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# PATTERNS IN THE DISTRIBUTION CAPACITY OF THIN PLATES UNDER DIFFERENT CONDITION FOR THEIR RESTING ON SUPPORTS

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***This paper reports a study into the distribution capacity of a flexible plate in different cross-sections exposed to the external vertical concentrated forces applied in any place of its area. A plate with one pinched side and a series of racks arranged at any distance from the pinching has been considered. In terms of the theory of elasticity and mathematics, solving this problem poses significant difficulties. This has study found that a lateral distribution coefficient could be used to simplify calculations aimed at determining the stressed-strained state of the system. In determining the stressed-strained state of the plate, the calculation method described in work [1] was applied. The plate is cut into a series of longitudinal strips that represent, from the standpoint of construction mechanics, a console strip with one pinched end and resting on a stationary support located at any distance from the pinching. It has been revealed that the distribution capacity of the examined plate in the same cross-section depends insignificantly on the point of application of the concentrated load along the length of the longitudinal strip (between 2.6 and 6.7 %). The distribution capacity in different cross-sections does differ greatly (in the range of 10 to 30 %). The result of this study is the proposed unified and easy-to-implement method of calculating plates under any conditions for their resting on supports and when exposed to any external loads. There is also no difficulty in calculating the plates backed by edges in both directions. Other estimation methods in these cases require a different mathematical approach, and, for the case of a series of external loads, or under difficult plate rest conditions, the issue relating to the stressed-strained state of the system remains open***

***Keywords: longitudinal strip, transverse strip, fictitious pinching, system of equation, lateral distribution coefficient***

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## 1. Introduction

The calculation of plates with the considered conditions for resting the sides poses significant difficulties in terms of solving the problems within the theory of elasticity, as well as in mathematical terms. The methods developed for calculating thin plates with different boundary conditions are based on a different approach in terms of the theory of elasticity and mathematics. The resulting solutions are mathematically very complex, so, for plates with different boundary conditions, it is necessary to devise a separate calculation program. This task is even more complicated when loading plates not all over their entire area, that is, when loading with concentrated forces or moments, piecewise evenly distributed loads. If the intensity of distributed loads is not linear, the problem in some cases cannot be solved at all using the proposed methods. There are no solutions to problems in determining the stressed-strained state of plates resting on the racks and loaded anywhere with concentrated forces, moments, or piecewise distributed loads.

A universal approach is proposed to determine the stressed-strained state of thin plates with any condition for the resting of sides loaded with any external loads, which produces acceptable results for designing. Resolving these issues is not a problem at all as all the solutions are reduced to solving the system of equations. The use of a mixed method of construction mechanics makes it easy to, "based on a single formula", to determine single movements and free terms included in the system of equations, exposed to any external loads. The possibility to build the lines of influence of the forces applied to longitudinal strips makes it possible to quickly and visually characterize the distribution capability of the system in any cross-section. This has allowed designers to pay attention to that the distribution capacity in the same cross-section varies and depends not only on the ratio of relative rigidities in the transverse and longitudinal directions but also on the type of external load and the point of its application. In addition, the number of equations in a system depends solely on the number of longitudinal strips; their number is not related to the conditions for the resting of the plate's sides.

The relevance of this work is due to the development of a single method of calculating thin plates (including those supported by edges) under any boundary conditions for their resting and exposed to any external loads. This, in turn, greatly simplifies the programming of calculations because all calculations are reduced to solving the system of canonical equations.

## 2. Literature review and problem statement

There are very few studies aimed at determining the stressed-strained state of the systems under consideration, so investigating such thin plates is of significant theoretical and practical interest.

In determining the stressed-strained state of plates with different boundary conditions for their sides' rest, various mathematical solutions have been used. For example, work [2] applies a matrix calculus; paper [3] – a tensor calculus. The difference methods [4] and a finite-element method are outlined in studies [5–9]. Variation methods were used quite widely (the Ritz-Timoshenko method [10, 11], the Raleigh-Ritz method [12, 13], the Galerkin-Bubnov method [14], and several others). A variation method, based on minimizing the expression of the elastic potential of the system relative to the nodal values of a bend function, is outlined in paper [15]. Initially, differential equations in particular derivatives were used in determining the SSS of thin plates, which, in some cases, were reduced to solving regular differential equations or integral-differential equations. The solutions derived from these equations were greatly complicated if the plates were loaded with concentrated forces, moments, or a piecewise distribution load. In this regard, it was quite often suggested that thin plates' deflections should be described by different series: Fourier's series [16], mixed series, or special-type series [17]. In this case, it was convenient to decompose the external load into the series as well.

However, there are still unresolved issues related to studying the work of thin plates, which rest in an arbitrary place on one or more racks. The plates with some patterns of their resting on racks have already been studied in works [18–20]; but the plates with the conditions under consideration have not been examined in them.

The above suggests that it is appropriate to conduct a study on the development of a single and easy-to-implement method for calculating plates (even backed by edges) under any conditions of their resting on supports exposed to any external loads. Other estimation methods in these cases require a different mathematical approach, and, for the case of a series of external loads or under complicated conditions for resting the plates, the issue relating to the stressed-strained state of the system remains open.

It is proposed to use the method described in work [1] to calculate the span structures of bridges in order to analyze the stressed-strained state of the system in question.

## 3. The aim and objectives of the study

The aim of this study is to establish patterns of change in the stressed-strained state of the considered plates when they are exposed to any external loads.

To accomplish the aim, the following tasks have been set:

– to devise a method of calculating thin plates by dividing them into a series of longitudinal and transverse strips using a mixed method of construction mechanics;

– to test the possibility of using the proposed method to calculate thin plates with one pinched side and simultaneously resting on a row of racks arranged at any distance from the pinching;

– to investigate the distribution capacity of the system in different cross-sections.

## 4. Method of calculating thin plates with the considered boundary conditions

### 4. 1. Building the lines of efforts' influence in cross-sections

A plate is cut into a series of longitudinal and transverse strips (Fig. 1). The longitudinal strips of width $d=b/n$, where $n$ is the number of racks, are cut along the side $l$ so that the racks are located under the middle width of the strip $d$. Thus, the number of longitudinal strips $n$ equals the number of racks in the transverse row.

These strips are statically a console strip, one end of which is pinched at point $A$, and the other is free (point $C$, Fig. 2). Between points $A$ and $C$, the strip rests on a support, that is, on a rack (point B). In the presence of longitudinal edges, they should be arranged under the middle of the longitudinal strips, that is, the edges must be pinched at point $A$ and must rest on the racks. For the case of a monolithic merging of the plate with the racks, a longitudinal element should be considered as a flat frame. The crossbar of this frame is a console strip with a pinched one end (Fig. 2), connected monolithically to a rack of height $h$.

A transverse strip of width $b'$=1 m is proposed to be cut in the cross-section along the length of the longitudinal strip where the distribution capacity of the plate is to be determined. In Fig. 1, the transverse strip is cut in the middle of the span $l_1$. In static terms, a transverse strip is a system on elastic-subsiding supports (Fig. 1). The role of the elastic-subsiding supports belongs to the longitudinal strips, which are sagged under the influence of external loads. We introduce to the calculation the rigidity of the transverse cross-section of the transverse strip of width $b'$=1 m and height δ, equal to the thickness of the plate. In the presence of transverse edges, we introduce the rigidity reduced to one linear meter, which is determined considering the transverse edges.

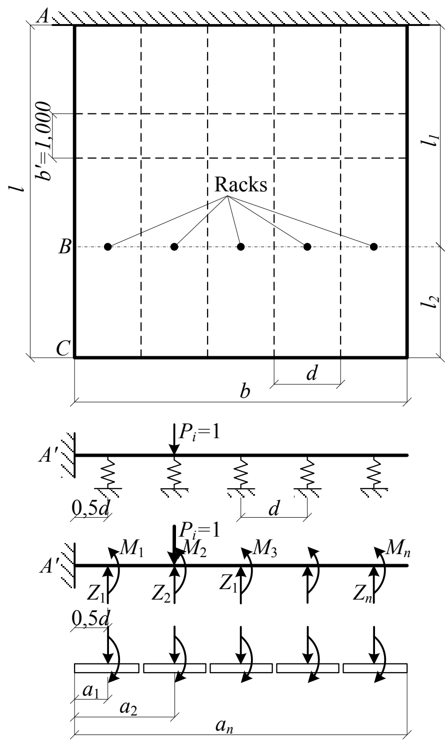


Fig. 1. The estimation and principal schemes of a transverse strip

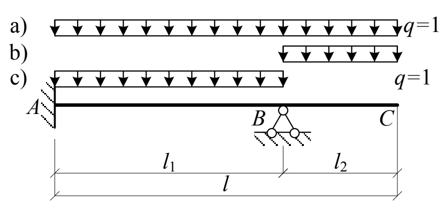


Fig. 2. The estimation scheme of a longitudinal strip

When calculating a transverse strip, we introduce a fictitious pinching of its left end (point $A'$, Fig. 1). The main system of a transverse strip is the console system loaded with forces $Z_i$. This produces the results, which are acceptable in practice, from determining the distribution capacity of the plate in a series of cases.

It is proposed to build the lines of influence of the efforts transferred by the transverse strip to the longitudinal elements. By loading the lines of influence with an external load, we shall determine the lateral distribution coefficients. To build the lines of influence, one should sequentially, at different points of the transverse strip, apply single external transverse forces $P_i$=1 (Fig. 1). At each position of the force, it is necessary to determine the efforts $Z_i$, which will be the ordinates of the lines of the efforts' influence. To implement the problem, we have applied a mixed method of construction mechanics, that is, the unknowns include the efforts $Z_i$, the turning angle $\varphi_{A'}$, and its deflection $y_{A'}$ at the point of the fictitious pinching. To find the unknowns, a system of canonical equations (1) that include ($n$+2) equations should be solved.

$$\begin{cases} \delta_{11}^{(Z)} \cdot Z_1 + ... + \delta_{1n}^{(Z)} \cdot Z_n + a_1 \cdot \varphi_{A'} + y_{A'} + \Delta_{1P} = 0 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \delta_{n1}^{(Z)} \cdot Z_1 + ... + \delta_{nn}^{(Z)} \cdot Z_n + a_n \cdot \varphi_{A'} + y_{A'} + \Delta_{nP} = 0, \\ Z_1 + Z_2 + ... + Z_n = 1 \\ a_1 \cdot Z_1 + ... + a_n \cdot Z_n - a_i = 0 \end{cases} \quad (1)$$

where $\delta_{11}^{(Z)}$ are the single movements of the transverse strip due to the forces $Z_i$=1 (Fig. 1); $a_i$ is the distance from a fictitious pinching to the $i$-th elastic-subsiding support (Fig. 1); $\Delta_{iP}$ is the free term; $i$ and $k$ are the numbers of the elastic-subsiding supports ($i$=1, 2...$n$, $k$=1, 2...$n$).

The single movements $\delta_{ik}^{(Z)}$ are easily determined because the diagrams of the bending moments in a console strip, which is, as mentioned, a transverse strip, will be triangular. In this case, the Vereshchagin rule (the rule of multiplication of the diagrams of bending moments) can be applied instead of the Maxwell-Mohr integral, which, in turn, makes it possible to derive a single formula (2) to determine single movements $\delta_{ik}^{(Z)}$. By denoting $\delta_{ik}^{(Z)} = V_{ik}^{(Z)}$, we obtain

$$\delta_{ik}^{(Z)} = V_{ik}^{(Z)} = \frac{d^3\left(1-\nu_n^2\right)}{6E_n I_n} \cdot w_{ik}, \quad (2)$$

where $d$ is the width of a longitudinal strip (Fig. 1); $\nu_n$ is the Poison ratio of the material of the transverse strip; $E_n I_n$ is the bending rigidity of the transverse strip of width 1 m. If there are transverse edges, $E_n I_n$ is, as previously agreed, the bending rigidity of the transverse strip reduced to one linear meter.

The transverse strip's deflection $w_{ik}$ resulting from the multiplication of the triangular diagrams of bending moments due to the single efforts $Z$, applied at points $i$ and $k$, is determined from the following formula

$$w_{ik} = \left(\frac{a_i}{d}\right)^2 \cdot \left(3\frac{a_k}{d} - \frac{a_i}{d}\right), \quad (3)$$

where $a_i/d$ and $a_k/d$ are the relative distances from a fictitious pinching to the points of application of single efforts $Z_i$=1 and $Z_k$=1 (Fig. 1).

Formula (3) holds at $a_k \geq a_i$. If $k<i$, one should swap the indices in formula (3). Formula (3) is much easier at $a_i=a_k$.

$$w_{ii} = 2\left(\frac{a_i}{d}\right)^3. \quad (4)$$

The deformation of the elastic supports $y_{ii}$ should be taken into consideration in determining the main single movements $\delta_{ii}^{(Z)}$. Then

$$\delta_{ii}^{(Z)} = y_{ii} + V_{ii}. \quad (5)$$

Since the role of elastic supports in the system belongs to the longitudinal strips, the movement $y_{ii}$ is a deflection of the longitudinal strip due to the distribution load of intensity $q$=1 of a certain length (Fig. 2). The deflection of the longitudinal strip in the same cross-section will depend on the length and location of the applied single distributed load. Consequently, the distribution capacity of the system, even in the same cross-section, will be different when the longitudinal strips are exposed to the loads that differ in application points. Thus, the distribution capacity of the examined plate in the same cross-section will depend not only on its geometric size but also on the location of the external transverse load applied to the system.

The free terms $\Delta_{iP}$ should also be determined using the Vereshchagin rule as the diagrams of bending moments in determining the lines of influence due to $Z_i$=1 and $P_i$=1 will be triangular.

To make it easier to determine the single movements, multiply the first $n$ of in the system of equations (1) by the quantity $1/y_{ii}$. Then the single movements, increased by $1/y_{ii}$ times, will be equal to

$$\delta_{ik}^{'(Z)} = \alpha \cdot w_{ik}; \quad (6)$$

$$\delta_{ii}^{'(Z)} = 1 + \alpha \cdot w_{ii}, \quad (7)$$

where α is the system's flexibility indicator.

$$\alpha = \frac{d^3\left(1-\nu_n^2\right)}{6E_n I_n \cdot y_{ii}}. \quad (8)$$

The free terms $\Delta_{iP}$ should also be increased by $1/y_{ii}$ times, to fit the system in the form of $\Delta_{iP}^{'}$.

The movements $a_i\varphi_i$, increased by $1/y_{ii}$ times, will take the form of $\frac{a_i}{y_{ii}} \cdot \varphi_{A'}$. Multiply the numerator and denominator of this expression by $d$ and, denoting

$$\frac{d}{y_{ii}} \cdot \varphi_{A^{'}} = \varphi_{A^{'}}^{'},$$

we obtain $\frac{a_i}{d} \cdot \varphi_{A^{'}}$. Thus, at the unknown $\varphi_{A^{'}}^{'}$, one should add the coefficients, which are the relative distances from a fictitious pinching to the $i$-th elastic support (to the middle of the $i$-th longitudinal strip), that is, the values 0.5; 1.5;...; ($n$–0.5).

Denote the movement $\left(\frac{y_{A^{'}}}{y_{ii}}\right)$ of the fictitious pinching, increased by $1/y_{ii}$ times, through $y_{A^{'}}^{'}$.

Divide the last equation from system (1) by $d$. Then the coefficients for the unknown $Z_i$ and the free term will represent the relative distances.

If the transverse strip is cut in the supporting cross-section (above the rack), then the deflection $y_{ii}$=0; then formula (5) will be simplified in determining the main single movements. In this case, the first n of the system's equations (1) should not be multiplied by the quantity $1/y_{ii}$ as it would be equal to infinity. The lines of influence of the efforts, transferred by the transverse strip to the longitudinal ones, will be built using the method of leverage.

When system (1) is solved by the above technique, that is, when the α indicator is introduced, it should be introduced very large, say $\alpha=10^5$ or $\alpha=10^6$.

**4. 2. The effect of torque on distribution capacity**

The distribution capacity of the system is significantly influenced by the moments $M_i$ (Fig. 1), which are, for longitudinal strips, torques. When taking into consideration the moments $M_i$, $n$ additional equations should be included, that is, the system (9) that includes 2($n$+1) equations should be considered.

$$\begin{cases} \delta_{11}^{(Z)} \cdot Z_1 + \ldots + \delta_{1n}^{(Z)} \cdot Z_n + \delta_{11}^{(M)} \cdot M_1 + \ldots + \delta_{1n}^{(M)} \cdot M_n + a_1 \cdot \phi_{A'} + y_{A'} + \Delta_{1P} = 0, \\ \ldots\ldots\ldots \\ \delta_{n1}^{(Z)} \cdot Z_1 + \ldots + \delta_{nn}^{(Z)} \cdot Z_n + \delta_{n1}^{(M)} \cdot M_1 + \ldots + \delta_{nn}^{(M)} \cdot M_n + a_n \cdot \phi_{A'} + y_{A'} + \Delta_{nP} = 0, \\ \theta_{11}^{(Z)} \cdot Z_1 + \ldots + \theta_{1n}^{(Z)} \cdot Z_n + \theta_{11}^{(M)} \cdot M_1 + \ldots + \theta_{1n}^{(M)} \cdot M_n + \phi_{A'} + \theta_{ip} = 0, \\ \ldots\ldots\ldots \\ \theta_{n1}^{(Z)} \cdot Z_1 + \ldots + \theta_{nn}^{(Z)} \cdot Z_n + \theta_{n1}^{(M)} \cdot M_1 + \ldots + \theta_{nn}^{(M)} \cdot M_n + \phi_{A'} + \theta_{np} = 0, \\ Z_1 + Z_2 + \ldots + Z_n = 1, \\ a_1 \cdot Z_1 + \ldots + a_n \cdot Z_2 + M_1 + \ldots + M_n - a_i = 0, \end{cases} \quad (9)$$

where $M_i$ is the torque for longitudinal stripes and bending moments for the transverse strip; $\delta_{ik}^{(M)}$ is the single vertical transverse strip movements at the $i$-th point due to the single moment $M_k$=1, applied to the $k$-the point; $\Theta_{ik}^{(Z)}$ is the single turning angle of the transverse strip at the $i$-th point due to the single force $Z_k$=1, applied to the $k$-$th$ point; $\Theta_{ik}^{(M)}$ is the single turning angle of the transverse strip at the $i$-th point due to the single moment $M_k$=1, applied to the $k$-th point; $\Theta_{ip}$ is the turning angle of the $i$-th point of the transverse strip due to the single force $P_k$=1, applied to the point $k$.

We shall use formula (10) to determine the single movements $\delta_{ik}^{(M)}$.

$$\delta_{ik}^{(M)} = \frac{d^2\left(1-\nu_n^2\right)}{6E_n I_n} \cdot w_{ik}^{'}, \quad (10)$$

where, at $k \geq i$

$$w_{ik}^{'} = 3\left(\frac{a_i}{d}\right)^2, \quad (11)$$

and, at $k<i$

$$w_{ik}^{'} = 3\frac{a_k}{d}\left(2\frac{a_i}{d} - \frac{a_k}{d}\right). \quad (12)$$

It should be noted that

$$\delta_{ik}^{(M)} = \Theta_{ki}^{(Z)}. \quad (13)$$

The single turning angles

$$\Theta_{ik}^{(M)} = \frac{d\left(1-\nu_n^2\right)}{6E_n I_n} \cdot w_{ik}^{''}, \quad (14)$$

where, at $k \geq i$

$$w_{ik}^{''} = 6\frac{a_i}{d}, \quad (15)$$

at $k<i$

$$w_{ik}^{''} = 6\frac{a_k}{d}. \quad (16)$$

The main single movements (turning angles) $\Theta_{ii}^{(M)}$ should include the twisting angle of the longitudinal strip $\lambda_{ii}$, and the turning angle of the transverse strip $\Theta_{ii}$, that is

$$\Theta_{ii}^{(M)} = \Theta_{ii} + \lambda_{ii}. \quad (17)$$

The turning angle $\Theta_{ii}$ at point $i$ of the transverse strip should be determined from formulae (14) and (15); the angle of the torsion of the longitudinal strip $\lambda_{ii}$ – from formula (18)

$$\lambda_{ii}=\frac{C}{G_{tor}I_{tor}^{long}}=\frac{C}{0{,}4E_{long}\cdot I_{tor}^{long}},\qquad(18)$$

where $C$ is the quantity that depends on the way a longitudinal strip is fixed against twisting; $G_{tor}$ is the module of elasticity of the material of the longitudinal strip at torsion; can be taken equal to $G_{tor}$=0.4$E_{long}$; $E_{long}$ is the module of elasticity of the material of the longitudinal strip at bending; $I_{tor}^{long}$ is the moment of inertia of the cross-section of the longitudinal strip at torsion.

To make it easier to determine the single movements, we shall, in formula (9), multiply the first $2n$ equations by the quantity $1/y_{ii}$. Then the movements $\delta_{ik}^{'(Z)}$, increased by $1/y_{ii}$, should be determined from formulae (6) and (7). The increased movements

$$\delta_{ik}^{'(M)}=\alpha_1\cdot w_{ik}^{'},\qquad(19)$$

and the increased single turning angles

$$\Theta_{ii}^{'(M)}=\alpha_2\cdot w_{ik}^{'},\qquad(20)$$

where

$$\alpha_1=\frac{\alpha}{d};\qquad(21)$$

$$\alpha_2=\frac{\alpha}{d^2}.\qquad(22)$$

The main single turning angles, increased by $1/y_{ii}$ times, should be determined from formula (23)

$$\Theta_{ii}^{'(M)}=\alpha_2\cdot w_{ik}^{''}+\alpha_3,\qquad(23)$$

where

$$\alpha_3=\frac{C}{0{,}4E_{long}\cdot I_{tor}^{long}\cdot y_{ii}}.$$

Deriving the $\alpha_1$…$\alpha_3$ coefficients makes it much easier to determine single movements when building a system of equations, which, in turn, simplifies the methodology for compiling a calculation program.

## 5. Testing the proposed method for calculating a plate with the predefined dimensions and load

Consider the work of a thin plate with dimensions in the plan of 7.5×7.5 m, resting on a row of racks located at a distance of 2.5 meters from the console (Fig. 3). The plate is loaded with the concentrated force $P$=100 kN, applied over the second longitudinal strip. The thickness of the plate $\delta$=0.3 m, the grade of concrete is V30, the Poisson coefficient $\nu_{tr}=\nu_{long}$=0.2, the elasticity module of concrete $E_{tr}=E_{long}=E_b$=34.5·10$^5$ MPa.

We shall determine the distribution capacity of the plate in the middle cross-section of a span $l_1$ under the action of force P; to this end, we shall cut the strip of width $b'$=1 m in this cross-section (Fig. 3). To calculate the bending moments at points $A$ (pinching) and at point $D$ (the middle of the span $l_1$) (Fig. 4), it is necessary to build the lines of influence of the forces transmitted by the considered transverse strip to the longitudinal strips.

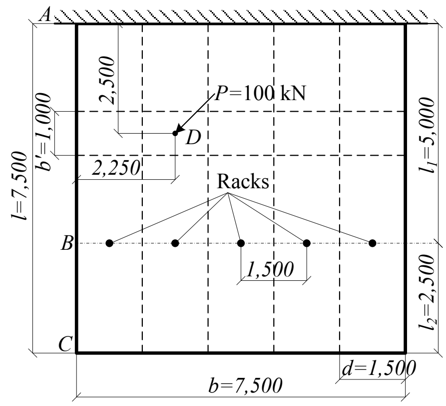


Fig. 3. A slab loaded with force $P$=100 kN at point $D$

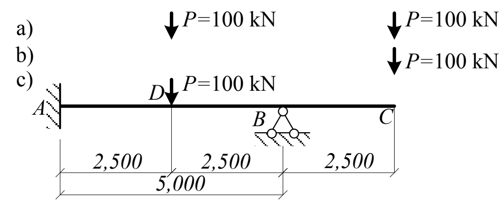


Fig. 4. Plate loading scheme

The deflection $y_{ii}$ of the longitudinal strip at point $D$, which is part of formula (8), should be determined based on the single distributed load $q$=1, applied only lengthwise of the span $l_1$ (Fig. 2, $c$). It is equal to

$$y_{ii}=\frac{625}{96}\cdot\frac{1-\nu_{long}^2}{E_{long}\cdot I_{long}},$$

where $I_{long}$=0.84375·10$^{-1}$ m$^4$ is the moment of inertia of the cross-section of the longitudinal strip.

The moment of inertia of the cross-section of the transverse strip of width $b'$=1 m is $I_{tr}$=0.25·10$^{-1}$ m$^4$. In this example, the influence of moments $M_i$ (Fig. 1) is not taken into consideration in building the efforts' lines of influence.

After fitting these values to formula (8), we determine the flexibility magnitude α, which is 0.3.

The ordinates of the line of influence are derived after solving the system of equations in formula (1) at the resulting indicator of the flexibility of the system α=0.3.

## 6. Exploring the system's distribution capacity at a change in the load location

### 6. 1. The distribution capacity of the plate in the middle cross-section of the span $l_1$

Two cases of loading the plate with concentrated forces were considered to investigate the distribution capacity in the cross-section located in the middle of the span $l_1$. Initially, the system was considered when the force was applied in the middle of the span $l_1$, and then when two forces were applied in the middle of the span and at the end of the console.

After loading the efforts' lines of influence with force $P$ applied over the second longitudinal strip (Fig. 3, 4, $c$), the lateral distribution coefficients (LDC) were calculated. They are given in Table 1.

Table 1

Values of the lateral distribution coefficients when loading the span $l_1$ with a single distributed load (Fig. 2, *c*)

| Longitudinal strip | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 0.2718 | 0.4311 | 0.2716 | 0.0762 | –0.0507 |

The values of the bending moments in longitudinal strips under this loading scheme (Fig. 4, *c*) are calculated on the basis of the derived LDCs (Table 1). They are given in Table 2.

Table 2

The values of bending moments due to force $P$=100 kN, applied in the middle of the span $l_1$, kNm

| Cross-section | Longitudinal strip | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| At pinching (point *A*) | –25.48 | –40.42 | –25.46 | –7.14 | 4.75 |
| In the middle of the span $l_1$ (point *D*) | 21.23 | 33.68 | 21.22 | 5.95 | –3.96 |

We apply the two concentrated forces $P$=100 kN over the second longitudinal strip in the middle of the span $l_1$ (at point *D*) and at the end of the console (at point *C*, Fig. 4, *a*). In determining the flexibility indicator of the system α from formula (8), the longitudinal strip's deflection $y_{ii}$ should be determined on the basis of the single distributed load $q$=1. The distributed load $q$ is applied along the entire length of the longitudinal strip (Fig. 2, *a*). After fitting this deflection, equal to

$$y_{ii} = \frac{625}{128} \cdot \frac{1-\nu_{long}^2}{E_{long} \cdot I_{long}},$$

to formula (8), we obtain an indicator of the flexibility of the system α≈0.4. Given this indicator α, we have solved the system of equations in formula (1) and determined the ordinates of the efforts' lines of influence, transferred by the transverse strip to the longitudinal ones. After loading the lines of influence with the concentrated forces *P*, we have calculated the lateral distribution coefficients (Table 3).

Table 3

Values of the lateral distribution coefficients when a single distributed load is applied along the entire length of the plate (Fig. 2, *a*)

| Longitudinal strip | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 0.2547 | 0.4569 | 0.2726 | 0.0651 | –0.0494 |

An analysis of the LDC values in the middle cross-section of the span $l_1$ (Tables 1, 3) has revealed that these coefficients depended on the location of the application of the concentrated forces; however, insignificantly.

The values of the bending moments at points *A*, *B*, and *D* (Fig. 4), calculated considering the lateral distribution coefficients, are given in Table 4.

Table 4

The values of bending moments due to the two concentrated forces $P$=100 kN applied in the middle of the span $l_1$ and at the end of the console, kNm

| Cross-section | Longitudinal strip | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| At pinching (point *A*) | 7.96 | 14.28 | 8.52 | 2.03 | –1.54 |
| In the middle of the span $l_1$ (point *D*) | 3.98 | 7.14 | 4.26 | 1.02 | –0.77 |
| Over the support (point *B*) | –63.68 | –114.22 | –68.15 | –16.27 | 12.35 |

An analysis of Tables 2, 4 reveals that the distribution capacity of the considered plate in the same cross-section depends insignificantly on the location of the concentrated load application along the length of the longitudinal strip (between 2.6 and 6.7 %).

**6. 2. The distribution capacity of the plate in the cross-section at the edge of the console**

We shall determine the distribution capacity of the plate at the edge of the console part; to this end, we shall cut a transverse strip of width *b*'=1 m at the end of the console. In determining the flexibility indicator of the system α (8), the deflection of the longitudinal strip $y_{ii}$ should be determined on the basis of the single distributed load $q$=1, applied only within the length of the console (Fig. 2, *b*).

The deflection at the end of the console is

$$y_{ii} = \frac{1875}{128} \cdot \frac{1-\nu_{long}^2}{E_{long} \cdot I_{long}}.$$

After fitting the deflection $y_{ii}$ to formula (8), we shall derive a value of α≈0.1.

By building, at α=0.1, the efforts' lines of influence, and loading them with a concentrated load *P*, applied at the end of the second longitudinal strip (Fig. 4, *b*), we shall obtain the LDC values (Table 5).

Table 5

Values of the lateral distribution coefficients when a console is exposed to the single distributed load (Fig. 2, *b*)

| Longitudinal strip | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 0.3328 | 0.3581 | 0.2479 | 0.0988 | –0.0376 |

The values of the bending moments at points *A*, *B* and *D* (Fig. 4), calculated considering the lateral distribution coefficients, are given in Table 6.

Table 6

Values of the bending moments due to the concentrated force $P$=100 kN, applied at the end of the console, kNm

| Cross-section | Longitudinal strip | | | | |
|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 |
| At pinching (point *A*) | 41.60 | 44.76 | 30.99 | 12.35 | –4.70 |
| In the middle of the span $l_1$ (point *D*) | –20.80 | –22.38 | –15.49 | –6.175 | 2.35 |
| Over the support (point *B*) | 83.20 | 89.525 | 61.975 | 24.70 | –9.40 |

Thus, by analyzing the data from Tables 1, 3, 5, one can conclude that the distribution capacity of the considered plate changes significantly along its length.

## 7. Discussion of results of studying the system's distribution capacity when a load location changes

The task was to devise a method for calculating one of the thin plates, namely a plate with one pinched side, simultaneously resting on a series of racks arranged at any distance from the pinching. It involves compiling a system of equations to build the efforts' lines of influence in any transverse cross-section of the system (1), (9), and determining the distribution capacity of the plate in these cross-sections (5÷8). The systems of equations are built in such a way that they do not change significantly when studying the stressed-strained state of plates in different transverse cross-sections and when the plate is loaded with any external loads. This makes it easier to compile a calculation program and significantly save machine time. The proposed method of calculation, based on dividing the system into a series of longitudinal and transverse strips and on the application of a mixed method of construction mechanics, eliminates the difficulties associated with solving integral-differential equations in particular derivatives.

The proposed method could be used for the system in question as this relates to determining the deflection of the longitudinal strip $y_{ii}$ (5) in the considered cross-section due to a single evenly distributed load. The deflections of the transverse strip $w_{ik}$ are determined from the unified formulae (3), (4).

The results of our study into the distribution capacity of the system in the same cross-section have demonstrated that it depends slightly on the location of the concentrated load along the length of the longitudinal strip (Tables 1, 3). For different beams, the lateral distribution coefficient ranges from 2.6 to 6.7 %. If one takes into consideration the efforts due to the constant load, the difference in the values for the lateral distribution coefficients can be neglected in determining the internal efforts and deformations from all types of external load. The distribution capacity of thin plates in different transverse cross-sections is significantly different (Tables 1, 3, 5); for different beams, the lateral distribution coefficient varies from 10 to 30 %. Using a single system of equations (a single solution approach) makes it possible to determine the carrying capacity of the system in different transverse cross-sections (including over the location of the racks).

The advantage of the proposed method is that under any conditions for the resting of sides and at any external loads, the problem is reduced to solving the system of equations. The number of equations in the system depends solely on the number of longitudinal strips.

We have examined the system only when a plate is divided into five longitudinal strips. Further research should tackle determining the optimal number of longitudinal strips, which affects the accuracy of the results.

The proposed method needs to be refined regarding the calculation program, which takes into consideration the impact of torques and other internal efforts on the distribution capacity.

## 8. Conclusions

1. A single method of calculation for thin plates has been developed, based on dividing the systems into a series of longitudinal and transverse strips using a mixed method of construction mechanics. This eliminates the use of complex mathematical approaches in solving similar problems.

2. We have proven the possibility to apply the proposed method to analyze the stressed-strained state of thin plates with one pinched side, simultaneously resting on a series of racks arranged at any distance from the pinching. For these plates, the number of equations depends only on the number of longitudinal strips.

3. An analysis of the system's distribution capacity has revealed:

– the distribution capacity of the considered plate in the same cross-section depends slightly on the application point of the concentrated load along the length of the longitudinal strip (between 2.6 and 6.7 %);

– the values of the lateral distribution coefficients in different cross-sections along the length of longitudinal strips vary significantly (between 10 and 30 %).

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