Physical models of the formation of ballistic and muzzle waves generated during an artillery shot have been developed and investigated. A promising method for assessing the degree of wear of artillery barrels is the acoustic non-contact method. However, its implementation requires separate records of the ballistic and muzzle waves. A series of physical models have been developed to assess the possibility of such a recording. A model for calculating parameters of a ballistic wave accompanying an artillery shot has been built. The proposed model features replacement of the problem of spatial axisymmetric streamlining the shell surface by the problem of plane streamlining the wedge. The model makes it possible to determine the value of the angle of inclination of the oblique shock to the direction of the oncoming flow depending on the Mach number. Calculation of pressure of the powder gases flowing from the muzzle section of the barrel behind the shell is based on the application of the law of energy conservation for compressed powder gases. This avoids solving the complex modified Lagrange problem. Calculations show that the muzzle wave pressure changes in the range (30...300) MPa. A physical model of the muzzle wave propagation at the initial stage of the outflow of powder gases from the bore was proposed. During propagation of the muzzle wave, a situation is possible at an initial stage in which this wave reaches the recording point before the ballistic wave. This situation can occur if the range angles and the wedge taper are small. This phenomenon can be avoided by appropriate angle selection. The proposed model determines the law of propagation of the muzzle wave and makes it possible to estimate the rate of its attenuation. It has been established that measuring microphones recording the actual ballistic wave can be located at distances of 50÷500 m from the barrel. The developed models are useful in practice. It is possible to estimate the initial speed of the shell and the degree of barrel wear by separate recording the ballistic and muzzle waves

Keywords: artillery shot, ballistic wave, muzzle wave, recording of acoustic signals, microphone

UDC 519.63
DOI: 10.15587/1729-4061.2020.209847

DEVELOPMENT OF PHYSICAL MODELS FOR THE FORMATION OF ACOUSTIC WAVES AT ARTILLERY SHOTS AND STUDY OF THE POSSIBILITY OF SEPARATE REGISTRATION OF WAVES OF VARIOUS TYPES

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Received date 21.05.2020
Accepted date 30.07.2020
Published date 31.08.2020

1. Introduction

Powerful acoustic fields are formed with each artillery shot. These fields are mainly formed by two waves: ballistic (accompanying the shell flight) and muzzle (created by powder gases flowing out of the gun barrel under high pressure). To determine coordinates of the firing gun, acoustic waves can be recorded by a system of spaced microphone sensors. In particular, the sonic artillery reconnaissance which has been widely used in practice for over a hundred years is based on this principle [1]. Another trend that has emerged in the last decade implies the recording of ballistic and muzzle waves formed by firing at fairly short distances (less than 500 m) from a gun firing position. Analysis of temporal and spectral characteristics of the thus recorded acoustic signals makes it possible to obtain a kind of «acoustic portrait» (or «acoustic signature») of the «artillery gun – shell» system using acoustic fields formed by shots [2]. According to [3], analysis of acoustic portraits of artillery systems can be used to assess the gun type, the shell caliber, and other data characterizing the gun. This, in turn, opens up the opportunity to form a new economical technology for assessing the artillery barrel wear [4]. Methods of acoustic assessment of the barrel state are simple, economical, requiring no expensive equipment and applicable in field conditions.

However, all methods of analyzing signals of ballistic and muzzle waves from a shot to assess the level of barrel wear require separate recording of waves by means of a recording device. There are detailed monographs on the formation of acoustic fields during firing [5] and fairly detailed studies on this topic [6]. Nevertheless, there are no adequate models of the formation of ballistic and blast waves and the dynamics of their behavior during the first seconds after the shot. This makes it impossible to develop a pattern of arrayal of recording microphones in such a way as to avoid the superposition of acoustic signals and have a clear idea of the nature of the measured audio signals. Therefore, the study of physical models of the formation of acoustic waves in artillery firing and the possibility of their separate recording is a rather urgent scientific and applied problem.

2. Analysis of published data and problem statement

Assessment of the degree of wear of an artillery barrel is an important scientific and applied problem [7]. The barrel is a gun part constantly experiencing extreme dynamic loads. Therefore, timely assessment of reaching its maximum permissible wear level is a pre-requisite for maintaining combat capability and effective use of an artillery unit [8]. Calculation methods widely used today in assessing barrel wear are based on determining the residual barrel life using statistical models and history of the gun's combat use [9]. However, computational methods are rather laborious [10] and the reliability of their results in relation to a specific gun was not confirmed [11]. An option for overcoming these disadvantages implies the application of instrumental methods of actual examination of the inner bore surface in order to assess wear using special gauges [12]. At the same time, instrumental methods of wear monitoring are characterized by high labor intensity and insufficient accuracy of wear assessment [13]. In the last decade, modern high-tech endoscopic methods of barrel inspection have been developed and brought to a robotic level [14]. These methods make it possible to obtain complete and accurate information on the degree of wear of the barrel bore, however, they have disadvantages: the high cost of required laser equipment and considerable labor intensity. In addition, all of the abovementioned methods have a major drawback: their result depends on the level of training of the person making the decision about the wear degree, that is, the qualifications of the operator making measurements. Existing automatic methods for assessing barrel wear are based on assessing the barrel condition by accurate measuring of the initial speed of the shell at the time point of its departure using specialized radar devices. These devices are centimeter- or millimeter-wave Doppler radars which provide a high-precision prompt measurement of initial shell speed. The main disadvantage of ballistic radars is associated with the high equipment cost [15, 16]. The above analysis allows us to establish the following. The existing methods for assessing the degree of wear of artillery barrels are diverse, however, the methods available for widespread use in the troops are ideologically outdated, provide low assessment accuracy, and are practically unsuitable for automation. The high-tech methods require the use of expensive equipment which is an obstacle to their practical implementation in military units. An alternative procedure for assessing the degree of the barrel wear includes a method based on assessing the initial shell speed derived from the acoustic fields accompanying the shot [4]. The method is fast, inexpensive, and easily automated. However, for its practical implementation, the measuring sensors (microphones) must be spaced in a special pattern [4]. First, a ballistic blast wave and then (already in the Mach cone formed by the ballistic wave) a muzzle blast wave that has not yet degenerated into a sound wave should be recorded. To assess the wear of the artillery barrel at the stage of development of a real measuring system, it is necessary to determine at what distance from the gun in the fire line it is necessary to place measuring microphones. Therefore, to ensure acoustic (more precisely, blast-acoustic) measurements, it is necessary to construct theoretically physical models of gas outflow from the barrel bore at relative proximity (up to 500 m) to the gun in the fire line within a short (up to 0.5 s) time interval after the shot. It is also necessary to estimate amplitudes of the blast waves generated during the shot.

3. The study objective and tasks

The study objective implies the construction of physical models of gas outflow from the gun barrel during the shot and formation of ballistic and muzzle waves. This will make it possible to obtain adequate estimates of ballistic and blast wave parameters. This will also make it possible to develop requirements for the layout of the microphones recording these waves in order to escape the overlapping of acoustic signals.

To achieve thy щиоусешму, the following tasks were set:

– obtain a theoretical estimate of the ballistic wave parameters;

– calculate the pressure of powder gases flowing from the muzzle section of the barrel after the shell to determine initial parameters of the muzzle wave;

– model the process of propagation of the muzzle wave to determine its decay rate as a blast wave in order to calculate time from the time point of the shot and the distance from the gun at which this wave acquires properties of a linear acoustic wave.

4. Theoretical estimation of the ballistic wave parameters

A shot from an artillery gun is accompanied by a number of acoustic phenomena. Let us assume that the obturation problem is solved, that is, there is no breakthrough of powder gases between the barrel wall and the shell (otherwise, the powder gas overtaking the shell is a significant source of sound at the shot).

In this case, two physical processes play the main role.

1. An air blast wave generated by the shell movement at supersonic speed (about 1000 m/s) and called a ballistic wave. The blast wavefront is a surface on which all parameters of the air medium abruptly change their values. A blast wave arises under a powerful impulsive loading of the medium [17]. When formed at the time point of the shot, the amplitude of the ballistic blast wave is not very high (since the speed of the shell exceeds the speed of sound by only 2...4 times). As the shell flies, the amplitude of the ballistic blast wave attenuates in a proportion to a square of the Mach number *M* of the shell. However, the ballistic wave remains a blast wave as long as the shell continues to move at the supersonic speed together with the shell. In this case, the kinetic energy of the shell movement «feeds» the blast wave. Acoustic signal of a ballistic wave is an *N*-shaped pulse lasting for 2...5 ms and its energy spectrum is broadband in the frequency range from 10 Hz to 500...700 Hz. A ballistic wave can only be detected within the Mach cone formed by the shell flying at the supersonic speed.

2. The sound pulse generated by the powder gases escaping from the barrel immediately after the shell forms a wave which is called a muzzle wave [18].

The muzzle wave has a very high intensity at the moment of its formation (pressure of powder gases at the muzzle section of the barrel is measured in hundreds of atmospheres). Hereinafter, an off-system unit of pressure measurement, an atmosphere, will be used to demonstrate magnitudes of the blast wave amplitudes in comparison with normal atmospheric pressure. However, it weakens very quickly and turns first into a linear sound wave and then fades out. Excessive muzzle pressure gradually drops to atmospheric pressure. The muzzle wave in the linear section of propagation has the form of a damped oscillatory process with a duration of 20...50 ms and an energy spectrum width of 40...60 Hz.

The gas-dynamic flow formed as a result of the appearance and propagation of a muzzle wave and its interaction with a ballistic wave is of very complex nature. A detailed calculation of such a movement is hardly possible even with the use of the fastest modern computers. However, since the nature of the physical processes occurring during the shot is quite clear, a number of specific calculations can be carried out.

First of all, the parameters of the ballistic wave were studied. It has been taken into account that the shell head has a conical shape (Fig. 1). This reduces air resistance. The gas streamlining of the shell is axisymmetric.

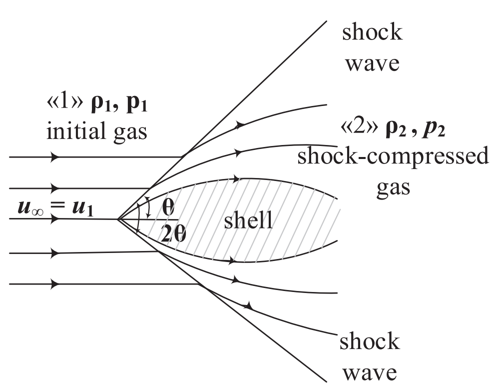

Fig. 1. The pattern of air streamlining the shell

The problem of a vortex-free axisymmetric streamlining a circular cone has been solved in principle [19] but the model with the help of which analytical solutions can be obtained is cumbersome. The solution can be simplified by replacing the round cone with a flat wedge with an appropriate angle of taper (Fig. 2).

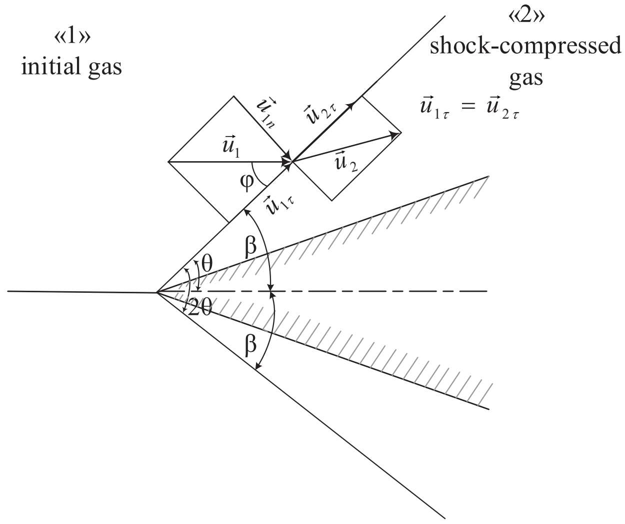

Fig. 2. Streamlining of a wedge and an oblique shock

In Fig. 2, speed $\vec{u}_1$ is the speed of airflow climbing on the wedge in the moving coordinates associated with the wedge. When applied to the problem under consideration, $u_1=|\vec{u}_1|$ is actually the magnitude of the shell speed. Speed $\vec{u}_2$ is the speed of air streamlining the wedge behind the surface of oblique shock (the vector $\vec{u}_2$ is directed along the wedge side). The vectors $\vec{u}_1$ and $\vec{u}_2$ can be decomposed into normal and tangent components (with respect to the surface of the oblique shock). In this case, $u_{1\tau}=u_{2\tau}$, that is tangential components of velocities $\vec{u}_1$ and $\vec{u}_2$ are equal to each other. This is the basis of the standard pattern of calculating parameters of the oblique shock that occurs when supersonic airflow is streamlining a wedge.

Such a wedge is a simplified model of a longitudinal plane section of the shell. Streamlining of a shell (which has the most streamlined shape) is still different from streamlining of a circular cone and the pattern of such streamlining is more complicated. However, this simplification seems to be acceptable for the following reasons. First, for further calculations and reasoning, only parameters of oblique shock formed on the shell «nose» during its movement are of interest and not the detailed picture of the shell streamlining. Secondly, the simplifications made below in calculation of propagation of the muzzle wave are more «coarse» than replacement of the shell streamlining by a round cone streamlining with a subsequent replacement of the spatial axisymmetric flow by a plane flow and the cone by a wedge.

From equation (1) [20]:

$$\sin\beta\left[\sin\beta-\frac{\gamma+1}{2}\frac{\sin\theta}{\cos(\beta-\theta)}\right]=\frac{1}{M_1^2}, \tag{1}$$

where angle θ is a half-angle of the wedge taper and the shell «nose»; $M_1=u_1/a_1$ is the Mach number ($M_1>1$); a_1 is the speed of sound in the air not compressed by the wave; u_1 is the speed of the airflow surging the shell (or the shell speed in the air); γ is the air polytropic index (γ=1.4).

It is possible to determine the value of the angle of inclination β of the oblique shock with respect to the direction of the surging flow. For example, under normal conditions (pressure p_1=1 atm=101,325 Pa, temperature T_1=0 °C=273 K), shell speed u_1=1000 m/s and angle θ=15° for angle β, the estimate of $\hat{\beta}=33°$. is right for the angle β.

Pressure p_2 and density ρ_2 behind the front of the oblique shock are determined from the relations using classical formulas for calculating parameters of an oblique shock [21]:

$$\frac{p_2}{p_1}=\frac{2\gamma}{\gamma+1}M_1^2\sin^2\beta-\frac{\gamma-1}{\gamma+1}, \tag{2}$$

$$\frac{\rho_2}{\rho_1}=\frac{\gamma+1}{2}M_1^2\sin^2\beta\left(1+\frac{\gamma-1}{2}M_1^2\sin^2\beta\right)^{-1}. \tag{3}$$

For the above values of p_1, T_1, u_1 we have: p_2=298,700 Pa≈≈3 atm and ρ_2=2.09 kg/m^3 (at ρ_1=1.29 kg/m^3). Thus, the ballistic wave is a blast wave of not too high intensity. The considered model in the form of expressions (1)–(3) makes it possible to determine the parameters of the ballistic wave. Thus, the first task of the study can be considered solved.

Note that the above method for calculating parameters of a ballistic wave (1)–(3) cannot be applied in cases when $\theta>\theta_{max}$ where θ_{max} is a certain angle that depends on M_1. The smaller the M_1, the smaller the region of permissible angles 2θ of the wedge taper at which an oblique shock occurs resting on this wedge tip. However, in the considered situation of the shell flight, magnitudes of M_1 are quite large since shells are intentionally manufactured in such a way that the condition $\theta\leq\theta_{max}$ is observed for practically all firing conditions. If this condition is violated for some reason or the shell head is dull (rounded), then the blast wave deviates from the shell creating a very strong resistance to its movement. Calculation of parameters of such a blast wave is a much more complicated problem that can only be solved numerically and under a number of additional assumptions [22].

5. Calculation of pressure of powder gases flowing from the muzzle section of the barrel behind the shell

The muzzle wave is formed as early as the shell separates from the gun barrel when the compressed powder gas held back until this moment by the rear surface of the shell is ejected into the atmosphere. To study the process of propagation of the muzzle wave, it is necessary to know the muzzle pressure p_d, i. e., pressure of the powder gases at the muzzle section at the moment of complete exit of the shell from the barrel. In turn, the determination of the muzzle pressure p_d requires solving the problem of internal ballistics.

Let us consider the problem of internal ballistics in the following statement. The barrel is modeled by a half-open bore with a numerical axis Ox directed along the bore axis (Fig. 3).

Cross-sectional area S of the bore is constant along its entire length. Section of the bore corresponding to the segment $[-L, 0]$ is filled with compressed powder gases at pressure p_* and temperature T_*. In front of the shell, the bore is filled with air at pressure p_1 and temperature T_1 that can be considered equal to the corresponding values of atmospheric air outside the bore. It can be assumed that the barrel was preheated by previous shots and temperature and accordingly air pressure in the barrel space ahead of the shell are slightly higher than those of the normal atmosphere. However, as it will be shown below, this has practically no effect on the further solution to the problem.

Suppose that the shell has a mass m and the current coordinate of the rear surface of the shell is X_s. Then $\dot{X}_s\equiv dx/dt$ is the speed of the shell and $\ddot{X}_s\equiv d^2x/dt^2$ is its acceleration.

Let us consider the forces acting on the shell as it moves.

The shell's driving force is the force of pressure of powder gases equal to pS at the initial time point, $p=p_*$. The shell movement is impeded by the air resistance force $-p_1S$ (the sign «–» indicates the direction in which the force is acting). Add these two forces to have an expression $(p-p_1)$ S. It is obvious that we have an inequality $p\gg p_1$. This is true since the forcing pressure (the minimum pressure at which the shell starts its movement) is approximately equal to 250...500 atm. It depends primarily on the design and materials of the shell belt and the barrel rifling. The pressure p_1 is close to 1 atm, so it can be neglected. When the movement of the shell along the bore gets supersonic, a blast wave is formed in front of the shell and then transformed into the above described ballistic wave when the shell exits the barrel. Moreover, if this blast wave slightly departs from the shell head and the air resistance pressure increases to the magnitude of pressure of the shock-compressed gas p_2, then, according to the above calculations, p_2 exceeds p_1 by only 2...4 times, so this pressure can also be neglected in the expression $(p_*-p_2)S$. Thus, we can assume that the shell moves under the action of the pressure force p_*S of the powder gases without a counterpressure.

Movement of the shell is significantly hampered by the force $-F_{fr}$ (F_{fr}>0 is the absolute value of this force). This force results from cutting off the leading belt into the bore rifling (a kind of «frictional force» of the shell against the inner surface of the barrel).

Thus, the equation of motion of the shell along the bore takes the form:

$$m\ddot{X}_s=pS-F_{fr}, \tag{4}$$

where p is the pressure of powder gases on the moving rear surface of the shell, $x=X_s$.

In this case, initial conditions are as follows:

$$X_s(0)=0,\ \dot{X}_s(0)=0. \tag{5}$$

Obviously, the problem with initial conditions (5) for a second-order ordinary differential equation (4) cannot be solved by itself since the pressure p is a function of X_s and time, and F_{fr} is also a function of X_s. But if the magnitude of F_{fr} can be considered with a small error as an averaged constant value, then

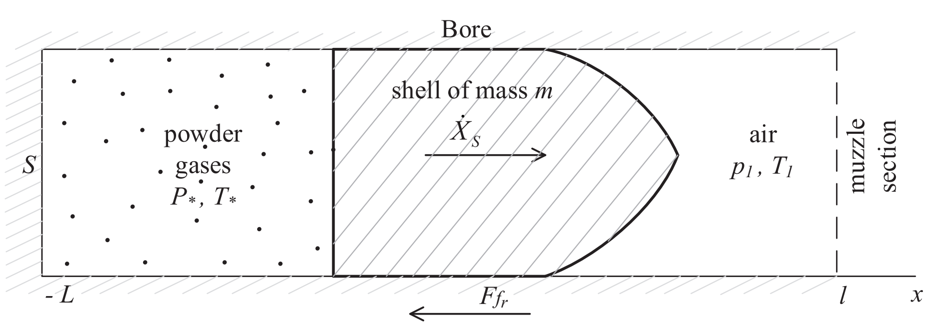

Fig. 3. Movement of the shell along the bore

the pressure p behind the shell changes continuously and significantly.

Under the assumption that the thermodynamic process in the powder gases behind the shell is adiabatic one, the gas flow in the region $0 \le x \le X_s(t)$ is described by a system of one-dimensional differential equations of continuity and the Euler equation, as well as the Poisson adiabatic equation. This system takes the form

$$\begin{cases} \dfrac{\partial \rho}{\partial t} + u\dfrac{\partial \rho}{\partial x} + \rho\dfrac{\partial u}{\partial x} = 0, \\ \dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + \dfrac{1}{\rho}\dfrac{\partial p}{\partial x} = 0, \\ \dfrac{p}{\rho^{\gamma}} = \dfrac{p_*}{\rho_*^{\gamma}}, \end{cases} \tag{6}$$

where $p(t, x)$, $\rho(t, x)$, γ=const are the pressure, density, and ratio of heat capacities of the gas (the index of polytrope of the mixture of powder gases), respectively.

Initial conditions for the system (6) take the form

$$p(0,x) = p_*, \;\; \rho(0,x) = \rho_*, \;\; u(0,x) = 0$$

at

$$-L \le x \le 0. \tag{7}$$

Boundary conditions:

$$u(t,-L) = 0, \tag{8}$$

$$u(t,X_s) = \dot{X}_s. \tag{9}$$

Boundary condition (8) is a typical condition of speed equality to zero on a rigid wall.

Boundary condition (9) is met if the shell does not accelerate so quickly that a vacuum zone is formed between it and the powder gases. Formally, this option is possible when solving the classical problem of pushing the piston out of the gas, but in fact, it cannot be realized in this problem for two reasons. First, for the formation of a vacuum zone between the shell and the powder gases, the shell must accelerate to a speed $2a/(\gamma-1)$, where a is the speed of sound in the powder gases formed by an explosion at their temperature about 3000...3500 K. Therefore, the value of a exceeds the speed of sound 3 times or more under normal conditions. Taking into account the coefficient $2/(\gamma-1)$, the shell speed should exceed the normal speed of sound by about 15 times which is obviously impossible. Second, it is assumed in the classical problem of piston motion that the piston can move spontaneously, according to any law, and with any speed and movement of the shell is supported by the movement of powder gases expanding behind it. Therefore, given the strong resistance to movement of the shell along the bore, if the shell even accelerated to speed $2a/(\gamma-1)$, it would have to «slow down».

Problem (4)–(9) is a closed joint problem of theoretical mechanics and gas dynamics. At the same time, a number of simplifications were made. The main one of them is that explosive combustion of powder causing the formation of compressed gases in the powder chamber is instantaneous after which the charge starts to move. In reality, when the shell begins to move in the bore, powder continues to burn, and the pressure of the powder gases increases. This factor could be taken into account by changing initial conditions (5) of the Cauchy problem for conditions:

$$X_s(t_0) = l_0, \;\; \dot{X}_s(t_0) = U_0, \tag{10}$$

where t_0 is the time point of complete powder combustion; l_0 is the distance covered by the shell from the moment it starts moving t=0 to the time point t_0; U_0 is the speed of the shell at the time point t_0. However, next, it is necessary to separately determine values of t_0, l_0 and U_0 additionally replacing initial conditions (7) in the gas-dynamic problem by the conditions:

$$p(t_0,x) = p_{**}, \;\; \rho(t_0,x) = \rho_{**}, \;\; u(t_0,x) = U_0$$

at

$$-L \le x \le l_0. \tag{11}$$

In this case, values of p_{**}, ρ_{**} (some values of density and pressure averaged over x in the interval $[-L, l_0]$ at the time point t_0) must also be calculated in advance. To correctly determine conditions (10) and (11), it is necessary to solve a problem even more complicated than the problem (4) to (9).

Speaking about other simplifications in the problem statement (4) to (9), exclusion from consideration of such phenomena as removal of powder gases and rollback of the gun or its barrel during the shot should be mentioned.

The assumption that the powder gas expands according to the adiabatic law $p/\rho^{\gamma} = \text{const}$ is completely correct since the shell passes extremely quickly the section of the path from the movement start to the muzzle section (the shot lasts only for 0.01...0.1 s), so the heat losses into the barrel walls during the shell movement can be neglected.

The problem (4) to (9) is a somewhat modified Lagrange problem [23, 24]. This problem can be solved analytically under a number of assumptions but taking into account just one reflection of a centered rarefaction wave (whose head initially moves through the compressed powder gases at the speed of sound) from the rear wall of the charge chamber $x=-L$ [25]. However, such a solution is not very suitable for real calculations since the rarefaction wave is actually repeatedly reflected from the rear wall of the charge chamber and the rear surface of the shell more than once. Interaction of forward and backward waves is very complicated, so the problem can be solved only by numerical methods of gas dynamics and it is very difficult to obtain such a solution. The main problem is in the fact that the right boundary of the gas flow region $L \le x \le X_s(t)$ for which calculations are performed and on which the boundary condition (9) is satisfied is mobile and the law of this motion is determined by the gas flow itself in this area.

However, what is important in the problem (4) to (9) as the final result, it is not the general picture of the gas flow in the space between the rear surface of the charge chamber and the rear surface of the shell. The main interest is the magnitude of pressure of the powder gases p_d in the muzzle section at the time point when the shell leaves the gun barrel and the magnitude of density ρ_d corresponding to this pressure. The approximate value of pressure p_d can be estimated without solving the problem (4) to (9) from simple energy considerations.

Indeed,

$$E_* = \frac{p_*}{\rho_*(\gamma-1)}\rho_* SL = \frac{p_* SL}{\gamma-1} \tag{12}$$

is the total energy of the powder gases at the initial time point (it coincides with the value of the internal energy of the gases).

At the time point when the shell completes its exit from the gun barrel, the internal energy of the powder gases E_i is approximately calculated from the following formula:

$$E_i = \frac{p_d S(L+l)}{\gamma - 1}, \tag{13}$$

where p_d is the sought pressure.

At the same time, the powder gases acquire kinetic energy E_k. This is defined as a product of the mass of powder gases by half the square of their speed. Since it is assumed that there are no breakthroughs of the powder gases between the barrel wall and the shell and removal of the powder gases is not taken into account, it can be assumed that the mass of the powder gases remains constant and equal to $\rho_* SL$. Then kinetic energy E_k of the powder gases at the time point when they leave the barrel after the shell is approximately calculated as follows:

$$E_k = \frac{\rho_* SL u_d^2}{2}, \tag{14}$$

where u_d is the shell speed upon exiting the barrel. The speed of the powder gases is equal to the shell speed since the boundary condition (9) must be met and the vacuum zone between the powder gases and the rear surface of the shell is not formed as mentioned above.

Since the total energy of powder gases at the moment the shell leaves the barrel is equal to the sum of their internal energy E_i and kinetic energy, that is,

$$E = E_i + E_k, \tag{15}$$

then the following relation takes place:

$$E = \frac{p_d S(L+l)}{\gamma - 1} + \frac{\rho_* SL u_d^2}{2}. \tag{16}$$

It is quite obvious that $E < E_*$. The initial total energy of the gases formed as a result of the combustion of gunpowder is spent on performing work to overcome the action of the force F_{fr} (this work magnitude is $F_{fr}l$) and impart kinetic energy $mu_d^2/2$. to the initially resting shell. The kinetic energy of its rotational motion which is small compared with the kinetic energy of translational motion is not taken into account.

Thus, taking into account equations (12) and (16), the following balance equation expressing the energy conservation law can be set up:

$$\frac{p_* SL}{\gamma - 1} - \left[\frac{p_d S(L+l)}{\gamma - 1} + \frac{\rho_* SL u_d^2}{2}\right] = F_{fr} l + \frac{m u_d^2}{2} \tag{17}$$

or

$$\frac{p_* SL}{\gamma - 1} = \frac{p_d S(L+l)}{\gamma - 1} + F_{fr} l + \frac{(m - \rho_* SL) u_d^2}{2}. \tag{18}$$

The following is obtained after simple transformations:

$$p_d = \frac{p_* L}{L+l} - \frac{(\gamma - 1) F_{tr} l}{S(L+l)} - \frac{(\gamma - 1)(m - \rho_* SL) u_d^2}{2S(L+l)}. \tag{19}$$

If the mass of the powder gases $\rho_* SL$ which is small in comparison with the mass of the shell m is neglected, then equation (19) takes the form:

$$p_d = \frac{p_* L}{L+l} - \frac{(\gamma - 1) F_{tr} l}{S(L+l)} - \frac{(\gamma - 1) m u_d^2}{2S(L+l)}. \tag{20}$$

Values of F_{fr} and u_d can be considered known from the solution of internal ballistics problems.

Density ρ_d is easily determined from the condition that the flow is isentropic. Since:

$$\frac{p_d}{\rho_d^\gamma} = \frac{p_*}{\rho_*^\gamma}, \tag{21}$$

then

$$\rho_d = \rho_* \left(\frac{p_d}{p_*}\right)^{\frac{1}{\gamma}}. \tag{22}$$

Calculations using relation (20) show that pressure p_d depends on the powder charge which determines the value of p_* and ρ_*. It also depends on the shell characteristics (primarily, on its mass and the leading belt) and the barrel (caliber and rifling that determine F_{fr}). Pressure p_d can vary in a range of 300...3,000 atm.

Thus, the obtained equation (20) in combination with formula (22) makes it possible to determine initial pressure and density of powder gases in the muzzle wave. Therefore, the second task of the study can also be considered solved.

6. Modeling the muzzle wave and determining the rate of its attenuation

At the first moment after a complete exit of the shell from the bore, the powder gas escaping into the atmosphere can be modeled as a cylindrical jet of compressed gas. This jet with pressure p_d is surrounded by atmospheric air and hits the rear surface of the shell. Due to this, the speed of the shell continues to increase for some time (for the period of aftereffect of gases on the shell). It is obvious that this gas cylinder becomes a source of a powerful blast wave propagating into the atmosphere. In this case, rarefaction waves travel along the cylinder itself both from the side of the rear surface of the shell (which plays the role of a piston exiting the gas) and from the side of atmospheric air compressed by the ballistic wave.

The flow of gas at the muzzle section is extremely complex in the first time point after the shell release. It can be assumed that the cylinder of initially immobile compressed gas having an infinite length decomposes into a vacuum. Certainly, the counterpressure p_2 (the pressure in the ballistic wave) can be neglected in this case, because it is very difficult to calculate such a movement even in the absence of a shell. The reason is that the problem is not self-similar: it has a characteristic dimension: diameter of the gas cylinder determined by the bore caliber. The presence of a moving shell and movement of the gas jet behind it violate the problem of uni-dimensionality. In addition, rotation of the shell begins to play a noticeable role at this stage creating vortices in the gas flow. Rifling of the barrel bore from which the powder gases flow out causes turbulence as well.

Gradually, a cloud of compressed gas is formed at the muzzle section becoming spherical in its shape. Later, the movement of the gas is determined by the law of explosion in a finite volume which (at a considerable distance from the gun) can be modeled as a point explosion.

Consequently, propagation of the muzzle wave is determined at the initial stage by the law of a point explosion [26]. In this case, the following can be taken as an estimate of the explosion energy:

$$E_d = \frac{p_d \rho_* SL}{(\gamma - 1)\rho_d}, \tag{23}$$

which was obtained by multiplying the mass of the powder gases by their specific internal energy $p_d/((\gamma-1)\rho_d)$ at the moment of the gas escape from the barrel.

It should be borne in mind that the muzzle wave propagates through the gas with parameters p_2, ρ_2 already compressed by the ballistic blast wave, and not through the gas with normal atmospheric parameters and p_1, ρ_1. However, a point explosion, in this case, can still be considered at the initial stage as an explosion without backpressure (p_d>>p_2).

The law of motion of the front of the muzzle blast wave, in this case, takes the following form [27]:

$$r = \left(\frac{E_d}{\alpha \rho_2}\right)^{1/5} t^{2/5}, \tag{24}$$

and its speed D is determined from the following formula:

$$D = \frac{2}{5}\left(\frac{E_d}{\alpha \rho_2}\right)^{1/5} t^{-3/5}, \tag{25}$$

where r is the distance of the front of the muzzle blast wave from the muzzle section and the constant α is calculated as follows provided that the flow behind the blast wave is adiabatic [27]:

$$\alpha = 0.31246(\gamma - 1)^{-(1.409+0.11735\lg(\gamma-1))}, \quad (1.2 \le \gamma \le 2). \tag{26}$$

Pressure p at the front of the muzzle blast wave falls according to the law:

$$p = \frac{2\rho_2 D^2}{\gamma + 1}, \tag{27}$$

because $D \sim t^{-6/5}$.

Formulas (24) to (27) are only valid in relative proximity to the point of origin of the muzzle blast wave, under the assumption that the blast wave is so strong that the counterpressure can be neglected. However, as it follows from the same formulas, the speed of the wave propagation and its intensity decrease very quickly. Counterpressure plays an increasing role, the muzzle blast wave gradually reaches the asymptotic damping mode and transforms into a linear sound wave.

During movement of the muzzle wave, a situation is possible at the initial stage in which this wave «overtakes» the ballistic one (that is, it arrives at the observation point simultaneously with the latter) since $D>u_1\sin\beta$. Physically, this is understandable: the muzzle wave propagates through the gas already compressed by the ballistic blast wave. Moreover, the pressure p_d which initiates the muzzle wave is very high. Whether the muzzle wave and the ballistic wave overlap at the observation point or not depends on a number of factors: in addition to thermo-dynamic and gas-dynamic parameters, geometric factors also play a significant role: the range angle and the angle θ. Obviously, this phenomenon definitely takes place if these angles are small. In the case when the muzzle wave is superimposed on the ballistic one, it is necessary to separately solve the problem of interaction of two waves. It is extremely complicated because of the fact that the front of the ballistic wave has a plane shape, and the front of the muzzle wave is close to a spherical shape. However, it is obvious that this phenomenon can be avoided by an appropriate selection of the range angle (the angle θ is determined by the shell shape).

In the absence of interaction between the muzzle and ballistic waves, propagation of the muzzle wave can be modeled as follows:

– until the time point t_{st} when the value of pressure p at the front of the muzzle wave is formally reached such that $p>p_d$ (the criteria for fulfilling this inequality can be different), the muzzle wave is calculated using formulas (24) to (27);

– starting from the moment of time t_{st} (at the radius of the wavefront r_{st}), the blast muzzle wave is considered weak and decays according to the well-known law [21]:

$$\frac{p - p_1}{\rho_1 a_1^2} = \sqrt{\frac{4}{\gamma+1}\frac{Q}{r_{st}\rho_1}}\frac{r_{st}}{r}\frac{1}{\sqrt{\ln\frac{r}{r_{st}}}}, \tag{28}$$

where

$$Q = \int_0^{+\infty} \rho u(r_{st}, t)\mathrm{d}t = \text{const} > 0, \tag{29}$$

ρ, u are speed and density distributions in the muzzle wave;

– starting from the moment of time t_{ac} when the muzzle wave can be considered linear acoustic (but this acoustic wave has a sufficiently high amplitude).

Note that parameters of the gas ahead of the muzzle wavefront in equation (28) have the index «1». This gas corresponds to atmospheric air under conditions close to normal. The index «2» corresponds to the air compressed by the ballistic wave since the action of the ballistic wave gone ahead is no longer felt up to the time point t_{st}.

The model represented by expressions (24) to (29) solves the third problem of the study: it determines the law of propagation of the muzzle wave and makes it possible to estimate the rate of its decay.

To carry out concrete calculations, let us establish one more relation. It immediately follows from equations (24), (25) that:

$$r = r_0\left(\frac{t}{t_0}\right)^{2/5} = r_0 t_0^{-2/5} t^{2/5}. \tag{31}$$

Let us consider for example a shot from a 76 mm gun with the charge chamber length L=304 mm=0.304 m, the barrel length l=3,680 mm=3.68 m and the chamber volume 3.56 dm^3=3.56·10^{-3} m^3. It is assumed that the mass of the powder charge is 1.08 kg, and the heat of powder combustion is 3.8·10^6 J/kg. The shell mass m is 7.1 kg, and the shell speed u_d at the exit from the barrel is 710 m/s.

In this case, when calculating parameters of the ballistic wave using formulas (1) to (3), the following estimates were obtained for normal conditions and the angle θ=15°; β=42°; ρ_2=3.875 kg/m^3.

It follows from equation (20) that the pressure of the powder gases in the barrel was p_d=3.35·10^7 Pa≈334 atm. In this case, explosion energy E_d is equal to 2.397 MJ=2.397·10^6 J according to (23).

Taking $p=p_d$ and $D=D_0$ in formula (27), D_0=10^7 m/s can be taken as an estimate of initial muzzle wave propagation speed.

The formation of a muzzle wave takes a certain time. The time t_* from the moment when the shell leaves the gun muzzle to the formation of a spherical muzzle blast wave can be estimated as follows:

$$t_* = \frac{l}{u_d}, \tag{32}$$

since the powder gases will flow out of the gun barrel around this time. For the above values of l and u_d, $t_* = 5\cdot 10^{-3}$ s was obtained. During this time, the shell moving at a speed of 710 m/s, traveled approximately 3.6 m and the shell nose was at a distance of 4.1 m from the muzzle section of the gun (the shell length was taken to be 0.5 m).

Let us assume that the speed of the spherical muzzle wave is still D_0=10^7 m/s by the time of its formation. The substitution of this value into formula (25) enables calculation of the value of t_0 corresponding to the time point of formation of the muzzle wave with a corresponding radius if the time is counted from the time point $t_* = 5\cdot 10^{-3}$ s. This value was $t_0 = 4.34\cdot 10^{-11}$ s. The muzzle wave radius corresponding to the time point t_0 and speed D_0 calculated from formula (24) was r_0=0.001 m.

From considerations of the self-similarity of a solution to the problem of a point explosion [9], the following was obtained:

$$\frac{D}{D_0} = \frac{r}{r_0}\left(\frac{t}{t_0}\right)^{-1}, \tag{32}$$

$$D = D_0\left(\frac{t}{t_0}\right)^{-3/5} = D_0\left(\frac{r}{r_0}\right)^{-2/5}. \tag{33}$$

The results of calculations using formulas (31), (33) are given in Table 1.

Table 1

Results of calculations of the radius of propagation of the muzzle wave *r* and its propagation speed *D vs.* time *t*

t (s)	r (m)	D (m/s)
4.34·10^{-11}	0.001	10^7
2.02·10^{-10}	0.05	10^6
9·10^{-8}	0.023	10^5
4·10^{-6}	0.109	10^4
2.02·10^{-4}	0.67	10^3
6.3·10^{-4}	1.06	500

Analysis of Table 1 shows that the muzzle wave cannot be superimposed on the ballistic wave at the recording point. By the time point when the speed of propagation D of the muzzle wave becomes equal to 500 m/s, that is, less than the speed u_d of the shell exit from the muzzle section of the barrel, its radius is only ≈1 m. In this case, already by the moment of the muzzle wave formation, the ballistic wave extending from the shell nose was found in the front of the muzzle wave at about 4.1 m.

Subsequently, the muzzle wave speed decreases very rapidly and the blast wave degenerates into a linear sound wave.

Another approach to the calculation of propagation of a muzzle blast wave is also possible. Let us assume that at the moment of complete exit of the shell from the bore taken as the initial time point, a spherical muzzle blast wave arises with its center in the middle of the muzzle section of the barrel. Then a formal calculation using formulas (24), (25) has given the following results: the wave radius r=0.24 m and D=9.6 m/s at t=10^{-2} s. That is, the muzzle blast wave degenerated into a sound wave faster than in 0.01 s and, again, it did not superimpose on the ballistic wave at the recording point.

8. Discussion of the results obtained in studying the physical models of the formation of acoustic waves during an artillery shot

The problem of the definition of parameters of the ballistic wave accompanying the artillery shot has been solved. The problem solution was more simple in comparison with the known models. The problem of spatial axisymmetric streamlining of a shell surface was replaced by a problem of plane streamlining of a wedge. This greatly simplified solution of the problem. Calculations have been carried out using the constructed model under normal atmospheric conditions and typical parameters of the shell and the shot. They have shown that the ballistic wave in the vicinity of the gun was a blast wave of not too high intensity with an excess pressure not exceeding 3 atm.

To calculate the pressure of powder gases exiting from the muzzle section of the barrel behind the shell, the problem of internal ballistics has been solved. In the proposed statement, the flow model makes it possible to avoid the solution of a complicated modified Lagrange problem. The resulting equations (20) to (22) have made it possible to determine the initial pressure and density of the powder gases in the muzzle wave. Calculations based on the obtained ratios have shown that amplitude of the muzzle wave pressure at the stage of formation can be very high varying in the range (300…3,000) atm.

It should be taken into account that weak acoustic disturbances determined by the nature of the movement of the shell and its design features can propagate inside the Mach cone formed during a supersonic flight of the shell. For example, the secondary (rather weak) blast waves can even form at the shell belt which, however, are close to linear sound waves. This phenomenon was not taken into account in this study.

The results of calculations based on the constructed model have confirmed that the muzzle blast wave decays rather quickly although it has a very high intensity during its formation. Equation (20) in combination with formula (22) makes it possible to determine initial pressure and density of the powder gases in the muzzle wave. The obtained expressions (24) to (29) represent a physical model of the process of the muzzle wave attenuation which makes it possible to estimate time from the moment of shot and distance from the muzzle section of the gun at which the blast wave degenerates into a sound wave.

When it was assumed that the thermal energy of powder gases during combustion in the charge chamber of the gun was

3…6 MJ, the energy of a point explosion was E_d=1.5…5 MJ. In this case, the time of conversion of the blast muzzle wave into acoustic one was 0.001…0.1 s and this occurred at a distance of 10…50 m from the muzzle section of the gun.

These results are useful in the practical creation of systems for assessing barrel wear by acoustic fields of shots [4]. It follows from them that measuring microphones recording the muzzle wave as a blast wave should be located at a distance of at least 30…50 m from the muzzle section but no farther than 300…500 m. In this case, these microphones will always record the ballistic wave first and then the muzzle one.

Thus, parameters of the ballistic and muzzle (blast) waves obtained from the presented model of the flow of powder gases were valid in the case when the muzzle and ballistic waves did not interact with each other.

The study results are not exhaustive. In particular, in the course of the study, the issues of taking into account the state of the bottom layer of the atmosphere were practically not touched upon, although it is known that the speed of sound in the air significantly depends on meteorological parameters. In addition, the frequency-dependent attenuation of acoustic waves in the air was not taken into account at this stage of the study. Evaluation of the effect of air state in the area of the shot on the possibility of superimposing ballistic and muzzle waves at the recording point can be the subject of further studies. Also, it is planned for the future to improve the developed models for larger, up to 203 mm, gun calibers.

8. Conclusions

1. A problem of determining parameters of a ballistic wave accompanying an artillery shot has been solved. The proposed solution of the problem features replacement of the problem of spatial axisymmetric streamlining surface of the shell with a pointed «nose» by the problem of a plane streamlining a wedge with a corresponding taper angle. The wedge is a simplified model of the longitudinal plane axial section of the shell. The model makes it possible to determine the value of the inclination angle β of the oblique shock to the direction of the ongoing flow depending on the Mach number and the gas-dynamic parameters behind the plane of the oblique shock. Concrete calculations were carried out for normal atmospheric conditions and typical parameters of the shell and the shot. They allow us to conclude that the ballistic wave of powder gases in the vicinity of the gun is a blast wave of not too high intensity: the overpressure is about 3 atm.

2. Calculation of pressure of powder gases escaping the muzzle section of the barrel behind the shell at the moment of firing is based on the application of the law of energy conservation to compressed powder gases in the charge chamber. This makes it possible to avoid solving the complex modified Lagrange problem (4) to (9). The calculation results show that depending on the energy of the combustion of the powder charge, characteristics of the shell, and parameters of the barrel, pressure varies in the range 30…300 MPa. Thus, the muzzle wave at the moment of its occurrence is a blast wave of extremely high intensity.

3. A physical model of the muzzle wave propagation at the initial stage of the outflow of powder gases from the bore was proposed. During propagation of the muzzle wave, a situation is possible at the initial stage in which this wave reaches the recording point before the ballistic wave. This situation can occur if the range angles of the gun and the wedge taper are small. However, this phenomenon can be avoided by an appropriate selection of the range angle (the taper angle of the wedge is determined by the shell shape).

In the absence of interaction between the muzzle and ballistic waves, the proposed model determines the law of propagation of the muzzle wave and makes it possible to estimate the rate of its attenuation. Measuring microphones recording the actual ballistic wave can be located at distances of 50…500 m from the muzzle section. At the same time, the initial speed of the shell and the level of barrel wear can be actually estimated with a separate recording of the ballistic wave and the muzzle wave.

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