A mathematical model has been built of the subway car on two double-axle bogies with an axial characteristic of 2_0–2_0, whose spring suspension's central link is equipped with springs and electromechanical dampers. A special feature of the model is its integration of such components as 17 differential equations of the second order, which describe the operation of the mechanical part «carriage-rail track», as well as 8 differential equations of the first order that describe the operation of 4 electromechanical shock absorbers. The model is complemented with three polynomials of orders 32 and 63 describing the state of the magnetic field of electromechanical shock absorbers and their electromagnetic force, as well as 4 algebraic coupling equations.

The mathematical model of the «carriage-rail track» system equipped with electromechanical shock absorbers takes into consideration the following components:

– the longitudinal and transverse oscillations by wheelsets of the car bogies and body;

– the parameters of a rail track;

– the electromagnetic features of electric shock absorbers;

– the excitation arising from a track irregularity;

– the path parameters, as well as the properties of other elements in a spring suspension.

This paper reports a study into the operation of a subway car's spring suspension that travels over a track with a sinusoidal irregularity. The study has established that the electromechanical processes in electric shock absorbers can be divided into three parts. The oscillation free mechanical components, free components, and the forced electromagnetic components. The duration of action, the amplitudes, and nature of the oscillations' components have been determined. The oscillation amplitude varies considerably with the increased speed: from 0.01 A and 2 V at 40 km/h up to 0.9 A and 115 V at 100 km/h. The oscillations are harmonious. The frequency of oscillations corresponds to the frequency of the track irregularity. The electric power of the electric shock absorber increases from 0.018 W at 40 km/h to 98 W at 100 km/h

Keywords: electromechanical shock absorber, subway car, spring suspension, running gear, spatial kinematic scheme

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BUILDING A MATHEMATICAL MODEL OF THE OSCILLATIONS IN SUBWAY CARS EQUIPPED WITH ELECTROMECHANICAL SHOCK ABSORBERS

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1. Introduction

The existence and efficiency of oscillation damping systems predetermine the smooth run of a vehicle on a road section. One of the basic elements in the running gear of a subway car is the system of oscillation damping, which is typically equipped with two spring suspension links whose parameters differ. Reducing the impact of track irregularities and oscillations exerted by the running gear on a crew part of the electric rolling stock is ensured by the presence of shock absorbers (oscillation dampers).

The most important element that defines the safety of movement and the speed of electric rolling stock is the dynamic indicators of shock absorbers. This applies to both urban and main rail transport. The indicators of shock absorbers are predetermined by the type and design of the shock absorber. In terms of smooth running, the best performance is demonstrated by pneumatic shock absorbers [1]; their use, however, requires an additional pneumatic power system, compressors, which reduce the overall energy efficiency of electric rolling stock [2]. Another approach to improving the dynamic properties of electric rolling stock is the use of electromechanical shock absorbers [3]. Shock absorbers of this type do not consume additional energy but they are able to partially recuperate the energy of oscillations. Such capabilities of electromechanical shock absorbers make it possible to improve the energy efficiency of rolling stock in general and subway specifically [3].

2. Literature review and problem statement

As noted in [4], in world practice, the most common is the spring suspension of bogies, consisting of the journal and central parts. The same system is used by subway cars in Ukraine. Spring suspension aims to reduce the effect of impacts exerted on the rolling stock by track irregularities and when traveling over curved sections. The oscillations that occur, in this case, in the central suspension of bogies are damped by hydraulic shock absorbers [4].

The analysis of oscillation damping systems in urban rail vehicles, reported in work [5], revealed that the most common technical solution is the friction oscillation dampers, supplemented with hydraulic dampers. Such combined systems are installed in the second link of the spring suspension.

As noted in [6], the E-KM series cars at PAT «Kryukiv car-building plant» (Kremenchuk, Ukraine) are equipped with new bogies, model 68-7054, with central pneumatic spring suspension, which is supplemented with hydraulic shock absorbers. The use of pneumatic springs makes it possible to adjust the parameters of damping depending on the loading of the car and the value of external influences; the pneumatic suspension, however, leads to additional energy losses and the increased power of the pneumatic system [2].

The analysis of different types of electromechanical shock absorbers [2] proves that among the currently known types such as induction, synchronous, electromagnetic [7], and DC [8], the most appropriate option is the latter. This type of electromechanical energy converter provides for the required constant tractive [9] (mechanical) characteristic for the electromechanical shock absorber, which changes only by the speed of the armature.

Electromechanical shock absorbers are increasingly used in road vehicles by being integrated into the suspension of the McPherson type [10] in combination with a spring. Adding an electric shock absorber to the unified onboard circuit of a vehicle is addressed in [11]; the authors consider the possibility of recuperating the energy of oscillations, converted into electric energy, in order to improve the energy efficiency of the system.

Work [12] assessed the energy efficiency of systems that recuperate the energy of mechanical oscillations of KRAZ-65055 vehicles (Russia), KAMAZ-45143 (Russia), Gazelle-3302 (Russia), VAZ 2101 (Russia), Renault Megane (France), Toyota Camry (Japan), based on the results from previously conducted mathematical modeling of such systems.

The analysis of complex oscillation processes in the mechanical system of the vehicle's running gear is reported in work [13]. Its authors devised basic approaches to modeling mechanical oscillations based on constructing a generalized mathematical model.

To model complex electromechanical systems, study [14] proposed building a unified mathematical model of the electromechanical system; the authors, however, take into consideration the influence of an electromechanical component only as a function that excites additional oscillations in the system. This approach does not make it possible to account for the impact exerted by the mechanical system on electromagnetic processes.

The spring suspension of subway cars is modeled in work [1]. It is noted that the use of pneumatic springs in the central link of suspension is better in terms of the dynamic indicators than the most common spring suspension.

Work [15] focuses on the selection of the design and parameters for the journal link of spring suspension. A mathematical model of interaction between the wheel and rail was constructed. For this purpose, a multi-mass spatial kinematic scheme of the rail track in a subway tunnel was built; it contains such elements as «tunnel – rail track – the surrounding soil». Even stiffness in the contact between the wheel and rail was taken into consideration. The design of the upper structure of the track was adopted for the following: P65 rails with metal pads, reinforced concrete sleepers. Special polymer gaskets were placed between them and metal pads, under the rails, to eliminate noise.

Study [5] proposes a design of the electromechanical shock absorber for a subway car and gives its main characteristics. The basic approaches to the construction of a mathematical model of the electromechanical shock absorber of this type are reported in the cited study; solving a Lagrange equation involving the electromechanical system helped devise a methodology for modeling an electromechanical shock absorber based on the mathematical model. Special features of the model are as follows. The model takes the form of a Cauchy problem, which is acceptable for use in modeling the processes of shock absorber operation. Two generalized coordinates (the charge and armature movement) were selected. The components of the Lagrange equation were identified.

It follows from our analysis [1–15] that up to now the issue has been resolved partially as demonstrated by the results obtained: the time and effectiveness of oscillation damping; the possibility of constructing a generalized mathematical model of the spring suspension in a subway car and the electromechanical shock absorber. However, the models built do not make it possible to investigate the work of electromechanical shock absorbers as part of the running gear of a subway car. Another task that must be solved relates to determining the amount of oscillation energy capable of recuperating electromechanical shock absorbers depending on the speed of movement over a track section.

3. The aim and objectives of the study

The aim of this study is to build a mathematical model of the oscillatory process in the body of a subway car equipped with systems that damp the oscillations electromechanically. This would make it possible to model the processes of subway car movement when running over a track with different types of track irregularities.

To accomplish the aim, the following tasks have been set:

– to build a kinematic scheme and the equations of oscillations of the subway car model and the rail track in a tunnel;

– to identify the parameters for the mathematical model of a subway car;

– to simulate the spring suspension operation in a subway car when running over a sinusoidal irregularity.

4. Modeling the dynamic processes of the electromechanical system of a subway car equipped with electric shock absorbers

4. 1. The kinematic scheme and the equations of oscillations of a subway car model and the rail track in a tunnel

To solve the task set in this work, it is required to build a spatial kinematic scheme of the dynamic system «carriage-rail track» for a subway car whose axle formula is 2_0–2_0. A similar generalized scheme that makes it possible to use springs and hydraulic dampers, installed in parallel to them, as well as pneumatic springs, in the central link of a spring suspension was constructed in [1, 15]; it could also be applied for electromechanical suspension. The scheme from the cited works was amended because in a subway car model, in contrast to the model of a railbus, the masses of the first and second bogies, their moments of inertia relative to the y and x axes are the same. This also applies to the wheelset of bogies.

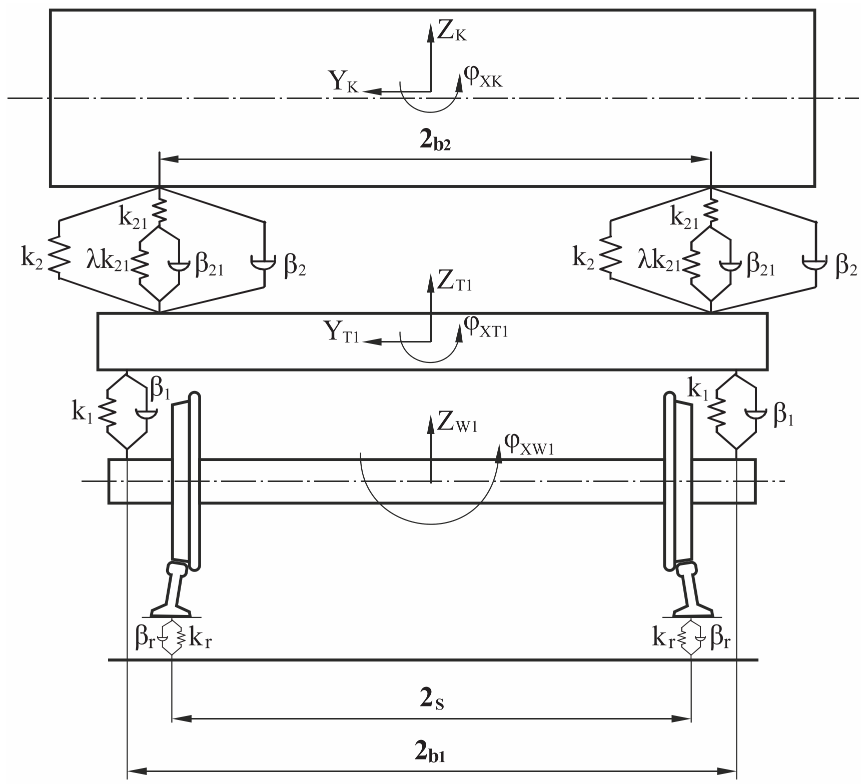

Fig. 2. Spatial kinematic scheme of a subway car model (front view)

The generalized kinematic scheme is shown in Fig. 1, 2. This diagram shows a generalized mechanical model of the central link in a spring suspension.

The elastic elements, k_{21}, λ_{k21}, and the dissipative element β_{21} belong to the scheme with an electromechanical suspension; k_2 and β_2 – to the scheme with a spring central suspension. When installing an electromechanical shock absorber, the functions of the dissipative element β_{21} are performed by electric shock absorbers; and k_2 is a standard spring.

We have built the kinematic scheme and the equation of oscillations in the subway car model and the rail track in a tunnel. According to Fig. 1, 2, the body rests on two bi-axle bogies via a central link of the spring suspension; each bogie, via a journal link, on two wheelsets.

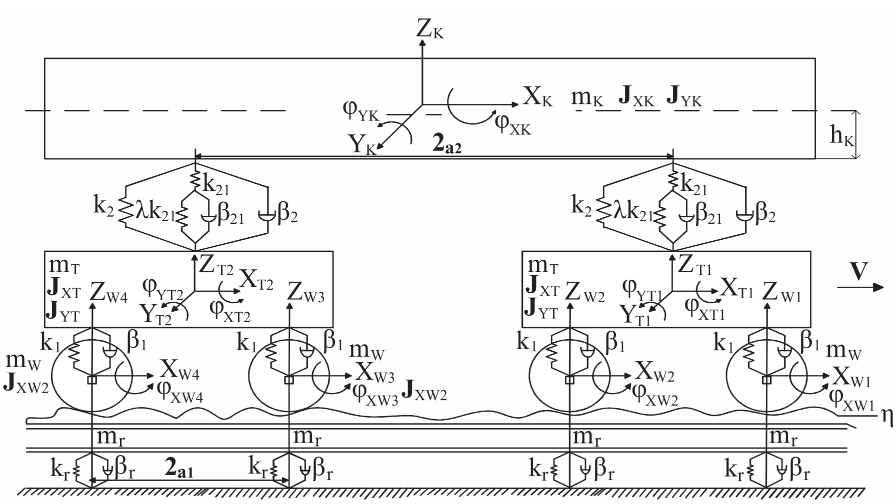

Fig. 1. Spatial kinematic scheme of a subway car model (side view)

When drawing up a spatial kinematic scheme of the dynamic system «carriage-rail track», we assumed the following:

– the body, bogie frames, and wheelsets are considered to be absolutely solid as their rigidity is much higher than the rigidity of the elastic elements connecting them;

– the centers of gravity of these solids coincide with their geometric centers;

– only the bouncing oscillations, galloping oscillations, and lateral wobbling oscillations are considered; in this case, the fluctuations of twitching, drifting, and swaying are considered conditionally unrelated to them, and are not taken into consideration;

– all movements are considered small (compared to the linear sizes of the bodies);

– the magnitudes of rigidity and damping coefficients are accepted the same for the corresponding elements in the spring suspension of various bogies and wheelsets;

– the elastic and dissipative forces are considered to be acting along the axis of the corresponding elastic and dissipative element;

– the spring and its parallel hydraulic damper are considered to be fixed at one point;

– the rigidity in the contact between the wheels and rail is not taken into consideration;

– the movement of the carriage over a track is considered to be rectilinear and uniform;

– the wheelset and the track masses applied to it move inseparably;

– the perturbation accepted in the study of the forced vertical oscillations was the equivalent geometric irregularities of the left and right rails.

To build a single mathematical model of a subway car, we considered part of the kinematic scheme with the electric shock absorber included in it. The system of equations describes the oscillations of a carriage model, consisting of three masses, the kinematic scheme of which is shown in Fig. 3, *b*. The mechanic-mathematical model of electromechanical shock absorbers is shown in Fig. 3, *a*. This kinematic scheme takes into consideration the oscillations of the body z_K, the bogie z_T, and the wheelset z_w. Fig. 3, *b* shows an electric shock absorber installed between the body and bogie.

The built equations take into consideration that the electric shock absorber receives the oscillations of the body z_K and bogie z_T. The system of mechanical equations that describes the oscillations of this model takes the following form:

$$m_K\ddot{z}_K+4F_{ek}+4k_{21}z_K=0;$$

$$m_T\ddot{z}_T+(4\beta_{21}+\beta_1)\dot{z}_T-4F_{ek}-\beta_1\dot{z}_W-k_1z_W+$$
$$+4k_{21}z_T+k_1z_T=0;$$

$$(m_W+m_r)\ddot{z}_W+(\beta_1+\beta_2)\dot{z}_W+(k_1+k_r)z_W-\beta_1\dot{z}_T-k_1z_T=$$
$$=m_r\ddot{\eta}+\beta_r\dot{\eta}+k_r\eta. \quad (2)$$

The system of mechanical equations was supplemented with the electric equations of electromechanical shock absorbers similar to those reported in [3] (Fig. 4) according to the scheme of the electric shock absorber in Fig. 5 (the symbol *k* corresponds to the number of the electric shock absorber):

$$\frac{di_k}{dt}=\frac{U_{ck}-i_k\cdot R-K_d\cdot v_k}{L_d},$$

$$\frac{dU_{ck}}{dt}=-\frac{i_k}{C}. \quad (3)$$

The adequacy of the mathematical model is confirmed by the principles of building mathematical models «carriage – track» for a subway car whose axle formula is 2_0–2_0, given in works [1, 15], and the principles of building mathematical models for linear systems [8].

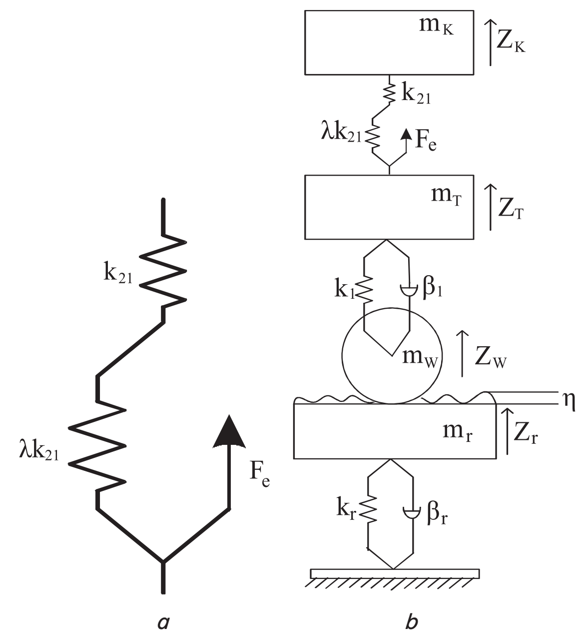

Fig. 3. The scheme of installing an electromechanical shock absorber: *a* – mechanical-mathematical model; *b* – estimated kinematic scheme

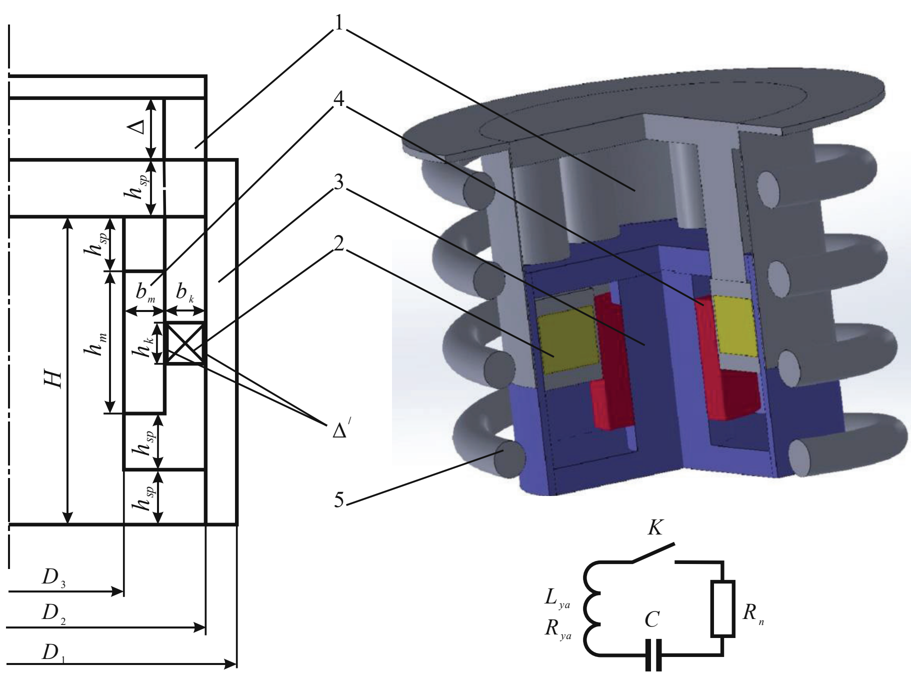

Fig. 4. Electromechanical DC shock absorber:
1 – armature; 2 – armature winding; 3 – bed; 4 – permanent magnet; 5 – spring

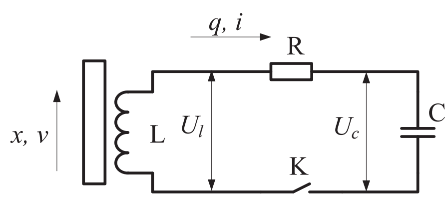

Fig. 5. Electromechanical shock absorber circuit

4. 2. The identification of parameters for a subway car mathematical model

The system of equations is supplemented with expressions that determine the differential induction, the coefficient at a counteracting electro-motive force, and the electromagnetic force of a shock absorber, calculated in line with a procedure given in [3]:

$$L_d = \frac{\partial \Psi(i,x)}{\partial i} = \sum_{j=0}^{J}\sum_{k=0}^{K}\left(AL_{jk} j\, MJ\,(MJ\cdot i + ZJ)^{j-1}(MK\cdot x + ZK)^{k}\right). \qquad (4)$$

$$K_d = \frac{\partial \Psi(i,x)}{\partial x} = \sum_{j=0}^{J}\sum_{k=0}^{K}\left(AL_{jk} k\, MK\,(MJ\cdot i + ZJ)^{j}(MK\cdot x + ZK)^{k-1}\right). \qquad (5)$$

$$F_e = \sum_{j=0}^{JF}\sum_{k=0}^{KF}\left(AF_{jk}(MJF\cdot i + ZJF)^{j}(MKF\cdot N + ZKF)^{k}\right), \qquad (6)$$

where AL_{jk}, AF_{jk} is the polynomial regression coefficient, which approximates flux coupling and force; *MJ, MK, MJF, MKF* are the scaling coefficients for the current and gap of, respectively, flux coupling and force; *ZJ, ZK, ZJF, ZKF* are the shift for the current and gap of, respectively, flux coupling and force; *J, K, JF, KF* are the polynomial powers for the current and gap, which approximates the flux coupling and force, respectively.

Equations (2) and (3) are related through the algebraic expressions for the following coordinates:

$$x_1 = z_k + a_2\varphi_{yk} - b_2\varphi_{xk} - z_{t1} + a_1\varphi_{yt1} - b_1\varphi_{xt1} + x_n,$$

$$x_2 = z_k + a_2\varphi_{yk} + b_2\varphi_{xk} - z_{t1} + a_1\varphi_{yt1} + b_1\varphi_{xt1} + x_n,$$

$$x_3 = z_k - a_2\varphi_{yk} - b_2\varphi_{xk} - z_{t2} - a_1\varphi_{yt2} - b_1\varphi_{xt2} + x_n,$$

$$x_4 = z_k - a_2\varphi_{yk} + b_2\varphi_{xk} - z_{t2} - a_1\varphi_{yt2} + b_1\varphi_{xt2} + x_n, \qquad (7)$$

and speeds:

$$v_1 = \dot z_k + a_2\dot\varphi_{yk} - b_2\dot\varphi_{xk} - z_{t1} + a_1\dot\varphi_{yt1} - b_1\dot\varphi_{xt1},$$

$$v_2 = \dot z_k + a_2\dot\varphi_{yk} + b_2\dot\varphi_{xk} - \dot z_{t1} + a_1\dot\varphi_{yt1} + b_1\dot\varphi_{xt1},$$

$$v_3 = \dot z_k - a_2\dot\varphi_{yk} - b_2\dot\varphi_{xk} - \dot z_{t2} - a_1\dot\varphi_{yt2} - b_1\dot\varphi_{xt2},$$

$$v_3 = \dot z_k - a_2\dot\varphi_{yk} + b_2\dot\varphi_{xk} - \dot z_{t2} - a_1\dot\varphi_{yt2} + b_1\dot\varphi_{xt2}. \qquad (8)$$

Considering [14, 15], we have recorded the equations of oscillations in the spatial model of a subway car in the central link of suspension. They were built on the basis of the D'Alembert principle. For the equations of body movement to take the form of equilibrium equations, the forces acting on the body and the moments of forces are supplemented with the forces of inertia: the main vector and the main moment.

Considering expressions (1) to (8), the generalized mathematical model takes the following form:

$$\begin{aligned}&m_K\ddot z_K + F_e(x_1,i_1) + F_e(x_2,i_2) + F_e(x_3,i_3) + F_e(x_4,i_4) +\\&+k_{21}(z_K + a_2\varphi_{YK} - b_2\varphi_{XK} - z_{T1} + a_1\varphi_{YT1} - b_1\varphi_{XT1}) +\\&+k_{21}(z_K + a_2\varphi_{YK} + b_2\varphi_{XK} - z_{T1} + a_1\varphi_{YT1} + b_1\varphi_{XT1}) +\\&+k_{21}(z_K - a_2\varphi_{YK} - b_2\varphi_{XK} - z_{T2} - a_1\varphi_{YT2} - b_1\varphi_{XT2}) +\\&+k_{21}(z_K - a_2\varphi_{YK} + b_2\varphi_{XK} - z_{T2} - a_1\varphi_{YT1} + b_1\varphi_{XT2}) = 0,\end{aligned}$$

$$\begin{aligned}&J_{YK}\ddot\varphi_{YK} + a_2\begin{pmatrix}F_e(x_1,i_1) + F_e(x_2,i_2) -\\ -F_e(x_3,i_3) - F_e(x_4,i_4)\end{pmatrix} +\\&+k_{21}a_2(z_K + a_2\varphi_{YK} - b_2\varphi_{XK} - z_{T1} + +a_1\varphi_{YT1} - b_1\varphi_{XT1}) +\\&+k_{21}a_2(z_K + a_2\varphi_{YK} + b_2\varphi_{XK} - z_{T1} + a_1\varphi_{YT1} + b_1\varphi_{XT1}) -\\&-k_{21}a_2(z_K - a_2\varphi_{YK} - b_2\varphi_{XK} - z_{T2} - a_1\varphi_{YT2} - b_1\varphi_{XT2}) -\\&-k_{21}a_2(z_K - a_2\varphi_{YK} + b_2\varphi_{XK} - z_{T2} - a_1\varphi_{YT1} + b_1\varphi_{XT2}) = 0,\end{aligned}$$

$$\begin{aligned}&J_{XK}\ddot\varphi_{XK} + b_2\begin{pmatrix}-F_e(x_1,i_1) + F_e(x_2,i_2) -\\ -F_e(x_3,i_3) + F_e(x_4,i_4)\end{pmatrix} -\\&-k_{21}b_2(z_K + a_2\varphi_{YK} - b_2\varphi_{XK} - z_{T1} + a_1\varphi_{YT1} - b_1\varphi_{XT1}) +\\&+k_{21}b_2(z_K + a_2\varphi_{YK} + b_2\varphi_{XK} - z_{T1} + a_1\varphi_{YT1} + b_1\varphi_{XT1}) -\\&-k_{21}b_2(z_K - a_2\varphi_{YK} - b_2\varphi_{XK} - z_{T2} - a_1\varphi_{YT2} - b_1\varphi_{XT2}) +\\&+k_{21}b_2(z_K - a_2\varphi_{YK} + b_2\varphi_{XK} - z_{T2} - a_1\varphi_{YT1} + b_1\varphi_{XT2}) = 0,\end{aligned}$$

$$\begin{aligned}&m_{T1}\ddot z_{T1} + 4\beta_1\dot z_{T1} - k_{21}b_2\varphi_{XT1} + k_{21}b_2\varphi_{XT1} +\\&+4k_1z_{T1} - 2\beta_1(\dot z_{W1} + \dot z_{W2}) - 2k_1(z_{W1} + z_{W2}) = 0,\end{aligned}$$

$$\begin{aligned}&m_{T2}\ddot z_{T2} + 4\beta_1\dot z_{T2} - k_{21}b_2\varphi_{XT2} + k_{21}b_2\varphi_{XT2} +\\&+4k_1z_{T2} - 2\beta_1(\dot z_{W3} + \dot z_{W4}) - 2k_1(z_{W3} + z_{W4}) = 0,\end{aligned}$$

$$\begin{aligned}&J_{YT}\ddot\varphi_{YT1} + 4\beta_1a_1^2\dot\varphi_{YT1} + 4k_1a_1^2\varphi_{YT1} +\\&+2\beta_1a_1(\dot z_{W2} - \dot z_{W1}) + 2k_1a_1(z_{W2} - z_{W1}) = 0,\end{aligned}$$

$$\begin{aligned}&J_{YT}\ddot\varphi_{YT2} + 4\beta_1a_1^2\dot\varphi_{YT2} + 4k_1a_1^2\varphi_{YT2} +\\&+2\beta_1a_1(\dot z_{W4} - \dot z_{W3}) + 2k_1a_1(z_{W4} - z_{W3}) = 0,\end{aligned}$$

$$\begin{aligned}&J_{XT}\ddot\varphi_{XT1} + 4\beta_1b_1^2\dot\varphi_{XT1} + (2k_{21}b_2^2 + 4k_1b_1^2)\varphi_{XT1} +\\&+2\beta_1b_1^2(\dot\varphi_{XW1} + \dot\varphi_{XW2}) - 2k_1b_1^2(\varphi_{XW1} + \varphi_{XW2}) = 0,\end{aligned}$$

$$\begin{aligned}&J_{XT}\ddot\varphi_{XT2} + 4\beta_1b_1^2\dot\varphi_{XT2} + (2k_{21}b_2^2 + 4k_1b_1^2)\varphi_{XT2} +\\&+2\beta_1b_1^2(\dot\varphi_{XW3} + \dot\varphi_{XW4}) - 2k_1b_1^2(\varphi_{XW3} + \varphi_{XW4}) = 0,\end{aligned}$$

$$\begin{aligned}&(m_{W1} + 2m_r)\ddot z_{W1} + (2\beta_1 + 2\beta_r)\dot z_{W1} + (2k_1 + 2k_r)z_{W1} -\\&-2\beta_1\dot z_{T1} - 2k_1z_{T1} - 2\beta_1a_1\dot\varphi_{YT1} - 2k_1a_1\varphi_{YT1} =\\&= m_r(\ddot\eta_{L1} + \ddot\eta_{P1}) + \beta_r(\dot\eta_{L1} + \dot\eta_{P1}) + k_r(\eta_{L1} + \eta_{P1}),\end{aligned}$$

$$\begin{aligned}&J_{XW}\ddot\varphi_{XW1} + (2\beta_1b_1^2 + \beta_rs^2)\dot\varphi_{XW1} + (2k_1b_1^2 + k_rs^2)\varphi_{XW1} -\\&-2\beta_1b_1^2\dot\varphi_{XT1} - 2k_1b_1^2\varphi_{XT1} = \beta_rs(\dot\eta_{P1} - \dot\eta_{L1}) + k_rs(\eta_{P1} - \eta_{L1}),\end{aligned}$$

$$(m_{W1}+2m_r)\ddot z_{W2}+(2\beta_1+2\beta_r)\dot z_{W2}+(2k_1+2k_r)z_{W2}-$$
$$-2\beta_1\dot z_{T1}-2k_1z_{T1}-2\beta_1a_1\dot\varphi_{YT1}-2k_1a_1\varphi_{YT1}=$$
$$=m_r(\ddot\eta_{L2}+\ddot\eta_{P2})+\beta_r(\dot\eta_{L2}+\dot\eta_{P2})+k_r(\eta_{L2}+\eta_{P2}),$$

$$J_{XW}\ddot\varphi_{XW2}+(2\beta_1b_1^2+\beta_rs^2)\dot\varphi_{XW2}+(2k_1b_1^2+k_rs^2)\varphi_{XW2}-$$
$$-2\beta_1b_1^2\dot\varphi_{XT2}-2k_1b_1^2\varphi_{XT2}=\beta_rs(\dot\eta_{P2}-\dot\eta_{L2})+k_rs(\eta_{P2}-\eta_{L2}),$$

$$(m_{W2}+2m_r)\ddot z_{W3}+(2\beta_1+2\beta_r)\dot z_{W3}+(2k_1+2k_r)z_{W3}-$$
$$-2\beta_1\dot z_{T2}-2k_1z_{T2}-2\beta_1a_1\dot\varphi_{YT2}-2k_1a_1\varphi_{YT2}=$$
$$=m_r(\ddot\eta_{L3}+\ddot\eta_{P3})+\beta_r(\dot\eta_{L3}+\dot\eta_{P3})+k_r(\eta_{L3}+\eta_{P3}),$$

$$J_{XW}\ddot\varphi_{XW3}+(2\beta_1b_1^2+\beta_rs^2)\dot\varphi_{XW3}+(2k_1b_1^2+k_rs^2)\varphi_{XW3}-$$
$$-2\beta_1b_1^2\dot\varphi_{XT2}-2k_1b_1^2\varphi_{XT2}=\beta_rs(\dot\eta_{P3}-\dot\eta_{L3})+k_rs(\eta_{P3}-\eta_{L3}),$$

$$(m_{W2}+2m_r)\ddot z_{W4}+(2\beta_1+2\beta_r)\dot z_{W4}+(2k_1+2k_r)z_{W4}-$$
$$-2\beta_1\dot z_{T2}-2k_1z_{T2}-2\beta_1a_1\varphi_{YT\,2}-2k_1a_1\varphi_{YT2}=$$
$$=m_r(\ddot\eta_{L4}+\ddot\eta_{P4})+\beta_r(\dot\eta_{L4}+\dot\eta_{P4})+k_r(\eta_{L4}+\eta_{P4}),$$

$$J_{XW}\ddot\varphi_{XW4}+(2\beta_1b_1^2+\beta_rs^2)\dot\varphi_{XW4}+(2k_1b_1^2+k_rs^2)\varphi_{XW4}-$$
$$-2\beta_1b_1^2\dot\varphi_{XT2}-2k_1b_1^2\varphi_{XT2}=\beta_rs(\dot\eta_{P4}-\dot\eta_{L4})+k_rs(\eta_{P4}-\eta_{L4}),$$

$$L(x_1,i_1)\cdot\dot i_1=U_{C2}-i_1\cdot R-K(x_1,i_1)\times$$
$$\times(\dot z_K+a_2\dot\varphi_{YK}-b_2\dot\varphi_{XK}-z_{T1}+a_1\dot\varphi_{YT1}-b_1\dot\varphi_{XT1}),$$

$$C\dot U_{C1}=-i_1,$$

$$L(x_2,i_2)\cdot\dot i_2=U_{C2}-i_2\cdot R-K(x_2,i_2)\times$$
$$\times(\dot z_K+a_2\dot\varphi_{YK}+b_2\dot\varphi_{XK}-\dot z_{T1}+a_1\dot\varphi_{YT1}+b_1\dot\varphi_{XT1}),$$

$$C\dot U_{C2}=-i_2,$$

$$L(x_3,i_3)\cdot\dot i_3=U_{C3}-i_2\cdot R-K\ (x_3,i_3)\times$$
$$\times(\dot z_K-a_2\dot\varphi_{YK}-b_2\dot\varphi_{XK}-\dot z_{T2}-a_1\dot\varphi_{YT2}-b_1\dot\varphi_{XT2}),$$

$$C\dot U_{C3}=-i_3,$$

$$L(x_4,i_4)\cdot\dot i_4=U_{C4}-i_4\cdot R-K(x_4,i_4)\times$$
$$\times(\dot z_K-a_2\dot\varphi_{YK}+b_2\dot\varphi_{XK}-\dot z_{T2}-a_1\dot\varphi_{YT2}+b_1\dot\varphi_{XT2}),$$

$$C\dot U_{C4}=-i_4. \qquad (9)$$

The system of equations (9) is a differential system of equations of the second order for a bogie of a subway car. The system takes into consideration the joint operation of four electromechanical shock absorbers installed in the second link of spring suspension, the excitation arising from a track irregularity, the track parameters, and the properties of other elements in a spring suspension. To link the parameters, system (9) is supplemented with the algebraic coupling expressions (4) to (8).

To identify the parameters of the mathematical model of a subway car, we substantiated the choice of a scheme and the parameters of a spring suspension for promising subway cars [1, 15], which were accepted as the basic structure. The mass, inertial, and geometric characteristics of the studied mathematical model of a subway car were adopted in accordance with the recommendations given in [1, 15]. These characteristics correspond to the parameters of a subway car the type of «Rusich» (VAT «Metrovagonmash» (Mytishchi Machine-Building Plant, Russia). The rigidity of the central and journal links of the spring suspension when using springs in these links was taken, similar to [1, 15], based on the manufacturer's specifications. The damping coefficients $\beta_{1,2}$ in the journal and central links of the suspension were determined as follows:

$$\beta_{1,2}=n_{1,2}\beta_{CR1,2},$$

where $n_{1,2}$ is the relative damping coefficient, (n_1=0.3, n_2=0.2); $\beta_{CR1,2}$ is the critical damping coefficient.

To model the modes of spring suspension operation, we considered the work of a spring suspension over a sinusoidal irregularity.

The irregularity parameters are determined from the expressions that complement system (9):

$$\eta_{L1}=A\cos(\omega t),$$
$$\eta_{P1}=A\cos(\omega t-\theta_4),$$
$$\eta_{L2}=A\cos(\omega t-\theta_1),$$
$$\eta_{P2}=A\cos(\omega t-\theta_4-\theta_1),$$
$$\eta_{L3}=A\cos(\omega t-\theta_2),$$
$$\eta_{P3}=A\cos(\omega t-\theta_4-\theta_2),$$
$$\eta_{L4}=A\cos(\omega t-\theta_2-\theta_3),$$
$$\eta_{P4}=A\cos(\omega t-\theta_4-\theta_2-\theta_3), \qquad (10)$$

where A is the amplitude of an irregularity, which is calculated from the expression chosen on the recommendations from [16]; θ_1, θ_2, θ_3, θ_4 are the shifts of oscillation phases between the first and second wheelset, between the second and third wheelset, as well as on the right and left wheels, respectively, ω is the angular oscillation frequency, determined from the following expression:

$$\omega=\frac{2\pi}{L}V, \qquad (11)$$

where L is the rail length; V is the movement speed.

4. 3. Modeling the work of a subway car spring suspension when running over a sinusoidal irregularity

We modeled the spring suspension operation at speeds of 40 km/h, 60 km/h, 80 km/h, and 100 km/h.

The results of modeling at speeds of 80 km/h for the oscillations in wheelsets are shown in Fig. 6; the body and bogies – Fig. 7; the electromechanical shock absorbers – Fig. 8. The oscillation processes' modeling shown lasted over 30 s.

The entire time of oscillations could be divided into two main stages. During the time from the beginning to about 0.15 s, there are free oscillations in the mechanical part. The oscillation period does not depend on the speed of movement and is about 0.02 s; the maximum amplitude value is 0.011 m for wheelset 1 and 3, and 0.01 m for wheelset 2 and 4. The oscillations of wheelset 1 and 2 are close in phases; wheelset 3 and 4 are in the anti-phase to them. The maximum speed is 2.8 m/s for wheelset 1 and 3, and 2.3 m/s for wheelset 2 and 4.

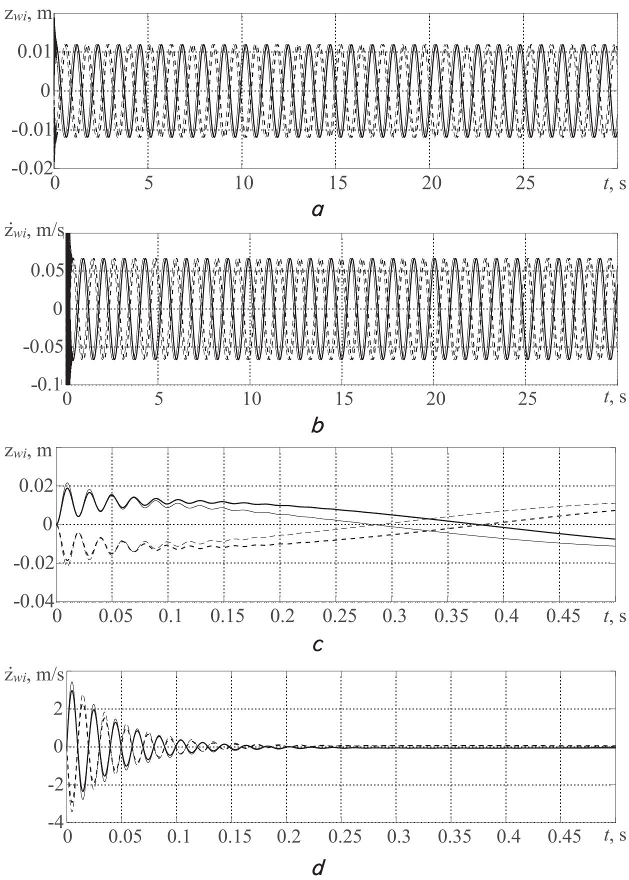

Fig. 6. Results of modeling the wheelset oscillations at a speed of 80 km/h: *a* — the linear movements on a segment from the beginning of modeling to second 30; *b* — the linear speeds from the beginning of modeling to second 30; *c* — the linear movements on the segment from the beginning of modeling to second 0.5; *d* — the linear speeds from the beginning of modeling to second 0.5; —— — Wp1, —— — Wp2, ----- — Wp3, -·-·- — Wp4

The forced oscillations in wheelsets have a much longer period of oscillations, which is due to the speed of movement of a subway car. It decreases with a speed increase from 3 s at a speed of 40 km/h to 1 s at a speed of 100 km/h. The amplitude of oscillations increases from 0.01 to 0.013 m. The amplitude of speed increases from 0.03 m/s to 0.1 m/s while increasing the speed from 40 km/h to 100 km/h. Such processes are due to the increasing frequency of changes in the track irregularity.

Similar to the oscillations in wheelsets, during the period from the beginning to about 0.15 s, there are free oscillations in bogies. The period of oscillations does not depend on the speed of movement and is about 0.02 s; their amplitude is about 0.001 m, and the linear speed has a maximum amplitude of 0.2 m/s. In addition to the oscillations caused by the free oscillations in wheelsets, there are also the free oscillations of bogies, which are observed to 0.25 s from the beginning of modeling, with a maximum amplitude of 0.02 m, and a maximum amplitude of speed fluctuation of 0.4 m/s. The period of these oscillations is 0.15 s, its dependence on speed is absent because it depends only on the structure of a bogie. From second 2 to second 30 of the modeling process, there are forced oscillations in the bogies and body. Their frequency is due to the speed of movement of a subway car, as well as the frequency of oscillations in wheelsets. Similar to the oscillations in wheelsets, during the time from the beginning to about 0.15 s, there are free oscillations in bogies. The period of oscillations does not depend on the speed of movement and is about 0.02 s; their amplitude is about 0.001 m, and the linear speed has a maximum amplitude of 0.2 m/s. The linear speed also increases from 0.025 m/s at 40 km/h to 0.07 m/s at 100 km/h, due to the growth of the dynamic forces over the track irregularities.

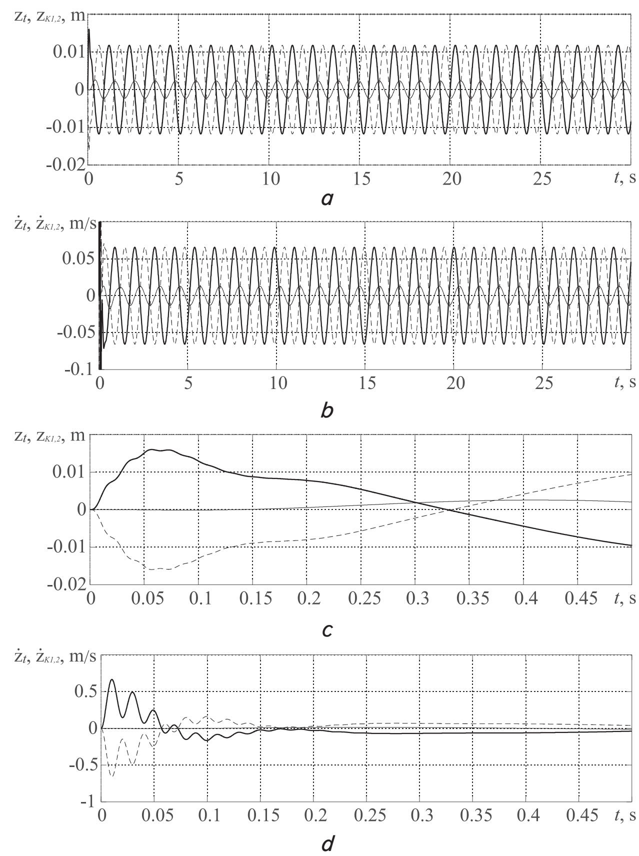

Fig. 7. Results of modeling the body and bogie oscillations at a speed of 80 km/h: *a* — the linear movements on a segment from the beginning of modeling to second 30; *b* — the linear speeds from the beginning of modeling to second 30; *c* — the linear movements on a segment from the beginning of modeling to second 0.5; *d* — the linear speeds from the beginning of modeling to second 0.5; —— — Body, —— — Bogie, ----- — Bogie 2

The amplitude of the forced body oscillations is much smaller than that of bogies due to the higher body weight; it is 0.002 m at 40 km/h. These oscillations are also harmonious and are offset into half a phase relative to bogie 1. The amplitude of oscillations increases with a speed growth to 0.002 m/h at 100 km/h; the linear speed from 0.01 m/s at 40 km/h to 0.02 m/s at 100 km/h. Free body oscillations are insignificant.

The electromechanical processes in electric shock absorbers could be divided into three parts. From second 0 to second 0.5, there are free oscillations in the mechanical part, due to the oscillations in the bogies and body. The maximum amplitudes of current are 0.32 A, voltages – 38 V. This component is characterized by damping. The second part is a free component of the electromagnetic oscillations, which is observed from second 0 to second 3. The nature of oscillations is harmonious. The third part is the forced oscillations in current and voltage. The amplitude of these oscillations varies considerably with the growth of speed: from 0.01 A and 2 V at 40 km/h to 0.9 A and 115 V at 100 km/h. The oscillations are harmonious. The frequency of oscillations corresponds to the frequency of the track irregularity.

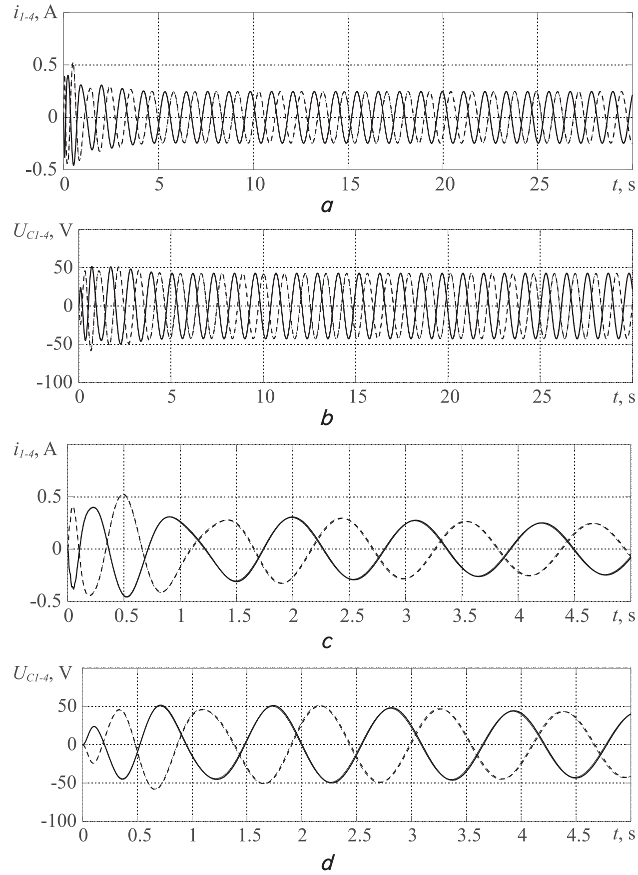

Fig. 8. Results of modeling the electromechanical shock absorbers at a speed of 80 km/h: *a* – the current on a segment from the beginning of modeling to second 30; *b* – the voltage from the beginning of modeling to second 30; *c* – the current on a segment from the beginning of modeling to second 5; *d* – the voltage from the beginning of modeling to second 5; —— – EA1, —— – EA2, ------ – EA3, - - - – EA4

The electric power of the electric shock absorber increases from 0.018 W at 40 km/h to 98 W at 100 km/h.

5. Discussion of results of modeling the work of a spring suspension in a subway car running over a sinusoidal irregularity

The results of our research are due to the following. The nature of the free component of oscillations in the wheelset and bogies depends on the eigenfrequencies of the elements of the structure of the spring suspension (Fig. 6, 7). The oscillation phase depends on the distance of the location of wheelsets and their position in the bogies of a subway car.

The nature of forced oscillations is due to the frequency of an irregularity that generates oscillations and directly depends on the speed of movement – (10) and (11). As the speed increases, the oscillation frequency increases.

The first component of the electromechanical free oscillations in electric shock absorbers depends on the parameters of the running gear and is due to their dimensions (Fig. 8). The second component is predetermined by the parameters of the electric circuit of the winding in electric shock absorbers and the load resistance. The frequency of the forced component of the oscillations in current and voltage, as well as mechanical movements, depends on the speed of movement – (10) and (11).

The increase in the amplitude of oscillations in the elements of the running gear and in the power of the electric shock absorber is due to the increase in energy delivered to the running gear from the irregularities of the track.

Owing to the construction of a unified mathematical model of body oscillation processes (9) in a subway car equipped with systems that damp the oscillations electromechanically, it was possible to determine the impact of the nature of irregularities on the electromagnetic processes in the electromechanical shock absorber. We have established the dependences of current and voltage in the shock absorber, as well as its power.

The mathematical model built, in contrast to those proposed in [2], takes into consideration the impact of track irregularities and the joint operation of shock absorbers. Applying a procedure for determining differential inductivity, the coefficient for a counteracting electromotive force, and the electromagnetic force of the shock absorber, determined by the methodology from [3], (4) to (6), makes it possible to take into consideration the geometry of the magnetic circle of the electric shock absorber and the saturation of its parts.

The devised model holds for the rolling stock whose axle characteristic is 2_0–2_0, equipped with a two-link spring suspension. The identification of the shock absorber parameters is limited to the structure given in [2, 3].

The originality of the resulting mathematical model is due, first of all, to a combination of the mathematical models of a subway car and a rail track in a tunnel with the mathematical models that take into consideration the electromechanical systems of the electric shock absorber. The latter are built on the basis of fairly accurate magnetic field calculations, which consider the geometry of magnetic systems, using a finite-element method.

The mathematical model could be further advanced by synthesizing the perturbing track irregularities depending on the track type, as well as by the generalized analysis of the operational quality of a subway car running gear based on the generalized evaluation criteria.

6. Conclusions

1. Based on the devised spatial kinematic scheme of the dynamic system «carriage – track» for a subway car, whose axle formula is 2_0–2_0, and for a rail track in a tunnel, we have built a mathematical model of the subway car whose central link of the spring suspension employs springs and electromechanical oscillation dampers. The model integrates the following:

– 17 differential equations of the second order, which describe the work of the mechanical part «carriage – track»;

– 8 differential equations of the first order, which describe the work of 4 electromechanical shock absorbers;

– 3 polynomials of orders 32 and 63 describing the state of the magnetic field of electromechanical shock absorbers and their electromagnetic force;

– 4 algebraic coupling equations.

The mathematical model of the system «carriage – track» equipped with an electromechanical shock absorber takes into consideration the following. The longitudinal and transverse oscillations in the wheelsets of bogies and the body of the car. The parameters of a rail track, the electromagnetic features of electric shock absorbers, and the excitation arising from the unevenness of the track. The track parameters and the properties of other spring suspension elements.

2. We have identified the model parameters by determining the differential inductiveness, the coefficient for counteracting electromotive force, and the electromagnetic force of the shock absorber. Algebraic relations between the mechanical component of the model (2) and the electromechanical equations of electric shock absorbers (3) have been established. The mathematical model is supplemented with expressions that determine the irregularity of a track (10) and generate the oscillations of a subway car body.

3. We have modeled the spring suspension operation at speeds of 40 km/h, 60 km/h, 80 km/h, and 100 km/h. It was determined that the entire time of oscillations could be divided into two main stages. During the time from the beginning to about second 0.15, there are free oscillations in the mechanical part. The oscillation period does not depend on the speed of movement and is about 0.02 s, and the maximum amplitude value is 0.011 m for wheelset 1 and 3, and 0.01 m for wheelset 2 and 4. The forced oscillations in wheelsets have a much larger period of oscillations, which is due to the speed of movement of a subway car. It decreases with a speed growth from 3 s at a speed of 40 km/h to 1 s at a speed of 100 km/h. The electromechanical processes in electric shock absorbers could be divided into three parts. From second 0 to second 0.5, there are free oscillations in the mechanical part caused by the oscillations in the bogies and body. The maximum amplitudes of current are 0.32 A, voltages – 38 V. This component is characterized by attenuation. The second part is a free component of the electromagnetic oscillations observed from second 0 to second 3. The oscillations are harmonious. The third part is the forced oscillations in current and voltage. The amplitude of these oscillations varies considerably with the growth of speed from 0.01 A and 2 V at 40 km/h to 0.9 A and 115 V at 100 km/h. The oscillations are harmonious. The frequency of oscillations corresponds to the frequency of track irregularity. The electric power of the electric shock absorber increases from 0.018 W at 40 km/h to 98 W at 100 km/h.

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