

*Solving the problems of setting requirements to the reliability of complex technical systems for various purposes presupposes their classification according to the features characterizing the purpose, modes of use, etc. According to the modes of use, systems are divided into objects of continuous long-term use, repeated cyclic use, and single-use. The objects of repeated cyclic use include the systems operating in cycles. Durations of the periods of work and pause in the cycle are considered deterministic values. Technological and/or technical maintenance is carried out in pauses between the operation periods.*

*In addition to the known classification, it was proposed to introduce a group of systems of repeated use with a complex operating mode. A complex mode is understood as a mode that includes waiting for a request of the system use and executing the request after it arrives at a random time.*

*An analytical model of reliability of such a system has been developed in the form of a ratio for a non-stationary total coefficient of operational readiness. This model describes the processes of the system functioning in the intervals of waiting and use. In this case, the duration of the intervals of waiting and/or execution of the request are random values.*

*Ratios for this indicator were obtained for three options of specifying the functions of distribution of durations of waiting in a turn-on condition and fulfilling the request for use.*

*The developed model makes it possible to set requirements for reliability and maintainability of the systems with a complex operating mode.*

*The results of modeling the dependences of the operational indicators of reliability on parameters of the functions of distribution of durations of waiting and executing the request were obtained for different distributions. Recommendations were formulated concerning the substantiation of the requirements to reliability and maintainability of the systems under consideration*

*Keywords: intended use, non-stationary coefficient of operational readiness, complex technical system*

# DEVELOPING THE MODEL OF RELIABILITY OF A COMPLEX TECHNICAL SYSTEM OF REPEATED USE WITH A COMPLEX OPERATING MODE

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Received date 13.09.2020

Accepted date 23.10.2020

Published date 30.10.2020

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## 1. Introduction

A systems approach to designing complex technical systems (CTS) provides for consideration of their operating conditions. This necessitates disclosure of the uncertainties associated with the stochastic nature of functioning (random

environmental conditions, internal properties of the system). This can be implemented with a probabilistic approach to the design process the essence of which is reduced to a problem of reliability standardization [1–3]. The solution to this problem requires the development of methodological regulations for selecting nomenclature of the standardized reliability

indicators of the CTS for various purposes, substantiation, and confirmation of their numerical values.

From the point of view of costs for checking, evaluating, and confirming the specified reliability indicators (RI) during designing, manufacture, and operation, the RI number should be minimal. At the same time, the nomenclature of the given RIs should fully characterize the object's reliability at all stages of the life cycle. In order to minimize the number of the set RIs, especially for restored CTSs, complex RIs are used.

Setting the requirements to the reliability of such CTSs assumes their classification according to the features characterizing the purpose, possible consequences of failures, modes of use (functioning), the possibility of restoration of operable condition (ROC), etc. According to the conditions of use (functioning), the CTSs are subdivided into objects of continuous long-term use, repeated cyclic use (RCU), and single-use.

In addition to the aforementioned groups of objects, the CTSs of repeated use (RU) with a complex operating mode are known. The complex operating mode of CTS means a mode that includes waiting until a signal is received to bring to readiness for the intended use (IU), waiting for a request for IU in the turn-on condition of the CTS. Products of such a group are used as intended in a deterministic or random time interval after a certain waiting interval  $[0, t]$  in the turn-on condition which can be deterministic or random  $[t, t+\tau]$ .

It is essential for them that at the end of the waiting interval determined by the arrival of a signal to bring the CTS to readiness for use, the system may be found in an operable or inoperable condition. In a case of detection of an inoperable condition, restoration of operability of the CTS is carried out. At the end of the waiting interval in the turn-on condition, the CTS must be efficient and function without failure during the IU interval. In this case, if the system is inoperable at time  $t$ , its use is refused. Note that the waiting interval can be forced, control, or both control and forced. The forced waiting interval is determined by random moments of arrival of a signal for bringing the system to readiness for IU or a request for IU. The control waiting interval is determined by the need to check the quality of the product functioning before use at a predetermined (planned) moment  $t$  while the duration of the waiting interval is a random value for the forced interval and a deterministic value for the control interval.

When specifying requirements to reliability and maintainability of the systems considered in [2] as the objects of MCU, the model of the stationary coefficient of operational readiness (COR) [2, 4, 5] is used, as a rule. This model does not take into account the above-mentioned features of the operation of the CTS of RU.

In this regard, the development of a model of reliability of a system of RU with a complex operating mode is relevant for its application in substantiating the requirements.

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## 2. Literature review and problem statement

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In studies [3, 5], when specifying requirements to the CTS reliability, operational and technical RIs are distinguished. Operational indicators include indicators that characterize the impact of reliability on the performance of main CTS functions and technical indicators that determine the CTS reliability as its own property regardless of the use specifics. At the same time, the basis for choosing a rational nomenclature (minimum necessary and sufficient) of RI [2, 5] of the CTS incorporates the principle of including the indi-

cators that determine functioning effectiveness. However, the studies [3, 5] do not consider the scientific basis for specifying requirements for the reliability of multipurpose CTS.

Reliability models are considered in [6, 7] only for the products of continuous long-term use. Reliability models are considered in [8] only for the products of repeated cyclic use. A stationary COR is used in [2, 9] as an operational indicator of the reliability of recoverable objects of MCU. The model of this coefficient does not consider the CTS functioning at the stage of waiting before the arrival of a signal for bringing the system to an operable condition and assumes its long-term functioning in the turn-on condition until the application for IU is received. This does not take into account the CTS reliability in the turn off condition when turning on and off as well as reliability in the intervals of waiting before a signal arrives to bring the CTS to readiness to IU, etc. Stationary or nonstationary coefficients of operational readiness (NCOR) the models of which more accurately describe the CTS condition immediately before IU are proposed in [5] as operative indicators. At the same time, reliability models for the products of repeated use in complex operating conditions are not studied in [2, 5–9].

The studies [10, 11] consider the development of models of reliability of complex systems with time redundancy for calculating RI, including stationary CR and COR of such systems. Issues of analyzing the reliability of technical systems at the operational stage with the development of diagnostic systems based on stationary CRs are considered in [12]. A model of reliability of a mechatronic system as a complex technical system operating in a stationary mode and characterized by a stationary readiness coefficient was developed in [13]. A model of reliability of a complex technical system with cold standby which makes it possible to estimate the stationary readiness coefficient of the system taking into account priorities of its devices for intended use and recovery is considered in [14]. However, the models in [10–14] take into account only one mode of the system operation and cannot be used for the systems with a complex operation mode.

A model of predicting fatigue life and assessing the reliability of high-pressure engine components in the design process is proposed in [15]. This model takes into account the influence of the random load and voltage changes in the motor and allows one to estimate the probability of its failure. The scope of use of the model [15] is limited only to the objects of repeated cyclic use with a simple operating mode.

An analytical method is proposed in [16] for assessing the reliability of modular multi-level devices with various redundancy schemes and making it possible to substantiate their structure based on calculating the reliability of various design options. The approach used in [16] is applicable only for non-recoverable systems of continuous long-term use with a simple operation mode.

A method for predicting the reliability of general-purpose technical products, considered in [17], is based on a continuous process of Markov degradation and making it possible to calculate reliability indicators of such objects in the course of designing. The proposed method [17] is applicable only for non-recoverable objects of continuous long-term use.

A model of reliability of a complex technical system of continuous long-term action including spatially distributed components was presented in [18]. The model makes it possible to obtain estimates of the level of the system reliability taking into account emergency conditions. However, the model [18] does not ensure the obtaining of quantitative estimates of the reliability indicators of such systems.

A model of reliability of a recoverable technical system was developed in [19]. It takes into account hardware failures and the failures caused by operator errors. The model [19] is used to analyze systems based on the model (or experimental) data and can be used to set requirements to the operator and equipment reliability according to known requirements to the system as a whole. At the same time, issues of setting requirements to the reliability of the system as a whole were not considered.

In works [1, 4, 5, 20] devoted to the study of stationary and non-stationary CORs, specification of requirements to the CTS reliability [2, 3, 5, 21–23], the time of waiting,  $t$ , and intended use,  $\tau$ , were considered only as deterministic values. Ratios were given in [3] only for the probability of no-failure operation (PNFO) at a random duration of job performance in an interval  $[0, \infty]$  for the law of exponential and normal distribution with a small coefficient of variation (no more than 0.36). A number of CTSs are characterized by random durations of waiting and intended use with large coefficients of variation and a large number of operating modes that must be taken into account when specifying and confirming requirements to their reliability. In this case, the interval of possible values of IU duration is characterized by minimum (other than zero) possible duration and a corresponding maximum duration determined by the CTS destination. The interval of possible values of the waiting time in a turn-on condition is limited by the maximum duration of continuous stay of the CTS in the turn-on condition or other features of its operation.

For correct specification of reliability requirements, in addition to the well-known CTS classification according to the modes of use [2], it is necessary to investigate the objects of RU with a complex operating mode and develop an appropriate reliability model.

In a general case, this model should be characterized by the random duration of waiting for IU and task performance, take into account the CTS reliability when turning on and off, waiting in the turn-off condition, etc.

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### 3. The aim and objectives of the study

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The study objective is to develop methodological provisions for setting requirements to reliability and maintainability of complex technical systems of repeated use in a complex mode of operation through the development of a reliability model for such systems.

To achieve the objective, the following tasks were set:

- to develop a mathematical model of reliability of complex technical systems of repeated use with a complex operating mode;
- to study the mathematical model of reliability of complex technical systems of repeated use in a complex operating mode;
- to develop methodological provisions for substantiating requirements to the reliability of complex technical systems of repeated use.

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### 4. Development of a mathematical model of reliability of complex technical systems of repeated use with a complex operating mode

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For a correct specification of requirements, the CTS reliability model should take into account the most signifi-

cant factors of the CTS operation, its typical operation cycle (TOC), the possibility of ROC, etc.

In this regard, let us first consider the development of a mathematical model of reliability in relation to the restorable CTSs of RU with a complex mode of operation at TOC including the following intervals:

- the first interval: waiting until the signal is received to bring the system to readiness to IU;
- the second interval: waiting in the turn-on condition before the receipt of a request for the IU;
- the third interval: the IU.

In the first interval, the CTS operation is organized as follows. The CTS is on duty in working condition and ready for its use. Control of functioning for a duration  $\tau_{cf}$  is performed with a regular period  $T_{cf}$ . When the CTS is turned on, its operation monitored, and then turned off, failures may occur during the stay in the turn-off condition. The failures are detected during functioning control. In this case, the CTS goes into a condition of recovery. Restoration of operability is completed with functioning control followed by the CTS turn off in the case of its operability.

In the second interval, the CTS is in the standby mode in the enabled operable condition or, in the case of the inoperable condition, restoration of the operable condition is performed.

Further (in the third interval), the CTS is used for its intended purpose provided that it is operational by the time when a request for IU has arrived. The CTS use for its intended purpose will be successful in the event of its trouble-free functioning during the IU period.

For the CTS of RU with a complex operating mode (COM), the duration of the first interval significantly exceeds the periodicity of operation monitoring, the mean time between failures, the mean time  $T_r$  of ROC, and other parameters of the process under consideration. As a rule, the duration of the second interval for such a CTS is less than the corresponding parameters of the recovery process. In this regard, when developing a reliability model, it is assumed that the process under consideration must be characterized by stationary characteristics in the first interval and non-stationary ones in the second interval.

The developed reliability model is an analytical relationship for a non-stationary total operational readiness factor (NSTORF) with known densities of distribution of random durations of the intervals of waiting  $\varphi(t)$  and intended use  $\omega(\tau)$ :

$$K_{orf}(\varphi(t), \omega(\tau)) = (P_1 P_{on} + P_2) K_{or}(\varphi(t), \omega(\tau)) + [1 - (P_1 + P_2)] \overset{\circ}{K}_{or}(\varphi(t), \omega(\tau)), \quad (1)$$

where  $P_1, P_2$  are stationary probabilities of the CTS being in the first operating interval in operable turn off and turn on conditions, respectively;  $P_{on}$  is the probability of trouble-free turning on;  $K_{or}(\varphi(t), \omega(\tau)), \overset{\circ}{K}_{or}(\varphi(t), \omega(\tau))$  are NCOR at the initial (at the moment  $t=0$ ) operable and inoperable condition, respectively.

Options of the CTS of RU with COM are possible when the waiting time or the IU duration are considered as deterministic values for which relation (1) takes the form:

$$K_{orf}(t, \omega(\tau)) = (P_1 P_{on} + P_2) K_{or}(t, \omega(\tau)) + [1 - (P_1 + P_2)] \overset{\circ}{K}_{or}(t, \omega(\tau));$$

$$K_{orf}(\varphi(t), \tau) = (P_1 P_{on} + P_2) K_{or}(\varphi(t), \tau) + [1 - (P_1 + P_2)] \overset{\circ}{K}_{or}(\varphi(t), \tau).$$

In the case of an MCU object with COM (deterministic durations of the intervals of waiting and intended use), relation (1) takes the form:

$$K_{orf}(t, \tau) = (P_1 P_{on} + P_2) K_{or}(t, \tau) + [1 - (P_1 + P_2)] \overset{\circ}{K}_{or}(t, \tau), \tag{2}$$

where  $K_{or}(t, \tau)$ ,  $\overset{\circ}{K}_{or}(t, \tau)$  are the NCOR for deterministic  $t$  and  $\tau$  durations.

For the MCU objects with a simple operation mode, a long stay in the turn-on condition is provided in the typical operating cyclogram until the request for use is received and relation (2) takes the well-known form [1, 3]:

$$K_{or}(\tau) = K_r \cdot P(\tau), \tag{3}$$

where  $K_r = T_o / (T_o + T_r)$ ;  $P(\tau)$  is PNFO of the CTS for the duration of IU,  $\tau$ .

This particular case testifies to the convergence and reliability of the developed model and, in fact, is a test for these characteristics. In this case, it is assumed that by the time the request for IU arrives at some arbitrary time  $t \rightarrow \infty$ , the CTS is characterized by a stationary recovery process with the parameters  $T_o$  (mean time between failures) and  $T_r$ .

Then, for objects of RU with a simple mode of operation, a long stay in turn on condition is provided for in the typical operating cyclogram until a request for an IU is received and relation (1) takes the form:

$$K_{or}(\omega(\tau)) = K_r \cdot P(\omega(\tau)), \tag{4}$$

where  $P(\omega(\tau))$  is the IU of the CTS for the random duration of the IU with distribution density  $\omega(\tau)$ .

Let us find analytical relations for the NSTORF components (1), (2). The relations for calculating the stationary probabilities  $P_1, P_2$  will be obtained using the mathematical apparatus of the theory of semi-Markov processes [3, 4, 21]. Let us determine the CTS condition in the first interval:

- $E_1$ : the CTS is off, operational, and waiting for IU;
- $E_2$ : the CTS is on; operable and functioning control is carried out;
- $E_3$ : the CTS is on, inoperable and restoration of operability is in progress;
- $E_4$ : the CTS is off, inoperable, considered to be operational;
- $E_5$ : the CTS is on; inoperable and operation control is being carried out.

In accordance with the above organization of the CTS operation process, possible transitions in a set of conditions  $E = \{E_i, i = 1..5\}$  are shown in Fig. 1. When developing a reliability model, it is assumed that the deterministic values of  $T_{cf}, \tau_{cf}$ , failure rates  $\lambda_1, \lambda_2$  in conditions  $E_1$  and  $E_2$ , the probability of failure-free turning on,  $P_{on}$ , and turning off,  $P_{off}$ , the average time of recovering working condition  $T_r$  are known. In this case, the function of distribution of the operating time between failures is assumed to be exponential with parameters  $\lambda_1$  in the  $E_1$  condition and  $\lambda_2$  in the  $E_2$  condition, and the ROC time is of a general form with mathematical expectation (ME)  $T_r$ .

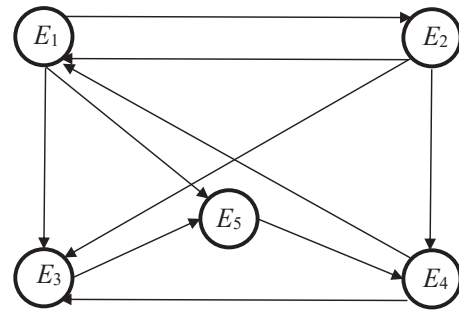


Fig. 1. The graph of conditions and transitions of the CTS in the first interval of the TOC

Stationary probabilities for the  $i$ -th condition of the CTS are found from the relation [4]:

$$P_i = \frac{\bar{t}_i \cdot \pi_i}{\sum_{i=1}^5 \pi_i \cdot \bar{t}_i}, \quad i = \overline{1,5}, \tag{5}$$

where  $\pi_i$  are stationary probabilities of the Markov chain embedded in the semi-Markov process under consideration.

$\bar{t}_i$  is the mean time of staying the semi-Markov process in the  $i$ -th condition.

To find the probabilities  $\pi_i$ , find the probabilities  $p_{ij}$  of the transition of the semi-Markov process from the condition  $E_i$  to the condition  $E_j$  at the moments of jump (transition). Determination of  $\pi_i$  is reduced to solving the system of equations

$$\pi_i = \sum_{j \in E} \pi_j p_{ji}, \quad i = \overline{1,5}, \quad \sum_1^5 \pi_i = 1.$$

As a result, the following relations are obtained:

$$\pi_1 = \left( 1 + \sum_{i=2}^5 a_i \right)^{-1}; \quad \pi_i = a_i \cdot \pi_1, \quad i = \overline{2,5};$$

$$a_2 = p_{12} = P_{on} \cdot e^{-\lambda_1 T_{cf}};$$

$$a_3 = p_{31}^{-1} \cdot (1 - p_{12} p_{21}), \quad p_{31} = P_{off}, \quad p_{21} = P_{off} \cdot e^{-\lambda_2 \tau_{cf}};$$

$$a_5 = a_3 - p_{12} \cdot p_{23}, \quad p_{23} = 1 - e^{-\lambda_2 \tau_{cf}};$$

$$a_4 = a_5 - p_{15}, \quad p_{15} = (1 - P_{on}) \cdot e^{-\lambda_1 T_{cf}}.$$

The following relationships were obtained for mean durations  $\bar{t}_i$ :

$$\bar{t}_1 = \lambda_1^{-1} \cdot (1 - e^{-\lambda_1 T_{cf}}); \quad \bar{t}_2 = \lambda_2^{-1} \cdot (1 - e^{-\lambda_2 \tau_{cf}});$$

$$\bar{t}_3 = T_r; \quad \bar{t}_4 = T_{cf} - \bar{t}_1; \quad \bar{t}_5 = \tau_{cf}.$$

The above relations make it possible to calculate required stationary probabilities  $P_1$  and  $P_2$  in models (1), (2).

The process of CTS functioning in a standby mode in a turn-on condition can be described by an alternating recovery process [1, 4, 5, 20]. Let  $F(t)$  and  $G(t)$  denote the functions of distribution (f.d.) of failure-free operation duration and ROC, respectively. Then the probability that the CTS will be operational and work without fail in the interval  $[t, t+\tau]$  by the time moment  $t$  provided that the product was operational at the time  $t=0$  is described by the expression

$$K_{or}(t, \tau) = \bar{F}(t + \tau) + \int_0^t \bar{F}(t + \tau - x) dH_1(x), \quad (6)$$

and provided that the product was inoperable at time  $t=0$ :

$$\overset{\circ}{K}_{or}(t, \tau) = G(t) \bar{F}(\tau) + \int_0^t \bar{F}(t + \tau - x) d\overset{\circ}{H}_1(x), \quad (7)$$

where

$$\bar{F}(t) = 1 - F(t),$$

$$H_1(t) = \sum_{k=1}^{\infty} (F * G)^{*k}(t),$$

$$\overset{\circ}{H}_1(t) = \sum_{k=1}^{\infty} G * (F * G)^{*k-1}(t),$$

$(F * G)^{*k}$  is the convolution of distribution functions  $G(t)$  and  $F(t)$  of the  $k$ -th order.

Formulas (6), (7) can be written differently:

$$K_{or}(t, \tau) = K_r(t) \cdot \bar{F}(\tau), \quad (8)$$

$$K_r(t) = \bar{F}(t) + \int_0^t \bar{F}(t - x) dH_1(x), \quad (9)$$

$$\overset{\circ}{K}_{or}(t, \tau) = \overset{\circ}{K}_r(t) \cdot \bar{F}(\tau), \quad (10)$$

$$\overset{\circ}{K}_r(t) = G(t) + \int_0^t \bar{F}(t - x) d\overset{\circ}{H}_1(x), \quad (11)$$

where  $K_r(t)$ ,  $\overset{\circ}{K}_r(t)$  is the probability that the CTS will be operable at time  $t$  provided that it was in an operable or inoperative condition, respectively, at time  $t=0$ .

Let us generalize these models for the CTS of RU with COM, that is, for random durations of waiting and use.

Let  $\omega(\tau)$  be the density of distribution of duration  $\tau$  of the CTS use at  $\tau \in [\tau_1, \tau_2]$  where  $\tau_1$  и  $\tau_2$  are the minimum and maximum durations of use. The probability that duration of intended use will belong to the interval  $[\tau, \tau + d\tau]$  is equal to  $\omega(\tau)d\tau$  and the conditional IU of the CTS during the interval  $[0, \tau]$  is  $\bar{F}(\tau)$ . Then, using the formula of total probability, unconditional PNFO for a random duration of the task execution will be obtained:

$$\bar{F}(\omega(\tau)) = \int_{\tau_1}^{\tau_2} \bar{F}(\tau) \omega(\tau) d\tau. \quad (12)$$

The calculation formulas for the PNFO at various functions of distribution of the CTS use durations obtained from formula (12) at  $\bar{F}(\tau) = \exp(-\lambda\tau)$ , (where  $\lambda$  is the CTS failure rate in the IU mode) are given below:

1)  $\omega(\tau)$  is a truncated normal distribution with parameters of the original distribution  $\tau_0$  and  $\sigma$ .

$$\bar{F}(\omega(\tau)) = c \cdot e^{-\lambda\tau_0 + \frac{\lambda^2\sigma^2}{2}} \left[ \Phi\left(\frac{\tau_2 - \tau_0}{\sigma} + \lambda\sigma\right) - \Phi\left(\frac{\tau_1 - \tau_0}{\sigma} + \lambda\sigma\right) \right], \quad (13)$$

where  $A = \left[ \Phi\left(\frac{\tau_2 - \tau_0}{\sigma}\right) - \Phi\left(\frac{\tau_1 - \tau_0}{\sigma}\right) \right]^{-1}$ ,  $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{y^2}{2}} dy$ ;

2)  $\omega(\tau)$  is the uniform distribution:

$$\bar{F}(\omega(\tau)) = \frac{1}{\lambda(\tau_2 - \tau_1)} \cdot (e^{-\lambda\tau_1} - e^{-\lambda\tau_2}); \quad (14)$$

3)  $\omega(\tau)$  is truncated exponential distribution with a parameter of the original distribution  $\gamma = (\tau_0)^{-1}$ :

$$\bar{F}(\omega(\tau)) = c \cdot \frac{\gamma}{\lambda + \gamma} \cdot (e^{-(\lambda+\gamma)\tau_1} - e^{-(\lambda+\gamma)\tau_2}), \quad (15)$$

where  $c = (e^{-\gamma\tau_1} - e^{-\gamma\tau_2})^{-1}$ .

Note that in the particular case for  $\tau \in [0, \infty]$ , relations (15) and (13) take the well-known form [3] under an additional condition  $\sigma/\tau_0 \leq 0.36$ .

Then, calculation interrelations for the non-stationary factor of readiness (NSFR) with a random waiting duration will be obtained. Let  $\varphi(t)$  be the density of distribution of duration  $t$  of waiting by the product for its intended use at  $t \in [t_1, t_2]$ . Then, the following will be obtained upon the reasoning similar to those given above

$$K_r(\varphi(t)) = \int_{t_1}^{t_2} K_r(t) \cdot \varphi(t) dt, \quad (16)$$

$$\overset{\circ}{K}_r(\varphi(t)) = \int_{t_1}^{t_2} \overset{\circ}{K}_r(t) \cdot \varphi(t) dt.$$

In (16), formulas for  $K_r(t)$  and  $\overset{\circ}{K}_r(t)$  for the functions of distribution take the form [1, 4]:

$$K_r(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t},$$

$$\overset{\circ}{K}_r(t) = \frac{\mu}{\lambda + \mu} (1 - e^{-(\lambda + \mu)t}). \quad (17)$$

The NSFR calculation formulas for various functions of distribution of  $\varphi(t)$  of the waiting time  $t \in [t_1, t_2]$  obtained from formulas (16), (17) are given below:

1)  $\varphi(t)$  is truncated normal distribution with parameters of the original distribution of  $t_w$  and  $\sigma_w$ :

$$K_r(\varphi(t)) = \frac{\mu}{\lambda + \mu} + \frac{\lambda \cdot c}{\lambda + \mu} \cdot e^{-(\lambda + \mu)t_w + \frac{(\lambda + \mu)^2 \sigma_w^2}{2}} \times \left[ \Phi\left(\frac{t_2 - t_w}{\sigma_w} + (\lambda + \mu)\sigma_w\right) - \Phi\left(\frac{t_1 - t_w}{\sigma_w} + (\lambda + \mu)\sigma_w\right) \right], \quad (18)$$

$$\overset{\circ}{K}_r(\varphi(t)) = \frac{\mu}{\lambda + \mu} \left[ \left( 1 - c e^{-(\lambda + \mu)t_w + \frac{(\lambda + \mu)^2 \sigma_w^2}{2}} \times \left[ \Phi\left(\frac{t_2 - t_w}{\sigma_w} + (\lambda + \mu)\sigma_w\right) - \Phi\left(\frac{t_1 - t_w}{\sigma_w} + (\lambda + \mu)\sigma_w\right) \right] \right) \right], \quad (19)$$

where  $c = \left[ \Phi\left(\frac{t_2 - t_w}{\sigma_w}\right) - \Phi\left(\frac{t_1 - t_w}{\sigma_w}\right) \right]^{-1}$ ;

2)  $\varphi(t) = (t_2 - t_1)^{-1}$  is the uniform distribution:

$$K_r(\varphi(t)) = \frac{\mu}{\lambda + \mu} + \frac{\lambda \cdot (e^{-(\lambda+\mu)t_1} - e^{-(\lambda+\mu)t_2})}{(\lambda + \mu)^2 \cdot (t_2 - t_1)}; \quad (20)$$

$$\overset{\circ}{K}_r(\varphi(t)) = \frac{\mu}{\lambda + \mu} \left( 1 - \frac{e^{-(\lambda+\mu)t_1} - e^{-(\lambda+\mu)t_2}}{(\lambda + \mu)(t_2 - t_1)} \right); \quad (21)$$

3)  $\varphi(t) = \frac{v e^{-vt}}{e^{-vt_1} - e^{-vt_2}}$  is truncated exponential distribution

with the parameter of the original distribution  $v = (t)^{-1}$ :

$$K_r(\varphi(t)) = \frac{\mu}{\lambda + \mu} + \frac{\lambda \cdot v \cdot (e^{-(\lambda+\mu+v)t_1} - e^{-(\lambda+\mu+v)t_2})}{(\lambda + \mu) \cdot (\lambda + \mu + v) \cdot (e^{-vt_1} - e^{-vt_2})}. \quad (22)$$

$$\overset{\circ}{K}_r(\varphi(t)) = \frac{\mu}{\lambda + \mu} \left( 1 - \frac{v(e^{-(\lambda+\mu+v)t_1} - e^{-(\lambda+\mu+v)t_2})}{(\lambda + \mu + v)(e^{-vt_1} - e^{-vt_2})} \right). \quad (23)$$

Since the random operating time of the CTS between failures and durations of ROC are assumed to be independent, then formulas (8)–(23) are used in the NCOR calculation, that is, the values  $\bar{F}(\omega(\tau))$ ,  $K_r(\varphi(t))$ ,  $K_{or}(\varphi(t), \omega(\tau))$ ,  $\overset{\circ}{K}_r(\varphi(t))$  and  $\overset{\circ}{K}_{or}(\varphi(t), \omega(\tau))$  are calculated sequentially (1), (2), (4).

The known results for deterministic durations of waiting and IU [3, 5] are a special case of the results presented in this article for the corresponding distribution densities  $\varphi(t) = \delta(t - t_w)$ ;  $\omega(\tau) = \delta(\tau - \tau_0)$ ,  $\omega(\tau) = \delta(\tau - \tau_0)$  where  $\delta(x)$  is a delta function,  $t_w$  and  $\tau_0$  are deterministic times of waiting and IU, respectively.

On the basis of the developed reliability model of a recoverable CTS RU with COM, a model of reliability of an unrecoverable CTS is obtained. In the first interval of operation, such an object is in the condition of waiting for a signal of bringing to the IU readiness in the turn off condition and gets into the turn-on condition when the signal arrives. In the second and third intervals, such an object is in the condition of waiting for a request of use in the turn-on condition and the IU.

Reliability of the CTS of RU with COM is characterized by the probability of failure-free turning on in the first interval, the IU during a random time of waiting for the IU  $P_w(\varphi(t))$  in the second interval, and the IU during the random duration of the IU  $P(\omega(\tau))$  in the third interval. If the operating conditions and modes of the CTS functioning in the second and third intervals are the same, then the PNFO during the total random waiting time and IU is the indicator of failure-free operation. This random variable is characterized by convolution of densities of distribution  $\varphi(t)$  and  $\omega(\tau)$ , and the PNFO is characterized by the probability  $P(\varphi(t) * \omega(\tau))$  (\* – convolution operation).

For an irrecoverable CTS of MCU with COM, reliability in the second interval is characterized by the PNFO in the standby mode  $P_w(t)$  and by the PNFO for the duration of the IU  $P(\tau)$  in the third interval. Under the same operating conditions and operating modes of the CTS functioning in the second and third intervals, it is possible to use the PNFO for the total waiting time and IU, that is,  $P(t + \tau)$ .

Then the non-recoverable CTS of MCU with a simple mode of operation is characterized by the «probability of failure-free turning on» and the mean time to failure  $T_m$  and the non-recoverable CTS of RU with a simple mode of operation is characterized by  $P_{on}$  and  $P(\omega(\tau))$ . In this case, the PNFO

for the duration of the IU  $P(\tau)$  can be used instead of the  $T_m$  indicator. The model for calculating the PNFO for a random duration was discussed above.

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### 5. Investigation of the model of reliability of complex technical systems of repeated use with a complex operating mode

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When specifying and confirming the requirements for the CTS reliability, the random waiting durations and IU are replaced by their maximum durations. Another option is to replace these random variables with their mathematical expectations. In addition, it is necessary to assess the mode of operation (simple or complex) for the analyzed CTS of RU or MCU in the second interval of operation.

The assignment of such systems to a subgroup of the objects with a complex or simple operating mode is determined by the time of establishing  $t_{est}$  by the NSTORF its stationary value. Therefore, it is necessary to estimate the duration of the establishment by non-stationary complex RIs of their stationary values. In this regard, the following studies were carried out:

- influence of the types and parameters of the laws of distribution of random waiting durations and intended use of the CTS on the operational reliability indicators;
- errors in calculating the operational RIs of CTS when random durations of waiting and IU are replaced by their maximum durations and their mathematical expectations;
- dependence of the value of NSTORF on the waiting duration.

The study of the influence of the types and parameters of various laws of distribution  $\varphi(t)$  and  $\omega(\tau)$  on quantities  $P(\omega(\tau))$ ,  $K_{or}(\varphi(t), \omega(\tau))$ ,  $K_{or}(\varphi(t), \omega(\tau))$  was performed for nine options of their distributions. Options of distributions  $\omega(\tau)$  and  $\varphi(t)$  are denoted as NN; NR; NM; RN; RR; RM; MN; MR; MM (where N, M, R are normal, exponential, and uniform distributions, respectively). The study was performed using a high-level language and an interactive environment for numerical computations, visualization, and programming, MATLAB. At the same time, the following restrictions and assumptions were used. The functioning of the CTS in the first interval is characterized by a stationary semi-Markov process. Parameters of this process:  $T_{cf} = 12$  h,  $\tau_{cf} = 0.083$  h,  $\lambda_1 = 0.001$  h<sup>-1</sup>,  $\lambda_2 = 0.1$  h<sup>-1</sup>,  $P_{on} = 0.98$ ,  $P_{off} = 0.99$ ,  $\mu = 1$  h<sup>-1</sup>. Other parameters of the reliability model are shown below in examples of the dependency graphs. Mathematical expectations of random waiting times (distributed according to the law  $\varphi(t)$  ( $t \in [t_1, t_2]$ )) and the IU time (distributed according to the law  $\omega(\tau)$  ( $\tau \in [\tau_1, \tau_2]$ )) are taken to be the same and equal to the values of  $t$  and  $\tau$ , respectively.

Dependences of these operational RIs of such CTSs on mathematical expectations or maximum values of random times of waiting or intended use were considered.

Examples of plots of dependences of  $P(\omega(\tau))$  on the value of ME of intended use (minutes) at  $\lambda = 0.1$  h<sup>-1</sup>, three options of distributions  $\omega(\tau)$  as well as the dependence of PNFO on the deterministic value of  $\tau$  (Fig. 2, a) are given below. Fig. 2, b shows the graphs of dependences of errors in calculating the IU when replacing the random duration of the IU with its mathematical expectation.

Comparison of these dependences shows that, with such a replacement, the value of  $P(\tau)$  gives an estimate of  $P(\omega(\tau))$  «from below», that is,  $P(\tau) \leq P(\omega(\tau))$ . In this case, relative estimation errors for the given  $t$  and  $\tau$  do not exceed 4.3 %.

What is below is examples of graphs of dependences of the NSTORF  $K_{orf}(\varphi(t), \omega(\tau))$  on ME of waiting times and intended use at  $\lambda=0.1 \text{ h}^{-1}$  in the second and third intervals,  $\mu=0.1 \text{ h}^{-1}$  and three variants of distributions  $\omega(\tau)$  and  $\varphi(t)$  (Fig. 3). For comparison, this Figure shows a graph of dependence of the NSTORF  $K_{orf}(t, \tau)$  on the corresponding deterministic values of  $\tau$  and  $t$ . Fig. 4 shows the graphs of dependences of relative errors in the calculation of the NSTORF  $\delta K_{orf}(\varphi(t), \omega(\tau))$  on replacement of random durations of waiting and IU with their mathematical expectations.

It can be seen from the comparison of dependences  $K_{orf}(\varphi(t), \omega(\tau))$  and  $K_{orf}(t, \tau)$  (Fig. 3) that the value of  $K_{orf}(t, \tau)$  gives an estimate of  $K_{orf}(\varphi(t), \omega(\tau))$  «from below», that is,  $K_{orf}(t, \tau) \leq K_{orf}(\varphi(t), \omega(\tau))$ . The relative error of such an estimation (Fig. 4) does not exceed 4.45 %.

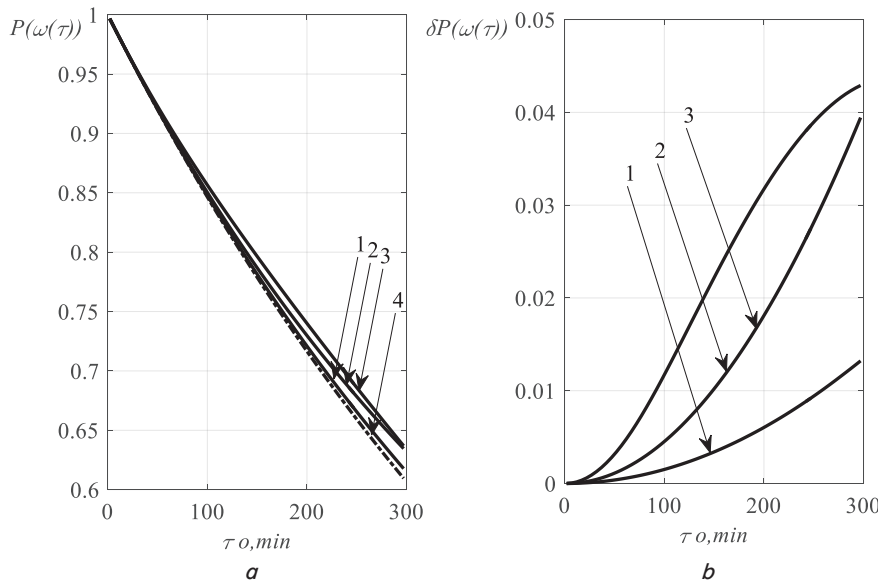


Fig. 2. The graphs of dependences of PNFO  $P(\omega(\tau))$  (a) and the relative error of its calculation  $\delta P(\omega(\tau))$  (b) on the value of ME of intended use at different options of distributions  $\omega(\tau)$ : 1 – N; 2 – R; 3 – M; 4 – deterministic value

The results of the study of relative errors in calculating the PNFO of CTS when replacing the random duration of IU with its mathematical expectation or its maximum value for the conditions considered above are given in Table 1.

It is seen from Table 1 and Fig. 2, b, and Fig. 4 that these errors do not exceed a few percent when replacing random variables with their mathematical expectations and tens of percent with their maximum values. Fig. 2–4 also show that the changes in the values of operational RI with changes in the types of distribution densities  $\varphi(t)$  and  $\omega(\tau)$  do not exceed a few percent.

In the study of the dependence of the NSTORF value on the duration of waiting, analysis of relations (1) and (2) was carried out. It has been established that the duration of establishment of  $t_{est}$  of the NSTORF of their stationary values is determined by corresponding characteristics for their NSFR, namely: duration of establishment of  $K_r(t)$ ,  $K_r(t)$  for the MCU objects and  $K_r(\varphi(t))$ ,  $K_r(\varphi(t))$  for the RU objects.

At a constant mean time between failures and mean ROC time, the stationary value of NSFR  $K_r(t)$  is set to the level  $K_r$ , the slower the smaller the sum of dispersions of the operating time and the ROC time [1, 3, 4]. In the general case, if densities  $f(t)$  and  $g(t)$  tend exponentially to 0, then the NSFR of  $K_r(t)$  converges to  $K_r$  at an exponential rate. For the particular case under consideration,  $f(t) = \lambda e^{-\lambda t}$ ,  $g(t) = \mu e^{-\mu t}$  it follows from formulas (17) that

$$K_r(t) - K_r = \left| \overset{o}{K_r}(t) - K_r \right| < \frac{\mu}{\lambda + \mu} e^{-(\lambda + \mu)t} < e^{-(\lambda + \mu)t}.$$

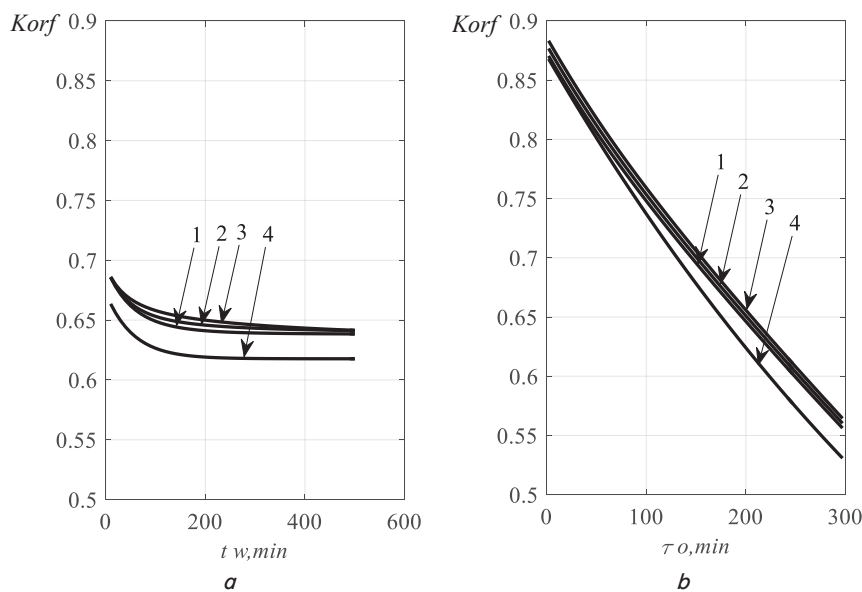


Fig. 3. Graphs of dependences of NSTORF on the value of ME of waiting time (a) at ME of the IU duration equal to 200 min, and on the value of ME of IU duration (b) at ME of waiting time equal to 140 min and various options of distributions  $\omega(\tau)$  and  $\varphi(t)$ : 1 – MN; 2 – MR; 3 – MM; 4 – deterministic  $t$  and  $\tau$

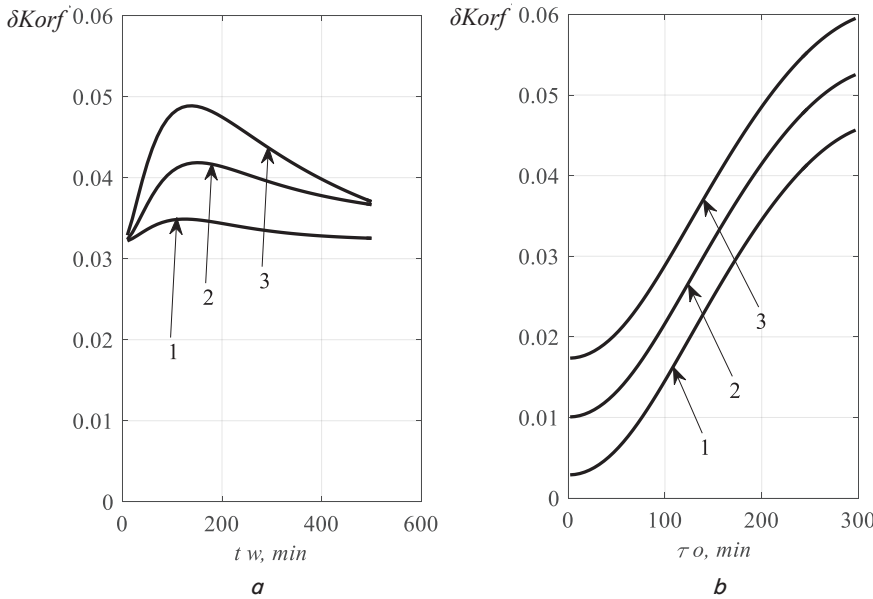


Fig. 4. Graphs of the dependences of relative errors in the calculation of the NSTORF on the value of ME of the waiting time (a) with ME of IU duration the equal to 200 min and on the value of ME of IU duration (b) with ME of waiting time equal to 140 min and variants of distributions  $\omega(\tau)$ ,  $\varphi(t)$ : 1 – MN; 2 – MR; 3 – MM

Table 1

Relative errors of  $\delta P(\omega(\tau))$  in calculating the PNFO of the CTS

Mathematical expectation or maximum value of the CTS IU duration $\tau$ , min	Relative error of $\delta P(\omega(\tau))$ in replacement of random duration of IU with...	
	mathematical expectation	maximum value
100	0.012	0.15
200	0.032	0.30

If accuracy is set at  $10^{-4}$ , then it is necessary that the inequality

$$e^{-(\lambda+\mu)t} < \frac{1}{2} 10^{-4}$$

is fulfilled or

$$t > -(\lambda + \mu)^{-1} \ln\left(\frac{1}{2} 10^{-4}\right) \approx 10(\lambda + \mu)^{-1}.$$

In this case, values of  $K_r(t)$ ,  $\overset{\circ}{K}_r(t)$  and  $K_r$  coincide, at least in the first 4 decimal places. Then, for the CTS of MCU too, a similar relation follows for duration  $t_{est}$  of the NSTORF  $t_{est} > 10 \min(T_o, T_r)$  and, with accuracy  $10^{-4}$ , values of  $K_{orf}(t, \tau)$  and  $K_{orf}(\infty, \tau)$  and coincide after  $t > 10T_o$  at  $T_o < T_r$  or after  $t > 10T_r$  at  $T_o > T_r$ .

The corresponding estimates for the durations  $t_{est}$  of the NSFR  $K_r(\varphi(t))$ ,  $\overset{\circ}{K}_r(\varphi(t))$  for the CTS of RU with different distribution densities of the waiting time  $\varphi(t)$  can be found by solving corresponding nonlinear inequalities (obtained from relations ((16) to (23)) with respect to  $t$ , for example, for the exponential distribution:

$$\frac{\lambda v}{(\lambda + \mu)(\lambda + \mu + v)} \left| \frac{1 - e^{-(\lambda + \mu + v)t}}{1 - e^{-vt}} - 1 \right| < 10^{-4}.$$

For the parameters  $\lambda=0.1 \text{ h}^{-1}$ ,  $\mu=1 \text{ h}^{-1}$ ,  $v=0.2 \text{ h}^{-1}$  specified above, estimates of durations  $t_{est}$  of the NSTORF of their stationary values are 10 h for MCU objects and 4.22 h for RU objects with an exponential distribution of time of waiting for IU. Then, if, in accordance with the object purpose, it is necessary to study its functioning at waiting intervals less than  $t_{est}$ , then such a CTS should be attributed to the objects with COM.

In the general case (for arbitrary distributions  $\varphi(t)$ ,  $\omega(\tau)$ ), an estimate of the value of  $t_{est}$  of NSTORF can be found using graphs of corresponding dependences of  $K_{orf}(\varphi(t), \omega(\tau))$  on ME of the duration of the waiting interval. An example of such dependences is shown in Fig. 3, a.

The developed reliability model is applicable for the CTSs, the typical operating model of which is characterized by a Multi-stage process of intended use with a stationary mode of operation at the first stage and a non-stationary one at others.

### 6. Development of methodological provisions for substantiating the requirements to the reliability of complex technical systems of repeated use

In accordance with [2], setting the requirements for the CTS reliability is reduced to the choice of a rational nomenclature of RI and the determination of their values. Based on the study results and recommendations for restorable CTS of RU and MCU, the following options for specifying the RI nomenclature are possible. The first option is one complex RI, the second is one complex indicator and  $\ell$  single ones from  $n$  reliability and maintainability indicators defining it  $\ell = 1, n-1$ , the third is a set of all single indicators defining the complex RI.

When choosing a rational nomenclature from the point of view of monitoring the fulfillment of requirements, the first option is the most preferable, the third one is the least preferable. At the same time, in addition to the complex indicator, one or several single indicators of reliability and maintainability can be set. The simultaneous setting of the complex indicator and all single CTS that define it is not allowed.

Based on the results of the studies performed, it is advisable to use the NSTORF or its particular cases as a complex indicator for the CTS of RU and MCU in accordance with their proposed classification according to the modes of use. At the same time, depending on the operating conditions and the CTS functioning modes in the second and third intervals, it is proposed to supplement the group of CTS of MCU [2] with a group of CTS of RU.

If it is necessary to take into account random times of waiting and (or) IU, the analyzed CTS should be attributed to the RU group, otherwise, to the MCU group. Further, depending on the operating conditions in the second interval and the need to take into account the non-stationarity of the recovery process for these CTS groups, it was proposed to



choose subgroups of objects with a simple or complex operating mode according to the following rule. If, when specifying the requirements, it is necessary to consider the functioning of the CTS in the second interval with a duration less than  $t_{est}$ , then such a CTS must be attributed to the subgroup of objects with COM, otherwise, with a simple operating mode.

For an unrecoverable CTS of RU or MCU, the following options for specifying the RI nomenclature are possible:

- one operational RI of «the probability of performing a typical task» type or «the probability of performing a typical IU cycle» type. In this case, it is necessary to additionally define the «typical task» and «typical IU cycle» terms;

- a set of all single indicators defining the operational RI characterizing reliability in relation to the intervals (stages) of the IU.

If it is possible to carry out confirmatory assessments at the design stages, statistical assessments during special tests, or according to the data of operational observations, it is preferable to set all single indicators that determine the operational RI.

Then Table B1 in [2] which defines rules of choosing the nomenclature of indicators of reliability and maintainability in terms of the CTSs which, according to their mode of use, were attributed to objects of MCU can be presented in the form of Table 2.

In Table 2, for non-recoverable CTSs with COM attributed to the objects of MCU, indicators  $P_w(t)$  and  $P(\tau)$  are replaced by one indicator  $P(t+\tau)$  and indicators  $P(\varphi(t))$ ,  $P(\omega(\tau))$  for the RU objects are replaced with indicator  $P(\varphi(t)*\omega(\tau))$  (\* – convolution operation).

Table 2

Choice of the nomenclature of reliability and maintainability indicators or complex indicators of the objects of a concrete purpose

Classification of the objects based on the features defining selection of the reliability indicators			
Based on the use (functioning) mode		Based on the possibility of recovering the operability	
		Recoverable	Irrecoverable
MCU	with a simple operating mode	$K_{or}(\tau), T_r$	$P_{on}, T_m$
	with COM	$K_{or}(t, \tau), T_r, P_{on}$	$P_{on}, P_w(t), P(\tau)$
MU	with a simple operating mode	$K_{or}(\omega(\tau)), T_r$	$P_{on}, P(\omega(\tau))$
	with COM	$K_{or}(\varphi(t), \omega(\tau)), T_r, P_{on}$	$P_{on}, P_w(\varphi(t)), P(\omega(\tau))$

Nomenclature of RIs for the objects of MCU in Table B1 [2] coincides with that proposed in Table 2 by the RI nomenclature for the objects of MCU with a simple operating mode. The nomenclature of other RIs for the objects of MCU with COM and for the objects of RU with simple and complex modes of operation is new. The rules for choosing the RI nomenclature for objects of RU and MCU proposed in Table 2 are a development of the well-known ones and Table 2 itself can be considered as a fragment of Table B1 in a regulatory document [2].

Quantitative requirements for the RIs of the selected nomenclature are set based on the permissible decrease in the CTS efficiency. In this case, a generalized ratio for operational RI is used:

$$P = W[W_0]^{-1},$$

where  $P$  is a generalized operational RI which in special cases takes the form of NSTORE, COR, PNFO during the task performance, CR, etc.,  $W$  is the efficiency of the CTS taking into account the actual reliability of the equipment;  $W_0$  is the efficiency of the CTS corresponding to the operable state at the moment  $t$  and trouble-free operation of the CTS for the duration of IU.

In practice, to set the requirements, it is enough to know the relative value of a decrease in efficiency  $\Delta W/W_0$ , where  $\Delta W = W_0 - W$ . The value of the operational RI found from the permissible value of a decrease in the CTS efficiency is considered as its minimum permissible value.

For a correct setting of quantitative requirements to RI of the CTS of RU, it is advisable to use the developed reliability model which quite fully describes features of functioning of such CTSs and the study results. At the same time, in order to reduce costs of checking, estimation, and monitoring the RI based on experimental data, it was proposed to replace the random times of waiting and IU with their ME. The option of replacing these random variables with their maximum values known in the practice of the CTS design is not recommended because of large errors in determining the operational (complex) RIs with such a replacement. Such a replacement leads to a significant overestimation of requirements to RI and an unjustified waste of financial, material, and other resources.

With a given value of the operational (complex) RI, it is possible to find the required values of the unit reliability and maintainability indicators that determine it. As an example, graphs of cross-sections of complex indicators determined by relations (1) to (4) at a level of 0.85 are shown in Fig. 5. Analysis of these graphs indicates a convergence of the simulation results obtained by means of the known and developed models.

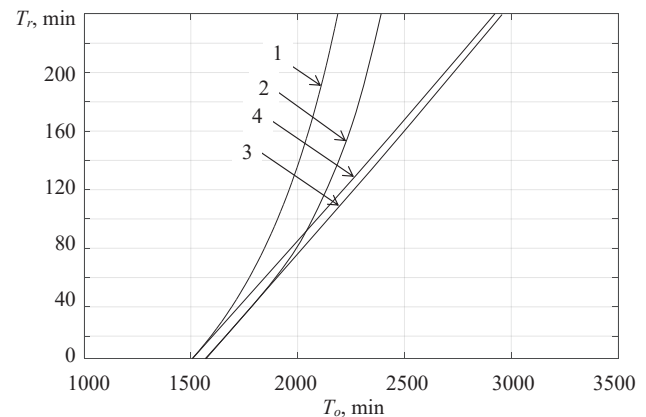


Fig. 5. Cross-section of the graphs of functions  $K_{orf}(T_{or}, T_r)$  by planes  $K_{orf}(T_{or}, T_r) = K_{orf_d}$ ,  $K_{orf_d} = 0.85$  with waiting time of 140 min, IU time of 200 min and  $P_{on} = 0.99$ : 1 – formula (1); 2 – formula (2); 3 – formula (3); 4 – formula (4)

It follows from the graphs in Fig. 5 that the following required values of mean recovery time correspond to the selected required mean time between failures of 36 hours. This time was no more than 97 min and 106 min for the objects of MCU and RU with simple operating modes, respectively. For the same objects with a complex operating mode, this time is no more than 160 min and 211 min, respectively. In this case,

if the CTS of RU with a complex mode of operation is assigned to the object of RU with a simple mode of operation in accordance with [2], then requirements to the average recovery time will be overestimated (97 min will be set instead of 211 min).

This will lead to the need to build a more expensive CTS and a system for its maintenance including overestimated requirements to the indicators of the sufficiency of operational spare part kits.

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### 7. Discussion of the results obtained in the study of the developed reliability model and recommendations for its use

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When specifying requirements to the reliability of complex technical systems, it is imperative to correctly classify the systems under study into pre-established groups as follows from the results obtained (Fig. 2–5, Tables 1, 2). For example, the three groups of the CTSs (continuous long-term use, repeated cyclic use, and single-use), highlighted by the regulatory documents on the modes of use, do not enable a correct setting of requirements to the reliability of the CTSs of repeated use. This is determined by peculiarities of their functioning, namely, the need to use them at random times and random durations of their intended use.

Attributing of such systems to the group of MCU in accordance with [2] leads, under certain conditions (Table 2), to an incorrect choice of the RI nomenclature. Application of the known models [1, 2, 4, 5] to the selected RI nomenclature leads to overestimated requirements to their values (Fig. 5) and an unjustified expenditure of substantial resources.

In this regard, a new classification group of «RU» objects (by modes of use) and their subgroup with simple and complex operating modes was proposed (Table 2).

When solving these problems, the developed reliability model (1), (5), (12) to (23), which is a generalization of known models and takes into account peculiarities of functioning of objects of the proposed classification group, is of great importance. The developed model, in comparison with the known ones, is based on a probabilistic approach to taking into account the operating conditions and describes a multi-stage process of intended use. This process is characterized by the intervals of duty, waiting for a request for use in the turn-on condition and, proper, use as intended. In this case, modes of the CTS operation, as a rule, differ significantly. It makes it possible to substantiate the choice of a rational nomenclature of the CTS reliability indicators and quantitative requirements to them and implement a systematic approach to the CTS design. In special cases, e. g., for an object of MCU with a simple operation mode, this model is reduced to the known one.

The studies have shown that to reduce the cost of checking, assessing, and monitoring the RIs in the course of CTS design process, it is possible to replace times of random waiting and intended use of the values for their intended purpose with their ME. The results of the study of the dependence of the NSTORF value on the waiting time have made it possible to formulate the rule of attributing products to subgroups of the objects with a simple or complex mode of operation.

Features of application of the developed model to specifying requirements to the reliability of such CTSs were shown. It is advisable to apply it at the stage of the life cycle «proposing a concept and justifying the need for development» to set reliability requirements to a sample (system).

The developed methodological approach is advisable to use when substantiating requirements to samples of military and civilian equipment which can be classified as CTS of RU or MCU with complex or simple operating modes, e. g., to samples of anti-aircraft missile weapons (anti-aircraft missile system), electronic equipment (radar system) or power units of nuclear power plants.

It should be noted that the developed mathematical model takes into account the peculiarities of functioning of restorable and unrestorable objects and does not take into account peculiarities of the serviced objects. In addition, some of the relationships of the reliability model were obtained for the exponential distribution of the operating time between failures and the duration of restoration of the operable condition. This imposes certain restrictions on the scope of the developed model and is its disadvantage. Further studies related to their elimination should be directed to the development of models of reliability of serviced and recoverable objects at different laws of distribution of operating time between failures and recovery time. Such a model will have a wider field of application for specifying requirements to the CTS reliability and the qualitative indicators of their maintenance system.

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### 8. Conclusions

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1. The developed mathematical model of reliability of the CTS as an object of repeated use with a complex operating mode, in contrast to the known ones, describes the processes of functioning at the stages of operation highlighted in accordance with the typical cyclogram. It is characterized by random durations of waiting and intended use in specified intervals of operation takes into account the reliability of the CTS when switching on and off in standby modes and other features of the object. Its particular cases for a CTS of repeated cyclic use with simple and complex operating modes were considered. Moreover, the known result for the deterministic duration of these intervals was considered as a particular case.

2. The errors in calculating the operational RIs of CTSs when replacing random durations with corresponding deterministic values do not exceed a few percent when replaced with their mathematical expectations and tens of percent with maximum values. This replacement gives an estimate of the operational indicator of the reliability of the CTS of RU «from below». In this case, the errors in estimating values of the operational CTS RI with a change in types of distribution densities  $\varphi(t)$  and  $\omega(t)$  do not exceed a few percent. Estimates of the duration of the establishment of own stationary values by non-stationary operational RIs of the CTSs were obtained.

3. It was proposed to introduce a new classification group of reusable objects with simple or complex modes of operation in addition to the well-known CTS classification by modes of their use. At the same time, it was recommended to select subgroups of objects with simple or complex modes of operation in a known group of objects of repeated cyclic use. The rules of assigning the CTS to the proposed groups and subgroups and the rules of choosing the nomenclature of reliability and maintainability indicators or complex indicators were formulated. It is recommended to replace the random times of waiting and intended use with their ME to reduce costs of checking, estimating, and monitoring their RIs when designing such systems.

4. The developed model, in comparison with the known ones, is based on a probabilistic approach to taking into account the operating conditions and describes a multi-stage process of intended use. This process is characteri-

zed by intervals of duty, waiting for a request for use in the turn-on condition and, proper, intended use. In this case, modes of the CTS operation, as a rule, differ substantially.

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