of using modern numerical algorithms and the role of experimental studies in the practice of designing actual structures. To a lesser extent, attention is paid to the interaction between the local and overall buckling modes of bulging in the analysis of carrying capacity.

Paper [6] outlines the results of an experimental study into the stability of a cylinder-cone structure under the action of internal pressure involving the analysis of axisymmetric and asymmetric forms of a stability loss based on the method of finite elements, which could be used to study the ribbed structures of complex configuration. Work [7] reports a study of the energy and deformed states of shells of the hemispheric and ogive shapes, taking into consideration the plastic deformations of the material. The results of analyzing the effect exerted by an impact load on the behavior of certain types of shells are relevant for the compound shell structures.

A method to analyze the dynamic stability of cylindrical shells made from a composite material, taking into consideration the geometric nonlinearity and shear deformations, is proposed in [8]; the issue of influence exerted by the force elements supporting the structure remains relevant. An experimental study and a finite-element analysis of the strength of a three-layer shell of a launch vehicle module are given in study [9] without taking into consideration the effect exerted by the discrete arrangement of intermediate frames.

As regards the application of the analytical-numerical methods for calculating the stability of compound structures, paper [10] proposed a hybrid asymptotic approach based on the WKB method for analyzing the cylinder-cone structure supported by frames under combined loading. The influence of geometric imperfections of the median surface of the compartments of a compound shell structure remains to be studied by researchers.

A new approach to solving the optimization problem in terms of the strength of the wafer shell sections of a launch vehicle is proposed in work [11], based on a finite-element method. However, the problem of optimal design reported in the work does not tackle the impact exerted by the meridian curvature of the median surface of the structure.

Paper [12] derived the governing equations of stability for a compound "barrel-ogive" structure under external pressure. Of interest is the solution to the stability problem of the specified shell structure under the joint action of external pressure and axial compression efforts in order to analyze possible ways to increase critical loads and reduce material consumption of compound shell systems. The specified type of force shell system could be effective in terms of rational design, especially when building aerospace equipment. It is the approach outlined in work [12] that is applied in the current paper. In this case, the focus is on the analysis of compound shell structures under the joint action of external loads that could cause a loss of stability. This allows us to argue about the relevance of the present study into the stability and rational design of the barrel-ogive-type structures under combined loading conditions. This relates to the fact that determining the rational rigidity characteristics of frames could significantly reduce the overall mass of a shell structure, while the effective choice of geometric characteristics would significantly improve its stability under combined loading.

3. The aim and objectives of the study

The aim of this work is to study the elastic stability and rational design of the supported shell compartments of the positive curvature of the meridian of a compound structure under the action of static axial compression and uniform external pressure.

To achieve the set aim, the following tasks have been solved:

– to derive the governing differential equations of stability of the thin-walled shells of the "barrel" and "ogive" shape, taking into consideration the curvature of the meridian of the middle surface;

– to modify a finite-difference method to study the stability of the conjugated shells, considering the discrete arrangement of intermediate frames;

– to propose an approach to choosing the rational parameters for a compound structure and to analyze the impact of the frame rigidity parameters on the effect of the equal stability of spans under combined loading;

– to analyze the effect of a meridian curvature parameter on the magnitude of critical load and the character of a stability loss by the supported shells.

4. Governing equations of stability

We consider a compound shell rotation structure, exposed to the influence of the normal external pressure q

and the axial compressive force T (Fig. 1). It is assumed that the structure has the following characteristics: thickness, h; elasticity module, E; a Poisson coefficient, v. The structure consists of two paired spans, the "barrel" and "ogive" types, whose axis of rotation is denoted through Oz. Compartments could be supported by the discretely arranged frames, including docking ones. We solve both the problems on the local stability of individual sections and the overall stability of the structure in general.

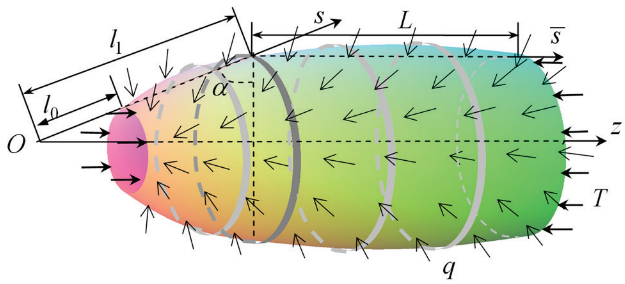


Fig. 1. Schematic of a compound shell structure

It is assumed that the relative height of deviations from the cylinder and cone is less than one-fifth of its smallest linear size. This assumption makes it possible to choose, as the coordinate lines of the structures under study, the coordinate lines for the cylinder and cone. In this case, we introduce the following designations: $\bar{s}$ and s – a linear meridional coordinate along the cylinder or cone generatrix, respectively, y – the arc coordinate of the cylinder, φ – the angular coordinate along the parallel circle of the conical shell.

A medium-length shell is considered, which is exposed to the combined effects of loads, provided the prevailing influence of external pressure, which makes it possible to use a "semi-moment-less" theory of thin shells. In this regard, such a combination of external loads is considered at which one half-wave is formed, when the stability is lost, along the meridian, and in the circumferential direction – n waves, and $n^2 \gg 1$.

It is assumed that the "barrel-shaped" section, as a surface of rotation, is approximated by the following function of the radius of a parallel circle in the cross-sections perpendicular to the axis of rotation:

$$r(z) = R\left(1 + C_{bar}\sin\frac{\pi z}{L}\right), \quad (1)$$

where L and R is the distance between the bases and the radius of the end cross-section of the barrel, respectively, C_{bar} is the relative elevation of the barrel lift.

For the "ogive" shell, the following designations are introduced (Fig. 1): l_0 and l_1 – a distance along the Os axis to the smaller and larger bases, respectively, α – the angle between the Os and Oz axes. It is assumed that the shell takes the following form of a parallel circle radius function in the cross-section perpendicular to the rotation axis:

$$r(z) = z\,\mathrm{ctg}\,\alpha + C_{og}R_{1,og}\sin\left[\frac{\pi}{l_1 - l_0}\left(\frac{z}{\sin\alpha} - l_0\right)\right], \quad (2)$$

where $z = s\sin\alpha$, $R_{1,og} = l_1\cos\alpha$, C_{og} is the relative elevation of the "ogive" meridian.

In accordance with the above assumptions, the following restrictions are imposed on the parameters of the "barrel-shaped" and "ogive" shells [12]:

$$C_{og} \le 1/5, \quad C_{og} \le \frac{1}{5}\cdot\frac{l_0}{l_1},$$

as well as

$$\frac{L}{R} \ge \frac{1}{2}, \quad \frac{l_0}{l_1 - l_0}\cos\alpha \le \frac{1}{\pi}.$$

These inequalities make it possible to obtain the approximate values of the curvature radii:

– for a "barrel-shaped" compartment:

$$\tilde{R}_1 = -\frac{\left(1+(r')^2\right)^{3/2}}{r''} \approx \frac{L^2}{RC_{bar}\pi^2\sin\frac{\pi\bar{s}}{L}},$$

$$\tilde{R}_2 = r\sqrt{1+(r')^2} \approx R\left(1 + C_{bar}\sin\frac{\pi\bar{s}}{L}\right); \quad (3)$$

– for an "ogive" compartment:

$$\tilde{\tilde{R}}_1 \approx \frac{(l_1 - l_0)^2}{C_{og}l_1\pi^2\cos\alpha\sin\alpha\sin\Omega},$$

$$\tilde{\tilde{R}}_2 \approx \mathrm{ctg}\,\alpha\left(s + C_{og}l_1\sin\Omega\right), \quad (4)$$

as well as the approximate distribution of stresses in the main state, described by a moment-less solution for the "barrel-shaped" compartment

$$\tilde{N}_{10} \approx -qRC_{bar}\sin\frac{\pi\bar{s}}{L} - \frac{T}{2\pi\left(R + C_{bar}\,\mathrm{in}\frac{\pi\bar{s}}{L}\right)}, \quad (5)$$

$$\tilde{N}_{20} \approx -qR\left(1 + C_{bar}\sin\frac{\pi\bar{s}}{L}\right) + T\frac{\pi RC_{bar}}{2L^2}\sin\frac{\pi\bar{s}}{L} \quad (6)$$

and the "ogive" compartment

$$\tilde{\tilde{N}}_{10} \approx -\frac{q\left(s^2 - l_0^2\right)\mathrm{ctg}\,\alpha}{2\left(s + C_{og}l_1\sin\Omega\right)} - qC_{og}l_1\,\mathrm{ctg}\,\alpha\sin\Omega - \\ -\frac{T}{2\pi\sin\alpha\cos\alpha\left(s + C_{og}l_1\sin\Omega\right)}, \quad (7)$$

$$\tilde{\tilde{N}}_{20} \approx -q\left(\begin{array}{c} s\,\mathrm{ctg}\,\alpha + C_{og}l_1\mathrm{ctg}\,\alpha\times \\ \times\sin\Omega\left(1 + \frac{\pi^2\cos^2\alpha\left(s^2 - l_0^2\right)}{2\left(l_1 - l_0\right)^2}\right)\end{array}\right) + \\ +T\frac{\pi l_1 C_{og}}{2\left(l_1 - l_0\right)^2}\sin\Omega, \quad (8)$$

where

$$\Omega = \frac{\pi\left(s - l_0\right)}{l_1 - l_0}.$$

The circumferential and meridional efforts contain the terms that depend on an axial compressive force, with opposite signs. This is due to that for a convex shell the axial compression leads to an increase in the stretching circumferential stresses.

Based on curvature radii (3) and (4), the coefficients of the first quadratic surface forms and the differentiation operators are calculated. The found characteristics, including (7), (8), are fitted to the system of basic equations as the particular derivatives from the theory of thin shells.

For a "barrel-shaped" shell, the following designations are introduced:

$$\varepsilon_1=\frac{h}{R},\quad K=\frac{L}{R},\quad \omega=12\left(1-\nu^2\right),$$

the dimensionless coordinates $\bar{x}=\bar{s}/L$, $\bar{\varphi}=y/R$, the dimensionless external normal pressure $\chi_{bar}=\dfrac{qR^2}{Eh^2}$ and the axial compressive force $\eta_{bar}=\dfrac{T}{2\pi Eh^2}$. In this case, the function of radial movements and the function of efforts allow the following representations

$$w_{bar}\left(\bar{x},\bar{\varphi}\right)=W_{bar}\left(\bar{x}\right)\cdot\cos\left(n\bar{\varphi}\right),$$

$$f_{bar}\left(\bar{x},\bar{\varphi}\right)=E\,h^2\,\Phi_{bar}\left(\bar{x}\right)\cdot\cos\left(n\bar{\varphi}\right). \tag{9}$$

For the thin shells of a medium length, $K^2>1$, $n^2\gg 1$, therefore, it is possible to exclude the terms of the order above $\dfrac{1}{K^2n^2}$. This makes it possible to exclude the function $\Phi_{bar}\left(\bar{x}\right)$ from the system and derive a governing differential equation of the main stressed state relative to the deflection function of a "barrel-shaped" shell:

$$a_4W_{bar}^{IV}\left(\bar{x}\right)+a_3W_{bar}'''\left(\bar{x}\right)+a_2W_{bar}''\left(\bar{x}\right)+$$
$$+a_1W_{bar}'\left(\bar{x}\right)+a_0W_{bar}\left(\bar{x}\right)=0, \tag{10}$$

where

$$a_0=\pi^3n^2\varepsilon_1C_{bar}K^{-2}\sin\pi\bar{x}\left(C_{bar}\sin\pi\bar{x}+1\right)^2\eta_{bar}-$$
$$-\pi\,n^2\varepsilon_1\left(C_{bar}\sin\pi\bar{x}+1\right)^3\chi_{bar}+\pi\,n^4\varepsilon_1^2\omega^{-1}+$$
$$+2\,\pi^3n^2\varepsilon_1^2C_{bar}\omega^{-1}K^{-2}\left(C_{bar}\sin^2\pi\bar{x}-\sin\pi\bar{x}-2C_{bar}\right)+$$
$$+\pi^5K^{-4}C_{bar}\left(C_{bar}\sin\pi\bar{x}+1\right)^3\times$$
$$\times\left[\begin{array}{l}C_{bar}\sin^2\pi\bar{x}\left(C_{bar}\sin\pi\bar{x}+1\right)+\\ +n^{-2}\left(\begin{array}{l}9C_{bar}^2\sin^3\pi\bar{x}+8C_{bar}\sin^2\pi\bar{x}-\\ -6C_{bar}^2\sin\pi\bar{x}+\sin\pi\bar{x}-4C_{bar}\end{array}\right)\end{array}\right];$$

$$a_1=K^{-2}\cos\pi\bar{x}\left(C_{bar}\sin\pi\bar{x}+1\right)\times$$
$$\times\left\{\begin{array}{l}\varepsilon_1C_{bar}\pi^2\left(C_{bar}\sin\pi\bar{x}+1\right)^3\chi_{bar}-\\ -K^{-2}\varepsilon_1C_{bar}^2\pi^4\sin\pi\bar{x}\left(C_{bar}\sin\pi\bar{x}+1\right)^2\eta_{bar}+\\ +2\varepsilon_1^2n^2\omega^{-1}C_{bar}\pi^2+\pi^4K^{-2}C_{bar}\times\\ \times\left[\begin{array}{l}\varepsilon_1^2\omega^{-1}\left(C_{bar}^2-1\right)-\\ -\dfrac{2}{n^2}\left(3C_{bar}\sin\pi\bar{x}+1\right)\left(C_{bar}\sin\pi\bar{x}+1\right)^3\end{array}\right]\end{array}\right\};$$

$$a_2=K^{-2}\left(C_{bar}\sin\pi\bar{x}+1\right)^2\times$$
$$\times\left\{\begin{array}{l}\varepsilon_1C_{bar}\pi\sin\pi\bar{x}\left(C_{bar}\sin\pi\bar{x}+1\right)^2\chi_{bar}+\\ +\varepsilon_1\pi\left(C_{bar}\sin\pi\bar{x}+1\right)\eta_{bar}-\\ -2\varepsilon_1^2n^2\omega^{-1}\pi+\pi^3K^{-2}C_{bar}\times\\ \times\left[\begin{array}{l}-\varepsilon_1^2\omega^{-1}\left(C_{bar}\sin^2\pi\bar{x}+2\sin\pi\bar{x}+C_{bar}\right)-\\ -2n^{-2}\sin\pi\bar{x}\left(C_{bar}\sin\pi\bar{x}+1\right)^3-\\ -3n^{-4}\sin\pi\bar{x}\left(\begin{array}{l}3C_{bar}\sin^2\pi\bar{x}+\\ +\sin\pi\bar{x}-2C_{bar}\end{array}\right)\times\\ \times\left(C_{bar}\sin\pi\bar{x}+1\right)^2\end{array}\right]\end{array}\right\};$$

$$a_3=\frac{2\pi^2C_{bar}}{n^4K^4}\cos\pi\bar{x}\left(C_{bar}\sin\pi\bar{x}+1\right)^3\times$$
$$\times\left(3C_{bar}^2\sin^2\pi\bar{x}+6C_{bar}\sin\pi\bar{x}+\frac{n^4\varepsilon_1^2}{\omega}+3\right);$$

$$a_4=\pi n^{-4}K^{-4}\left(C_{bar}\sin\pi\bar{x}+1\right)^4\times$$
$$\times\left(C_{bar}^2\sin^2\pi\bar{x}+2C_{bar}\sin\pi\bar{x}+\varepsilon_1^2\omega^{-1}n^4+1\right).$$

At $C_{bar}=0$, we obtain a cylindrical shell, and

$$W_{bar}\left(x\right)=W_{cyl}\left(x\right),$$

with the equation taking the following form

$$\frac{1}{K^4}\left(\frac{\varepsilon_1n^2}{\omega}+\frac{1}{n^2\varepsilon_1}\right)W_{cyl}^{IV}\left(\bar{x}\right)+\frac{n^2}{K^2}\left(\eta_{cyl}-\frac{2\varepsilon_1n^2}{\omega}\right)W_{cyl}''\left(\bar{x}\right)+$$
$$+n^4\left(\frac{n^2\varepsilon_1}{\omega}-\chi_{cyl}\right)W_{cyl}\left(\bar{x}\right)=0, \tag{11}$$

which is an approximate equation to the one given in [5].

For an "ogive" shell, the following designations are introduced:

$$\delta=\frac{n^2}{\cos^2\alpha},\quad p=\varepsilon\delta,\quad \varepsilon=\sqrt{\frac{h\,\mathrm{ctg}\,\alpha}{l_1\sqrt{12\left(1-\nu^2\right)}}},\quad K_c=\frac{l_0}{l_1},$$

the dimensionless coordinate $x=\dfrac{s}{l_1}$ and the efforts

$$\chi_{og}=\frac{q\,l_1}{Eh\,\varepsilon^3\,\mathrm{tg}^3\alpha},$$

$$\eta_{og}=\frac{T\cos\alpha}{2\pi Eh\,\varepsilon^2l_1\sin^3\alpha}.$$

The functions of radial displacements and efforts are recorded in the following form

$$w_{og}\left(x,\varphi\right)=W_{og}\left(x\right)\mathrm{tg}\,\alpha\cdot\cos\left(n\varphi\right),$$

$$f_{og}\left(x,\varphi\right)=\Phi_{og}\left(x\right)\varepsilon^2l_1Eh\,\mathrm{tg}^2\alpha\cdot\cos\left(n\varphi\right). \tag{12}$$

Since for the thin conical medium-length shells $\varepsilon\ll 1$, $n^2\gg 1$, such terms are retained in the equations whose order does not exceed ε. The exclusion from system (12) of the function $\Phi_{og}\left(x\right)$ yields a governing differential equation for the stability of an "ogive" shell:

$$b_4 W_{og}^{IV}(x)+b_3 W_{og}'''(x)+$$
$$+b_2 W_{og}''(x)+b_1 W_{og}'(x)+b_0 W_{og}(x)=0, \qquad (13)$$

where

$$b_0=\left[\begin{pmatrix}0{,}5\pi^2\cos^2\alpha C_{og}\sin\Omega\left(K_c^2-x^2\right)-\\ -\left(C_{og}\sin\Omega+x\right)\left(1-K_c\right)^2\end{pmatrix}\times \\ \times\chi_{og}+\pi^2 C_{og}\sin\Omega\cos^2\alpha\,\eta_{og}\right]\times$$
$$\times p^3\left(C_{og}\sin\Omega+x\right)^2\left(1-K_c\right)^2+\delta\,\pi^2\cos^2\alpha C_{og}\times$$
$$\times\left(C_{og}\sin\Omega+x\right)^3\begin{bmatrix}-4x\,\pi^2 C_{og}-\\ -4\pi\left(2C_{cone}\sin\Omega+x\right)\times\\ \times\left(1-K_c\right)\cos\Omega+9\pi^2 C_{og}^2\times\\ \times\sin^3\Omega+8x\pi^3 C_{og}\sin^2\Omega-\\ -\sin\Omega\left(\pi^2\left(6C_{og}^2-x^2\right)+2\left(1-K_c\right)^2\right)\end{bmatrix}+$$
$$+\delta^2\,\pi^4\cos^4\alpha C_{og}^2\sin^2\Omega\left(C_{og}\sin\Omega+x\right)^4+p^4\left(1-K_c\right)^4;$$

$$b_1=\delta\pi^2\cos^2\alpha\,C_{og}\left(C_{og}\sin\Omega+x\right)^3\left(1-K_c\right)\times$$
$$\times\begin{bmatrix}p\varepsilon\sin\Omega\left(K_c-1-\pi\,C_{og}\cos\Omega\right)\eta_{og}+\\ +2\left(C_{og}\sin\Omega+x\right)\begin{pmatrix}-2K_c\sin\Omega+1{,}5\pi\,C_{og}\times\\ \times\sin2\Omega+2\sin\Omega+\pi x\cos\Omega\end{pmatrix}\end{bmatrix};$$

$$b_2=\left(C_{og}\sin\Omega+x\right)^3\left(1-K_c\right)^2\times$$
$$\times\left\{\begin{bmatrix}\left(1-K_c\right)^2\left(p^2\eta_{og}+6\left(C_{og}\sin\Omega+x\right)\right)+\\ +3C_{og}\pi\left(C_{og}\sin\Omega+x\right)\times\\ \times\begin{pmatrix}\pi\left(2C_{og}-3C_{og}\sin^2\Omega-x\sin\Omega\right)+\\ +4\cos\Omega\left(1-K_c\right)\end{pmatrix}\end{bmatrix}-\\ -2\delta\,\pi^2\cos^2\alpha\,C_{og}\sin\Omega\left(C_{og}\sin\Omega+x\right)^2\right\};$$

$$b_3=6\left(C_{og}\sin\Omega+x\right)^5\left(1-K_c\right)^3\left(\pi C_{og}\cos\Omega-K_c+1\right);$$

$$b_4=\left(C_{og}\sin\Omega+x\right)^6\left(1-K_c\right)^4.$$

At $C_{og}=0,$ we obtain $W_{og}(x)=W_{cone}(x),$ and the governing equation of stability (13) transforms into the equation for a conical shell given in work [5]:

$$W_{cone}^{IV}(x)+\frac{6}{x}W_{cone}'''(x)+\left(\frac{6}{x^2}+\frac{\eta_{cone}p^2}{x^3}\right)W_{cone}''(x)-$$
$$-\left(\frac{\chi_{cone}\,p^3}{x^3}-\frac{p^4}{x^6}\right)W_{cone}(x)=0. \qquad (14)$$

5. A modified finite-difference method to study the stability of conjugated shells

Initially, we introduce a designation for the "barrel" type structure in general $[a;b]=[x_0;1],$ for an "ogive" structure – $[a;b]=[0;1].$ A problem on the stability of a compound structure is solved by the finite-difference method. At each internal point $x_k=a+kH,\quad k=\overline{1,N-1}$ of the uniform splitting of the segment $[a;b]$ at an increment $H=\dfrac{b-a}{N},$ the derivatives from the W function in the governing equations are represented as the central finite differences of the second order, for the boundary conditions – the first-order differences.

For the case when the side-ends rest on hinges, the corresponding equations could be written in the following form:

$$W_0=0,\;\; W_{-1}=-W_1;\;\; W_N=0,\;\; W_{N+1}=-W_{N-1}. \qquad (15)$$

Equalities (15) make it possible to exclude the W_{-1}, W_{N+1}, W_0, W_N variables from the system of difference equations. The characteristic equation of the connection between the critical efforts and the wave-forming parameters in the case of a stability loss $\chi_{bar/og},\ \eta_{bar/og},\ n,$ or $q_{bar/og},\ T_{bar/og},\ n$ is the consequence of equating to zero of the determinant of the derived system.

Conjugating the compartments of the "barrel" and "ogive" shapes. The study into the conjugation of shells of the examined types begins by determining values for the lift parameters C_{bar} and C_{og} based on the condition of harmonizing the angular coefficients of the generatrixes of the paired sections:

$$C_{bar}R\frac{\pi}{L}=\operatorname{ctg}\alpha-C_{og}R\frac{\pi}{\left(l_1-l_0\right)\sin\alpha}. \qquad (16)$$

Condition (16) in the vicinity of the conjugation cross-section assigns the locally cone shape of the structure with an angle at the base

$$\beta^*=\operatorname{arcctg}\frac{\pi R C_{bar}}{L}. \qquad (17)$$

Conditions for the shell's sections conjugation through an intermediate frame.

It is assumed that the supporting frame possesses stiffness at bending, both in the plane and from the plane of the initial curvature. A change in the stresses-deformed state when crossing a frame on the cylinder and cone was studied in works [5, 12] where, specifically, the following dependences are used:

$$W_{left}(t_1)=W_{right}(t_2),$$
$$W_{left}'(t_1)=W_{right}'(t_2),$$
$$W_{left}''(t_1)+G_2 W_{left}'(t_1)=W_{right}''(t_2),$$
$$W_{left}'''(t_1)-G_1 W_{left}(t_1)=W_{right}'''(t_2), \qquad (18)$$

where $t_{1,2}$ are the coordinates of a conjugation point corresponding to the adjacent sections; G_1 and G_2 are the parameters of the dimensionless rigidities of the frames, which support a cylindrical or conical shell, represented by the following dependences:

$$G_{cyl,1}=G_1^*\frac{R^3}{r_{ring}^3},\;\; G_{cyl,2}=G_2^*\frac{R^3}{r_{ring}^3}; \qquad (19)$$

$$G_{cone,1}=G_1^*\frac{R^3 L_{cone}}{\cos^2\beta\, r_{ring}^4},\;\; G_{cone,2}=G_1^*\frac{R^3 L_{cone}}{r_{ring}^4}; \qquad (20)$$

$$G_1^* = \frac{n^4\left(n^2-1\right)^2 (EJ)_x^{ring}}{Eh\,R^3},\ G_2^* = \frac{n^2\left(n^2-1\right)^2 (EJ)_z^{ring}}{EhR^3\left(n^2+1\right)}. \qquad (21)$$

In formulae (19) to (21), J_x^{ring}, J_z^{ring} are the momenta of inertia when bending the frame in the plane of the initial curvature and from the plane, respectively; r_{ring} is the radius of a frame; L_{cone} is the length of a "local cone" on which a frame is fixed, at an angle at the base of β.

In the specific conjugation cases, we obtain:

1) for an intermediate frame on the "barrel-shaped" shell $t_1=t_2$ corresponds to the coordinate of the frame's position; in formula (19), one selects

$$r_{ring} = R\left(1 + C_{bar}\sin\pi t_1\right);$$

2) for an intermediate frame on the "ogive" shell, in formula (20), the following be chosen:

$$t_1 = t_2,\ r_{ring} = R\left(t_1 + C_{og}\sin\Omega\big|_{x=t_1}\right),\ L_{cone} = \frac{r_{ring}}{\cos\beta},$$

$$\beta = \text{arcctg}\left(\text{ctg}\,\alpha\left[1 + C_{og}\frac{\pi\, l_1\cos\Omega\big|_{x=t_1}}{l_1 - l_0}\right]\right); \qquad (22)$$

3) for a docking frame, the use of ratios (17), (19) to (21) leads to the following equalities:

$$t_1 = 1,\ \ t_2 = 0,\ \ r_{ring} = R,\ \ L_{cone} = \frac{R}{\cos\beta},\ \ \beta = \beta^*. \qquad (23)$$

Let us consider the way in which a system of difference equations is modified when conjugating the "ogive" and "barrel-shaped" sections. The derivatives in equations (10) and (13) are represented by the central finite differences of the second order. In equation (10), for a structure of the "ogive" type, the natural numbering at step $H_1 = \frac{1-x_0}{N}$. is retained. The boundary conditions are assigned by the first pair of equalities (15). In equation (13), for a barrel-type structure, the numbering of internal points is set from N+4 to 2N+2 at step $H_2 = \frac{1}{N}$.

At the boundary (2N+3)-th point, the equalities are recorded that are similar to the second pair of equalities (15).

The finite differences for conjugation conditions (18) through a frame could be represented in the following form:

$$W_N = W_{N+3};$$

$$\frac{1}{6H_1}\left(W_{N-2} - 6W_{N-1} + 3W_N + 2W_{N+1}\right) = \\ = \frac{1}{6H_2}\left(-2W_{N+2} - 3W_{N+3} + 6W_{N+4} - W_{N+5}\right);$$

$$\frac{1}{H_1^2}\left(W_{N-1} - 2W_N + W_{N+1}\right) + \\ + \frac{G_2}{6H_1}\left(W_{N-2} - 6W_{N-1} + 3W_N + 2W_{N+1}\right) = \\ = \frac{1}{H_2^2}\left(W_{N+2} - 2W_{N+3} + W_{N+4}\right);$$

$$\frac{1}{H_1^3}\left(-W_{N-2} + 3W_{N-1} - 3W_N + W_{N+1}\right) - G_1 W_N = \\ = \frac{1}{H_2^2}\left(-W_{N+2} + 3W_{N+3} - 3W_{N+4} + W_{N+5}\right). \qquad (24)$$

The conjugation of frames' spans with an increased number of the discretely arranged frames is carried out by using ratios (22), (23), or similar to (24). The characteristic equation regarding the parameters of the $q_{bar/og}$, $T_{bar/og}$, n wave formation is derived in accordance with the above. For each axial compressive effort value $T_{bar/og}$, the characteristic equation determines the number of waves of a stability loss in the circumferential direction that corresponds to the lowest value of the critical external pressure $q_{bar/og}$.

6. Choosing the rational parameters for a compound structure. Influence of the frames' rigidity parameters on the spans' equi-stability effect

A numerical analysis of stability was carried out for a compound shell structure with the following characteristics: h=0.3 cm, E=7·10^5 kg/cm^2, ν=0.32. A structure of the "ogive" section was chosen with the following parameters: l_1=182 cm, l_0=182 cm, α=75°; of the "barrel-shaped" section – L=2.5R. The calculations were performed for the case of the boundary conditions corresponding to the hinge-supported ends.

In accordance with the algorithm proposed in this paper, Fig. 2 illustrates the effect of the relative loft height of unsupported structures of the barrel C_{bar} and "ogive" C_{og} types (denoted via the "C" axis) on the critical pressure $q_{bar/og}$ [kg/cm^2] (denoted via "q"). In this case, we established an increase in critical pressure with the growth of the meridian curvature of the middle surface of the examined structures.

The equi-stability of shell sections was investigated in accordance with the proposed algorithm.

In the first stage of the calculation, the hinge-supported sections (a "barrel" and an "ogive") were considered separately. By using the diagrams in Fig. 2, we selected one of the possible pairs C_{bar} and C_{og}, matched with equal critical pressure values. The pair found was then refined based on formula (16). For a shell with the following parameters

$$C_{og} = 0.0626;\ C_{bar} = 0.137 \qquad (25)$$

the "ogive" section had a critical pressure value of q_{og} = = 2,98 kg/cm^2, the "barrel-shaped" – q_{bar} = 2,97 kg/cm^2, the compound structure – q_{constr} = 2,3 kg/cm^2.

When arranging an intermediate frame in the "ogive" section of the compound structure, we initially determined its most rational position (in terms of local stability) on a conical (without lifting a meridian) shell. If the generatrix length is divided in the following ratio

$$L_{left} : L_{right} = 1{,}809 : 1, \qquad (26)$$

counting from the smaller base of the cone, the critical pressure of the "left" and "right" sides is equal to, respectively, q_{left} = 3,13 kg/cm^2 and q_{right} = 3,11 kg/cm^2. In this case, the cone in general lost stability at the critical pressure of q_{cone} = 1,68 kg/cm^2.

For the convenience of analysis, the following designations of the dimensionless efforts are introduced:

$$\widehat{T}_{bar/og}=\frac{T}{T_{classic,cyl/cone}},\ \widehat{q}_{bar/og}=\frac{q}{q_{classic,cyl/cone}}, \qquad (27)$$

$$T^*=\frac{T}{Eh_0^2},\ q^*=\frac{q}{q_{constr}}. \qquad (28)$$

In the designations (27), we accepted the classic critical effort values for the cylinder and cone in the form of the following ratios:

$$T_{classic,\,cyl}=0{,}605\,E\,h^2,\ \ q_{classic,\,cyl}=0{,}92\,E\left(\frac{h}{R}\right)^{5/2}\frac{R}{L},$$

$$T_{classic,\,cone}=\frac{2\pi\,E\,h^2\sin^2\alpha}{\sqrt{3\left(1-\nu^2\right)}},$$

$$q_{classic,\,cone}=C_1E\left(\frac{h}{l_1}\right)^{5/2}\frac{(\mathrm{tg}\,\alpha)^{3/2}}{\left(1-\nu^2\right)^{3/4}},\ \ C_1\approx 3. \qquad (29)$$

Graphic representation of the results of the impact of the frames' rigidities on the stability of shells.

Below are the results of numerical calculations for various cases of supporting the structures by the dependences of dimensionless normal critical pressures $\widehat{q}_{bar/og}$ or q on the parameters of the frame rigidity G_1^* in the plane of the initial curvature at the fixed values $G_2^*=0$ or $G_2^*=10$ of rigidities from the plane of the initial curvature:

1) Fig. 3 – for an "ogive" shell, supported by a single intermediate frame, arranged to satisfy ratio (26); in this case, the lift values were chosen that equal $C_{og}=0$ (blue lines), $C_{og}=0{,}06$ (red lines), $C_{og}=0{,}09$ (green lines);

2) Fig. 4, *a*, *b* – for a "barrel-shaped" shell, supported by one and two intermediate frames, dividing the length of the "barrel" into equal segments; in this case, the lift values are $C_{bar}=0$ (blue lines), (red), $C_{bar}=0{,}05$ (green lines), $C_{bar}=0{,}15$ (orange lines);

3) Fig. 5 – for a "barrel-ogive" structure with the parameters satisfying (25) and (26), supported by frames as shown in the figure.

In Fig. 3–5, the wavenumbers n are denoted in the circumferential direction at a stability loss.

The darker lines correspond to the value $G_2^*=0$, the lighter ones – $G_2^*=10$. As the specified dependences indicate, the critical loads that match the value of the rigidity parameter from the plane $G_2^*=10$, are higher than the value $G_2^*=0$. Solid blue lines in Fig. 3, 4 correspond to sections without curvature of the meridian of the median surface, that is, conical, or cylindrical shells.

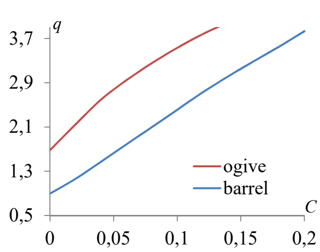


Fig. 2. Critical pressure dependences on the relative curvature of the meridian of unsupported "barrel" and "ogive" structures

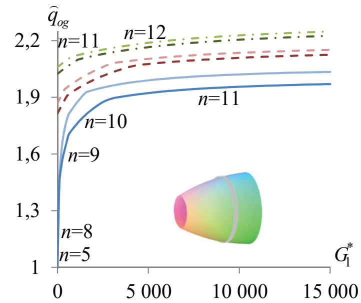
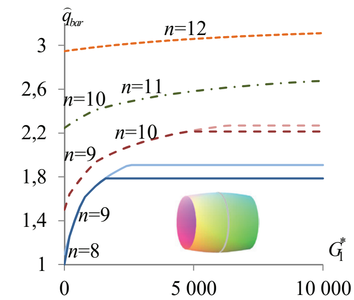
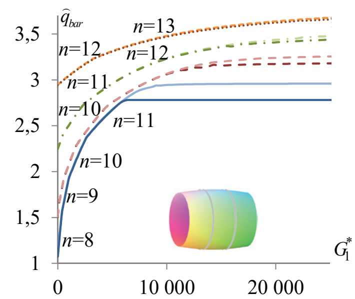


Fig. 3. The dependence of the critical pressure $\widehat{q}_{og}$ of the "ogive" shell on the rigidity of the intermediate frame G_1^* at the assigned parameters G_2^*



a



b

Fig. 4. The dependence of the critical pressure $\widehat{q}_{bar}$ on the rigidity of intermediate frames G_1^* at the assigned parameters G_2^*: *a* – one frame; *b* – two frames

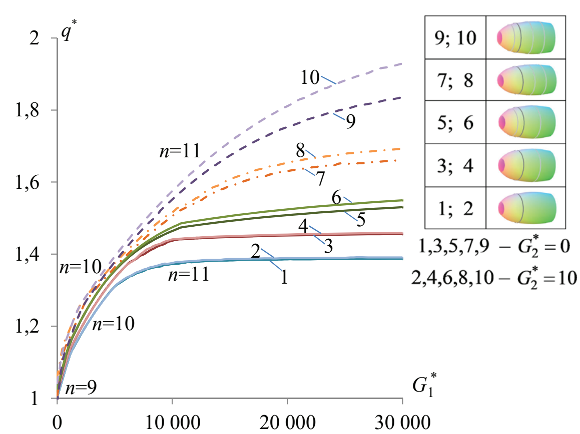


Fig. 5. The dependence of the critical pressure q^* of a "barrel-ogive" structure on the frames' rigidity parameter G_1^*

The dependences in Fig. 2–5 make it possible to assess the effect of the curvature of the median surface of the structure's compartments and the rigidity characteristics of intermediate frames on stability under the action of external pressure. This defines the rational parameters of the rigidity of frames, ensuring the stability of a structure with respect to the local and overall buckling modes.

7. Numerical analysis of the effect of the meridian curvature parameter on the magnitude of critical load and the character of a stability loss by the supported shells

Fig. 6–8 show the dependences of the dimensionless parameters of the critical pressure $\hat{q}_{bar/og}$ on the axial compressive force $\hat{T}_{bar/og}$ or T^*:

1) Fig. 6, *a*, *b* – for the unsupported "barrel-shaped" and "ogive" shells at different lift values C_{bar} and C_{og}, respectively, (in Fig. 6, their values are denoted via "C");

2) Fig. 7 – for the supported structures of the "barrel" and "ogive" types at different values of C_{bar} and C_{og}, and at various rigidities of the frames in the G_1^* plane and from the G_2^* plane (in Fig. 7, they are denoted via "G1" and "G2");

3) Fig. 8 – for a supported structure of the "barrel-ogive" type with the parameters satisfying (25) and (26), which is supported by frames as shown in Fig. 8.

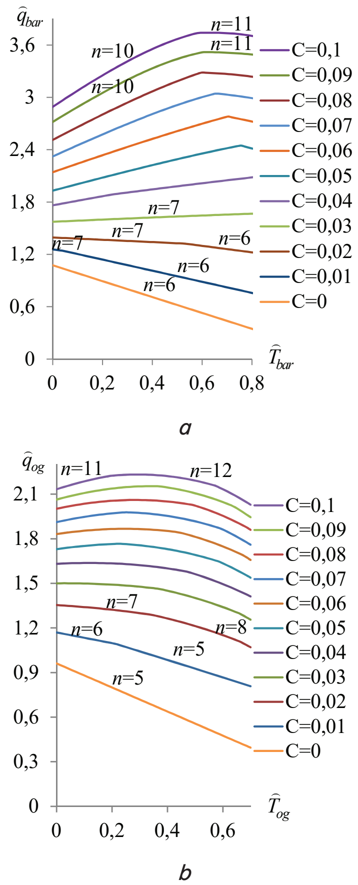


Fig. 6. The dependence of critical external pressure on the magnitude of the axial compressive force: *a* – "barrel" compartment; *b* – "ogive" compartment

It follows from Fig. 6–8 that the presence of a positive curvature of the meridian of the median surface of the examined structures could lead to a significant increase in the critical external pressure at an overall positive effect of compression efforts in terms of carrying capacity.

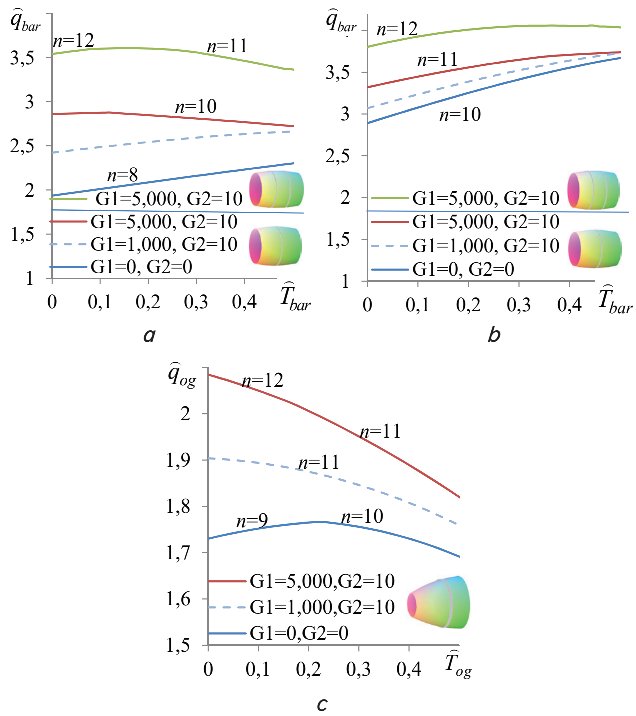


Fig. 7. The dependence of critical external pressure on the magnitude of the axial compressive force: *a* – "barrel, $C_{bar} = 0{,}05$; *b* – "barrel", $C_{bar} = 0{,}1$; *c* – "ogive", $C_{og} = 0{,}05$

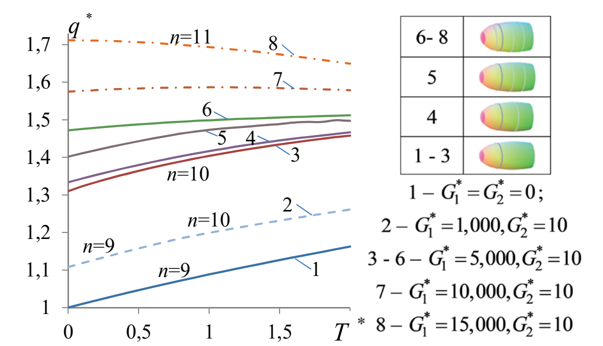


Fig. 8. The dependence of the critical pressure of a "barrel-ogive" structure on the axial compressive force for various supporting techniques

8. Discussion of results of studying the stability and rational design of the supported shell compartments of the barrel-ogive type

The presence of frames leads to an increase in the stability of shells under external pressure. In this case, with the growth of rigidity parameters in the plane of the initial curvature G_1^*, the critical pressure stabilizes. This effect suggests that it is possible to determine the rational rigidity characteristics of supportive frames, which ensure the equi-stability with respect to the local and overall buckling modes. It should be noted (Fig. 3, 4) that it is possible to specify such a limit of the rigidity parameter $\bar{G}_1^*$, depending on the number of the supporting elements and the curvature of the mid-surface meridian, at which, for $G_1^* > \bar{G}_1^*$, the magnitude of the critical pressure $q\left(G_1^*\right)$ is not much different from $q\left(\bar{G}_1^*\right)$. A similar situation is for the compound structure (Fig. 5). However, the magnitude of the critical pressure depends on the location of the supporting elements along the axial coordinate. In addition, there is a question about the limits of the applicability of the orthotropic theory of shells for the case of compound structures.

As regards the curvature parameters of the median surface and the rigidity of the supporting elements from the plane of the initial curvature, in some of the examined cases their impact may be significant, evidenced by Fig. 3–5.

When analyzing the compartments and a compound structure of the "barrel-ogive" type, it was found the difference in the form of a stability loss by convex shells compared to the shells of a zero Gaussian curvature. The character of the loss of stability is associated with an increase in the wave number in the circumferential direction and, accordingly, an increase in critical pressure at the initial stage of loading by axial compressive efforts leading to the stretching internal annular efforts.

The identified effect for unsupported shells holds for the supported shells at the certain combinations of shell lift values and frame rigidity (Fig. 7, 8). Note that for the shells of a small curvature of the meridian the effect of the frames is more significant than the effect exerted by stretching efforts in the annular direction caused by axial compression.

The validity of the study results was tested by comparing numerical results with known data obtained by other methods and by comparing the results of the analysis of compartments and a whole structure in the absence of supporting elements. Thus, at zero lift, in extreme cases, the characteristic equations corresponding to the equations for cylindrical and conical shells were obtained. The agreement in critical pressure values with known results [5] amounted to 10 %. For a single section of the conjugation, the deviation was up to 4 %, for two – up to 7 %. The error when the number of break points (according to a numerical calculation method) along the axial coordinate of each shell span was up to 4 %.

It should be noted that the results obtained are correct only for the medium-length shells with a deviation of the meridian curvature parameter from the ideal state not exceeding 15 **%** and 20 **%** for the "ogive" and "barrel-shaped" compartment, respectively.

The proposed approach to solving a given problem could be used in the future to study the stability of compound wafer, anisotropic, and multi-layered shells of a complex configuration, supported by the discretely arranged longitudinal and transverse force kits under a combined static and static-dynamic loading.

9. Conclusions

1. We have built governing equations for the stability of the "barrel-shaped" and "ogive" compartments, taking into consideration the curvature of the median surface and the peculiarities of the influence of axial force on the circumferential stresses; an algorithm for calculating the compound structure has been proposed.

When deriving the governing equations for individual sections, it was noted that, in contrast to the shell of a zero Gaussian curvature, the convex shell has an axial compressive effort resulting in the occurrence of internal stretching stresses in the circumferential direction.

The compound structure calculation algorithm uses the conditions for harmonizing the geometric characteristics of the compartments, the discretion, and the location of the intermediate frames.

2. The modified method of finite differences was proposed for the numerical analysis of the main governing equations, allowing the stability assessment of the conjugated shell compartments supported by frames. The specificity of modifying a given method [12] is in the construction of equations that correspond to the conjugation points of sections through an intermediate frame.

3. The stability of the compartments and the compound structure "barrel-ogive" was investigated, taking into consideration the discrete arrangement of intermediate frames, under a joint action of axial compressive forces and external pressure. For the different values of the meridian curvature parameter and the number of supporting frames, the effect of stabilizing the increase in critical pressure with the increase in the rigidity of the frames in the plane of their initial curvature has been illustrated.

4. We have analyzed the effect of the curvature of the middle surface of the compartments and a compound shell structure of the "barrel-ogive" type on the magnitude of critical efforts and the character of a stability loss by the examined system under a joint action of external pressure and axial compressive forces.

For the shell sections with a relative deviation of the meridian curvature value exceeding 4 %, at the growth of the axial compressing force, the effect of increasing critical pressure by 1.5–2 times in a certain range of change in the axial force has been detected.

Acknowledgments

This work was carried out within the framework of the State Budget Theme by Ukraine's Ministry of Education No. 1/18 "Mathematical and software maintenance of the automated design of aerospace equipment" (State Registration No. 0118U00021).

References

1. Bai, X., Tang, R., Zan, Y., Li, J. (2019). Stability analysis of a cylindrical shell with axially symmetric defects under axial compression based on the reduction stiffness method. Ocean Engineering, 193, 106584. doi: https://doi.org/10.1016/j.oceaneng.2019.106584
2. Bai, X., Xu, W., Ren, H., Li, J. (2017). Analysis of the influence of stiffness reduction on the load carrying capacity of ring-stiffened cylindrical shell. Ocean Engineering, 135, 52–62. doi: https://doi.org/10.1016/j.oceaneng.2017.02.034
3. Tafreshi, A., Bailey, C. G. (2007). Instability of imperfect composite cylindrical shells under combined loading. Composite Structures, 80(1), 49–64. doi: https://doi.org/10.1016/j.compstruct.2006.02.031
4. Teng, J. G., Barbagallo, M. (1997). Shell restraint to ring buckling at cone-cylinder intersections. Engineering Structures, 19 (6), 425–431. doi: https://doi.org/10.1016/s0141-0296(96)00087-9

5. Schmidt, H. (2018). Two decades of research on the stability of steel shell structures at the University of Essen (1985–2005): Experiments, evaluations, and impact on design standards. Advances in Structural Engineering, 21 (16), 2364–2392. doi: https://doi.org/10.1177/1369433218756273
6. Zhao, Y., Teng, J. G. (2003). A stability design proposal for cone–cylinder intersections under internal pressure. International Journal of Pressure Vessels and Piping, 80 (5), 297–309. doi: https://doi.org/10.1016/s0308-0161(03)00048-6
7. Iqbal, M. A., Tiwari, G., Gupta, P. K. (2016). Energy dissipation in thin metallic shells under projectile impact. European Journal of Mechanics - A/Solids, 59, 37–57. doi: https://doi.org/10.1016/j.euromechsol.2016.03.004
8. Amabili, M. (2018). Nonlinear vibrations and stability of laminated shells using a modified first-order shear deformation theory. European Journal of Mechanics - A/Solids, 68, 75–87. doi: https://doi.org/10.1016/j.euromechsol.2017.11.005
9. Akimov, D. V., Gryshchak, V. Z., Gomenyuk, S. I., Larionov, I. F., Klimenko, D. V., Sirenko, V. N. (2016). Finite-Element Analysis and Experimental Investigation on the Strength of a Three-Layer Honeycomb Sandwich Structure of the Spacecraft Adapter Module. Strength of Materials, 48 (3), 379–383. doi: https://doi.org/10.1007/s11223-016-9775-y
10. Degtyarenko, P. G., Grishchak, V. Z., Grishchak, D. D., Dyachenko, N. M. (2019). To equistability problem of the reinforced shell structure under combined loading. Space Science and Technology, 25 (6 (121)), 3–14. doi: https://doi.org/10.15407/knit2019.06.003
11. Degtyarev, M. A., Shapoval, A. V., Gusev, V. V., Avramov, K. V., Sirenko, V. N. (2019). Structural Optimization of Waffle Shell Sections in Launch Vehicles. Strength of Materials, 51 (2), 223–230. doi: https://doi.org/10.1007/s11223-019-00068-7
12. Degtyarenko, P. G., Gristchak, V. Z., Gristchak, D. D., Dyachenko, N. M. (2020). Statement and basic solution equationsof the stability problem for the shell-designed type “barrel-revived” under external pressure. Problems of Computational Mechanics and Strength of Structures, 1 (30), 33–52. doi: https://doi.org/10.15421/4219025