A methodology has been proposed for estimating the nonlinear effects in radio tracts of receiving and transmitting devices in radio-electronic means of mobile communication systems, based on using the nonlinear transfer functions of the higher-order Volterra series.

A procedure has been devised for obtaining the output responses from a nonlinear non-inertia circuit under the harmonious input action using a method for determining the transfer functions of higher orders obtained on the basis of the transfer functions of lower orders.

We have derived the analytical expressions for the output responses from a nonlinear system of different orders for three inputs for the case of representing a nonlinear system in the form of a nonlinear non-inertia circuit.

The values of the transfer functions of higher orders for a nonlinear non-inertia circuit were determined by using a state variable method.

This paper demonstrates the derivation of analytical expressions to calculate a harmonic coefficient based on the second and third harmonics using the nonlinear higher-orders transfer functions of a nonlinear non-inertia circuit.

It has been shown that the use of the nonlinear transfer functions to the fifth order inclusive allows a more accurate assessment of nonlinear effects in the form of the harmonious and intermodulation distortions in the radio tracts of radio-electronic means of mobile systems.

The outlined technique for determining the nonlinear transfer functions is invariant to the topology of a nonlinear electrical circuit, as well as to the quantity and type of nonlinear elements. Existing estimation procedures of electromagnetic compatibility related to the problems of calculating intermodulation interference can be improved by the introduction of the determined magnitudes of influence products.

The proposed methodology makes it possible to evaluate the set of nonlinear effects in the problems related to electromagnetic compatibility in the groups of radio-electronic means with the accuracy required by users

Keywords: nonlinear system, Volterra series, transfer function, nonlinear non-inertia circuit

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METHOD FOR DETERMINING THE RESPONSES FROM A NON-LINEAR SYSTEM USING THE VOLTERRA SERIES

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1. Introduction

Over the past decade, as part of the creation of standards for the new generations of mobile telecommunications technologies, their physical interface has undergone significant changes, providing opportunities for the development and rapid distribution of these technologies around the world. The growing demands of mobile network users are due to their obtaining quality services and applications in the mobility option.

In this regard, there is a multicriteria optimization issue to improve the quality of service provision together with the search for a free physical resource. At the same time, conditions for ensuring the electromagnetic compatibility of radio-electronic means in communication networks in a dynamic complex signal-interference environment must be met.

The main limitation of the air of mobile communication systems, in terms of its openness, is the issue of electromagnetic compatibility (EMC). This problem also occurs due to the fact that other radio-electronic means (REM) operate in the compatible frequency ranges [1].

The electromagnetic situation in mobile communication systems (MCS), which is determined at the input of radio receiver devices of subscriber and base stations, is characterized by the totality of influences along all receiving channels (main, side, and off-band) formed from many sources of radiation [2].

The impact of these radiations, which are undesirable signals, although they are conflicting but not antagonistic, defines the specificity of EMC tasks. It should be noted that the emergence of unwanted interference also occurs in the radio communication systems, where the use of the radiofrequency resource is very well planned. The essence of unwanted interference is due to the fact that the REM radiation can lead to negative consequences and create an unwanted effect in systems for which this radiation was not intended [3].

At individual facilities, offices, in a limited area, a large number of different radio means can be installed. This leads to the formation of multiple electromagnetic interactions between them and a sharp increase in the overall level of radiation, which leads to mutual interference.

The considerable destructive character is observed in the cases of the emergence of electromagnetic nonlinear influences that occur at different nodes in the receiving and transmitting equipment of a mobile communication system in general [4].

The manifestation of nonlinear properties implies the appearance of nonlinear effects such as blocking of useful signals, cross distortions, and intermodulation. Taking into consideration the high level of engineering solutions in the modern circuitry of radio equipment units in mobile communication systems, it should be noted that the phenomena of blocking effects and cross distortions in radio receiving tracts account for no more than 15 % of such cases [5]. Thus, the remaining proportion of the cases of mutual interference is due to the effect of intermodulation, which is the result of nonlinear effects arising in the input cascades of radio receivers. Susceptibility to intermodulation interference (II) is an important indicator of the electromagnetic compatibility of RES groups.

Many methods, procedures, theoretical justifications related to improving the electromagnetic situation (EMS) in the radio lines of mobile communication systems have been developed within the framework of the task to ensure EMC. It can be considered that under stationary conditions, especially in the dual consideration (transmitter-receiver), the task of EMC can be solved. The situation and the EMS itself are greatly complicated by the fact that this situation is affected by various, often random, factors that are difficult to predict. In these circumstances, it is not always possible to calculate EMS in advance and solve the problem of EMC with sufficient accuracy, and often simply impossible, due to the *a priori* uncertainty in the parameters of a signal-interference situation.

Thus, as part of the solution to the EMC problem, the grouping of mobile communication systems' REM can be represented as a complex nonlinear dynamic system, which can be described by generally adopted methods for studying the nonlinear dynamic systems. These methods include the direct integration of equations in the time domain, the equation of harmonious balance, as well as the functional series, each of which has its advantages and disadvantages [6].

It is possible to solve the problem of EMC in the nonlinear circuits of radio-electronic devices by approximating their characteristics using linearized methods.

Volterra series provides an opportunity for a thorough analysis of the physical phenomena of nonlinear distortions from the sources of different origins and to evaluate their contribution at the output from a nonlinear circuit.

The functional Volterra series is the most fruitful area in the study of nonlinear inertial and non-inertial systems, originated in the works by Wiener and Van Tris.

It is obvious that existing procedures of EMC assessment based on intermodulation do not make it possible to take into consideration the multiple characters of influences, so for a more accurate representation of the multiple nature of interactions, it is necessary to take into consideration the II of higher orders.

Therefore, it is an actual scientific task to develop a new approach to assess the manifestation of the nonlinear nature of electromagnetic interactions in the grouping of radio-electronic means in mobile communication systems. A given task aims to further clarify the parameters of output signals from radio receiving equipment with the typical nonlinearity and take into consideration the effects of interference of nonlinear origin when analyzing the EMC conditions at different MCS objects.

2. Literature review and problem statement

The task of determining the spectral composition of voltage at the output from a non-inertial nonlinear element (NNE) under the influence exerted at its output by the sum of harmonic fluctuations dates back over forty years [7–9]. Paper [7] reported the results to determine the amplitude of harmonic at any combination frequency (CF) at the NNE output with the characteristic described by an arbitrary function. An analysis of the nonlinear dynamic systems of this type, given the high commonality of the model used, is widespread in solving a large number of radio electronics issues [10–12].

Even though there are constantly published scientific works about new or modified methods of the mathematical analysis of nonlinear systems, the task of calculating the multifrequency modes of nonlinear systems is extremely difficult.

Study [13] reports an effective method of rapid evaluation of basic intermodulation products. The most accurate procedure for determining the products of intermodulation implies, first, finding a period of the stable signal, and, then, calculating its spectrum by using the rapid Fourier transform. The method employs the Volterra series in a simple multi-step algorithm that is compatible with the typical structure of the frequency part of chain simulators. However, this method requires a multi-operational numerical integration over many periods of a faster signal, even for the improved methods for finding a stable state.

Non-ideal implementation schemes cause a significant deterioration in the performance of time-interleaved analog-to-digital converters (TIADC) described in work [14]. The cited work gives a simulation model for TIADC based on the Volterra series and is offered for the simulation of dynamic nonlinearity in TIADC. The authors derived expressions of the pattern of behavior in the time and frequency domains based on the hybrid Volterra series. The findings provide a theoretical basis for applying the Volterra series with discrete-time to simulate the TIADC system in a mixed domain.

A distortion contribution analysis (DCA) is determined by the total contribution at the output of an analog electronic scheme as the sum of the contribution of distortions of its sub-schemes, reported in work [15]. DCA helps the designer define the actual source of distortion. Classically, DCA employs a Volterra theory to model the scheme and its sub-schemes. The DCA method considered was useful for simple circuits; however, for more complex schemes, the amount of contributions increases rapidly, making it difficult to interpret the results.

An analysis of nanoscale engine distortions (BD – bulk-driven) by the high-frequency amplifier CMOS, described in [16], is based on the Volterra series. The first three cores of Volterra were calculated; the expressions for the second and third order of harmonic distortions were derived.

These expressions produce greater accuracy in comparison with the results of simulation and can provide an understanding of the nonlinearity of the nanoscale BD amplifier.

Paper [17] reports a new computational procedure for analyzing distortions in nonlinear circuits. The new procedure applies to the same class of circuits, namely to the weakly nonlinear and time-variable circuits, as the periodic Volterra series. However, unlike the Volterra series, it does not require calculating the second and third derivative of the responses from device models. The new method is effective in calculating compared to a full multitone nonlinear stationary analysis such as harmonic balance. Moreover, the new procedure naturally makes it possible to calculate and characterize the contribution of individual components of the chain to the overall distortion of the scheme.

A method of the analysis of Volterra series distortions in the analysis of the overall radiation scheme is described in work [18]. The authors gave a model of the integrated circuit and assessed its nonlinearity. Based on the provided equivalent diagram and by using the Voltaire series method, the cores of the corresponding linear, square, and cube systems were developed. Changes in the IT operation mode in the frequency range affected by the input signal were determined considering the IT parameters and external elements.

As the general analysis of the reviewed works has revealed, one can note that when analyzing electronic devices represented by equivalent schemes in works [13–18], the analysis is usually carried out in the frequency domain for the nonlinearities of the third or even the second order [19, 20]. In general, there are very few studies that consider the nonlinear interaction of at least the fifth order [10, 11]. At the same time, when concentrating wireless systems of different standards at short distances from each other and as a result of multiple influences there is an increase in the likelihood of the formation of intermodulation interference (II) caused by a large number of REM. Known classical procedures of group evaluation suggest the separation and accounting of only three main types of interference from a variety of input influences and make it possible to calculate the levels of II of not higher than the third order.

Therefore, when analyzing electromagnetic multiple interactions in the REM grouping, as regards the accounting of nonlinear distortions, one should not be limited to considering the components of only the third order, as the significant level of influence is exerted by the components of higher orders. This primarily concerns the calculation of radio tract parameters, in particular the harmonic coefficient for individual harmonics. It is possible to obtain the nonlinear transfer characteristics of a radio tract, which links the input influence and a response from the nonlinear circuit in an apparent form, by using the apparatus of the functional Volterra series.

The experience in applying the method by researchers demonstrated its effectiveness and versatility for the analysis and synthesis of a wide class of radio tracts with a small degree of nonlinearity. A given method is of great practical interest for the receiving and transmitting devices in radio-electronic means of mobile communication systems.

3. The aim and objectives of the study

The aim of this study is to devise a procedure for evaluating nonlinear effects in radio tracts of the receiving and transmitting devices of radio-electronic means in mobile communication systems based on the use of the nonlinear transfer functions of higher orders.

To accomplish the aim, the following tasks have been set:

– to obtain the output responses from a nonlinear system (a nonlinear non-inertia circuit) under the harmonic input action using a method for determining the higher-order transfer functions, which are derived on the basis of the lower-order transfer functions;

– to obtain the expressions and determine the values for the higher-order transfer functions of a nonlinear non-inertia circuit;

– to obtain the analytical expressions on calculating the harmonic coefficient for individual harmonics using the nonlinear higher-order transfer functions of a nonlinear non-inertia circuit.

4. Deriving the output response from a nonlinear system using the higher-order transfer functions

The result of the development of an NS analysis method based on the apparatus of the functional Volterra series, we created convenient ways to obtain a nonlinear transfer function for the characteristics of the tract, which links the input influence and response in an apparent form.

We shall use the representation of the functional Volterra series in the form of the following expression [21, 22]:

$$y(t)=\sum_{n=1}^{\infty}\frac{1}{n!}\int_{-\infty}^{+\infty}\dots\int_{-\infty}^{+\infty}H_n\left(f_{1,\dots,}f_n\right)\prod_{i=1}^{n}X\left(f_i\right)\exp\left(i2\pi f_i t\right)\mathrm{d}f_i, \quad (1)$$

where $H_n\left(f_{1,\dots,}f_n\right)$ is the nonlinear transfer function of the n-the order; $X(f_i)$ is the Fourier transform of the input action $x(t)$.

Let the response at the output from a nonlinear non-inertia circuit be represented in a series

$$y(t)=\sum_{n=1}^{\infty}y_n(t), \quad (2)$$

where

$$y_n(t)=\frac{1}{n!}\int_{-\infty}^{+\infty}\dots\int_{-\infty}^{+\infty}H_n\left(f_{1,\dots,}f_n\right)\times\prod_{i=1}^{n}X\left(f_i\right)\exp\left(i2\pi f_i t\right)\mathrm{d}f_i. \quad (3)$$

Index n denotes the response's order, for example, the first-order response

$$y_1(t)=\int_{-\infty}^{+\infty}H_1X\left(f_i\right)\exp\left(i2\pi f_i t\right)\mathrm{d}f. \quad (4)$$

The second-order response

$$y_2(t)=\frac{1}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}H_2\left(f_{1,}f_2\right)X\left(f_1\right)X\left(f_2\right)\times$$
$$\times\exp\left[i2\pi\left(f_1+f_2\right)t\right]\mathrm{d}f_1\mathrm{d}f_2, \quad (5)$$

etc.

Consider three input influences in the studied system represented by a nonlinear circuit:

$$x(t)=A\cos\left(2\pi f_0 t\right), \quad (6)$$

$$x(t)=A^2\cos^2\left(2\pi f_0 t\right), \quad (7)$$

$$x(t)=A^3\cos^3(2\pi f_0 t). \quad (8)$$

Define the nonlinear circuit response expressions for the input actions (6) to (8).

The most convenient representation of input action (6) is the Fourier transform that takes the following form:

$$X(f)=\frac{A}{2}\left[\delta(f-f_0)+\delta(f+f_0)\right], \quad (9)$$

where $\delta(f-f_0)$ and $\delta(f+f_0)$ are the delta functions.

As part of this study, it should be noted that the transfer functions $H_n(f_1,\dots,f_n)$ are symmetrical, that is their value do not depend on the permutation of arguments, and are also even [8, 23], that is:

$$H_n\Big(\underbrace{f_0,\dots,f_0}_{k},\underbrace{-f_0,\dots,-f_0}_{m}\Big)=H_n\Big(\underbrace{-f_0,\dots,-f_0}_{k},\underbrace{f_0,\dots,f_0}_{m}\Big). \quad (10)$$

Given the above, one can fit a Fourier transform (9) in expression (3) to obtain the first-order response:

$$y_1(t)=\frac{1}{n!}\int_{-\infty}^{+\infty}H_1(f)\left[\delta(f-f_0)+\delta(f+f_0)\right]\cdot e^{i2\pi\cdot f\cdot t}\mathrm{d}f_0, \quad (11)$$

where n is the order of the response, which, in this case, is equal to unity, and the delta function filtering property is used:

$$\int_{-\infty}^{+\infty}f(x)\delta(x-x_0)\mathrm{d}x=f(x_0). \quad (12)$$

Considering the above, we obtain the result of the analytical expression for the first response:

$$\begin{aligned}y_1(t)&=\frac{1}{n!}\int_{-\infty}^{+\infty}H_1(f)\left[\delta(f-f_0)+\delta(f+f_0)\right]\cdot e^{i2\pi\cdot f\cdot t}\mathrm{d}=\\&=\frac{A}{2}\left[H_1(f_0)\cdot e^{i2\pi\cdot f_0\cdot t}+H_1(-f_0)\cdot e^{-i2\pi\cdot f_0\cdot t}\right]=\\&=AH_1(f_0)\cdot\cos 2\pi f_0 t, \quad (13)\end{aligned}$$

where

$$H_n(f)=H_n(-f).$$

By analogy, represent in detail the second and third responses:

$$\begin{aligned}y_2(t)&=\frac{1}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}H_2(f_1,f_2)\frac{A^2}{4}\begin{bmatrix}\delta(f_1-f_0)+\\+\delta(f_1+f_0)\end{bmatrix}\times\\&\quad\times\left[\delta(f_2-f_0)+\delta(f_2+f_0)\right]\exp\left(i2\pi(f_1+f_2)t\right)\mathrm{d}f_1\mathrm{d}f_2=\\&=\frac{1}{2}\int_{-\infty}^{+\infty}H_2(f_1,f_2)\frac{A^2}{4}\left[\delta(f_1-f_0)+\delta(f_1+f_0)\right]\begin{bmatrix}H_2(f_1,f_0)\exp\left(i2\pi(f_1+f_0)t\right)+\\+H_2(f_1,-f_0)\exp\left(i2\pi(f_1-f_0)t\right)\end{bmatrix}\mathrm{d}f_1=\\&=\frac{A^2}{8}\begin{bmatrix}H_2(f_0,f_0)\exp(i2\pi 2f_0t)+\\+H_2(-f_0,-f_0)\exp(-i2\pi 2f_0t)+\\+H_2(-f_0,f_0)\exp(0)+H_2(f_0,-f_0)\exp(0)\end{bmatrix}=\\&=\frac{A^2}{4}H_2(f_0,f_0)\cos(2\pi 2f_0t)+\frac{A^2}{4}H_2(f_0,-f_0). \quad (14)\end{aligned}$$

$$\begin{aligned}y_3(t)&=\frac{1}{6}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}H_3(f_1,f_2,f_3)\frac{A^3}{8}\begin{bmatrix}\delta(f_1-f_0)+\\+\delta(f_1+f_0)\end{bmatrix}\times\\&\times\begin{bmatrix}\delta(f_2-f_0)+\\+\delta(f_2+f_0)\end{bmatrix}\begin{bmatrix}\delta(f_3-f_0)+\\+\delta(f_3+f_0)\end{bmatrix}\times\\&\times\exp\left(i2\pi(f_1+f_2+f_3)t\right)\mathrm{d}f_1\mathrm{d}f_2\mathrm{d}f_3=\\&=\frac{A^3}{48}\begin{bmatrix}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left[\delta(f_1-f_0)+\delta(f_1+f_0)\right]\times\\\times\left[\delta(f_2+f_0)+\delta(f_2-f_0)\right]\times\\\times\begin{bmatrix}H_3(f_1,f_2,f_0)\times\\\times\exp\left(i2\pi(f_1+f_2+f_0)t\right)+\\+H_3(f_1,f_2,-f_0)\times\\\times\exp\left(i2\pi(f_1+f_2-f_0)t\right)\end{bmatrix}\mathrm{d}f_1\mathrm{d}f_2\end{bmatrix}=\\&=\frac{A^3}{48}\int_{-\infty}^{+\infty}\left[\delta(f_1-f_0)+\delta(f_1+f_0)\right]\times\\&\times\begin{bmatrix}H_3(f_1,f_0,f_0)\exp\left(i2\pi(f_1+2f_0)t\right)+\\+H_3(f_1,-f_0,f_0)\exp(i2\pi f_1t)+\\+H_3(f_1,f_0,-f_0)\exp(i2\pi f_1t)+\\+H_3(f_1,-f_0,f_0)\exp\left(i2\pi(f_1-2f_0)t\right)\end{bmatrix}\mathrm{d}f_1=\\&=\frac{A^3}{48}\begin{bmatrix}H_3(f_0,f_0,f_0)\exp(i2\pi 3f_0t)+\\+H_3(-f_0,f_0,f_0)\exp(i2\pi f_0t)+\\+H_3(f_0,-f_0,f_0)\exp(i2\pi f_0t)+\\+H_3(-f_0,-f_0,f_0)\exp(-i2\pi f_0t)+\\+H_3(f_0,f_0,-f_0)\exp(i2\pi f_0t)+\\+H_3(-f_0,f_0,-f_0)\exp(-i2\pi f_0t)+\\+H_3(f_0,-f_0,-f_0)\exp(-i2\pi f_0t)+\\+H_3(-f_0,-f_0,-f_0)\exp(-i2\pi 3f_0t)\end{bmatrix}=\\&=\frac{A^3}{24}H_3(f_0,f_0,f_0)\cos(i2\pi 3f_0t)+\frac{A^3}{8}\times\\&\times H_3(f_0,f_0,-f_0)\cos(i2\pi f_0t). \quad (15)\end{aligned}$$

As the calculations become more time-consuming, we shall omit further calculations for a shorter accumulation of formulae and provide the result for the higher-order responses [24]:

$$\begin{aligned}y_4(t)&=\frac{A^4}{192}H_4(f_0,f_0,f_0,f_0)\cos(2\pi 4f_0t)+\\&+\frac{A^4}{48}H_4(f_0,f_0,-f_0,-f_0)\cos(2\pi 3f_0t). \quad (16)\end{aligned}$$

$$\begin{aligned}y_5(t)&=\frac{A^5}{1{,}920}H_5(f_0,f_0,f_0,f_0,f_0)\times\\&\times\cos(2\pi 5f_0t)+\\&+\frac{A^5}{384}H_5(f_0,f_0,f_0,f_0,-f_0)\times\\&\times\cos(2\pi 3f_0t)+\\&+\frac{A^5}{192}H_5(f_0,f_0,f_0,-f_0,-f_0)\times\\&\times\cos(2\pi f_0t). \quad (17)\end{aligned}$$

$$y_6(t)=\frac{A^6}{46{,}080}H_6(f_0,f_0,f_0,f_0,f_0,f_0)\cos(2\pi 6f_0t)+$$
$$+\frac{A^6}{7{,}680}H_6(f_0,f_0,f_0,f_0,f_0,-f_0)\times\cos(2\pi 4f_0t)+$$
$$+\frac{A^6}{3{,}072}H_6(f_0,f_0,f_0,f_0,-f_0,-f_0)\cos(2\pi 3f_0t)+$$
$$+\frac{A^6}{1{,}536}H_6(f_0,f_0,f_0,-f_0,-f_0,-f_0)\times$$
$$\times\cos(2\pi 3f_0t). \qquad (18)$$

$$y_7(t)=\frac{A^7}{325{,}120}H_7(f_0,f_0,f_0,f_0,f_0,f_0,f_0)\cos(2\pi 7f_0t)+$$
$$+\frac{A^7}{92{,}160}H_7(f_0,f_0,f_0,f_0,f_0,f,-f_0)\times\cos(2\pi 5f_0t)+$$
$$+\frac{A^7}{30{,}720}H_7(f_0,f_0,f_0,f_0,f_0,-f_0,-f_0)\times$$
$$\times\cos(2\pi 3f_0t)+\frac{A^7}{9{,}216}\times$$
$$\times H_7(f_0,f_0,f_0,f_0,-f_0,-f_0,-f_0)\cos(2\pi f_0t). \qquad (19)$$

The first term of the series (2) to (13) contains the first harmonic of the frequency f_0; the second term of the series – $y_2(t)$ (14) – the second harmonic of the frequency f_0 and a constant component; third – $y_3(t)$ (15) – the third harmonic of the frequency f_0 and the first harmonic f_0.

It is proposed to represent analytical expressions for the first, second, and third responses for the input actions in the form of (7) and (8).

Before one calculates the responses to action (7), one must bring it to a more acceptable form:

$$x(t)=A^2\cos^2(2\pi f_0t)=\frac{A^2(1+\cos(4\pi f_0t)}{2}=$$
$$=\frac{A^2}{2}+\frac{A^2\cos(4\pi f_0t)}{2}. \qquad (20)$$

By reducing the degree of cosine, one can easily find the Fourier transform [24]:

$$\frac{A^2}{2}\cos(2\omega_0t)\to\frac{A^2}{2}\sqrt{2\pi}\frac{[\delta(\omega-2\omega_0)+\delta(\omega+2\omega_0)]}{2}, \qquad (21)$$

$$\frac{A^2}{2}\to\frac{A^2}{2}\sqrt{2\pi}\delta(\omega). \qquad (22)$$

Considering expressions (21) and (22), we can record the Fourier transform for action (20):

$$X(f)=\frac{\sqrt{2\pi}}{2\pi}\frac{A^2}{2}\left[\delta(f)+\frac{[\delta(f-f_0)+\delta(f+f_0)]}{2}\right]. \qquad (23)$$

To calculate the responses, consider that:

$$\int f(x)\delta(x)=f(0), \qquad (24)$$

$$\omega_0=2\pi f_0, \qquad (25)$$

$$\int x(t)e^{-j\omega_0t}dt=\frac{1}{2\pi}\int x(t)e^{-j2\pi f_0t}dt. \qquad (26)$$

Thus, the first-order response (20):

$$y_1(t)=\frac{1}{n!}\int_{-\infty}^{+\infty}\underline{H}_1(f)\frac{A^2}{2}\frac{\sqrt{2\pi}}{2\pi}\times$$
$$\times\left[\delta(f)+\frac{\delta(f-2f_0)+\delta(f+2f_0)}{2}\right]\cdot e^{i2\pi\cdot f\cdot t}df_0=$$
$$=\frac{A^2\sqrt{2\pi}}{4\pi}\underline{H}_1(2f_0)\cos(2\pi f_0t). \qquad (27)$$

The second-order response:

$$y_2(t)=\frac{1}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}H_2(f_1,f_2)\left(\frac{A^2\sqrt{2\pi}}{4\pi}\right)^2\times$$
$$\times\left[\begin{matrix}\delta(f_1)+\\+\dfrac{\delta(f_1-2f_0)+\delta(f_1+2f_0)}{2}\end{matrix}\right]\times$$
$$\times\left[\begin{matrix}\delta(f_2)+\\+\dfrac{\delta(f_2-2f_0)+\delta(f_2+2f_0)}{2}\end{matrix}\right]\times$$
$$\times\exp(i2\pi(f_1+f_2)t)df_1df_2=$$
$$=\frac{A^4}{32\pi}H_2(2f_0,2f_0)\cos(2\pi 4f_0t)+\frac{A^4}{32\pi}H_2(2f_0,-2f_0). \qquad (28)$$

And the third-order response:

$$y_3(t)=\frac{1}{6}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\underline{H}_3(f_1,f_2,f_3)\left(\frac{A^2\sqrt{2\pi}}{4\pi}\right)^3\times$$
$$\times\left[\begin{matrix}\delta(f_1)+\\+\dfrac{\delta(f_1-2f_0)+\delta(f_1+2f_0)}{2}\end{matrix}\right]\times$$
$$\times\left[\begin{matrix}\delta(f_2)+\\+\dfrac{\delta(f_2-2f_0)+\delta(f_2+2f_0)}{2}\end{matrix}\right]\times$$
$$\times\left[\begin{matrix}\delta(f_3)+\\+\dfrac{\delta(f_3-2f_0)+\delta(f_3+2f_0)}{2}\end{matrix}\right]\times$$
$$\times\exp(i2\pi(f_1+f_2+f_3)t)df_1df_2df_3=$$
$$=\frac{A^6\sqrt{2\pi}}{768\pi^2}H_3(2f_0,2f_0,2f_0)\cos(2\pi 6f_0t)+$$
$$+\frac{A^6\sqrt{2\pi}}{256\pi^2}H_3(-2f_0,2f_0,2f_0)\cos(2\pi 2f_0t). \qquad (29)$$

By analogy, we calculate the responses from action (8) by preliminarily reducing the degree of the following expression:

$$x(t)=A^3\cos^3(2\pi f_0t)=\frac{A^3}{4}[3\cos(\omega_0t)+\cos(3\omega_0t)]. \qquad (30)$$

Given expressions (25) and (26), one can record the Fourier transform:

$$X(f)=\frac{\sqrt{2\pi}}{2\pi}\frac{A^3}{4}\left[\begin{matrix}3[\delta(f-f_0)+\delta(f+f_0)]+\\+[\delta(f-3f_0)+\delta(f+3f_0)]\end{matrix}\right]. \qquad (31)$$

The first-order response for action (30) takes the following form:

$$y_1(t)=\frac{1}{n!}\int_{-\infty}^{+\infty}\underline{H}_1(f)\frac{3A^3}{4}\frac{\sqrt{2\pi}}{2\pi}\begin{bmatrix}\delta(f-f_0)+\\+\delta(f+f_0)\end{bmatrix}+$$
$$+\frac{A^3}{4}\frac{\sqrt{2\pi}}{2\pi}\begin{bmatrix}\delta(f-3f_0)+\\+\delta(f+3f_0)\end{bmatrix}\cdot e^{i2\pi\cdot f\cdot t}\mathrm{d}f_0=$$
$$=\frac{3A^3\sqrt{2\pi}}{4\pi}\underline{H}_1(f_0)\cos(2\pi f_0t)+$$
$$+\frac{A^3\sqrt{2\pi}}{4\pi}\underline{H}_1(3f_0)\cos(2\pi 3f_0t). \quad (32)$$

The second-order response for action (30) takes the following form:

$$y_2(t)=\frac{1}{2}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}H_2(f_1,f_2)\begin{bmatrix}\frac{3A^3\sqrt{2\pi}}{8\pi}\begin{bmatrix}\delta(f_1-f_0)+\\+\delta(f_1+f_0)\end{bmatrix}+\\+\frac{A^3\sqrt{2\pi}}{8\pi}\begin{bmatrix}\delta(f_1-3f_0)+\\+\delta(f_1+3f_0)\end{bmatrix}\end{bmatrix}\begin{bmatrix}\frac{3A^3\sqrt{2\pi}}{8\pi}\begin{bmatrix}\delta(f_2-f_0)+\\+\delta(f_2+f_0)\end{bmatrix}+\\+\frac{A^3\sqrt{2\pi}}{8\pi}\begin{bmatrix}\delta(f_2-3f_0)+\\+\delta(f_2+3f_0)\end{bmatrix}\end{bmatrix}\times$$
$$\times\exp\big(i2\pi(f_1+f_2)t\big)\mathrm{d}f_1\mathrm{d}f_2=$$
$$=\frac{25A^6}{48\pi}H_2(f_0,f_0)\cos(2\pi 2f_0t)+\frac{3A^6}{16\pi}\underline{H}_2(f_0,3f_0)\cos(2\pi 4f_0t)+$$
$$+\frac{A^6}{32\pi}\underline{H}_2(3f_0,3f_0)\cos(2\pi 6f_0t)+\frac{35A^6}{96\pi}H_2(f_0,-f_0). \quad (33)$$

The third-order response:

$$y_3(t)=\frac{1}{6}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}H_3(f_1,f_2,f_3)\begin{bmatrix}\left(\frac{3\sqrt{2\pi}A^3}{8\pi}\begin{bmatrix}\delta(f_1-f_0)+\\+\delta(f_1+f_0)\end{bmatrix}+\frac{\sqrt{2\pi}A^3}{8\pi}\begin{bmatrix}\delta(f_1-3f_0)+\\+\delta(f_1+3f_0)\end{bmatrix}\right)\times\\\times\left(\frac{3\sqrt{2\pi}A^3}{8\pi}\begin{bmatrix}\delta(f_2-f_0)+\\+\delta(f_2+f_0)\end{bmatrix}+\frac{\sqrt{2\pi}A^3}{8\pi}\begin{bmatrix}\delta(f_2-3f_0)+\\+\delta(f_2+3f_0)\end{bmatrix}\right)\times\\\times\left(\frac{3\sqrt{2\pi}A^3}{8\pi}\begin{bmatrix}\delta(f_3-f_0)+\\+\delta(f_3+f_0)\end{bmatrix}+\frac{\sqrt{2\pi}A^3}{8\pi}\begin{bmatrix}\delta(f_3-3f_0)+\\+\delta(f_3+3f_0)\end{bmatrix}\right)\end{bmatrix}\times$$
$$\times\exp\big(i2\pi(f_1+f_2+f_3)t\big)\mathrm{d}f_1\mathrm{d}f_2\mathrm{d}f_3=\frac{129\sqrt{2\pi}A^9}{768\pi^2}\underline{H}_3(f_0,f_0,-f_0)\cos(2\pi f_0t)+$$
$$+\frac{\sqrt{2\pi}A^9}{16\pi^2}\underline{H}_3(f_0,f_0,f_0)\cos(2\pi 3f_0t)+$$
$$+\frac{11\sqrt{2\pi}A^9}{256\pi^2}H_3(f_0,f_0,3f_0)\cos(2\pi 5f_0t)+$$
$$+\frac{3\sqrt{2\pi}A^9}{256\pi^2}H_3(-f_0,3f_0,3f_0)\cos(2\pi 7f_0t)+$$
$$+\frac{\sqrt{2\pi}A^9}{768\pi^2}H_3(3f_0,3f_0,3f_0)\cos(2\pi 9f_0t). \quad (34)$$

It may be noticed that the more the order of the response, the smaller the part of the contribution of its components into the overall product of nonlinearity, for example, the third-order harmonic in the response order of the third order to action (6) has the amplitude $\frac{A^3}{24}H_3(f_0,f_0,f_0)$, and in the response of the fifth order – $\frac{A^5}{384}H_5(f_0,f_0,f_0,f_0,-f_0)$.

5. Obtaining expressions and determining the values for the higher-order transfer functions of a nonlinear non-inertia circuit

Based on expressions (1) and (9), write down the response of the n-th order signal

$$y(t)=\sum_{n=1}^{\infty}\frac{1}{n!}\int_{-\infty}^{+\infty}\dots\int_{-\infty}^{+\infty}H_n(f_1,\dots,f_n)\times$$
$$\times\prod_{i=1}^{n}\begin{bmatrix}\delta(f-f_1)+\\+\delta(f-f_2)+\dots+\\+\delta(f-f_n)\end{bmatrix}\exp(i2\pi f_it)\mathrm{d}f_i. \quad (35)$$

The product of the sum of the delta functions produces the sum of all different terms in the following form

$$\delta(f-f_{k1})+$$
$$+\delta(f-f_{k2})+\dots+\delta(f-f_{kn}) \quad (36)$$

and each k_i index accepts a value from 1 to n. If each is represented in the product (45) by m_i times, we have

$$\frac{n!}{m_1!m_2!\dots m_n!}=$$
$$=(n;m_1,m_2,\dots,m_n), \quad (37)$$

terms, which, if one does not take into consideration the permutations between them, are identical to each other.

In ratio (37), the multiplicative coefficient is marked as (n; $m_1, m_2,\dots, m_n$). Act according to (35) and bring together the derived terms. Then the result can be represented in the following form:

$$y(t)=$$
$$=\sum_{n=1}^{\infty}\frac{1}{n!}\sum_m\frac{n!}{m_1!m_2!\dots m_n!}\times$$
$$\times H_n(f_1,\dots,f_n)\times$$
$$\times\exp\left[i2\pi(f_{k1}+\dots+f_{kn})t\right], \quad (38)$$

where m under the sum sign shows that the sum includes all different sets $\{m\}$, such as

$$m_i<m_{i+1} \text{ and } \sum_{i=1}^{n}m_i=n. \quad (39)$$

The inequality $m_i<m_{i+1}$ arranges the frequencies in $\{f_{mi}\}$ based on the index so that the frequency sets that differ only in permutations are not repeated.

Thus, we obtain that in (38) the coefficient at

$\exp[i2\pi(f_{k1}+\dots+f_{kn})t]$ is $n!H_n(f_1,\dots, f_n)$.

Hence, we can conclude that there is a recurrent method for determining all nonlinear transfer functions based on the equation describing the system.

A given method implies:

1) the system is initially "probed" by one exponential input action, resulting in finding $H_1(f)$.

2) then the sum of the two exponents is given to determine $H_2(f_1, f_2)$ via $H_1(f)$.

3) when this procedure is extended, one additional exponent is added at each step, up to the n-the order at which the input signal is the sum of n exponents with frequencies $(f_1,...,f_n)$.

Hence it follows that the nonlinear transfer function of the order n is built from all nonlinear transfer functions of lower orders (Fig. 1).

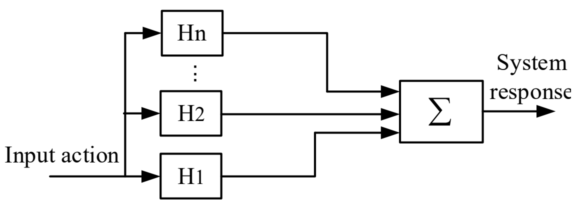

Fig. 1. A procedure for obtaining a nonlinear system response

Thus, knowing the parameters of the input signal $x(t)$ and the transfer function of the n-th order H_n, one can analyze the parameters of the output reaction of the system $y(t)$ and thereby solve the set problem.

To develop a method for calculating the nonlinear transfer functions of the higher orders for a nonlinear electric circuit, which makes it possible to calculate the coefficients for expanding into the Volterra series of function $[y(t)]^L$. L is the degree indicator, via the coefficients for expanding into the Volterra series of function $y(t)$.

As shown above, $y(t)$ can be presented as a series (38), in which the coefficient at $\exp(j2\pi(f_1+...+f_n))$ is $H_n(f_1,...,f_n)$, then $[y(t)]^L$ can be expanded into a similar series. In this series, the coefficient at $\exp(j2\pi(f_1+...+f_n))$ is $H_n^{(L)}(f_1,\ldots,f_n)$, where (L) denotes the order of the functions $H_n^{(L)}(f_1,\ldots,f_n)$, that are part of the Volterra series' cores.

Functions in the form $H_n^{(L)}(f_1,\ldots,f_n)$, are expressed through the nonlinear lower-order transfer functions from the Volterra series for $[y(t)]^L$, that is

$$H_n^{(L)}(f_1,...,f_n)=$$
$$=L!\sum_{\mu;L;n}\sum_N H_{\mu 1}(f_{\mu 1},...,f_{\mu 2})\times$$
$$\times H_{\mu 2}(f_{\mu 2+1},...,f_{\mu 1+\mu 2})\ldots H_{\mu L}(f_{\nu L},...,f_n). \quad (40)$$

This formula expresses the n-multiple Fourier transform of the kernel of the n-th order from the Volterra series for $[y(t)]^L$. Here, L is a positive integer and $1 \le L \le n$. At $L>n$, the value $H_n^{(L)}(f_1,\ldots,f_n)$ is convoluted to zero, and, at $L=n$:

$$H_n^{(L)}(f_1,...,f_n)=n!H_1(f_1)H_1(f_2)...H_1(f_n). \quad (41)$$

In ratio (40), $\nu=\mu_1+\mu_2+...+\mu_{L-1}+1=n-\mu_L+1$, and $(\mu; L; n)$ under the sign of the first sum represents the sum of integers of μ_i, so that

$$\sum_{i=1}^{L}\mu_i=n, \quad \mu_1\le\mu_2\le\ldots\le\mu_L. \quad (42)$$

The second sum in ratio (40) captures N "non-identical" sets obtained by permuting indexes of different f. The concept of "identity" is used in the sense that the combinations of f arguments are the same, that is, $H_2(f_1, f_2)$ matches $H_2(f_2, f_1)$, etc.

The number of terms of the second amount is equal to

$$N=\frac{n!}{\mu_1!\mu_2!...\mu_L!s_1!s_2!...s_k!}, \quad (43)$$

where s_1 is the number of inequalities in the first series of equal V from the group $\mu_1\le\mu_2\le...\le\mu_L$, s_2 is the number of equal μ in the second series of inequalities from the group $\mu_1\le\mu_2\le...\le\mu_L$, etc.

When the μ values are not equal, s does not appear.

For example, it is required to calculate $H_2^{(2)}(f_1,f_2)$ when n=2

$$L=2,\ \mu_1=\mu_2=1,\quad s_1=2,$$

$$N=\frac{2!}{1!1!2!}=1.$$

Thus,

$$H_2^{(2)}(f_1,f_2)=2!H_1(f_1)H_1(f_2). \quad (44)$$

By analogy, at n=3 and

$$L=2,\quad \mu_1=1,\quad \mu_2=2,$$

$$N=\frac{3!}{1!2!}=3.$$

Thus,

$$H_3^{(3)}(f_1,f_2,f_3)=2!\begin{bmatrix}H_1(f_1)H_2(f_2,f_3)+\\+H_1(f_2)H_2(f_1,f_3)+\\+H_1(f_3)H_2(f_1,f_2)\end{bmatrix}. \quad (45)$$

Therefore, based on the above transforms we have shown the possibility to obtain an analytical expression for the transfer function of any order.

Fig. 2 shows the circuit of a nonlinear non-inertia circuit, which contains the linear resistant element R_N and the nonlinear resistant element R_{NM}.

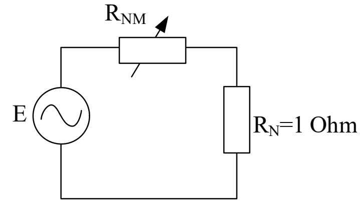

Fig. 2. A diagram of the nonlinear non-inertia circuit

Input action (5) has the amplitude A=1, that is, the harmonic signal generator has the function – $e(t)=\cos(2\pi f_0 t)$. A volt-ampere characteristic of the nonlinear resistant element is represented by the fifth-degree polynomial [23, 24]:

$$i_{R_{NM}}=u_{R_{NM}}+0.5U_{R_{NM}}^2+0.17U_{R_{NM}}^3+$$
$$+0.04U_{R_{NM}}^4+0.01U_{R_{NM}}^5. \quad (46)$$

The current $i(t)$ is similar to $x(t)$, the voltage U – to $y(t)$.

Papers [7–9] show that the method of state variables can be used to define the nonlinear transfer functions for both the inertial and non-inertial nonlinear electric circuits.

Under this scenario, a mathematical model for the circuit in a state variable method takes the following form [7–9]:

$$i_{R_N}(t)=\vec{B}_1\vec{X}(t)+\vec{B}_2\vec{X}_{IS}(t)+\vec{B}_3\vec{X}_N(t), \quad (47)$$

$$i_{R_{NM}}(t)=\vec{M}_1\vec{X}(t)+\vec{M}_2\vec{X}_{IS}(t)+\vec{M}_3\vec{X}_N(t), \quad (48)$$

where $\vec{X}(t)$ is the state vector that contains information about the voltage on the linear capacitive elements, and currents through the linear inductive elements; $\vec{X}_{IS}(t)$ is the vector of voltage and current from independent sources; $\vec{X}_N(t)$ is the vector of voltage and current in the nonlinear elements.

For the electric circuit shown in Fig. 2:

$$\vec{X}(t)=[\ ],\ \vec{X}_{IS}(t)=[e(t)],\ \vec{X}_N(t)=\left[u_{R_{NM}}(t)\right]. \quad (49)$$

Equation (47) is the equation of the resistive elements' currents [23], where

$$\vec{B}_1=[\ ],\ \vec{B}_2=\left[\frac{1}{R_N}\right] \text{ and } \vec{B}_3=\left[-\frac{1}{R_N}\right]. \quad (50)$$

Equation (48) is the equation of current and voltage on the nonlinear elements, where

$$\vec{M}_1=[\ ],\ \vec{M}_2=\left[\frac{1}{R_N}\right],\ \vec{M}_3=\left[-\frac{1}{R_N}\right]. \quad (51)$$

A nonlinear transfer function of the first order is the transfer function of the linearized circuit, which is obtained from the nonlinear circuit by replacing all nonlinear elements with their linear equivalents. This linear replacement scheme is shown in Fig. 3.

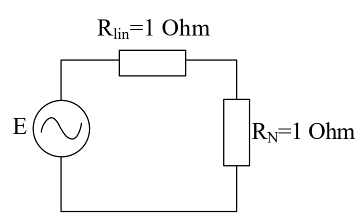

Fig. 3. A replacement scheme of the nonlinear circuit

It is known that the nonlinear transfer function of the first order $H_1(f)$ relative to the electrodes of the nonlinear element R_{NM} is 0.5, and, relative to the electrodes of the linear resistive element R_N, – $H_{R_N 1}(f)=0.5$ [7–9].

Fit into (48), instead of the vector $\vec{X}(t)$, an expression as the sum of the exponents: $\exp(i2\pi f_1 t)+\exp(i2\pi f_2 t)+\ldots+$ $+\exp(i2\pi f_n t)$, and, instead of the vectors $\vec{X}_{IS}(t)$ and $\vec{X}_N(t)$, the nonlinear transfer functions that are determined relative to the electrodes of the corresponding elements in the nonlinear circuit. Next, by equating the coefficients to the exponents with the same arguments in the right and left sides of (48), we derive an equation to calculate the nonlinear transfer functions of higher orders [21–24]. That is, the nonlinear second-order transfer function relative to electrodes relative to the electrodes of the nonlinear element R_{NM} is calculated through the nonlinear transfer function of the first order according to expression (48):

$$H_2(f_1,f_2)+0.5H_2^{(2)}(f_1,f_2)=0-\frac{1}{R_N}H_2(f_1,f_2), \quad (52)$$

where

$$H_2^{(2)}=2!H_1(f_1)H_1(f_2).$$

Thus

$$H_2(f_1,f_2)+0.5\left[2!H_1(f_1)H_1(f_2)\right]=-H_2(f_1,f_2), \quad (53)$$

and,

$$H_2(f_1,f_2)+H_1(f_1)H_1(f_2)=-H_2(f_1,f_2),$$

$$2H_2(f_1,f_2)=-H_1(f_1)H_1(f_2). \quad (54)$$

Thus,

$$H_2(f_1,f_2)=-(0.5\cdot 0.5)/2=-0.125.$$

The nonlinear third-order transfer function is calculated in a similar way through the nonlinear transfer functions of the first and second orders [7–9]:

$$H_3(f_1,f_2,f_3)+0.5H_3^{(2)}(f_1,f_2,f_3)+$$
$$+0.17H_3^{(2)}(f_1,f_2,f_3)=0-\frac{1}{R_H}H_3(f_1,f_2,f_3), \quad (55)$$

where

$$H_3^{(2)}=2!\begin{pmatrix}H_1(f_1)H_2(f_1,f_2)+\\+H_1(f_2)H_3(f_1,f_3)+H_1(f_3)H_2(f_1,f_2)\end{pmatrix}, \quad (56)$$

$$H_3^{(3)}=3!H_1(f_1)H_1(f_2)H_1(f_3). \quad (57)$$

Then

$$H_3(f_1,f_2,f_3)+H_1(f_1)H_2(f_1,f_2)+$$
$$+H_1(f_2)H_3(f_1,f_3)+H_1(f_3)H_2(f_1,f_2)+$$
$$+1.02H_1(f_1)H_1(f_2)H_1(f_3)=-H_3(f_1,f_2,f_3) \quad (58)$$

and,

$$2H_3(f_1,f_2,f_3)=-H_1(f_1)H_2(f_1,f_2)-$$
$$-H_1(f_2)H_3(f_1,f_3)-H_1(f_3)H_2(f_1,f_2)-$$
$$-1.02H_1(f_1)H_1(f_2)H_1(f_3). \quad (59)$$

Thus,

$$H_3(f_1,f_2,f_3)=\frac{\left[-0.5(-0.125)-0.5(-0.125)-0.5(-0.125)-1.02\cdot 0.5\cdot 0.5\cdot 0.5\right]}{2}=0.03.$$

Table 1 gives the results of calculating the nonlinear transfer functions of higher orders obtained by the above technique for a signal of frequency f_0. Because there are no frequency-dependent elements in the circuit, the values of the nonlinear transfer functions do not depend either on the frequency value or their set.

The values of the nonlinear transfer functions in Table 1 for the analyzed circuit relative to the electrodes of the linear resistant element R_{NM} carry a negative sign according to equation (46).

Table 1

Values of the nonlinear transfer functions of higher orders

Function order	Transfer function	Estimated value	Estimated value relative to the resistive element R_{NM}
1	$H_1(f_0)$	0.5	0.5
2	$H_2(f_0,f_0)$	–0.125	0.125
3	$H_3(f_0,f_0,-f_0)$	0.03	–0.03
4	$H_4(f_0,f_0,f_0,-f_0)$	–0.0121875	0.0121875
5	$H_5(f_0,f_0,f_0,f_0,-f_0)$	–0.03825	0.03825

In a general case, the nonlinear transfer functions are complex quantities [4]. However, in this case, since the electric circuit lacks both the linear and nonlinear reactive elements, the nonlinear transfer functions are the real numbers.

6. Obtaining analytical expressions on calculating the harmonic coefficient for individual harmonics using the nonlinear higher-order transfer functions for a nonlinear non-inertia circuit

After fitting the harmonic amplitudes from (13) to (18) to the expression for calculating the coefficient of the i-th harmonic, the formulae for harmonic coefficients on the second and third harmonics will take the following form:

$$K_{H_2}\approx$$
$$\approx\frac{\left|\frac{A^2}{4}H_2(f_0,f_0)+\frac{A^4}{48}\underline{H}_4(f_0,f_0,f_0,-f_0)+\frac{A^6}{1{,}536}H_6(f_0,f_0,f_0,f_0,-f_0,-f_0)\right|}{\left|AH_1(f_0)+\frac{A^3}{8}\underline{H}_3(f_0,f_0,-f_0)+\frac{A^5}{192}H_5(f_0,f_0,f_0,-f_0,-f_0)+\frac{A^7}{9{,}216}H_7(f_0,f_0,f_0,f_0,-f_0-f_0,-f_0)\right|},\quad(60)$$

$$K_{H_3}\approx$$
$$\approx\frac{\left|\frac{A^3}{24}H_3(f_0,f_0,f_0)+\frac{A^5}{384}H_5(f_0,f_0,f_0,f_0,-f_0)+\frac{A^7}{1{,}536}H_7(f_0,f_0,f_0,f_0,-f_0,-f_0,-f_0)\right|}{\left|AH_1(f_0)+\frac{A^3}{8}H_3(f_0,f_0,-f_0)+\frac{A^5}{192}H_5(f_0,f_0,f_0,-f_0,-f_0)+\frac{A^7}{9{,}216}H_7(f_0,f_0,f_0,f_0,-f_0-f_0,-f_0)\right|}.\quad(61)$$

Expressions (60) and (61) make it possible to obtain more accurate results than the results produced by the corresponding formulae in which the transfer functions in the nominator do not exceed the third order:

$$K_{H_2}\approx\frac{\left|\frac{A^2}{4}\underline{H}_2(f_0,f_0)\right|}{\left|A\underline{H}_1(f_0)\right|},\quad(62)$$

$$K_{H_3}\approx\frac{\left|\frac{A^3}{24}\underline{H}_3(f_0,f_0,f_0)\right|}{\left|A\underline{H}_1(f_0)\right|}.\quad(63)$$

Table 2 gives the results of calculating the harmonic coefficients of the studied electrical circuit based in expressions (60) to (63) and by using the MicroCAP 9.0 circuit modeling package.

Table 2

Values of the harmonic coefficients based on expressions (60) to (63)

Harmonic coefficient	Expression for calculation	Estimated values	Simulation data in the MicroCAP 9.0 package
$K_{Г2}$	(60)	0.062452	0.062346
	(62)	0.0625	
$K_{Г3}$	(61)	0.002396	0.002302
	(63)	0.0025	

Comparing the results of harmonic coefficients' values given in Table 2, one can see that:

– the discrepancy between the harmonic coefficients obtained from expressions (60), (61), compared to those obtained from (62), (63), is about 0.07 % and 3.9 %, respectively;

– the discrepancy between the harmonic coefficients' values obtained from expressions (60), (61), compared to those from the MicroCAP 9.0 circuit modeling package, does not exceed 0.17 % and 4 %, respectively.

7. Discussion of results of the methodology for calculating the nonlinear transfer functions in the Volterra series

The use of the nonlinear transfer functions up to the seventh order inclusively makes it possible to obtain the responses from a nonlinear non-inertia circuit for different types of influences and calculate a harmonic coefficient with a greater degree of accuracy.

The considered methodology makes it possible to evaluate the set of nonlinear effects in the form of harmonic and intermodulation distortions in the problems related to the electromagnetic compatibility of groups of radio-electronic means with the accuracy required by users.

A special feature of the procedure for calculating the transfer functions in the Volterra series is the recurrent method for determining the nonlinear transfer functions of any order n based on an equation describing the system and built from all nonlinear transfer functions of lower orders.

Once we determine the transfer function of the n-th order and the input signal parameters $x(t)$, one can obtain the output response function of the system $y(t)$ and analyze the parameters of a radio tract.

To compute the nonlinear transfer functions of higher orders for a nonlinear electric circuit, we have derived formula (40), which makes it possible to calculate the coefficients for expanding the Volterra series of the response function $[y(t)]^L$ of the L-th order.

The outlined technique of determining the nonlinear transfer functions is invariant to the topology of a nonlinear electrical circuit, as well as to the quantity and type of nonlinear elements.

The most common point of view is that calculating the Volterra kernels above the third order is a time-consuming procedure, so the analysis of highly nonlinear modes based on functional series becomes impractical. This can be denied by that all highly nonlinear problems, which are solved beyond the level of the third order, face the "curse of dimensionality". No systematic study of factors limiting the use of the Volterra series in highly nonlinear modes was conducted [6]. This indicates that comprehensive in-depth research into the specific aspects of the Volterra series application for essentially nonlinear modes remains relevant.

The limitation of this study is that calculating the values of transfer functions becomes more time-consuming in proportion to the growth of their degree. According to expression (40), one can see that when trying to calculate the transfer functions exceeding the fifth order, the complexity of computation increases significantly by adding the sum of the components' coefficients of lower orders. In general, it also affects the response order of the entire system (1), which eventually leads to an increase in the components of the formulae to calculate harmonic coefficients (60) and (61).

A subsequent procedure for obtaining the expressions of the output responses of a nonlinear system and determining the transfer functions and their values requires the application of the programmed implementation of the procedure in the form of software for mathematical modeling and circuit simulation.

This study could be advanced by developing algorithmic maintenance for simulating the nonlinear non-inertial and inertial electrical circuits. When considering inertial electrical circuits, it is worth noting that the electric circuit has the linear and nonlinear reactive frequency-dependent elements; the transfer functions are complex quantities, which would significantly complicate a procedure for determining system responses.

In order to cover more components that affect the accuracy of calculating the system output parameters, we argue about the prospect of combining a given method with the methods of differential equations. The methods of differential equations describe in detail the properties of a circuit from a time point of view.

9. Conclusions

1. By using the Volterra series, we have obtained the analytical expressions for the output responses of the first-third orders for 3 types of harmonic input actions in a nonlinear system. It is noted that representing an output signal from a nonlinear electric circuit in the form of the functional Volterra series has several advantages such as the explicit connection between an input action and response, the invariability relative to the types of the input action. A Fourier transform was used to represent an input action. It is shown that the main difficulty in determining the nonlinear transfer functions of higher orders, included in the kernels of the Volterra series, is related to the increased volume of computation by adding the exponential components of the transfer functions of lower orders. It is shown that the order of the transfer functions increases in proportion to the system's response order.

2. We have demonstrated the possibility to determine the transfer functions up to the fifth order for a nonlinear inertia circuit, under a harmonic input action, which are derived on the basis of the lower-order transfer functions. The use of the nonlinear transfer functions up to the fifth order inclusive allows a more accurate assessment of the nonlinear effects in the form of harmonic and intermodulation distortions regarding radio-electronic means of mobile systems.

3. The derivation of the analytical expressions has been demonstrated regarding the calculation of a harmonic coefficient for the second and third harmonics using the nonlinear transfer functions of higher orders for a nonlinear non-inertia circuit. We have computed the harmonic coefficient based on the second and third harmonics according to the derived expressions containing the responses from a nonlinear system of higher orders. It is shown that the discrepancy between the harmonic coefficients' values, obtained from expressions (60), (61), compared to those simplified from (62), (63), is about 0.07 % and 3.9 %, respectively, which increases the accuracy of the calculation. Thus, the proposed methodology makes it possible to evaluate the set of nonlinear effects in the EMC problems related to the groups of radio-electronic means with the accuracy required by users.

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References

1. Zaker, N. A., Alsaleem, N., Kashmoola, M. A. (2018). Multi-agent Models Solution to Achieve EMC In Wireless Telecommunication Systems. 2018 1st Annual International Conference on Information and Sciences (AiCIS). doi: https://doi.org/10.1109/aicis.2018.00061
2. Alsaleem, N. Y. A., Moskalets, M., Teplytska, S. (2016). The analysis of methods for determining direction of arrival of signals in problems of space-time access. Eastern-European Journal of Enterprise Technologies, 4 (9 (82)), 36–44. doi: https://doi.org/10.15587/1729-4061.2016.75716
3. Alsaleem, N. Y. A., Kashmoola, M. A., Moskalets, M. (2018). Analysis of the efficiency of spacetime access in the mobile communication systems based on an antenna array. Eastern-European Journal of Enterprise Technologies, 6 (9 (96)), 38–47. doi: https://doi.org/10.15587/1729-4061.2018.150921
4. Kashmoola, M. A., Alsaleem, M. Y. anad, Alsaleem, N. Y. A., Moskalets, M. (2019). Model of dynamics of the grouping states of radio electronic means in the problems of ensuring electromagnetic compatibility. Eastern-European Journal of Enterprise Technologies, 6 (9 (102)), 12–20. doi: https://doi.org/10.15587/1729-4061.2019.188976

5. Kolyadenko, Yu. Yu., Chursanov, N. A., Bondarenko, O. S. (2019). Model of electromagnetic interactions in LTE network. Radiotekhnika, 196, 46–50. doi: https://doi.org/10.30837/rt.2019.1.196.05
6. Bobreshov, A. M., Mymrikova, N. N. (2013). Problemy analiza sil'no nelineynyh rezhimov elektronnyh ustroystv na osnove ryadov Vol'terry. Voronezhskiy gosudarstvennyy universitet. Vestnik VGU. Seriya: fizika. Matematika, 2, 15–25.
7. Bussgang, D., Erman, L., Greyam, D. (1974). Analiz nelineynyh sistem pri vozdeystvii neskol'kih vhodnyh signalov. TIIER, 62 (8), 56–82.
8. Bedrosyan, E., Rays, S. O. (1971). Svoystva vyhodnogo signala sistem, opisyvaemyh ryadami Vol'terra (nelineynyh sistem s pamyat'yu), pri podache na vhod garmonicheskih kolebaniy i gaussova shuma. TIIER, 59 (12), 58–82.
9. Chong, E. The Volterra series and the direct method of distortion analysis. Available at: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.387.7283&rep=rep1&type=pdf
10. Kolding, T. E., Larsen, T. (1997). High-order volterra series analysis using parallel computing. International Journal of Circuit Theory and Applications, 25 (2), 107–114. doi: https://doi.org/10.1002/(sici)1097-007x(199703/04)25:2<107::aid-cta956>3.0.co;2-y
11. Heiskanen, A., Rahkonen, T. (2002). 5th order multi-tone Volterra simulator with component-level output. 2002 IEEE International Symposium on Circuits and Systems. Proceedings (Cat. No.02CH37353). doi: https://doi.org/10.1109/iscas.2002.1010293
12. Schreurs, D., ODroma, M., Goacher, A. A., Gadringer, M. (Eds.) (2008). RF Power Amplifier Behavioral Modeling. Cambrige University Press. doi: https://doi.org/10.1017/cbo9780511619960
13. Dobes, J. (2008). Using Volterra Series for an Estimation of Fundamental Intermodulation Products. Radioengineering, 17 (4), 59–64.
14. Anilionienė, J. (2011). The Volterra series of distortion analysis. Lietuvos Matematikos Rinkinys, 52. doi: https://doi.org/10.15388/lmr.2011.mt01
15. Cooman, A., Bronders, P., Peumans, D., Vandersteen, G., Rolain, Y. (2018). Distortion Contribution Analysis With the Best Linear Approximation. IEEE Transactions on Circuits and Systems I: Regular Papers, 65 (12), 4133–4146. doi: https://doi.org/10.1109/tcsi.2018.2834139
16. Yu, H., El-Sankary, K., El-Masry, E. I. (2015). Distortion Analysis Using Volterra Series and Linearization Technique of Nano-Scale Bulk-Driven CMOS RF Amplifier. IEEE Transactions on Circuits and Systems I: Regular Papers, 62 (1), 19–28. doi: https://doi.org/10.1109/tcsi.2014.2341116
17. Gourary, M. M., Rusakov, S. G., Ulyanov, S. L., Zharov, M. M., Mulvaney, B. J. (2011). Circuit Distortion Analysis Based on the Simplified Newton's Method. Journal of Electrical and Computer Engineering, 2011, 1–11. doi: https://doi.org/10.1155/2011/540305
18. Wei, W., Ye, P., Song, J., Zeng, H., Gao, J., Zhao, Y. (2019). A Behavioral Dynamic Nonlinear Model for Time-Interleaved ADC Based on Volterra Series. IEEE Access, 7, 41860–41873. doi: https://doi.org/10.1109/access.2019.2905365
19. Maas, S. A. (2003). Nonlinear Microwave and RF Circuits. Artech House, 608.
20. Schetzen, M. (2006). The Volterra and Wiener Theories of Nonlinear Systems. Krieger Pub Co, 595.
21. Doroshenko, T. V., Selivanov, K. A. (2006). Opredelenie parametrov vihidnogo signala priemnika signalov s pomoshch'yu ryadov Vol'terra. Radiotekhnika, 144, 182–186.
22. Moskalets, N. V., Selivanov, K. A., Nikitenko, T. V. (2011). Analiz nelineynyh iskazheniy v radiotrakte s primeneniem razlichnyh metodov otsenki nelineynosti. Problemy telekommunikatsiy, 2 (4), 150–161. Available at: https://openarchive.nure.ua/bitstream/document/469/1/112_selivanov_radio.pdf
23. Bogdanovich, B. M. (1980). Nelineynye iskazheniya v priemno-usilitel'nyh ustroystvah. Moscow: Svyaz', 280.
24. Baskakov, S. I. (1988). Radiotehnicheskie tsepi i signaly. Moscow: Vysshaya shkola, 446.