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RESEARCH OF ANTI-RESONANCE THREE-MASS VIBRATORY MACHINE WITH A VIBRATION EXCITER IN THE FORM OF A PASSIVE AUTO-BALANCER

A three-mass anti-resonance vibratory machine with a vibration exciter in the form of a passive auto-balancer has been analytically synthesized. In the vibratory machine, platforms 1 and 2 are visco-elastically attached to platform 3. Platform 3 is visco-elastically attached to the base. The motion of loads relative to the auto-balancer housing is hindered by the forces of viscous resistance.

A theoretical study has shown that the vibratory machine possesses three resonance frequencies and three corresponding forms of platforms' oscillations. Values for the parameters of supports that ensure the existence of an anti-resonance form of motion have been analytically selected. Under an anti-resonance form, platform 3 is almost non-oscillating while platforms 1 and 2 oscillate in the opposite phase.

In the vibratory machine, platform 1 can be active (working), platform 2 will then be reactive (a dynamic vibration damper), and vice versa. At the same time, the vibratory machine will operate when mounting a vibration exciter both on platform 1 and platform 2.

An anti-resonance form would occur when the loads get stuck in the vicinity of the second resonance frequency of the platforms' oscillations.

Given the specific parameters of the vibratory machine, numerical methods were used to investigate its dynamic characteristics. Numerical calculations have shown the following for the case of small internal and external resistance forces in the vibratory machine:

– theoretically, there are seven possible modes of load jam;

– the second (anti-resonance) form of platform oscillations is theoretically implemented at load jamming modes 3 and 4;

– jamming mode 3 is locally asymptotically stable while load jamming mode 4 is unstable;

– for the loads to get stuck in the vicinity of the second resonance frequency, one needs to provide the vibratory machine with the initial conditions close to the jamming mode 3, or smoothly accelerate the rotor to the working frequency;

– the dynamic characteristics of the vibratory machine can be controlled in a wide range by changing both the rotor speed and the external and internal forces of viscous resistance.

The results reported here are applicable for the design of anti-resonance three-mass vibratory machines for general purposes

Keywords: inertial vibration exciter, resonance vibrations, anti-resonance vibratory machine, auto-balancer, three-mass vibratory machine, Sommerfeld effect

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1. Introduction

Resonance vibratory machines are promising as regards the vibratory machines for various applications [1]. In them, vibration exciters of smaller mass excite oscillations with a greater amplitude, which improves the reliability and durability of vibratory machine operation.

The most effective and simple techniques to excite resonance oscillations are based on the use of the Sommerfeld effect [2–12]. Among these techniques, of special relevance

is a method that is based on using passive auto-balancers in the form of vibration exciters. The technique is applicable for single- and multi-mass vibratory machines [8–12] and is distinguished by the possibility of changing the characteristics of vibrations in a wide range.

It should be noted that two- and three-mass vibratory machines have been widely used in various industries [13–20]. The multi-mass structure makes it possible to design vibratory machines that almost do not transmit vibrations to the base. Such machines include the anti-resonance vibratory machines. In the three-mass anti-resonance vibratory machines [15–20], the intermediate platform is visco-elastically attached to the base. An active platform (working) and a reactive platform (a dynamic oscillation damper) are visco-elastically attached to the intermediate platform. The anti-resonance form of oscillations is also a resonance mode. However, the vibratory machine parameters are chosen so that the intermediate platform is almost stationary while the active and passive platforms oscillate in the opposite phase.

It is a relevant task to design, based on the results reported in [8–12], an anti-resonance three-mass vibratory machine with a vibration exciter in the form of a passive auto-balancer and to investigate its steady vibrations.

2. Literature review and problem statement

In the resonance-type vibratory machines, it is possible to excite vibrations with electromechanical vibration exciters [1]. However, when the mass of a vibratory platform changes, its resonance oscillation frequency changes as well. Therefore, it is necessary to constantly change the frequency of the disturbing force in order to achieve resonance. This requires a complex control system. In addition, electromechanical vibration exciters are less powerful than the inertial ones.

The easiest technique to excite resonance vibrations is based on a Sommerfeld effect [2]. The effect implies that the shaft of the electric motor with a pendulum mounted onto it (an unbalanced-mass rotor) cannot accelerate to the working frequency of rotation. The pendulum gets stuck at one of the resonance oscillation frequencies of the platform that hosts the electric motor. In a vibratory machine, this automatically excites the resonance vibrations of the vibratory platform. And when the mass of the vibratory platform changes, the frequency of the pendulum's rotation automatically adjusts to the changed resonance frequency [3]. The disadvantage of this technique of vibration excitation is the overloaded electrical circuits of the electric motor.

Instead of an electric motor, it is proposed in [4] to use an unbalanced impeller and an airflow. There is no overloaded electric motor when applying this technique. However, the efficiency of this technique is quite low.

The Sommerfeld effect is also evident in the operation of passive auto-balancers; – ball-type (roller) [5, 6], or pendulum-type [7], both with the rotor isotropic [5, 7] and anisotropic [7] supports. In auto-balancers, loads cannot accelerate to the frequency of rotor rotation and get stuck at one of the resonance frequencies of rotor vibrations. At the same time, the loads are brought together, which creates the greatest imbalance [8]. These loads act as a resonance inertial vibration exciter.

A technique to excite resonance vibrations using passive auto-balancers is theoretically explored in [9–12]. The equations of motion of single-mass, two-mass, and three-mass vibratory machines with a rectilinear translational motion of the platforms and a vibration exciter in the form of a passive auto-balancer were derived in [9]. Study [10] analytically examined the performance of a single-mass vibratory machine, [11] – a two-mass vibratory machine, [12] – a three-mass vibratory machine.

It should be noted that multi-mass vibratory machines possess a series of advantages over single-mass ones: the frequencies of platform oscillations are less dependent on the weight of a load [13]; it is possible to excite the anti-resonance oscillations at which the platforms' oscillations are not transferred to the base [14]. Therefore, three-mass resonance vibratory machines have been widely used in different industries. These machines include: vibratory polishing [15] and vibration lapping [16] machines; vibrating tables [17]; vibratory conveyors [18]; vibratory mills [19]; vibratory transporters [20], etc. These machines are also anti-resonance. In them, the fluctuations of the working platform are practically not transferred to the base.

Thus, the feasibility of the technique for exciting resonance vibrations by passive auto-balancer has not been investigated for the case of three-mass anti-resonance vibratory machines.

3. The aim and objectives of the study

The aim of this work is to study a three-mass anti-resonance vibratory machine with a vibration exciter in the form of a passive auto-balancer. This is necessary for the development and design of new anti-resonance three-mass vibratory machines.

To accomplish the aim, the following tasks have been set:
– to synthesize a three-mass anti-resonance vibratory machine;
– to investigate the dynamic properties of the vibratory machine at certain parameters by numerical methods.

4. Materials and methods to study the three-mass anti-resonance vibratory machine

To build a model of the anti-resonance three-mass vibratory machine, a generalized model is used, which was constructed in work [8]. The vibratory machine parameters are selected from the following conditions (for the existence of an anti-resonance frequency):
– the existence of some threshold frequency of unbalanced rotor rotation, at which the amplitude of an intermediate platform's oscillations is minimal;
– the match between the threshold frequency and one of the natural (resonance) oscillation frequencies of the vibratory machine.

A vibratory machine will operate at the rotor speed exceeding the threshold frequency. In this case, loads in the vibration exciter (under certain conditions) will get stuck at the appropriate resonance frequency, thereby exciting the anti-resonance form of the platform motion.

Our numerical experiment will be conducted using a procedure based on the idea of a parametric solution to the problem of finding frequencies at which loads get stuck and the bifurcation theory of motions [10–12].

5. 1. The synthesis of an anti-resonance three-mass vibratory machine

5. 1. 1. Description of the model of an anti-resonance three-mass vibratory machine

The generalized model of the anti-resonance three-mass vibratory machine is shown in Fig. 1 [9, 12]. The vibratory machine consists of three platforms weighing M_1, M_2, and M_3. The intermediate platform 3 is held by external elastic-viscous supports with a stiffness coefficient k_3 and a viscosity coefficient b_3. Platforms 1 and 3 (2 and 3) are connected via the internal elastic-viscous supports with a stiffness coefficient k_{13} (k_{23}) and a viscosity coefficient b_{13} (b_{23}).
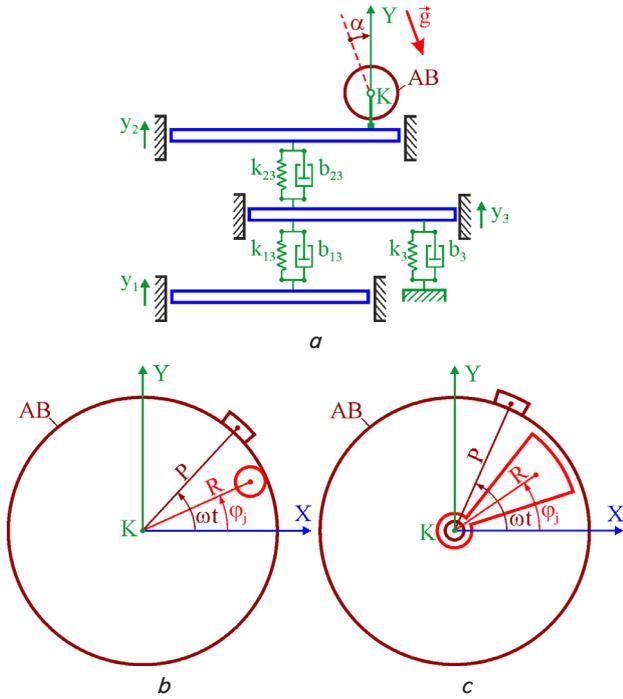
Fig. 1. Model of the anti-resonance three-mass vibratory machine (rotated at an angle α) [9, 12]: *a* — the kinematics of the platform motions, *b* — balls or rollers, *c* — pendulums

Note that in some works the platforms (masses) are termed an active, intermediate, and reactive platform, or a working body, an intermediate platform, and a dynamic vibration damper. In the designed vibratory machine, platform 1 or 2 can be active, and platform 2 or 1 – reactive.

The platforms can only move rectilinearly translationally owing to the fixed guides. The direction of the platforms' motion forms an angle α with the vertical. The platforms' coordinates y_1, y_2, y_3 originate from the positions of the platforms' static equilibrium. In Fig. 1, *a*, $\vec{g}$ is the vector of free fall acceleration near the Earth's surface.

The second platform hosts a passive auto-balancer [9, 12] – a ball, a roller (Fig. 1, *b*), or a pendulum (Fig. 1, *c*).

The auto-balancer housing rotates around the shaft, point K, at a constant angular velocity ω. Two mutually perpendicular axes X, Y originate from the point K and form the right-hand coordinate system.

The auto-balancer consists of N identical loads. The mass of one load is m. The center of the load mass can move along the circumference of radius R with the center at point K (Fig. 2, *a*, *b*). The position of load number j relative to the housing is determined by the angle φ_j, $/j = \overline{1, N}/$. The motion of the load relative to the auto-balancer housing is hin-

dered by the force of the viscous resistance, having a module $F_j = b_w v_j^{(r)} = b_W R |\phi_j' - \omega|$, $/j = \overline{1, N}/$. Here, b_W is the viscous resistance force factor, $v_j^{(r)} = R |\phi_j' - \omega|$ is the module of the speed of the motion of the center of the mass of load number j relative to auto-balancer housing. A bar behind the value denotes the time derivative t.

5. 1. 2. Differential equations of the anti-resonance three-mass vibratory machine motion

For the considered vibratory machine model (Fig. 1), the differential motion equations in a dimensionless form are as follows:

$$\ddot{v}_1 + 2h_{13}\left(\rho_1\dot{v}_1 - \rho_3\dot{v}_3\right) + n_{13}^2\left(\rho_1 v_1 - \rho_3 v_3\right) = 0,$$

$$\ddot{v}_2 + 2h_{23}\left(\dot{v}_2 - \rho_3\dot{v}_3\right) + n_{23}^2\left(v_2 - \rho_3 v_3\right) = -\ddot{s}_y,$$

$$\ddot{v}_3 + 2h_3\dot{v}_3 + n_3^2 v_3 - 2h_{13}\left(\rho_1\dot{v}_1 - \rho_3\dot{v}_3\right) - \\ -n_{13}^2\left(\rho_1 v_1 - \rho_3 v_3\right) - 2h_{23}\left(\dot{v}_2 - \rho_3\dot{v}_3\right) - \\ -n_{23}^2\left(v_2 - \rho_3 v_3\right) = 0,$$

$$\ddot{\phi}_j + \varepsilon\beta\left(\dot{\phi}_j - n\right) + \sigma\cos\left(\phi_j - \alpha\right) + \\ +\varepsilon\ddot{v}_2\cos\phi_j = 0,$$

$$/j = \overline{1, N}/. \tag{1}$$

In (1), a dot above the value denotes a derivative with respect to dimensionless time τ and:
 – the dimensionless variables and time

$$v_1 = y_1/\left(\rho_1\tilde{y}\right), \quad v_2 = y_2/\tilde{y}, \quad v_3 = y_3/\left(\rho_3\tilde{y}\right),$$

$$s_x = \frac{1}{N}\sum_{j=1}^{N}\cos\phi_j, \quad s_y = \frac{1}{N}\sum_{j=1}^{N}\sin\phi_j; \quad \tau = \tilde{\omega}t; \tag{2}$$

 – dimensionless parameters

$$n_{13}^2 = \frac{k_{13}}{M_{2\Sigma}\tilde{\omega}^2}, \quad n_{23}^2 = \frac{k_{23}}{M_{2\Sigma}\tilde{\omega}^2}, \quad n_3^2 = \frac{k_3}{M_3\tilde{\omega}^2},$$

$$h_{13} = \frac{b_{13}}{2M_{2\Sigma}\tilde{\omega}}, \quad h_{23} = \frac{b_{23}}{2M_{2\Sigma}\tilde{\omega}}, \quad h_3 = \frac{b_3}{2M_3\tilde{\omega}},$$

$$n = \frac{\omega}{\tilde{\omega}}, \quad \varepsilon = \frac{\tilde{y}}{\kappa R}, \quad \varepsilon\beta = \frac{b_W}{\kappa m\tilde{\omega}},$$

$$\beta = \frac{b_W}{\varepsilon\kappa m\tilde{\omega}}, \quad \delta = \frac{S_d}{\tilde{s}}, \quad \sigma = \frac{g}{\kappa R\tilde{\omega}^2}. \tag{3}$$

In turn, in (2), (3):
 – characteristic scales

$$\rho_1 = \frac{M_{2\Sigma}}{M_1}, \quad \rho_3 = \frac{M_{2\Sigma}}{M_3}, \quad \tilde{y} = \frac{\tilde{s}}{M_{2\Sigma}}, \quad \tilde{s} = NmR;$$

 – dimension parameters

$$S_d = \mu P, \quad M_{2\Sigma} = M_2 + Nm + \mu; \tag{4}$$

 – for a ball, a roller, a pendulum, respectively,

$$\kappa = \frac{7}{5}, \quad \kappa = \frac{3}{2}, \quad \kappa = 1 + J_C/\left(mR^2\right), \tag{5}$$

where J_C is the principal central axial moment of the pendulum inertia.

5. 1. 3. Selecting the vibratory machine parameters for executing anti-resonance oscillations

Under an anti-resonance regime, platform 3 (the intermediate platform) of the vibratory machine should be stationary ($v_3=0$). At the same time, platforms 1 and 2 should execute the anti-phase oscillations. A pure anti-resonance regime is possible only in the absence of resistance forces.

In the absence of resistance forces, with loads collected, the loads getting stuck at a constant rotation speed Ω, system (1) takes the following form

$$\ddot{v}_1 + n_{13}^2\left(\rho_1 v_1 - \rho_3 v_3\right) = 0,$$

$$\ddot{v}_2 + n_{23}^2\left(v_2 - \rho_3 v_3\right) = S_{\max}^{(AB)}\Omega^2 \sin \Omega\tau,$$

$$\ddot{v}_3 + n_3^2 v_3 - n_{13}^2\left(\rho_1 v_1 - \rho_3 v_3\right) - n_{23}^2\left(v_2 - \rho_3 v_3\right) = 0, \qquad (6)$$

where $S_{\max}^{AB}$ is the total unbalanced mass of the tightly pressed loads.

The frequency equation that determines the system's natural (resonance) oscillation frequencies is

$$\mathrm{Det}(p) =$$

$$= \begin{vmatrix} -p^2 + n_{13}^2\rho_1 & 0 & -n_{13}^2\rho_3 \\ 0 & -p^2 + n_{23}^2 & -n_{23}^2\rho_3 \\ -n_{13}^2\rho_1 & -n_{23}^2 & -p^2 + n_3^2 + \left(n_{13}^2 + n_{23}^2\right)\rho_3 \end{vmatrix} =$$

$$= b_0 - b_1 p^2 + b_2 p^4 - p^6, \qquad (7)$$

where

$$b_0 = n_3^2 n_{13}^2 n_{23}^2 \rho_1,$$

$$b_1 = n_3^2\left(n_{23}^2 + n_{13}^2\rho_1\right) + n_{13}^2 n_{23}^2\left(\rho_1 + \rho_3 + \rho_1\rho_3\right), \qquad (8)$$

$$b_2 = n_{13}^2\left(\rho_1 + \rho_3\right) + n_{23}^2\left(1 + \rho_3\right) + n_3^2.$$

Find the rotor rotation frequency at which the oscillations of intermediate platform 3 are absent. Let $v_3=0$. Equations (6) then take the following form

$$\ddot{v}_1 + n_{13}^2\rho_1 v_1 = 0,$$

$$\ddot{v}_2 + n_{23}^2 v_2 = S_{\max}^{(AB)}\Omega^2 \sin \Omega\tau,$$

$$-n_{13}^2\rho_1 v_1 - n_{23}^2 v_2 = 0. \qquad (9)$$

We find from the last equation in (9):

$$v_1 = -\frac{n_{23}^2}{n_{13}^2}\rho_1 v_2. \qquad (10)$$

Fitting (10) to the first equation in (9), after the transformation, we obtain

$$\ddot{v}_2 + n_{13}^2\rho_1 v_2 = 0. \qquad (11)$$

Deduct (11) from the second equation in (9); we obtain

$$\left(n_{23}^2 - n_{13}^2\rho_1\right)v_2 = S_{\max}^{(AB)}\Omega^2 \sin \Omega\tau,$$

$$v_2 = \frac{S_{\max}^{(AB)}\Omega^2}{n_{23}^2 - n_{13}^2\rho_1}\sin \Omega\tau. \qquad (12)$$

Fitting v_2 from (12) to (11), we obtain $-\Omega^2 + n_{13}^2\rho_1 = 0$, hence

$$\Omega = n_{13}\sqrt{\rho_1}. \qquad (13)$$

In order for it to be its natural (resonance) oscillation frequency of the vibratory machine, it is required that it should the root of frequency equation (7). Fitting (13) to (7), after the transformation, we obtain

$$\mathrm{Det}\left(n_{13}\sqrt{\rho_1}\right) = n_{13}^4\rho_1\rho_3\left(n_{13}^2\rho_1 - n_{23}^2\right) = 0. \qquad (14)$$

Hence, we find the following condition

$$n_{23} = n_{13}\sqrt{\rho_1}. \qquad (15)$$

By fitting (15) to (6), we obtain

$$\ddot{v}_1 + n_{13}^2\left(\rho_1 v_1 - \rho_3 v_3\right) = 0,$$

$$\ddot{v}_2 + n_{13}^2\rho_1\left(v_2 - \rho_3 v_3\right) = S_{\max}^{(AB)}\Omega^2 \sin \Omega\tau,$$

$$\ddot{v}_3 + \left[n_3^2 + n_{13}^2\left(1+\rho_1\right)\rho_3\right]v_3 - n_{13}^2\rho_1 v_1 - n_{13}^2\rho_1 v_2 = 0. \qquad (16)$$

By fitting (15) to frequency equation (7), we obtain:

$$\mathrm{Det}(p) = \left(\rho_1 n_{13}^2 - p^2\right) \times$$

$$\times\left\{p^4 - \left[\left(\rho_1 + \rho_3 + \rho_1\rho_3\right)n_{13}^2 + n_3^2\right]p^2 + \rho_1 n_{13}^2 n_3^2\right\} = 0. \qquad (17)$$

Find three resonance frequencies from (17):

$$n_r^{(i)} = n_{13}\sqrt{\rho_1}, \quad n_r^{(j/k)} = \sqrt{\frac{\left(\rho_1 + \rho_3 + \rho_1\rho_3\right)n_{13}^2 + n_3^2 \mp \sqrt{D}}{2}}, \qquad (18)$$

where

$$D = \left[\left(\rho_3 - \rho_1 + \rho_1\rho_3\right)n_{13}^2 + n_3^2\right]^2 + 4n_{13}^4\rho_1\rho_3\left(\rho_1 + 1\right) > 0. \qquad (19)$$

(18) shows that (13) is one of the three resonance frequencies. Since the discriminator D>0, the other two resonance frequencies always exist, and

$$n_r^{(k)} > n_r^{(j)} > 0. \qquad (20)$$

Arrange the resonance frequencies i, j, k from (18) in ascending order. Consider

$$\left[\left(n_r^{(j)}\right)^2 - \left(n_r^{(i)}\right)^2\right]\left[\left(n_r^{(k)}\right)^2 - \left(n_r^{(i)}\right)^2\right] =$$

$$= -\rho_1\rho_3\left(\rho_1 + 1\right)n_{13}^4 < 0. \qquad (21)$$

Given (20) and (21), arrange the resonance frequencies in ascending order as follows

$$n_r^{(1)} = \sqrt{\frac{\left(\rho_1 + \rho_3 + \rho_1\rho_3\right)n_{13}^2 + n_3^2 - \sqrt{D}}{2}}, \quad n_r^{(2)} = n_{13}\sqrt{\rho_1},$$

$$n_r^{(3)} = \sqrt{\frac{\left(\rho_1 + \rho_3 + \rho_1\rho_3\right)n_{13}^2 + n_3^2 + \sqrt{D}}{2}}. \qquad (22)$$

In order to set an anti-resonance oscillation form, the rotor speed must exceed the second resonance frequency. At the same time, loads in the auto-balancer should get stuck at a speed close to the second resonance frequency. Then the platforms will execute oscillations close to the anti-resonance regime (the second form of the platforms' natural oscillations).

5. 1. 4. The law of anti-resonance oscillations

The loads in a vibration exciter can only get stuck if there are viscous resistance forces in the system [11]. Under a jam mode, the loads are tightly pressed together and create a total unbalanced mass S_{max}^{AB}. For the balls or rollers [8]

$$S_{max}^{AB} = \frac{mR^2}{r\sin\left[N\arcsin(r/R)\right]}. \tag{23}$$

For the case of pendulums, additional information about the design of pendulums is needed to determine the greatest unbalanced mass S_{max}^{AB}.

In the presence of viscous resistance forces, the law of platform motion in a zero approximation ($\varepsilon=0$) takes the following form [12]

$$v_i(\tau) = X_{2i-1}(\Omega,S)\sin(\Omega\tau+\gamma_0) + \\ + X_{2i}(\Omega,S)\cos(\Omega\tau+\gamma_0), \ /i=\overline{1,3}/. \tag{24}$$

Here:
Ω is the load jam frequency;

$$S = S_{max}^{AB}/\tilde{s}; \tag{25}$$

$$\mathbf{X}(q,S) = \mathbf{A}^{-1}(q)\mathbf{B}(q,S), \tag{26}$$

where

$$\mathbf{A}(q) = \left\|a_{ij}(q)\right\|_1^6, \ \mathbf{X}(q,S) = \left\|X_i(q,S)\right\|_1^6, \\ \mathbf{B}(q,S) = \begin{pmatrix} 0 & 0 & Sq^2 & 0 & 0 & 0 \end{pmatrix}^{\mathrm{T}}. \tag{27}$$

In turn

$$a_{11}(q) = \rho_1 n_{13}^2 - q^2, \ a_{12}(q) = -2q\rho_1 h_{13}, \ a_{13}(q) = 0,$$

$$a_{14}(q) = 0, \ a_{15} = -n_{13}^2\rho_3, \ a_{16}(q) = 2qh_{13}\rho_3;$$

$$a_{21}(q) = -a_{12}(q), \ a_{22}(q) = a_{11}(q), \ a_{23}(q) = 0,$$

$$a_{24}(q) = 0, \ a_{25}(q) = -a_{16}(q), \ a_{26} = a_{15}(q);$$

$$a_{31} = 0, \ a_{32}(q) = 0, \ a_{33}(q) = n_{23}^2 - q^2,$$

$$a_{34}(q) = -2qh_{23}, \ a_{35} = -n_{23}^2\rho_3, \ a_{36}(q) = 2qh_{23}\rho_3;$$

$$a_{41}(q) = 0, \ a_{42}(q) = 0, \ a_{43}(q) = -a_{34}(q),$$

$$a_{44}(q) = a_{33}(q), \ a_{45}(q) = -a_{36}(q), \ a_{46} = a_{35}(q);$$

$$a_{51}(q) = -\rho_1 n_{13}^2, \ a_{52}(q) = 2q\rho_1 h_{13}, \ a_{53}(q) = -n_{23}^2,$$

$$a_{54}(q) = 2qh_{23}, \ a_{55}(q) = n_3^2 + (n_{13}^2 + n_{23}^2)\rho_3 - q^2,$$

$$a_{56}(q) = -2q\left[h_3 + (h_{13} + h_{23})\rho_3\right];$$

$$a_{61}(q) = -a_{52}(q), \ a_{62}(q) = a_{51}(q), \ a_{63}(q) = -a_{54}(q),$$

$$a_{64}(q) = a_{53}(q), \ a_{65}(q) = -a_{56}(q), \ a_{66} = a_{55}(q). \tag{28}$$

The platforms' oscillation amplitudes:

$$Amp_i(\Omega,S) = \sqrt{X_{2i-1}^2(\Omega,S) + X_{2i}^2(\Omega,S)}, \ /i=1,2,3/. \tag{29}$$

The frequencies at which loads can get stuck are determined as the actual roots of equation [12].

$$P(\Omega,n) = 2\beta(n-\Omega)\Delta(\Omega) + \Omega^2\Delta_4(\Omega,S) = 0, \tag{30}$$

where

$$\Delta(q) = \left|\mathbf{A}(q)\right|,$$

$$\Delta_4(q,S) = \\ = -Sq^2 \begin{vmatrix} a_{11}(q) & a_{12}(q) & a_{13}(q) & a_{15}(q) & a_{16}(q) \\ a_{21}(q) & a_{22}(q) & a_{23}(q) & a_{25}(q) & a_{26}(q) \\ a_{41}(q) & a_{42}(q) & a_{43}(q) & a_{45}(q) & a_{46}(q) \\ a_{51}(q) & a_{52}(q) & a_{53}(q) & a_{55}(q) & a_{56}(q) \\ a_{61}(q) & a_{62}(q) & a_{63}(q) & a_{65}(q) & a_{66}(q) \end{vmatrix}. \tag{31}$$

Equation (28) is a 13-power polynomial relative to Ω, which defies analytical examination. Therefore, further research is carried out by numerical methods.

5. 2. The numerical study into the dynamic properties of a vibratory machine

5. 2. 1. Procedure for studying the dynamic properties of a vibratory machine

We find from (30) the following solution to the equation of the frequencies of load jams in the parametric form

$$n(\Omega) = \Omega\frac{2\beta\Delta(\Omega) - \Omega\Delta_4(\Omega,S)}{2\beta\Delta(\Omega)}, \ \Omega\in(0,+\infty). \tag{32}$$

In the plane $(\Omega, n(\Omega))$, $\Omega\in(0,+\infty)$, we build a diagram of function $\Omega(n)$, $n\in(0,+\infty)$. At the points of bifurcation of motions, there is an origination or merging of a pair of jam frequencies. At the same time,

$$\frac{dn(\Omega)}{d\Omega} = \\ = \frac{1}{2\beta\Delta^2(\Omega)} \left\{ \begin{aligned} &2\beta\Delta^2(\Omega) - 2\Omega\Delta_4(\Omega,S)\Delta(\Omega) + \\ &+\Omega^2\begin{bmatrix} \Delta_4(\Omega,S)\dfrac{d\Delta(\Omega)}{d\Omega} - \\ -\Delta(\Omega)\dfrac{d\Delta_4(\Omega,S)}{d\Omega} \end{bmatrix} \end{aligned} \right\} = 0. \tag{33}$$

The procedure for studying the dynamic properties of a vibratory machine includes several stages [11] given below.

1. Equation (33) produces six bifurcation frequencies of load jamming, such as $0<\Omega_1<\Omega_2<\cdots<\Omega_6<n$.

2. Formula (32) yields six bifurcation angular velocities of the rotor rotation $n_i = n(\Omega_i)$, $/i=\overline{1,6}/$. For convenience, we shall number them and arrange them in order of ascending. When these velocities are reached, one pair of jamming modes occurs or disappears.

3. For each jam mode, formula (32) calculates in parametric form the corresponding rotor speed:

$$n_1(\Omega) = n(\Omega), \ \Omega \in [0, \Omega_1];$$

$$n_2(\Omega) = n(\Omega), \ \Omega \in [\Omega_1, \Omega_2]; \dots$$

$$n_6(\Omega) = n(\Omega), \ \Omega \in [\Omega_5, \Omega_6];$$

$$n_7(\Omega) = n(\Omega), \ \Omega \in [\Omega_6, +\infty). \tag{34}$$

Based on the calculation results, we build in the (n, Ω) plane the diagrams of seven possible modes of jamming $\left(n_i(\Omega), \Omega \right), \ /i = \overline{1,7}/$.

4. When assessing the stability of the possible jam modes, we are governed by the following rules:

– if there is only one mode of load jam at a certain rotor rotation speed, it is (globally or locally) asymptotically robust;

– if there are three or more modes of load jam at a certain rotor rotation speed, only the odd modes of jamming are locally asymptotically robust.

5. For each jam mode, formulae (29) are used to calculate, in the parametric form, the amplitudes of the slow platform oscillations

$$Amp_{i,1}(\Omega, S) = Amp_i(\Omega, S), \ \Omega \in [0, \Omega_1],$$

$$Amp_{i,2}(\Omega, S) = Amp_i(\Omega, S), \ \Omega \in [\Omega_1, \Omega_2], \dots$$

$$Amp_{i,6}(\Omega, S) = Amp_i(\Omega, S), \ \Omega \in [\Omega_5, \Omega_6],$$

$$Amp_{i,7}(\Omega, S) = Amp_i(\Omega, S), \ \Omega \in [\Omega_6, +\infty),$$

$$/i = 1,2,3/. \tag{35}$$

Based on the calculation results, we build in the (n, Amp) plane the diagrams of the amplitudes of the platforms' oscillations $\left(n_i(\Omega), Amp_{i,j}(\Omega) \right), \ /i = \overline{1,3}; \ j = \overline{1,7}/$.

5. 2. 2. The numerical study into the dynamic properties of a vibratory machine

All calculations involve dimensionless quantities. The results are also derived in a dimensionless form.

The estimation data (dimensionless parameters):

$$n_{13} = n_{23} = 1, \ n_3 = 0.5, \ h_{13} = h_{23} = 0.01, \ h_3 = 0.01,$$

$$\rho_1 = 1, \ \rho_3 = 0.25, \ F = 1, \ \beta = 2, \ \varepsilon = 0.02, \ \sigma = 0. \tag{36}$$

Fitting (36) to (22), we find three natural (resonance) frequencies of the system's oscillations in the absence of resistance forces

$$n_r^{(1)} = 0.39614, \ n_r^{(2)} = 1.0, \ n_r^{(3)} = 1.26217.$$

Six bifurcation frequencies of load jamming are found as the roots of equation (30):

$$\Omega_b^{(1-6)} = \begin{Bmatrix} 0.39637; \ 0.42316; \ 1.00016; \\ 1.09908; \ 1.26251; \ 1.41131 \end{Bmatrix}. \tag{37}$$

Fitting (37) to (32), we find six appropriate bifurcation speeds of the rotor. Arrange them in ascending order:

$$n_b^{(1-6)} = \begin{Bmatrix} 0.43984; \ 0.66525; \ 1.22507; \\ 1.51629; \ 5.16456; \ 7.26216 \end{Bmatrix}. \tag{38}$$

Fig. 2 shows the diagrams built for 7 possible load jamming modes (34).
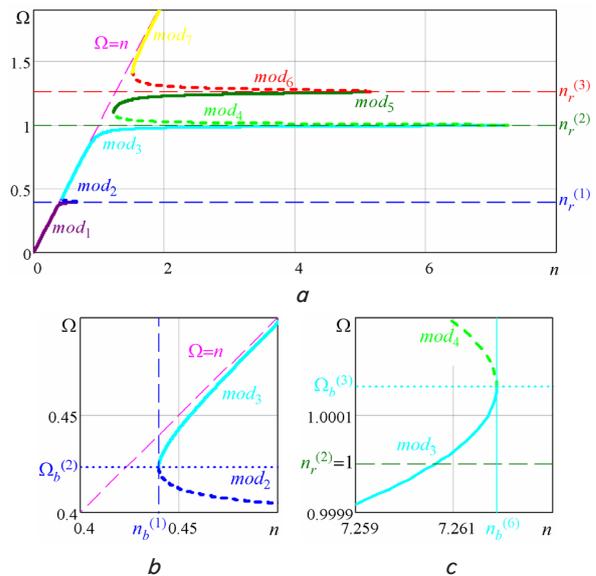
Fig. 2. Diagrams of possible modes of load jamming depending on the rotor speed: a — general view; b — in the vicinity of the origination of modes 2, 3; c — in the vicinity of the disappearance due to the merging of modes 3 and 4

In Fig. 2, solid lines show stable (odd) modes of jamming, dotted lines – unstable (even). Fig. 3 shows the diagrams built for possible amplitudes of platform oscillations depending on the frequency at which loads get stuck.
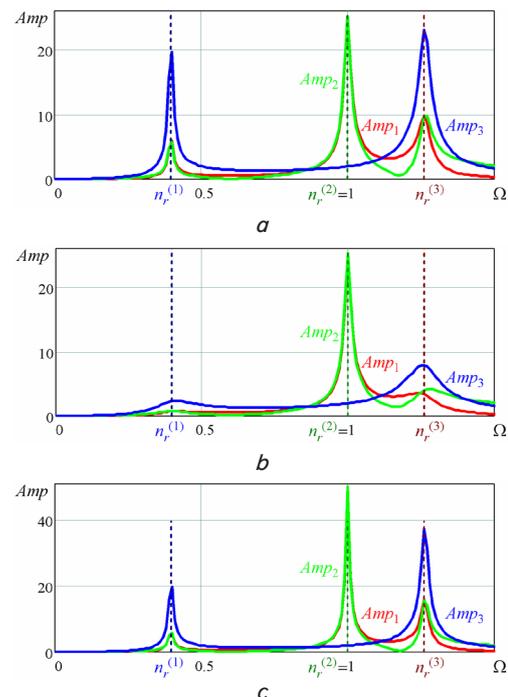
Fig. 3. Diagrams of possible amplitudes of platform oscillations depending on the frequency at which loads get stuck: a — h_{13}=0.01, h_3=0.01; b — h_{13}=0.01, h_3=0.1; c — h_{13}=0.005, h_3=0.01

Fig. 3 shows the following:

– increasing the external viscous resistance forces, acting on platform 3 (h_3), has little or no effect on the anti-resonance regime, and reduces the amplitudes of platform oscillations at the first and third resonances (Fig. 3, *b*);

– reducing the internal viscous resistance forces, acting between platforms 1, 3, and 2, 3 (h_{13}), has little effect on the amplitudes of platform oscillations at the first resonance and increases the amplitudes of platform oscillations at the second and third resonances (Fig. 3, *c*).

Fig. 4 shows a diagram of the amplitudes of platform oscillations depending on the frequency at which loads get stuck under a stable anti-resonance mode 3.
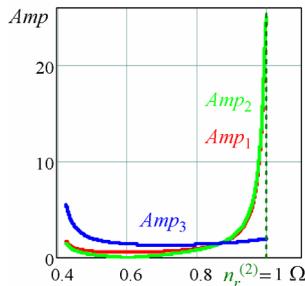
Fig. 4. Diagram of the platforms' oscillation amplitudes depending on the frequency at which loads get stuck under load jam mode 3 (h_{13}=0.01, h_3=0.01)

Fig. 5 shows the diagrams built for possible amplitudes of platform oscillations depending on the speed of rotor rotation.
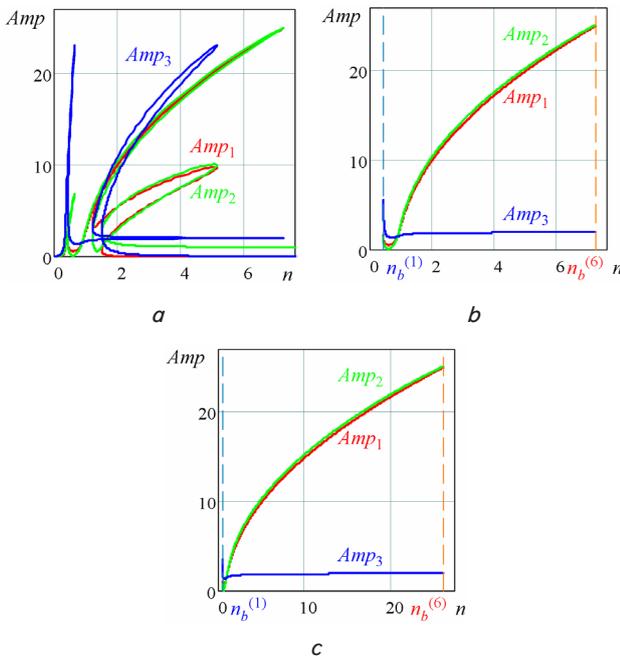
Fig. 5. Diagrams of the possible amplitudes of platform oscillations depending on the rotor speed: *a* — all modes of load jamming; *b* — stable anti-resonance mode 3 at β=2; *c* — stable anti-resonance mode 3 at β=0.5

A stable anti-resonance regime of the vibratory machine can be achieved in the range (n_{b1}, n_{b6}) of rotor speeds when implementing jam mode 3. At the same time, as the rotor speed increases, the oscillation amplitudes of platforms 1 and

2 increase. The oscillation amplitude of platform 3 is much smaller and is almost unchanged.

The forces of viscous resistance to the motion (β) of loads do not affect the minimum and maximum values of the amplitudes but narrow the range of rotor speeds over which there is an anti-resonance regime (Fig. 5, *b*, *c*). With larger resistance forces, a smaller change in the rotor speed results in a greater change in the oscillation amplitudes of platforms 1 and 2.

5. 2. 3. Integrating differential motion equations, assessing the stability of the anti-resonance regime

Differential motion equations (1) were integrated given the estimation data from (36). The initial conditions are close to the anti-resonance regime:

$$v_1 = 30, \quad \dot{v}_1 = 0, \quad v_2 = -30, \quad \dot{v}_2 = 0,$$

$$v_3 = 0, \quad \dot{v}_3 = 0, \quad \varphi = 0, \quad \dot{\varphi} = n_r^{(2)} = 1. \qquad (39)$$

Fig. 6 shows the diagrams of the platforms' coordinates and a diagram of the angular speed of load rotation at different rotor speeds after entering the mode of oscillations.
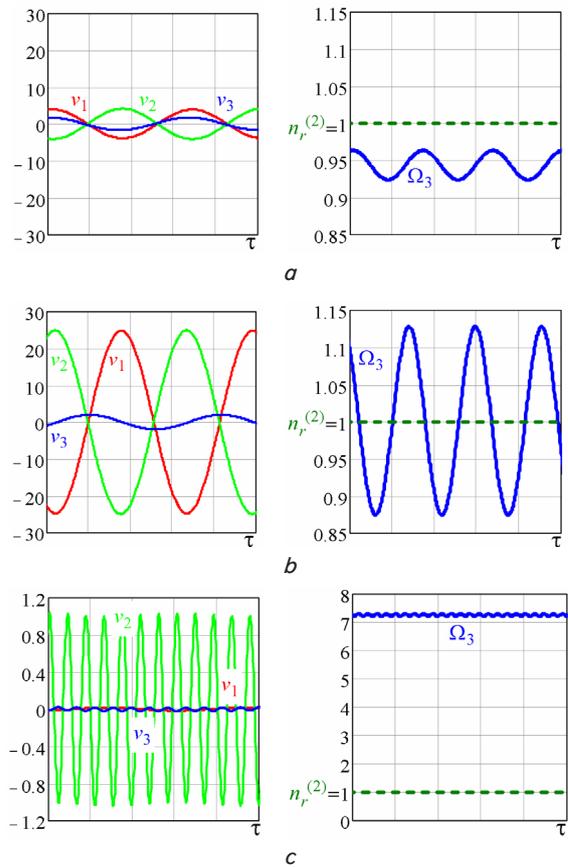
Fig. 6. Diagrams of platforms' coordinates (v_1, v_2, v_3) and a load rotation angular velocity diagram (Ω_3) at: *a* — *n*=1.1; *b* — *n*=7.2; *c* — *n*=7.3

The calculation results confirm that the anti-resonance regime does exist and is stable in the range of angular rotor speeds (n_{b1}, n_{b6}). Significantly, the stability is locally asymptotic. Thus, under zero initial conditions, the anti-resonance regime is entered before the rotor speed *n*=2. At *n*=2.1, there is the onset of mode 7 of load jam. It is obvious that mode 7

has a large area of attraction, while for mode 3 increasing the rotor speed reduces the area of attraction. In this regard, the onset of load jamming mode 3 can be enabled by smoothly accelerating the rotor to the working frequency.

7. Discussion of results of studying the three-mass anti-resonance vibratory machine

Our study has demonstrated the possibility of selecting parameters for a three-mass vibratory machine based on condition (15) that ensures the existence of an anti-resonance form of motion. The anti-resonance form corresponds to the second of the three resonance frequencies (22) of the vibratory machine oscillations. Under an anti-resonance form, platform 3, visco-elastically attached to the base, almost does not oscillate while platforms 1 and 2, attached to platform 3, oscillate in antiphase. The forces of viscous resistance in the supports prevent the anti-resonance regime from being ideal. The amplitude of platform 3 oscillations is not zero but an order of magnitude smaller than the oscillation amplitudes of platforms 1 and 2. However, the presence of viscous resistance forces in the supports and auto-balancer is necessary for the existence of modes at which loads get stuck in the auto-balancer.

In order to execute an anti-resonance form of oscillations, the rotor speed must exceed the second resonance frequency. At the same time, loads in the auto-balancer should get stuck at a speed close to the second resonance frequency. Then the platforms will execute fluctuations close to the anti-resonance form.

In the anti-resonance vibratory machine, platform 1 can act as active (working), then platform 2 will be reactive (a dynamic vibration damper), and vice versa. In this case, the vibratory machine will operate when mounting an auto-balancer on both platform 1 and platform 2.

The numerical calculations have shown the following for the case of small resistance forces in the vibratory machine:

– theoretically, there are seven possible modes of load jams (Fig. 2), with the first form of the resonance platform oscillations excited under modes 1 and 2, the second (anti-resonance) – 3 and 4, the third – 5, 6, and, under jamming mode 7, the frequency at which loads get stuck is slightly less than the frequency of rotor rotation;

– the odd jamming modes (1, 3, 5) are stable (implemented in practice);

– to excite the anti-resonance platform oscillations, jamming mode 3 must be set.

The properties of the anti-resonance form are significantly influenced by the external and internal forces of viscous resistance:

– increasing the external viscous resistance forces, acting on platform 3 (h_3), has little or no effect on the anti-resonance form, and reduces the amplitudes of platform oscillations at the first and third resonances (Fig. 3, b);

– reducing the internal viscous resistance forces, acting between platforms 1, 3 and 2, 3 (h_{13}), has little effect on the amplitude of platform oscillations at the first resonance, and increases the amplitudes of platform oscillations on the second and third resonances (Fig. 3, c);

– the forces of viscous resistance to the motion (β) of loads do not affect the minimum and maximum values of the amplitudes but narrow the range of rotor speeds over which there is an anti-resonance form (Fig. 5, b, c);

– with greater resistance forces to the motion of loads, a smaller change in the rotor speed results in a greater change in the oscillation amplitudes of platforms 1 and 2.

The anti-resonance form of platform motion is asymptotically stable over the entire range of rotor speeds. However, the area of attraction of this form decreases with the increasing rotor speed. Therefore, for the loads to get stuck in the vicinity of the second resonance frequency, one needs to provide the vibratory machine with initial conditions (38), close to jamming mode 3. A gradual increase in the rotation rate of the rotor to the working frequency is also possible.

It should be noted that the dynamic characteristics of the vibratory machine were investigated at a specific value of the parameters for the vibratory machine. However, the research methodology is also applicable for other parameters, however, at small viscous resistance forces. Increasing viscous resistance forces in the supports could lead to the disappearance of some modes of load jams. These cases have not been investigated given their practical insignificance.

In the future, it is planned to synthesize a two-mass anti-resonance vibratory machine and investigate its dynamic characteristics.

8. Conclusions

1. The theoretical study has shown that the three-mass vibratory machine possesses three resonance frequencies and three corresponding forms of platform oscillations. For such a vibratory machine, it is possible to select such parameters for the supports that would ensure the existence of an anti-resonance form of motion. Under an anti-resonance form, platform 3, visco-elastically attached to the base, almost does not oscillate, and platforms 1 and 2, attached to platform 3 oscillate in antiphase.

In the anti-resonance vibratory machine, platform 1 can act as active (working), then platform 2 will be reactive (a dynamic vibration damper), and vice versa. At the same time, the vibratory machine will operate when mounting an auto-balancer on both platform 1 and platform 2.

The anti-resonance form will occur when loads get stuck in the vicinity of the second resonance frequency of platform oscillations.

2. Our numerical calculations have demonstrated the following for the case of small resistance forces in the vibratory machine:

– theoretically, there are seven possible modes of load jamming;

– the second (anti-resonance) form of platform oscillations is theoretically implemented under load jamming modes 3 and 4;

– jamming mode 3 is locally asymptotically stable while load jamming mode 4 is unstable;

– for the loads to get stuck in the vicinity of the second resonance frequency, one needs to provide the vibratory machine with the initial conditions close to jamming mode 3, or smoothly accelerate the rotor to the working frequency;

– the dynamic characteristics of the vibratory machine can be controlled over a wide range by changing the rotor speed and the forces of viscous resistance.