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A method has been developed for the combined de-orbiting of large-size objects of space debris from low-Earth orbits using an electro-rocket propulsion system as an active de-orbiting means.

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A principal de-orbiting technique has been devised, which takes into consideration the patterns of using an electric rocket propulsion system in comparison with the sustainer rocket propulsion system.

A procedure for determining the parameters of the de-orbiting scheme has been worked out, such as the minimum total speed and the time of the start of the de-orbiting process, which ensures its achievement. The proposed procedure takes into consideration the impact exerted on the process of the de-orbiting by the ballistic factor of the object, the height of the initial orbit, and the phase of solar activity at the time of the de-orbiting onset. The actual time constraints on battery discharge have been accounted for, as well as on battery charge duration, and active operation of the control system.

The process of de-orbiting a large-size object of space debris has been simulated by using the combined method involving an electro-rocket propulsion system. The impact of the initial orbital altitude, ballistic coefficient, and the phase of solar activity on the energy costs of the de-orbiting process have been investigated. The dependences have been determined of the optimal values of a solar activity phase, in terms of energy costs, at the moment of the de-orbiting onset, and the total velocity, required to ensure the de-orbiting, on the altitude of the initial orbit and ballistic factor. These dependences are of practical interest in the tasks of designing the means of the combined de-orbiting involving an electric rocket propulsion system. The dependences of particular derivatives from the increment of a velocity pulse to the gain in the ballistic factor on the altitude of the initial orbit have been established. The use of these derivatives is also of practical interest to assess the effect of unfolding an aerodynamic sailing unit

Keywords: large-sized space debris, combined de-orbiting, electric rocket propulsion system, low orbits

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## 1. Introduction

The low-orbit satellite groups Starlink and Oneweb, launched over recent years, whose quantity is expected to exceed 12,000 units, will make the already busy space environment in low orbits much more difficult. According to [1], its condition is described by about 20,000 observed space objects (SOs). In addition, a relatively recent emergency involved the Starlink-44 and Aeolus satellites, which UDC 629.764

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# DEVELOPMENT OF THE COMBINED METHOD TO DE-ORBIT SPACE OBJECTS USING AN ELECTRIC ROCKET PROPULSION SYSTEM

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forced the latter to maneuver on February 9, 2020, to avoid a collision.

The large-sized non-functioning SOs are even more dangerous [2, 3]: spacecraft and the upper stages of launch vehicles. On the one hand, a collision with them is highly likely to lead to the shutdown of the existing satellite. On the other hand, the many debris created by the collision can lead to the beginning of an avalanche-like increase in the number of near-Earth SOs, called Kessler Syndrome [1, 4, 5]. That would greatly complicate the exploration and use of near-Earth space.

It follows from the above that clearing low-earth orbits from large-sized objects of space debris is one of the most important tasks for the coming decades.

### 2. Literature review and problem statement

At present, the following de-orbiting methods are used and developed to reduce the clogged near-Earth space [6-11]:

 – using a jet propulsion system (JPS) based on the sustainer propulsion (SJPS), low thrust, or electric rocket propulsion systems (ERPS);

- using the aerodynamic and solar sailing units;

- using electrodynamic cable systems;

- the remote braking of an object using directional ion and laser radiation, as well as the creation of an artificial atmosphere;

- using magneto-dynamic systems;

– collecting space debris.

Despite such a variety of methods for clearing the low orbits from debris, the most common of them for large-sized SOs, such as the launch vehicle orbital stages and spacecraft that have completed their mission, is the de-orbiting using the JPS [9–11]. This method makes it possible to remove SOs within a predefined time to the specified target orbits. The main drawback of the method is the significant cost of fuel components needed to enable the de-orbiting.

One possible solution that can be used to reduce the cost of the de-orbiting using JPS is to combine it with another method, such as an aerodynamic sailing unit. This combined method of de-orbiting was addressed in work [12]. JPS is used to form an orbit of de-orbiting with perigee in the dense layers of the Earth's atmosphere. In turn, the aerodynamic sailing unit provides a gradual cessation of existence over 25 years under the influence of the force of aerodynamic resistance of the Earth's atmosphere. The use of SJPS was proposed as a JPS. For the first time, the cited work set and solved the task of minimizing energy costs (the required velocity pulse and the mass of fuel components) depending on the phase of solar activity at the time of the de-orbiting onset. However, the issues of using JPS of other types, except for SJPS, which have their own features of operation, remained unresolved.

A possible option for the implementation of an active method of cleaning near-Earth space is the use of special garbage collectors. Their task is to ensure the de-orbiting of passive objects. Thus, to clean geostationary orbits, paper [13] proposed using a special active dispenser. The authors developed two schemes of de-orbiting objects from geostationary orbits. It was determined that the most effective scheme is the scheme of the de-orbiting of objects to the orbits of burial directly by the dispenser itself. After the de-orbiting, the dispenser returns for the next object to the geostationary orbit. It is shown that six such garbage collectors can solve in eight years the issue of the upper stages of launch vehicles in geostationary orbits. In turn, the authors of report [14] were among the first to propose the use of a garbage collector with EJPS for collecting space debris.

One of the areas of research is the development of cheap means and complexes to remove the objects of space debris. Worth noting among them is the development of a conceptual project of a cheap dispenser with a hybrid JPS for the active de-orbiting of large-sized objects of space debris [15]. The authors designed de-orbiting schemes, as well as the schemes of soft and rigid docking. The dispenser is planned to be operated in conjunction with the Vega rocket and space complex. A given dispenser makes it possible to execute active de-orbiting without additional use of an aerodynamic sailing unit. In addition, research is underway aimed at creating cheap autophase launch vehicles capable of bringing the means of cleaning the near-Earth space. Thus, article [16] addresses the development of the design and ballistic maintenance for the flight of an autophase launch vehicle, taking into consideration the peculiarities of its JPS operation. Works [17, 18] report the results of testing an auto-phase JPS. Research is also underway to find the optimal chemical composition of fuel for the autophase launch vehicles [19].

The most important task of reducing environmental pollution in the process of removing large-sized objects of space debris is to find ways to destroy them in dense layers of the atmosphere. One aspect of this issue related to the search for ways to increase the temperature of the object being diverted in the earth's dense atmosphere is considered in report [20].

The design of the de-orbiting tools using aerodynamic sailing systems is discussed in works [21–24]. Thus, the issue of the development of an aerodynamic sailing unit for the de-orbiting of the upper stages of launch vehicles is considered in [21]. The authors proposed a structural scheme of the sailing unit, found the optimal shape, and studied the influence of the atmosphere on the unit's shell. Paper [22] reports the results of research on the development of the aerodynamic sailing system, the IDEAS prototype, as part of the CNES MICROSSOPE project. The authors defined the optimal geometry of the sailing unit, which would ensure the termination of existence over 25 years. Article [23] addresses the development of the concept of an aerodynamic sailing unit consisting of racks with a membrane stretched between them. The optimal indicators of the unit were selected to stabilize the angular movement and achieve the predefined time of orbital existence. It is shown that the proposed device can ensure the de-orbiting of many objects of space debris. Work [24] considered the probability of the collision of the removed object with an aerodynamic sailing unit and the objects of space debris. The authors proposed a new approach in order to reduce the probability of the aerodynamic sailing unit hitting objects of space debris. It is based, on one hand, on improving the aerodynamic characteristics of the removed object. On the other hand, on the choice of the appropriate date to start the de-orbiting within the current cycle of solar activity. The proposed technique reduces the probability of collision by 5–10 %. One of the ways to stabilize the aerodynamically unstable structures in the process of de-orbiting is the use of controlling elements on permanent magnets [25]. The possibility of using controlling elements on permanent magnets to stabilize the aerodynamically unstable spacecraft was analyzed. The feasibility analysis was carried out and the effectiveness of the proposed solutions was evaluated. The issue was further developed in article [26]. It reports the synthesis of a controller on permanent magnets, providing the two-degree stabilization of an unstable spacecraft with an aerodynamic sailing unit. A controller with the width-pulse modulation was synthesized. The scope of application of permanent magnets for the two-stage stabilization of unstable aerodynamic structures was defined. The authors demonstrated the advantages of using permanent magnets in comparison with electromagnetic stabilization devices.

The de-orbiting of space debris using a solar sail was considered in works [27, 28]. Paper [27] studied the influence of sunlight pressure on a geostationary object of space debris. A mathematical model of the movement of the removed object under the influence of sunlight pressure was built. An approach to managing the orientation of the solar sail is proposed in order to achieve the greatest efficiency. The authors studied the effect of solar pressure on a change in the orbital parameters of the object being taken away. The effectiveness of the use of solar sailing units to remove space debris from geostationary orbits was confirmed. Thus, the removed object can leave the geostationary orbit, without the use of a jet propulsion system, in two years. Article [28] investigated a method of cleaning the middle near-Earth orbits from space debris using a solar sailing unit. The authors developed a strategy of de-orbiting, based on the joint influence of solar light pressure and the second zonal harmonic of the gravitational potential of the Earth. A mathematical model of movement was built. It is shown that for high-inclination orbits, the position of the Sun relative to the orbital plane and the longitude of an ascending node has a significant impact. The areas of the orbits for which a given method is most effective were identified.

Research into de-orbiting using electrodynamic cable systems is reported in works [29, 30]. Article [29] addresses the development of an energy-efficient scheme for managing the de-orbiting of space debris using an electrodynamic cable system. The authors proposed a two-contour control consisting of the circuit of autonomous control (without feedback) and the circuit of operational control (with feedback). Their numerical modeling confirmed that the proposed scheme could ensure the safe, cost-effective, and efficient de-orbiting of space debris into the orbit of the de-orbiting. The construction of a new method for analyzing the dynamics of space debris with a flexible electrodynamic tether is considered in [30]. The authors studied the effect of a moving electric field on the dynamics of the cable system. It is shown that the moving electric field has a significant impact on the dynamics of the tether system in the event of rocking and sagging.

The issues related to the process of de-orbiting using directional ion radiation are considered in works [31-34]. An analysis of the concept of building a de-orbiting means that brakes an object using directional ion radiation is given in [31]. Key aspects of design were highlighted. It was determined that to ensure the quality of operation it is necessary to use ion engines with a beam divergence of fewer than 15 degrees. Article [33] reported a study of the interaction between the removed object and a means of de-orbiting using directional ion radiation. The authors assessed the braking force and torque received by the removed object from the impact of ion radiation. They determined the difference in the electrical potentials between the means and object of the de-orbiting, the reverse scattering of ions, and the contamination of the means of de-orbiting. One option to reduce the energy costs of the de-orbiting involving directional ion radiation is to use a single compensatory engine [34]. To ensure the manageability of the de-orbiting process, the author proposed the introduction of a program of prowling for damping the out-of-plane oscillations. The modeling confirmed the possibility of implementing a given solution, although with an increase in an orbital lifetime.

The development of a methodology for the use of the remote braking method of the object using directed laser radiation is described in [35]. The authors built a mathematical model of the generation and distribution of the laser beam, as well as its impact on the surface of the removed object. A simulation of the de-orbiting process was carried out. The possibility was shown to lower the orbit of the removed object by 2.4 km over two hours of the de-orbiting means operation.

Article [36] proposes to use a magnetodynamic method to ensure the de-orbiting of small satellites. It is shown that the use of electromagnets installed on most spacecraft is sufficient for the process of de-orbiting. The study showed the possibility of stopping the existence of a satellite weighing 10 kg with electromagnetics with a magnetic field intensity of 100  $\text{Am}^2$  in three years.

It should be noted that of all the methods discussed above, only the active method involving JPS and the passive method involving an aerodynamic sail are currently used. In turn, our analysis of the scientific literature reveals that there remain unresolved issues related to the reduction of energy costs of the active method of de-orbiting. On the one hand, this can be achieved by using a combination of the active de-orbiting with the passive one, based on an aerodynamic sailing unit. On the other hand, it is possible to use ERPSs that are more optimal in terms of energy and size-mass characteristics. This suggests that it is a relevant task to study the construction of a method for the combined de-orbiting of large-sized space debris from low orbits using ERPS.

### 3. The aim and objectives of the study

The aim of this study is to develop a combined method for removing the large-sized SOs from low-Earth orbits using ERPS as an active means of de-orbiting.

To accomplish the aim, the following tasks have been set:

- to develop a scheme of the de-orbiting of large-sized SOs from low-Earth orbits by a combined method, taking into consideration the implementation of the features of the ERPS's functioning;

- to devise a methodology for determining the parameters of the de-orbiting scheme of large-sized SOs from low near-Earth orbits by a combined method using ERPS;

- to simulate the de-orbiting of large-sized SOs from low near-Earth orbits by a combined method using ERPS and the mathematical model of motion reported in work [12];

- to determine the impact of ballistic aspects on the parameters of the scheme of the combined method of de-orbiting involving ERPS.

### 4. Developing a scheme to remove the large-sized SOs from low near-Earth orbits by a combined method taking into consideration the implementation of the features of ERPS operation

The advantages of using ERPS in comparison with SJPS are the smaller dimensions, the ease of manufacturing and operation, greater reliability, the possibility to re-enable, lower energy costs, a larger value (by an order and more) of the specific pulse.

The shortcomings of these engines include the small amount of thrust force and the dependence of their characteristics on the power supply system of the removed object. These include the battery capacity and the power output of the solar panels. On the one hand, this is reflected in the limited time of one activation, determined by the maximum charge of the battery, and, accordingly, the magnitude of the velocity pulse. On the other hand, there is an engine downtime caused by the need to charge the battery with solar-powered energy. Therefore, the implementation of large increments of flight velocity using ERPS is possible only by repeatedly turning on the propulsion system with a long duration of downtime between them.

Let us assume the "perfectly regulated" ERPS. Consider a scheme of the de-orbiting of large-sized SOs by the combined method given in [12]. It is designed to implement the de-orbiting using SJPS, whose traction force can reach thousands of Newton. Its main feature is the use of the energy-optimal Homan coplanar two-pulse junction. In this case, the sustainer propulsion unit over a relatively short period of time, lasting several tens of seconds, changes the motion velocity of the removed object in the near-Earth orbit, thereby ensuring the formation of the de-orbiting orbit at the enabling point.

This is not feasible in the case of ERPS. Due to the small amount of the thrust force, giving a velocity pulse using these engines would require a longer enabling duration. This leads to the impossibility of implementing the Homan flight, at which velocity is given to the object instantly or in a short period of time. In addition, given the limited capacity of the power supply system, setting the speed would be pulsed in the form of repeated sequences of enabling and downtime. This, in turn, would lead to a significant increase in the time of an active flight. And, as a result, to the need to take into consideration the time limits on the active functioning of the entire control system (CS) in general.

Based on the above, the following factors should be taken into consideration in the design of the combined de-orbiting means involving ERPS:

- the characteristics of ERPS;

- power supply capabilities;

- the time of active functioning of the entire CS in general (it can vary within 15–20 years, or longer).

We shall modify the scheme of the de-orbiting of largesized SOs by the combined method, proposed in work [12], taking into consideration the above features of the ERPS's operation, and represent it in the form shown in Fig. 1.

The scheme consists of the following characteristic sections: 1) delivering a means of de-orbiting to the near-earth orbit;

2) a passive flight of the means of de-orbiting with the reorientation towards the targeted SO;

3) the section of following the targeted SO;

4) the section of capturing the targeted SO;

5) reorientation for the first ERPS enabling for de-orbiting;

6) the first ERPS enabling for de-orbiting;

7) a passive flight after the first ERPS enabling, the deployment of an aerodynamic sailing unit, charging a battery;

8) a passive flight, reorientation before the repeated ERPS enabling;

9) the repeated ERPS enabling;

10) a passive flight after the repeated ERPS enabling, battery charging;

11) a passive flight after reaching the term of the active CS operation.

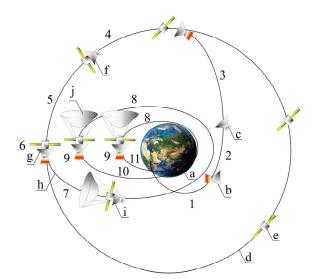


Fig. 1. The scheme of de-orbiting the large-sized SOs by a combined method using ERPS: a – Earth; b – a means of de-orbiting with the enabled propulsion system; c – a means of de-orbiting with the disabled propulsion system;
d – the orbit of the removed SO; e – removed SO; f – a means of de-orbiting with SO and disabled propulsion system;
g – a means of de-orbiting with SO and enabled propulsion system; h – the orbit of the de-orbiting; i – a means of de-orbiting with SO, an aerodynamic sailing unit, and disabled propulsion system; j – a means of de-orbiting with SO, an

aerodynamic sailing unit, and enabled propulsion system

In the case when the combined de-orbiting is implemented directly by SO (a spacecraft or the upper stage of the launch vehicle), the scheme excludes points 1 4.

The main difference between a given scheme and that reported in work [12] is the implementation of the section of the de-orbiting, starting the first enabling of the propulsion system.

## 5. Devising a methodology for determining the parameters of the de-orbiting scheme, which ensures the predefined time of existence of the removed object, taking into consideration the minimization of energy costs of ERPS

Based on the results of refining the de-orbiting scheme in comparison with the scheme reported in [12], we can conclude that it is necessary to devise a methodology for determining the parameters of the de-orbiting scheme.

As already noted in papers [12, 24], the existence of SO in the low near-Earth orbit depends on the following parameters:

- the altitude of the initial orbit;

- the characteristics of a propulsion system;

- the aerodynamic characteristics of SO;

- the phase of solar activity at the moment of the de-orbiting onset.

In order to implement the de-orbiting using ERPS, a given list needs to be supplemented with the following parameters:

- the velocity pulse gained in one enabling cycle;

- the number of enabling;

- the charge time of a battery;

- the time of active CS operation.

We represent the dependence of the time of existence of a large-sized SO on the energy costs and parameters of the combined de-orbiting scheme using ERPS in the form of the following function

$$t_{LT} = F(h_s, \sigma, v_s, \Delta V_{EW}, \tau_{BC}, \tau_{CS}, n_{EW}),$$
(1)

$$\sigma = \frac{C_X S}{2m},\tag{2}$$

where *F* is the functional dependence of the existence time on the characteristics of the de-orbiting process;  $h_s$  is the height of the initial orbit of SO;  $\sigma$  is the SO ballistic coefficient,  $v_s$  is the phase of solar activity at the time of the de-orbiting process onset;  $\Delta V_{EW}$  is the velocity pulse gained over one-time ERPS enabling;  $\tau_{BC}$  is the time of a single battery charge;  $\tau_{CS}$  is the time of CS active operation;  $n_{EW}$  is the number of times ERPS is enabled;  $C_x$  is the averaged coefficient of the strength of the aerodynamic resistance of the Earth's atmosphere; *S* is the characteristic SO area; *m* is the SO mass.

Introduce an assumption about the permanence of ERPS characteristics and the velocity pulse, gained over one enabling cycle. Determine from (1) the magnitude of the phase of solar activity at the de-orbiting onset, the pulse of the velocity gained over one enabling cycle, and the number of ERPS enabling cycles, ensuring the predefined time of existence taking into consideration the minimization of energy costs

$$\begin{bmatrix} \Delta V_{EWT}, n_{EWT}, \mathbf{v}_{ST} \end{bmatrix}^{l} = \\ = \arg \{ F(h_{S}, \sigma, \mathbf{v}_{S}, \Delta V_{EW}, \tau_{BC}, \tau_{CS}, n_{EW}) \}, \qquad (3) \\ \stackrel{t_{LT} = t_{LT} \max}{\Delta V_{EW} n_{EW} \to \min} \end{cases}$$

where  $\Delta V_{EWT}$  is the velocity pulse gained over one enabling cycle of ERPS, ensuring the predefined time of de-orbiting at minimal energy costs;  $n_{EWT}$  is the number of times ERPS is enabled, which ensures the predefined de-orbiting time at minimal energy costs;  $v_{ST}$  is the phase of solar activity at the de-orbiting process onset, which ensures the predefined de-orbiting time at minimal energy costs;  $t_{LT \max}$  is the maximum time of SO existence in the near-Earth orbit.

A procedure for solving (3) is represented in the form of the following steps:

1. Determine the time of existence during the passive section of the flight after reaching the term of CS active operation:

$$\tau_P = t_{LT\max} - \tau_{CS}.\tag{4}$$

2. Determine the height of the perigee of the orbit at the onset of the passive section of the de-orbiting for different values of the phase of solar activity at the de-orbiting process onset. Represent the dependence of the time of SO existence at the section of passive de-orbiting on the parameters of the initial orbit, the ballistic factor, and the phase of solar activity in the form

$$\tau_{LT} = H(h_{P_{\alpha}}, h_{P_{\pi}}, \sigma, \nu_{C}), \tag{5}$$

where  $\tau_{LT}$  is the SO existence time at the passive section of the de-orbiting orbit;  $h_{P\alpha}$  and  $h_{P\pi}$  is the height of apogee and perigee at the beginning of the passive section of the de-orbiting orbit; H is the functional dependence of the time of existence

at the passive section of the de-orbiting orbit on the parameters of the orbit, ballistic factor, and the phase of solar activity.

Since ERPS is enabled only to reduce the height of the perigee, thereby ensuring the optimal energy costs, we shall write

$$h_{P\alpha} = h_S, \tag{6}$$

The relationship between the height of the perigee at the beginning of the passive section of the de-orbiting orbit and the phase of solar activity at the de-orbiting onset is found by solving the following functional

$$h_{P\pi M}(\mathbf{v}_{C}) = \arg \left\{ H\left(h_{S}, h_{P\pi}, \mathbf{\sigma}, \mathbf{v}_{C}\right) \right\}$$
(7)  
$$\frac{\tau_{LT} = \tau_{P}}{h_{P\pi} \to \max}$$

using the methodology and mathematical model proposed in [12].

3. Determine the values of the velocity pulse gained over one enabling cycle and the number of times ERPS is enabled that make it possible to form the orbit at the beginning of the passive section of the de-orbiting for the different values of the phases of solar activity at the de-orbiting onset. Let us assume that the pulse of velocity is applied to SO transversely counter wise the movement. A change in the velocity is executed at the moment of the first passage of the apogee after the following condition is met

$$\Delta \tau_{BC} > \tau_{BC}, \tag{8}$$

where  $\Delta \tau_{BC}$  is the time that has elapsed since the last battery charge began. This makes it possible to minimize energy costs by reducing only the height of the perigee into the denser layers of the Earth's atmosphere.

The dependences of the velocity pulse gained over one enabling cycle and the number of times ERPS is enabled on the solar activity phase are to be found in the form of the following functions

$$\Delta V_{EW}(\mathbf{v}_{S}) = U_{V}(h_{S}, h_{P_{\pi M}}(\mathbf{v}_{C}), \mathbf{\sigma}, \mathbf{v}_{S}, \mathbf{\tau}_{BC}, \mathbf{\tau}_{CS}), \qquad (9)$$

$$n_{EW}(\mathbf{v}_{S}) = U_{N}(h_{S}, h_{P\pi M}(\mathbf{v}_{C}), \mathbf{\sigma}, \mathbf{v}_{S}, \mathbf{\tau}_{BC}, \mathbf{\tau}_{CS}), \qquad (10)$$

where  $U_V$  is the functional dependence of the velocity pulse gained over one enabling cycle on the phase of solar activity and source data;  $U_N$  is the functional dependence of the number of times ERPS is enabled on the phase of solar activity and source data.

(9) and (10) are solved numerically using the mathematical model, given in [12], accounting for the pulse change in velocity, in the apogee of the orbit after the battery is charged.

4. Determine the phase of solar activity at the de-orbiting onset, at which the minimum value of the total velocity pulse is achieved. It implies solving the following functional

$$\mathbf{v}_{ST} = \arg\left\{\min\left[\Delta V_{EW}(\mathbf{v}_S)n_{EW}(\mathbf{v}_S)\right]\right\}.$$
(11)

5. Determine the optimal values of the velocity pulse gained over one ERPS enabling cycle

$$\Delta V_{EWT} = \Delta V_{EW} \left( \mathbf{v}_{ST} \right), \tag{12}$$

$$n_{EWT} = n_{EW} (v_{ST}). \tag{13}$$

The resulting velocity pulse values from (12) and (13), gained over one enabling cycle, and the number of times ERPS is enabled fully characterize the energy costs of the de-orbiting process and do not depend on the characteristics of a particular propulsion system.

## 6. Simulation of the de-orbiting of large-sized SOs from low near-Earth orbits by a combined method involving ERPS

Consider the ballistic analysis of the combined de-orbiting based on ERPS and an aerodynamic sail (Fig. 1). Let us state the task of modeling. The source data used are the data from work [12], supplemented with the necessary parameters that take into consideration the specificity of ERPS operation.

Given:

- the initial orbits - equatorial circular, of altitude from 300 to 1,500 km;

- the weight of the de-orbiting means together with a target object is three tons;

– three ballistics ratios of 0.001, 0.01, and 0.1  $m^2/kg$  are considered (the first is typical of a large-sized space debris object without an aerodynamic sail and the second and third – with a sail);

- the time of active CS operation is five years;

– battery charging time is 24 hours.

It is required:

- to determine the velocity pulse required to form the de-orbiting means' orbit together with SO with the time of existence not exceeding 25 years, taking into consideration a change in the state of the atmosphere over the 11-year cycle of solar activity;

– to determine the dependence of the energy-optimal values of the solar activity phase and the total velocity required to ensure de-orbiting on the altitude of the target orbit and the SO ballistic coefficient;

- to determine the dependence of particular derivatives from a velocity pulse gain, gained over one ERPS enabling cycle, to gain the increment of the ballistic factor on the altitude of the initial orbit;

- to determine the dependence of particular derivatives from the total velocity gain, required to ensure de-orbiting, to the increment in the ballistic coefficient on the altitude of the initial orbit;

- to compare the results with those for the de-orbiting involving SJPS [12].

The phase of solar activity at the de-orbiting onset is accounted for by selecting the date of de-orbiting within the  $24^{\text{th}}$  cycle of solar activity. To this end, we shall choose 11 time points from 01.01.2009 00:00:00 to 01.01.2019 00:00:00 with a step in one year.

The result of the simulation of the process of de-orbiting of large-sized SO using (1) to (10), as well as the devised methodology, and the mathematical model proposed in [12] is the following:

- the dependence of the velocity pulse gained over one ERPS enabling cycle on the altitude of the initial orbit, the ballistic factor, and the phase of solar activity (Fig. 2, a);

- the dependence of the total velocity, required to ensure de-orbiting, on the altitude of the initial orbit, the ballistic factor, and the phase of solar activity (Fig. 2, *b*).

In this case, the orbital lifetime is no more than 25 years.

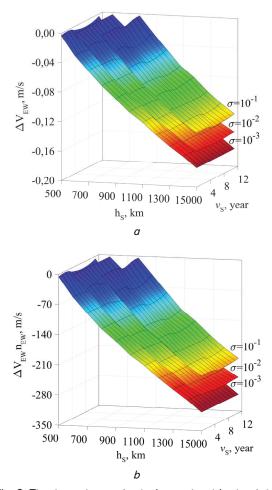


Fig. 2. The dependence of velocity on the altitude of the initial orbit, the ballistic factor, and the phase of solar activity: a – the dependence of the velocity pulse gained over one ERPS enabling cycle; b – the dependence of the total velocity needed to ensure the de-orbiting

It follows from the results obtained that the velocity increases (by module) with an increase in the altitude of the initial orbit, falls with the growth of the ballistic factor, and fluctuates periodically with a change in the phase of solar activity. The amplitude of oscillations is up to 6 m/s, reaching maximum values in the region of the purely passive de-orbiting. Afterward, with the altitude of the initial orbit, it gradually decreases. A similar dependence was observed for de-orbiting with the use of SJPS [12].

It should be noted that the energy costs (total velocity) to ensure the de-orbiting of large-sized SOs based on a combined method involving ERPS are higher than the cost of the de-orbiting using SJPS. On average, this magnitude (by module) is up to 10 m/s for a ballistic factor of  $0.1 \text{ m}^2/\text{kg}$ , up to  $4 \text{ m/s} - \text{for } 0.01 \text{ m}^2/\text{kg}$ , and up to  $3 \text{ m/s} - \text{for } 0.001 \text{ m}^2/\text{kg}$ . That is, the de-orbiting of large-sized SOs using ERPS leads to an increase in the total velocity required to ensure de-orbiting in comparison with SJPS, on average up to 10 %.

As a result of solving functional (3) using the developed methodology, we have determined the combinations of the phase of solar activity and total velocity, which provide for the optimal energy costs on de-orbiting. Fig. 3, a shows the dependence of the optimal phase of solar activity at the de-orbiting onset on the altitude of the initial orbit and the ballistic factor.

As one can see, a minimum of the energy costs corresponds to the de-orbiting onset in the second-third year of the solar activity cycle. This, for example, corresponds to 2021 and 2022 for the current cycle of solar activity. A similar pattern is observed for SJPS, the results for which are reported in article [12], but the range of the phases of solar activity is wider.

Fig. 3, b shows the dependence of the minimum (by module) value of the total velocity, required to ensure the de-orbiting, on the altitude of the initial orbit, and the ballistic factor.

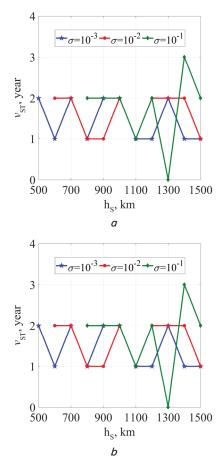


Fig. 3. Dependences of the optimal parameters of de-orbiting on the altitude of the initial orbit and the ballistic factor: a – the dependence of the phase of solar activity, providing minimal energy costs; b – the dependence of the total velocity needed to ensure the de-orbiting

Fig. 3, *b* defines the boundary between the areas of the passive de-orbiting using an aerodynamic sail and the combined de-orbiting, located on the axis of abscissas. Thus, the combined de-orbiting for a SO with a ballistic factor of  $0.001 \text{ m}^2/\text{kg}$  corresponds to a minimum altitude of about 500 km, for  $0.01 \text{ m}^2/\text{kg}$  – about 600 km, for  $0.1 \text{ m}^2/\text{kg}$  – about 800 km.

The dependences in Fig. 3, a, b are of practical interest. They are the source data for the design of the means for the combined de-orbiting of large-sized SOs from low near-Earth orbits.

Determine the particular derivatives from the increment of the velocity pulse, gained over one ERPS enabling cycle, and the total velocity, required to ensure the de-orbiting, based on the increment of the ballistic factor, from the following expressions

$$\frac{\partial \Delta V_{EW}}{\partial \sigma} = \frac{\Delta V_{EW}(\sigma_A) - \Delta V_{EW}(10^{-3})}{\sigma_A - 10^{-3}},$$
(15)

$$\frac{\partial (\Delta V_{EW} n_{EW})}{\partial \sigma} = \frac{\Delta V_{EW} (\sigma_A) n_{EW} (\sigma_A) - \Delta V_{EW} (10^{-3}) n_{EW} (10^{-3})}{\sigma_A - 10^{-3}}, \quad (16)$$

where  $\sigma_A$  is the ballistic object of the means of de-orbiting with SO and an unfolded aerodynamic sail (for the problem under consideration, it is  $10^{-2}$  and  $10^{-1}$  m<sup>2</sup>/kg). The resulting dependences of particular derivatives on the altitude of the initial orbit and the ballistic factor with an unfolded aerodynamic sail are shown in Fig. 4, *a*, *b*. The left regions in Fig. 4, where the dependence is missing, correspond to the de-orbiting without ERPS, only due to the force of aerodynamic braking of the Earth's atmosphere.

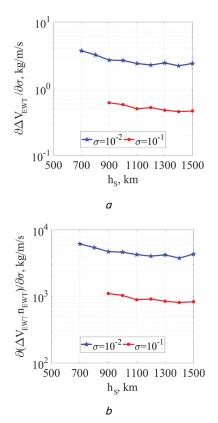


Fig. 4. Dependences of particular derivatives from the velocity increments based on the ballistic factor gain; a – the dependence of the velocity pulse gained over one ERPS enabling cycle; b – the dependence of the total velocity needed to ensure the de-orbiting

It follows from Fig. 4 that the particular derivatives monotonously decrease with an increase in the altitude of the initial orbit. This suggests that the effectiveness of the combined method decreases as the altitude of the target orbit increases. The resulting dependences are of practical interest, too. Their application makes it possible to determine the reduction (by module) of energy costs for the process of de-orbiting by a combined method due to the increase in the ballistic factor as the consequence of unfolding an aerodynamic sailing unit.

## 7. The impact of ballistic aspects on the parameters of the scheme for a combined de-orbiting method

The results above testify to the following:

1. An increase in the ballistic factor of a large-sized object of de-orbiting leads to a decrease by a module in both the magnitude of the velocity pulse, gained over one ERPS enabling cycle, and in the total velocity required to ensure the de-orbiting (the dynamics of the change is described by the obtained particular derivatives). Similar dependences were observed for SJPS [12].

2. The dependences of the velocity pulse, gained over one ERPS enabling cycle, and the total velocity required to ensure the de-orbiting, on the phase of solar activity at the de-orbiting process onset are similar to the harmonic dependence (Fig. 5, *a*). The magnitude of the relative amplitude of fluctuations is determined from the ratio

$$\delta \Delta V_{EW} = \frac{\Delta V_{EW} - \text{mean}(\Delta V_{EW})}{\text{mean}(\Delta V_{EW})} \cdot 100\%, \tag{17}$$

where mean( $\Delta V_{EW}$ ) is the average value of the velocity pulse, gained over one ERPS enabling cycle, for all the phases of solar activity under consideration.

The dependence of the relative amplitude of oscillations on the altitude of the initial orbit and the ballistic factor is shown in Fig. 5, b.

dependences are of practical interest in the tasks to design the means of the combined de-orbiting.

4. It has been shown that the minimum energy costs for the de-orbiting are mainly observed at the time of the de-orbiting onset, corresponding to the second-third year of the current cycle of solar activity.

5. In general, the resulting dependences are similar to those reported in [12] involving SJPS. It should be noted that the total velocity required to ensure de-orbiting in the scheme involving ERPS is higher than that when using SJPS. On average, this value does not exceed 10 m/s, and in relative units – 10 %.

## 8. Discussion of results of studying the combined method of removing large-sized space objects from low near-Earth orbits using an electric rocket propulsion system

One can see from the results obtained that the main feature of using ERPS in comparison with SJPS is the complexity of the de-orbiting scheme. ERPS has a greater value of the specific pulse and, accordingly, lower energy costs of the working body for the process of de-orbiting, which can exceed the similar value for SJPS by two orders of magnitude. However, at the same time, it requires a significant complication of the structure of the onboard and ground-based CS.

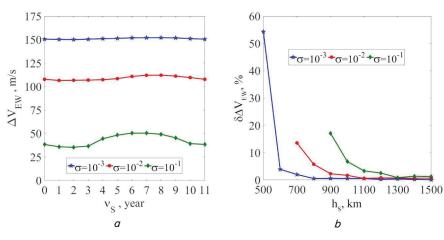


Fig. 5. Dependence of the relative amplitude magnitude on the parameters of the de-orbiting onset: a – change in the velocity pulse, gained over one ERPS enabling cycle, on the phase of solar activity and the ballistic factor for an altitude of the initial orbit of 900 km; b – dependence of the relative amplitude of change in velocity on the altitude of the initial orbit and the ballistic factor

Fig. 5, b shows that the amplitude of velocity fluctuation on the altitude of the initial orbit takes the shape of an exponent with a negative power degree. In this case, the most impact, exerted by the phase of solar activity at the de-orbiting onset, is on the velocity in the region of the border area with a purely passive method of de-orbiting involving an aerodynamic sail. Thus:

- for a ballistic factor of  $10^{-3}\,m^2/kg$  – at altitudes up to 700 km;

- for a ballistic factor of  $10^{-2} \text{ m}^2/\text{kg}$  to 1,000 km;
- for a ballistic factor of  $10^{-1}$  m<sup>2</sup>/kg to 1,200 km.

3. We have established the dependences of the optimal energy costs for the combined de-orbiting on the altitude of the initial orbit and the ballistic coefficient of SO. These

Using ERPS, on the one hand, requires the creation of a complex branched terrestrial segment to manage the de-orbiting process. That is, it is necessary to create an organizational and technical system consisting of the means of obtaining primary information about the movement of the removed object and a control center for the de-orbiting process. On the other hand, there is a need to develop and create a system of control over the de-orbiting object with the time of active existence up to two decades, which, given the current state of the scientific and technological progress, is an issue that needs to be solved in the future.

The results of our study showed the possibility of the practical implementation of the de-orbiting of large-sized SO from the low near-

Earth orbits using ERPS. However, at the same time, the tasks of finding the optimal parameters for the de-orbiting scheme become relevant, depending on the variations in:

- the altitude of the initial orbit;
- the ballistic coefficient;
- the time of the active SC existence;
- a battery charging time;

 the phases of solar activity at the de-orbiting process onset;

- the cost of creating an organizational and technical system to remove large-sized SOs from the low near-Earth orbits.

Comparing our results with those reported in [12], one can conclude that the dependences derived for SJPS are

observed in one way or another for ERPS. However, one should pay attention to the following. In the range of the altitudes of initial orbits up to 1,100 km, one observes, for a ballistic factor of  $0.001 \text{ m}^2/\text{kg}$ , the dominance of minimum energy costs of SJPS over ERPS, which had to be observed according to study [37]. However, in the range of altitudes exceeding 1,100 km, there is a reverse trend, the reasons for which need to be identified in the future.

#### 9. Conclusions

1. We have designed a scheme for the de-orbiting of large-sized SOs from the low near-Earth orbits using a combined method taking into consideration the implementation of the features of ERPS operation. Its feature, in comparison with the earlier developed scheme for SJPS, is the presence of repetitive sections of work and downtime of the propulsion system. This ensures the existing limitations of the operation of ERPS. They are related to the power supply system and the time of the active operation of CS.

2. A procedure has been devised for determining the parameters of the scheme of the de-orbiting of large-sized SOs from the low near-Earth orbits by a combined method involving ERPS. A feature of this procedure, in comparison with [12], is the need to take into consideration the operating conditions of ERPS, related to the limited capacity of the energy supply system and the time of active operation of CS.

3. We have performed the simulation of the impact of the initial orbital altitude, ballistic coefficient, and the phase of solar activity, on the energy costs of the de-orbiting process. It is shown that the energy costs are proportional to the altitude of the initial orbit, are inversely proportional to the ballistic factor, and change cyclically when the phase of solar activity changes. The total velocity, required to ensure the de-orbiting involving ERPS, for the considered values of

the active operating time of CS and the battery charge, is on average up to 10 m/s higher than that when using a sustainer rocket propulsion system [12].

4. The impact of the ballistic aspects on the parameters of the scheme of the combined method of de-orbiting involving ERPS has been determined. It was revealed that the minimum energy costs for the formation of the orbit of the de-orbiting with the time of existence of 25 years are observed for the de-orbiting onset in the second-third year of the current cycle of solar activity. We have derived the dependence of the optimal energy costs for the combined de-orbiting involving ERPS on the phase of solar activity, the altitude of the initial orbit, and the ballistic factor. It is of practical interest for the task of designing the means for the combined de-orbiting of large-sized SOs from the low near-Earth orbits using ERPS. In addition, this dependence determines the boundary between the regions of the passive de-orbiting using an aerodynamic sail and the combined de-orbiting. We have established the dependences of particular derivatives form the velocity pulse gain, gained over one ERPS enabling cycle, by the increment of the ballistic factor on the altitude of the target orbit. As well as the dependences of particular derivatives from the total velocity gain, required to ensure the de-orbiting, by the increment in the ballistic factor on the altitude of the target orbit. These dependences are also of practical interest in the task of designing the means of combined de-orbiting involving ERPS.

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