- 22. Nechyporenko, V. M., Salo, V. A., Litovchenko, P. I., Kovbaska, B. V., Verkhorubov, D. O. (2016). Using of the theory of R-functions for producing a rational interference fit. Zbirnyk naukovykh prats Natsionalnoi akademiyi Natsionalnoi hvardiyi Ukrainy, 2, 72–76.
- 23. Timoshenko, S., Woinowsky-Krieger, S. (1987). Theory of Plates and Shells. New York: McGraw-Hill Book Company, 580.
- 24. Salo, V. A. (2004). O kontsentratsii napryazheniy okolo otverstiya v uprugoy sfericheskoy obolochke. Voprosy proektirovaniya i proizvodstva konstruktsiy letatel'nyh apparatov, 37 (2), 66–72.

This paper reports the synthesized two-mass antiphase resonance vibratory machine with a vibration exciter in the form of a passive auto-balancer. In the vibratory machine, platforms 1 and 2 are viscoelastically attached to the stationary bed and are tied together viscoelastically. A passive auto-balancer is mounted on platform 2.

D-

-

It has been established that the vibratory machine has two resonant frequencies and two corresponding forms of platform oscillations. Such values for the supports' parameters have been analytically selected at which:

- there is an antiphase mode of motion at which platforms 1 and 2 oscillate in the opposite phase and the principal vector of forces acting on the bed (when disregarding the forces of gravity) is zero;

- the frequency of platform oscillations under an antiphase mode coincides with the second resonance frequency.

The antiphase mode occurs when the loads in an auto-balancer get stuck in the vicinity of the second resonance frequency, which is caused by the Sommerfeld effect.

The dynamic characteristics of a vibratory machine have been investigated by numerical methods. It has been established that in the case of small internal and external resistance forces:

- there are five theoretically possible modes of load jamming;

- the antiphase (second) form of platform oscillations is theoretically implemented under jamming modes 3 and 4;

- jamming mode 3 is locally asymptotically stable while jamming mode 4 is unstable;

- for the loads to get stuck in the vicinity of the second resonance frequency, the vibratory machine must be provided with the initial conditions close to jamming mode 3, or the rotor must be smoothly accelerated to the working frequency;

- the dynamic characteristics of the vibratory machine during operation can be controlled in a wide range by changing both the rotor speed and the number of loads in the auto-balancer.

The reported results are applicable for the design of resonant antiphase two-mass vibratory machines for general purposes

Keywords: inertial vibration exciter, resonant vibrations, antiphase vibratory machine, auto-balancer, twomass vibratory machine, Sommerfeld effect

Received date 16.10.2020 Accepted date 04.12.2020 Published date 14.12.2020

1. Introduction

Among the vibratory machines for various applications, resonance vibratory machines appear promising [1]. In them, vibration exciters of lower mass generate vibrations at a greater UDC 622.74: 621.928.235 DOI: 10.15587/1729-4061.2020.217628

SYNTHESIZING A RESONANCE ANTI-PHASE TWO-MASS VIBRATORY MACHINE WHOSE OPERATION IS BASED ON THE SOMMERFELD EFFECT

G. Filimonikhin

Doctor of Technical Sciences, Professor, Head of Department Department of Machine Parts and Applied Mechanics** E-mail: filimonikhin@ukr.net

V. Yatsun

PhD, Associate Professor Department of Road Cars and Building** E-mail: yvkr@i.ua

A. Kyrychenko Doctor of Technical Sciences, Professor, Dean** E-mail: amkyrychenko@gmail.com

A. Hrechka PhD, Associate Professor, Head of Department* E-mail: angre17@gmail.com

K. Shcherbyna PhD*

E-mail: kir2912s@ukr.net *Department of Metal Cutting Machines and Systems** **Central Ukrainian National Technical University Universytetskyi ave., 8, Kropyvnytskyi, Ukraine, 25006

Copyright © 2020, G. Filimonikhin, V. Yatsun, A. Kyrychenko, A. Hrechka, K. Shcherbyna This is an open access article under the CC BY license (http://creativecommons.org/licenses/by/4.0)

amplitude, thereby improving the energy efficiency, reliability, and durability of vibratory machine operation. The basic principles of designing such machines were considered in [1, 2].

The inertial vibration exciters of a lower mass excite more intense vibrations compared to electromagnetic vibration exciters [2]. Therefore, hereafter we consider vibratory machines only with inertial vibration exciters.

Vibratory machines can be single- and multi-mass. It should be noted that multi-mass vibratory machines possess a series of advantages over single-mass ones [3–11]. Thus, a multi-mass structure makes it possible to design vibratory machines that almost do not transmit vibrations to the foundation.

The most effective and simple ways to excite resonant oscillations are based on the use of the Sommerfeld effect [7–12]. Among these methods, there is a particular technique based on using passive auto-balancers in the form of vibration exciters [12, 17–21]. The technique is applicable for singleand multi-mass vibratory machines and is distinguished by the possibility to change the characteristics of vibrations in a wide range.

It is a relevant task to build on the results reported in [17–21] in order to synthesize a resonant antiphase twomass vibratory machine with a vibration exciter in the form of a passive auto-balancer and to investigate its steady state vibrations. In a given vibratory machine, the two platforms are viscoelastically attached to the bed and are tied together viscoelastically. A passive auto-balancer is mounted on one of the platforms. Under an antiphase mode, the platforms execute intense anti-phase resonance oscillations. At the same time, the total force with which the elastic supports act on the bed is almost zero.

2. Literature review and problem statement

Two-mass vibratory machines have a series of advantages over single-mass ones. Let us consider what these advantages provide for when designing vibratory machines.

In most two-mass vibratory machines, a working body (a working platform) is viscoelastically attached to a platform while the platform is similarly attached to the foundation. A vibration exciter is mounted on the platform.

It is known that the two-mass vibratory machine, in contrast to a single-mass one, has an anti-resonance mode of oscillations under which the platforms' oscillations are almost not transferred to the foundation [3–5]. Significantly, the anti-resonance regime is implemented in a wide region of the parameters of the vibratory machine [3]. At the same time, the region of the existence of an anti-resonance regime [4], as well as the frequency of platform oscillations [5], are less dependent on the mass of the load. Under an anti-resonance mode, the working body oscillates at a significant amplitude while the platform oscillates at a minimum amplitude.

However, anti-resonance two-mass vibratory machines are not resonant. They operate in the inter-resonance region or after the second resonance [6]. Therefore, these machines do not demonstrate the advantages of resonant machines.

The easiest technique to excite resonant vibrations in vibratory machines is based on the Sommerfeld effect [7]. There are examples of this technique's application for single-mass [8], two-mass [9, 10], and three-mass [11] vibratory machines. The effect is manifested in that the unbalanced rotor of a DC electric motor [9], or induction motor [8, 10], the unbalanced impeller [11] cannot accelerate to the working frequency and gets stuck at one of the resonant frequencies of platform vibrations.

The two-mass vibratory machine has two resonant frequencies and two corresponding forms of oscillations [3–6, 9, 10]. At a lower resonance frequency, the platforms oscillate in phase;

at a higher frequency – in antiphase. The antiphase mode of platform motion is of more interest in practical terms. Under this regime, the large amplitudes of platform oscillations are achieved at less perturbing forces operating on the foundation.

Study [12] proposed using a passive auto-balancer (a ball, a roller, or a pendulum) to excite resonant vibrations in single- and multi-mass vibratory machines. The technique is also based on the Sommerfeld effect. The technique employs the effect of the balls (rollers) [13] or pendulums [14] getting stuck in the auto-balancer at one of the resonant frequencies of the vibratory machine. The effect is manifested at small resistance forces in rotary systems with isotropic [13, 14] and anisotropic [15] supports. Platform vibration parameters vary widely by changing the rotor speed, the total mass of loads. There can be several loads [13–15], as well as one [16].

The technique for exciting resonant vibrations by passive auto-balancers is theoretically substantiated in studies [17–21]. Thus, [17] describes the generalized models of single-, two-, and three-mass resonant vibratory machines, as well as the derived differential motion equations. The analytical methods were applied to investigate the feasibility of this technique for a single-mass [18], two-mass [19], and three-mass [20] vibratory machine.

The results reported in [17–20] have made it possible for the authors of [21] to synthesize and investigate the dynamics of a three-mass anti-resonant vibratory machine with a vibration exciter in the form of a passive auto-balancer. The new vibratory machine is interesting because it almost does not transmit vibrations to the foundation. The vibratory machine consists of three viscoelastically connected platforms. The intermediate platform is viscoelastically attached to the foundation. One of the outside platform hosts a passive auto-balancer. Under an anti-resonance motion mode, the intermediate platform almost does not oscillate while the two outside platforms oscillate in opposite phases. The rigidity of the supports was chosen so that the frequency of platforms' oscillations under an anti-resonance mode was a resonant frequency of the vibratory machine's oscillations. In this case, due to the Sommerfeld effect, the loads in the auto-balancer can get stuck at the specified resonance frequency, which could excite the anti-resonance mode of motion.

It should be noted that the two-mass vibratory machines' structure is simpler than that of three-mass vibratory machines. However, up to now, no resonant two-mass vibratory machines with a vibration exciter in the form of a passive autobalancer that do not transmit vibrations to the foundation have been synthesized.

3. The aim and objectives of the study

The aim of this work is to synthesize and study the dynamics of a two-mass resonant anti-phase vibratory machine with a vibration exciter in the form of a passive auto-balancer. This is necessary for the development and design of vibratory machines of the specified structure, which would almost not transmit vibrations to the foundation.

To accomplish the aim, the following tasks have been set:

- to synthesize a two-mass resonant antiphase vibratory machine that almost does not transmit oscillations to the foundation;

to analytically find the laws of its platforms' oscillations;
 to investigate by numerical methods the dynamic properties of the vibratory machine at certain parameters.

) 0

4. The mechanical-mathematical model of vibratory machine

4.1. Description of the vibratory machine model

A model of the two-mass vibratory machine is depicted in Fig. 1 [17]. The vibratory machine consists of a bed, rigidly attached to the foundation, and platforms 1, 2 of, respectively, masses M_1 and M_2 . Platform number i is attached to the bed with an elastic-viscous support whose rigidity coefficient is k_i , and viscosity coefficient is b_i , i=1, 2/. The platforms are connected by an elastic-viscous support whose rigidity coefficient is k_{12} , and viscosity coefficient is b_{12} .



Fig. 1. The kinematics of a two-mass vibratory machine motion [17]: *a* – platform; *b* – a ball or roller; c - pendulum

The guides on the bed allow the platforms to execute only translational motion in the vertical direction. The positions of the platforms are determined by the y_1, y_2 coordinates, and the coordinates are counted from the positions of the static equilibrium of the platforms.

Platform 2 hosts a ball-, a roller- (Fig. 1, *b*), or a pendulum-type (Fig. 1, *c*) auto-balancer.

The body of the auto-balancer is mounted onto the shaft and rotates with a shaft at a constant angular velocity ω (around point *K*). The turning angle of the body is ωt , where *t* is the time.

The auto-balancer consists of N identical loads. The mass of one load is m. The center of the load's mass can move along the circumference of radius R with the center at point K (Fig. 1, b, c). The position of load number j is determined by angle φ_j , j = 1, N / .

The center of the mass of load number j moves relative to the body of the auto-balancer at a relative speed whose module is equal to $v_i^{(r)} = R | \varphi'_i - \omega |$. Hereafter, a stroke by the value denotes a time derivative t.

At the relative motion, the load is exposed to the force of viscous resistance whose module is:

$$F_{j} = b_{W} v_{j}^{(r)} = b_{W} R |\phi_{j}' - \omega|, \ /j = 1, N/,$$
(1)

where b_W is the viscous resistance force factor.

Further, we do not take into consideration the gravity forces.

4.2. Differential motion equations

The differential equations of the vibratory machine's motion take the following dimensionless form [19].

$$\begin{aligned} \ddot{v}_{1} + 2h_{1}\dot{v}_{1} + n_{1}^{2}v_{1} + 2h_{12}(\rho\dot{v}_{1} - \dot{v}_{2}) + n_{12}^{2}(\rho v_{1} - v_{2}) &= 0, \\ \ddot{v}_{2} + 2h_{2}\dot{v}_{2} + n_{2}^{2}v_{2} - \\ &- 2h_{12}(\rho\dot{v}_{1} - \dot{v}_{2}) - n_{12}^{2}(\rho v_{1} - v_{2}) + \ddot{s}_{y} = 0, \\ \ddot{\varphi}_{j} + \varepsilon\beta(\dot{\varphi}_{j} - n) + \varepsilon\ddot{v}_{2}\cos\varphi_{j} = 0, \quad /j = \overline{1, N} /. \end{aligned}$$

$$(2)$$

•) . 2 (

The following dimensionless quantities are introduced in (2):

– constants and time: (~)

$$v_{1} = y_{1} / (\rho y), \quad v_{2} = y_{2} / y,$$

$$s_{y} = \frac{mR}{\tilde{s}} \sum_{j=1}^{N} \sin \varphi_{j}, \quad \tau = \tilde{\omega}t;$$
(3)

– parameters:

$$n_{1}^{2} = \frac{k_{1}}{M_{1}\tilde{\omega}^{2}}, \quad n_{12}^{2} = \frac{k_{12}}{M_{2\Sigma}\tilde{\omega}^{2}}, \quad n_{2}^{2} = \frac{k_{2}}{M_{2\Sigma}\tilde{\omega}^{2}},$$
$$h_{1} = \frac{b_{1}}{2M_{1}\tilde{\omega}}, \quad h_{12} = \frac{b_{12}}{2M_{2\Sigma}\tilde{\omega}}, \quad h_{2} = \frac{b_{2}}{2M_{2\Sigma}\tilde{\omega}}, \quad \rho = \frac{M_{2\Sigma}}{M_{1}},$$
$$n = \frac{\omega}{\tilde{\omega}}, \quad \varepsilon = \frac{\tilde{y}}{\kappa R}, \quad \beta = \frac{b_{W}}{\varepsilon \kappa m \tilde{\omega}}, \quad \left(\varepsilon \beta = \frac{b_{W}}{\kappa m \tilde{\omega}}\right). \tag{4}$$

In (2), a point above the value denotes a derivative from the dimensionless time.

In turn, in (3), (4):

$$-M_{2\Sigma} = M_2 + Nm, \quad \tilde{y} = \tilde{s}/M_{2\Sigma}; \tag{5}$$

- for a ball, a roller, and a pendulum, respectively:

$$\kappa = 7/5, \ \kappa = 3/2, \ \kappa = 1 + J_C / (mR^2),$$
 (6)

where J_C is the main central axial moment of the pendulum inertia; $\tilde{\omega}$ is the characteristic scale of time; \tilde{s} is the characteristic scale of the unbalanced mass.

Note that the characteristic scales can be chosen arbitrarily depending on the problem under consideration.

From the side of the platforms' elastic supports, the bed is exposed to a variable perturbing force. The projection of the perturbing force onto the *y* axis is equal to:

$$R_{y} = k_{1}y_{1} + b_{1}y_{1}' + k_{2}y_{2} + b_{2}y_{2}'.$$
(7)

Given (3) and (4), the projection of the perturbing force is reduced to the following dimensionless form:

$$\bar{R}_{y} = R_{y} / (M_{2\Sigma} \tilde{\omega}^{2} \tilde{y}) = n_{1}^{2} v_{1} + 2h_{1} \dot{v}_{1} + n_{2}^{2} v_{2} + 2h_{2} \dot{v}_{2}.$$
(8)

This force makes the bed and foundation oscillate vertically. In a perfect anti-phase machine, this force should be zero.

5. The synthesis and study of the dynamics of an antiphase resonance two-mass vibratory machine

5.1. Synthesizing the antiphase resonant two-mass vibratory machine

5.1.1. Search for the angular velocity of load rotation at which the vibratory machine operates as an anti-phase machine

A purely antiphase mode of platform motion is possible only in the absence of resistance forces.

In the absence of resistance forces, the loads collected, the loads getting stuck at a constant rotation rate Ω , the system of equations (2) takes the form:

$$\ddot{v}_{1} + n_{1}^{2}v_{1} + n_{12}^{2}(\rho v_{1} - v_{2}) = 0,$$

$$\ddot{v}_{2} + n_{2}^{2}v_{2} - n_{12}^{2}(\rho v_{1} - v_{2}) = s\Omega^{2}\sin\Omega\tau.$$
 (9)

where *s* is the total dimensionless unbalanced mass of tightly pressed loads.

In the absence of viscous resistance forces, the condition for the perturbing force to equal zero (8) takes the form:

$$\overline{R}_{\mu} = n_1^2 v_1 + n_2^2 v_2 = 0.$$
⁽¹⁰⁾

Solve the system of equations (9) under constraint (10). Add equations in (9), we obtain:

$$\ddot{v}_1 + n_1^2 v_1 + \ddot{v}_2 + n_2^2 v_2 = s \Omega^2 \sin \Omega \tau.$$

Given the constraint (10), the latter equation takes the form:

 $\ddot{v}_1 + \ddot{v}_2 = s\Omega^2 \sin \Omega \tau.$

Then, given that the platforms oscillate near the positions of static equilibrium, we obtain:

$$\ddot{v}_2 = -\ddot{v}_1 + s\Omega^2 \sin \Omega \tau,$$

$$\dot{v}_2 = -\dot{v}_1 - s\Omega \cos \Omega \tau, \quad v_2 = -v_1 - s \sin \Omega \tau.$$
 (11)

Substituting (11) in equation (9), after the transforms, we obtain:

$$\ddot{v}_{1} + \left[n_{1}^{2} + n_{12}^{2} (\rho + 1) \right] v_{1} = -n_{12}^{2} s \sin \Omega \tau,$$

$$\ddot{v}_{1} + \left[n_{2}^{2} + n_{12}^{2} (\rho + 1) \right] v_{1} = -s \left(n_{12}^{2} + n_{2}^{2} \right) \sin \Omega \tau.$$
 (12)

In (12), subtract the second equation from the first equation, and obtain:

$$\left(n_1^2 - n_2^2\right)v_1 = n_2^2s\sin\Omega\tau$$

From the equation above and the last equation in (11), we find:

$$v_1 = \frac{n_2^2}{n_1^2 - n_2^2} s \sin \Omega \tau, \quad v_2 = -\frac{n_1^2}{n_1^2 - n_2^2} s \sin \Omega \tau.$$
(13)

Find at what angular velocity of load jamming Ω the laws of platform oscillations (13) could be the solution to the system of equations (9). Denote:

$$L_{1} = \ddot{v}_{1} + n_{1}^{2}v_{1} + n_{12}^{2}(\rho v_{1} - v_{2}) = 0,$$

$$L_{2} = \ddot{v}_{2} + n_{2}^{2}v_{2} - n_{12}^{2}(\rho v_{1} - v_{2}) - s\Omega^{2}\sin\Omega\tau = 0.$$
 (14)

Substituting (13) in (14), we obtain:

$$L_{2} = -L_{1} = \left(n_{2}^{2}\Omega^{2} - n_{1}^{2}n_{2}^{2} - n_{1}^{2}n_{12}^{2} - \rho n_{2}^{2}n_{12}^{2}\right) \frac{s\sin(\Omega t)}{n_{1}^{2} - n_{2}^{2}}.$$
 (15)

From (15), we find that two equations in (14) will simultaneously hold only at the following angular velocity:

$$\Omega_w = \sqrt{\left(n_1^2 n_2^2 + n_1^2 n_{12}^2 + \rho n_2^2 n_{12}^2\right) / n_2^2}.$$
(16)

This is the angular velocity of load rotation at which a vibratory machine operates as an ideal antiphase vibratory machine.

5. 1. 2. Conditions under which the antiphase vibratory machine becomes resonant

The frequency equation of the system of equations (9) takes the following form:

$$\Delta(p) = \begin{vmatrix} a_{11}(p) & a_{12}(p) \\ a_{21}(p) & a_{22}(p) \end{vmatrix} = = a_{11}(p)a_{22}(p) - a_{12}(p)a_{21}(p) = 0,$$
(17)

where

$$a_{11}(p) = n_1^2 + n_{12}^2 \rho - p^2, \quad a_{12}(p) = -n_{12}^2,$$

$$a_{21}(p) = -n_{12}^2 \rho, \quad a_{22}(p) = n_2^2 + n_{12}^2 - p^2.$$
(18)

Then

$$\Delta(p) = (n_1^2 + n_{12}^2 \rho - p^2)(n_2^2 + n_{12}^2 - p^2) - \rho n_{12}^4 = 0.$$
(19)

If Ω_w from (16) is a resonance frequency, then:

$$\Delta(\Omega_w) = \left(n_1^2 - n_2^2\right) \left[n_{12}^2 \left(n_1^2 n_2^2 + n_1^2 n_{12}^2 + \rho n_2^2 n_{12}^2\right)\right] / n_2^4 = 0.$$
 (20)

From (20), we find that Ω_w is a resonance frequency provided:

$$n_2 = n_1. \tag{21}$$

Let the condition (21) be met. Then the frequency equation (19) takes the form:

$$\Delta(p) = (p^2 - n_1^2) [p^2 - n_1^2 + n_{12}^2 (1+\rho)].$$
(22)

From equation (22), we find the following two resonance frequencies of the vibratory machine:

$$n_1^{(r)} = n_1, \quad n_2^{(r)} = \sqrt{n_1^2 + (1+\rho)n_{12}^2}, \quad (n_2^{(r)} > n_1^{(r)}).$$
 (23)

Thus, in the absence of resistance forces in the supports $(h_1, h_{12}, h_2=0)$, the vibratory machine has two resonance frequencies (23). They correspond to two forms of the platforms' resonance oscillation. The first oscillation form is dominated by the component at which the platforms oscillate in phase; the second – in antiphase.

Note that at $n_2 \rightarrow n_1$ the laws of platform motion (13) are incorrect as amplitudes of oscillations tend to infinity. The presence of viscous resistance forces limits the amplitude of platform fluctuations. However, the projection of the perturbing force onto the *y* axis is, in this case, no longer zero.

5.2. Analytical study of the dynamics of a resonant antiphase vibratory machine

5.2.1. The laws of steady state platform motions

Loads can get stuck in a vibration exciter only if there are viscous resistance forces [19]. Under a jam mode, the loads are tightly pressed together and generate the largest total (dimensional) unbalanced mass S_{max}^{AB} . For balls or rollers [19]:

$$S_{\max}^{AB} = mR^2 / \{r \sin[N \arcsin(r / R)]\}.$$
 (24)

For the case of pendulums, additional information about the design of pendulums is needed to determine the greatest unbalanced mass S_{max}^{AB} .

The steady state modes of platform motions are determined using the results reported in [19]. At ϵ =0:

$$v_{i}(\tau) = X_{2i-1}(\Omega, s)\sin(\Omega\tau) + + X_{2i}(\Omega, s)\cos(\Omega\tau), / i = \overline{1, 2} /.$$
(25)

where Ω is the constant frequency at which loads get stuck;

$$s = S_{\max}^{AB} / \tilde{s}; \tag{26}$$

$$X_{j}(\Omega,s) = \Delta_{j}(\Omega,s) / \Delta(\Omega), / j = 1,2,3,4 /.$$
⁽²⁷⁾

In turn, in (27):

$$\Delta(\Omega) = \begin{cases} a_{11}(\Omega)a_{33}(\Omega) - \\ -\rho[a_{13}^{2}(\Omega) - a_{14}^{2}(\Omega)] - a_{12}(\Omega)a_{34}(\Omega) \end{cases}^{2} + \\ + [2\rho a_{13}(\Omega)a_{14}(\Omega) - a_{12}(\Omega)a_{33}(\Omega) - a_{11}(\Omega)a_{34}(\Omega)]^{2}, \\ \Delta_{1}(\Omega, s) = \end{cases}$$

$$\begin{split} & \left| A_{4}(\Omega, s) \right| \left| \left| a_{13}(\Omega) \right| \left| \left| a_{13}^{2}(\Omega) + a_{14}^{2}(\Omega) \right| - a_{11}(\Omega) a_{33}(\Omega) + a_{12}(\Omega) a_{34}(\Omega) + a_{12}(\Omega) a_{34}(\Omega) + a_{12}(\Omega) a_{33}(\Omega) - a_{12}(\Omega) a_{33}(\Omega) - a_{12}(\Omega) a_{34}(\Omega) + a_{12}(\Omega) a_{33}(\Omega) + a_{12}(\Omega) a_{13}(\Omega) a_{14}(\Omega) + a_{12}(\Omega) a_{14}(\Omega) + a_{1$$

Finally, in (28):

$$a_{11}(\Omega) = n_1^2 + \rho n_{12}^2 - \Omega^2, \quad a_{12}(\Omega) = -2\Omega(h_1 + \rho h_{12}),$$

$$a_{13}(\Omega) = -n_{12}^2, \quad a_{14}(\Omega) = 2\Omega h_{12},$$

$$a_{33}(q) = n_1^2 + n_{12}^2 - \Omega^2, \quad a_{34}(\Omega) = -2\Omega(h_1 + h_{12}),$$

$$b_3(\Omega, s) = s\Omega^2.$$
(29)

Note that in reality ε is the small parameter and, therefore, the correction to law (25) does not exceed 2 %.

From (25), we find the amplitudes of platforms' oscillations:

$$Amp_{i}(\Omega,s) = \sqrt{X_{2i-1}^{2}(\Omega,s) + X_{2i}^{2}(\Omega,s)}, \quad /i = 1, 2/.$$
(30)

In the laws of platform motion (25), all possible values of the constant parameter Ω are determined from the following equation [19]:

$$P(\Omega, n) = 2\beta(n - \Omega)\Delta(\Omega) + \Omega^2 \Delta_4(\Omega, s) = 0.$$
(31)

In the presence of viscous resistance forces, the dimensionless projection of the perturbing force takes the form of (8). Given (25), this projection changes according to the following harmonic law:

$$\overline{R}_{f}(\tau,\Omega,s) = \begin{cases} n_{1}^{2}X_{1}(\Omega,s) + n_{2}^{2}X_{3}(\Omega,s) - \\ -2\Omega[h_{1}X_{2}(\Omega,s) + h_{2}X_{4}(\Omega,s)] \end{cases} \sin(\Omega\tau) + \\ + \begin{cases} n_{1}^{2}X_{2}(\Omega,s) + n_{2}^{2}X_{4}(\Omega,s) + \\ +2\Omega[h_{1}X_{1}(\Omega,s) + h_{2}X_{3}(\Omega,s)] \end{cases} \cos(\Omega\tau)$$
(32)

at amplitude:

(28)

$$A_{R}(\Omega,s) = \begin{pmatrix} \left\{ n_{1}^{2}X_{1}(\Omega,s) + n_{2}^{2}X_{3}(\Omega,s) - \\ -2\Omega[h_{1}X_{2}(\Omega,s) + h_{2}X_{4}(\Omega,s)] \right\}^{2} + \\ + \left\{ n_{1}^{2}X_{2}(\Omega,s) + n_{2}^{2}X_{4}(\Omega,s) + \\ + 2\Omega[h_{1}X_{1}(\Omega,s) + h_{2}X_{3}(\Omega,s)] \right\}^{2} \end{pmatrix}^{1/2} . (33)$$

At the antiphase form of platform motion, the amplitude (33) is a small quantity.

5. 2. 2. Procedure for the numerical study of steady state platform vibrations [19, 21]

In the absence of resistance forces in the supports, $\Delta_4(\Omega, s) = 0$. In this case, the component $2\beta(n-\Omega)\Delta(\Omega)$ that remained in (31) has five valid positive roots:

$$n_1^{(r)}, n_1^{(r)}, n_2^{(r)}, n_2^{(r)}, n_2^{(r)}, n_2^{(r)} > n_1^{(r)} >$$
 (34)

Therefore, for the case of small viscous resistance forces in the supports, the frequencies of load jamming are close to the resonance (natural) oscillation frequencies of the vibratory machine or to the frequency of rotor rotation.

From (31), we find the following solution to the equation of the frequencies of load jamming in the parametric form:

$$n(\Omega) = \Omega \begin{bmatrix} 2\beta\Delta(\Omega) - \\ -\Omega\Delta_4(\Omega, s) \end{bmatrix} / [2\beta\Delta(\Omega)], \ \Omega \in (0, +\infty).$$
(35)

In a general case, when one increases Ω from 0 to $+\infty$, the dimensionless angular velocity of rotor rotation *n* can take the local minimum or maximum values. Then at the points of a local minimum, there would occur (to disintegrate later) a pair of jamming frequencies, and, at the points of a local maximum, a pair of jamming frequencies would disappear through merging.

Thus, at the bifurcation points:

$$dn(\Omega)/d\Omega = 0. \tag{36}$$

In the plane $(\Omega, n(\Omega))$, $\Omega \in (0, +\infty)$, the evolution or disappearance of the jamming frequencies is illustrated by the chart of function $\Omega(n)$, $n \in (0, +\infty)$.

Our analysis allows us to use the following computational algorithm for studying the steady state vibrations of the vibratory machine [19, 21]:

1. Equation (36) is employed to find four bifurcation frequencies of load jamming, such as $0 < \Omega_b^{(1)} < \Omega_b^{(2)} < \Omega_b^{(3)} < \Omega_b^{(4)}$.

2. Applying formula (35) produces four bifurcation angular velocities of rotor rotation $n(\Omega_b^{(i)})$, /i = 1, 4. Number them in ascend order: $0 < n_b^{(1)} < n_b^{(2)} < n_b^{(3)} < n_b^{(4)}$. When a rotor passes the bifurcation speed, one pair of jamming modes emerges or disappears.

3. In the plane (n, Ω) , build the charts of five possible modes of load jamming $(n_j(\Omega), \Omega), /j = \overline{1,5}/$, where

$$n_{1}(\Omega) = n(\Omega), \quad \Omega \in \left[0, \Omega_{b}^{(1)}\right]; \quad n_{2}(\Omega) = n(\Omega),$$
$$\Omega \in \left[\Omega_{b}^{(1)}, \Omega_{b}^{(2)}\right]; \quad \dots, \quad n_{5}(\Omega) = n(\Omega), \quad \Omega \in \left[\Omega_{b}^{(4)}, +\infty\right). \tag{37}$$

It was established in [19, 21] that only the odd modes of jamming are locally asymptotically stable while even ones are always unstable.

4. For each jamming mode, the following is calculated in the parametric form:

- from formulae (30), the amplitudes of platform oscillations:

$$\begin{aligned} Amp_{i,1}(\Omega,s) &= Amp_i(\Omega,s), \quad \Omega \in \left[0,\Omega_b^{(1)}\right], \\ Amp_{i,2}(\Omega,s) &= Amp_i(\Omega,s), \quad \Omega \in \left[\Omega_b^{(1)},\Omega_b^{(2)}\right], \quad \dots, \\ Amp_{i,5}(\Omega,s) &= Amp_i(\Omega,s), \quad \Omega \in \left[\Omega_b^{(4)}, +\infty\right), \quad /i = 1,2/; \quad (38) \end{aligned}$$

– from formula (33), the amplitudes of oscillations of the dimensionless perturbing force:

$$\begin{aligned} A_{R,1}(\Omega,s) &= A_R(\Omega,s), \quad \Omega \in \left[0, \Omega_b^{(1)}\right], \\ A_{R,2}(\Omega,s) &= A_R(\Omega,s), \quad \Omega \in \left[\Omega_b^{(1)}, \Omega_b^{(2)}\right], \dots, \\ A_{R,5}(\Omega,s) &= A_R(\Omega,s), \quad \Omega \in \left[\Omega_b^{(4)}, +\infty\right). \end{aligned}$$
(39)

Based on the results of calculations in the (*n*, *Amp*) plane, the charts of the dimensionless oscillation <u>amplitudes</u> are built:

- platform $(n_j(\Omega), Amp_{i,j}(\Omega, s)), /i = \overline{1,2}; j = \overline{1,5}/;$ -<u>dimensionless</u> perturbing force $(n_j(\Omega), A_{R,j}(\Omega, s)),$

 $-\underline{\text{dimensionless perturbing force } (n_j(\Omega), A_{R,j}(\Omega, s)), /j = 1, 5 /.$

Note that the charts of dependences (38), (39), built in the (Ω, Amp) plane, are also informative. They can be used to study the influence of external and internal viscous resistance forces on the vibration characteristics of platforms [19].

5. 3. Numerical study of the dynamics of a resonant antiphase vibratory machine

All calculations involve dimensionless quantities. The results are also obtained in a dimensionless form.

The estimated data (dimensionless parameters) are:

$$n_1 = 1/3, \ n_{12} = 2/3, \ n_2 = 1/3, \ \rho = 1, \ s = 1,$$

 $\beta = 1, \ h_1 = 0.01, \ h_{12} = 0.01, \ h_2 = 0.01.$ (40)

Substituting (40) in (23), we find two resonance (natural) oscillation frequencies of the system in the absence of resistance forces:

$$n_r^{(1)} = 0.3333$$
, $n_r^{(2)} = 1$.

The bifurcation frequencies of load jamming are found as the roots of equation (31):

$$\Omega_b^{(1)} = 0.3339, \ \Omega_b^{(2)} = 0.3806,$$

 $\Omega_b^{(3)} = 1.0015, \ \Omega_b^{(4)} = 1.2154.$ (41)

Substituting (41) in (35), we find the corresponding bifurcation speeds of rotor. We arrange them in order of ascend:

$$n_b^{(1)} = 0.4141, \quad n_b^{(2)} = 0.7978,$$

 $n_b^{(3)} = 1.3932, \quad n_b^{(4)} = 5.1829.$ (42)

Fig. 2 shows the built charts of 5 possible load jamming modes (37).





In Fig. 2, solid lines show the stable modes of jamming, dotted lines – unstable.

Fig. 3 shows the dependences of the dimensionless amplitudes of platform oscillations and the dimensionless perturbing force on the frequency of load jamming.

Fig. 3 demonstrates that the platforms oscillate at significant amplitudes when loads get stuck in the vicinity

of the first and second resonance frequencies of platform oscillations. At the same time, in the vicinity of the first resonance frequency, the amplitude of the perturbing force is an order of magnitude greater than this amplitude in the vicinity of the second resonance frequency. The simultaneous increase in the number of loads and viscous resistance forces that impede the motion of loads leads to a proportional increase in the amplitude of platform oscillations and perturbing force.



Fig. 3. Dependences of the dimensionless amplitudes of platform oscillations and the dimensionless perturbing force on the frequency of load jamming: a – one load; b – three loads and three times the large resistance forces to the motion of the load

Note that the third mode of load jamming is stable in the range of rotor speeds $n \in (n_b^{(1)}, n_b^{(4)})$.

Fig. 4 shows the built charts of dependences of the following, under the third mode of load jamming, on the rotor speed: the dimensionless amplitude of platform oscillations; the amplitude of the dimensionless perturbing force.



Fig. 4. Dependence of the dimensionless amplitudes of platform oscillations and the amplitude of the dimensionless perturbing force on the rotor speed under the third mode of load jamming: *a* – one load; *b* – three loads and three times the large resistance forces to the motion of the load

Fig. 4 demonstrates the following:

– the antiphase mode of platform motion is manifested the stronger the larger the rotor speed in the range $(n_{k}^{(1)}, n_{k}^{(4)})$; – with an increase in the rotor speed there are increases in the amplitude of platform oscillations while the amplitude of the dimensionless perturbing force decreases;

- the simultaneous increase in the number of loads and viscous resistance forces that impede the motion of loads leads to a proportional increase in the amplitude of platform oscillations and the perturbing force.

The integration of the differential vibratory machine motion equations (2) confirms that the third mode of load jamming is locally asymptotically stable across the entire range $(n_b^{(1)}, n_b^{(4)})$.

Fig. 5, a-c shows, respectively, the charts of changes over the dimensionless time of the following dimensionless quantities: the platform coordinates v_1 , v_2 ; the frequency of load jamming (under the third mode) Ω_3 ; the dimensionless projection \bar{R}_y onto the y axis of the perturbing force at $n = n_{\mu}^{(1)} + 0.005$.



Fig. 5. The charts of change over the dimensionless time of the following dimensionless quantities: a - platform coordinates v_1 , v_2 ; b - f requencies of load jamming (under the third mode) Ω_3 ; c - p rojections \overline{R}_y onto the y axis of the perturbing force

Fig. 5 demonstrates that at the beginning of the range $\left(n_{b}^{(1)}, n_{b}^{(4)}\right)$ the antiphase form of platform motion is almost unobservable.

Fig. 6 shows the charts built for the same quantities as in Fig. 5, but at $n = n_b^{(4)} - 0.005$.



Fig. 6. The charts of change over the dimensionless time of the following dimensionless quantities: a - platform coordinates $v_1, v_2; b - f$ requencies of load jamming (under the third mode) $\Omega_3; c - p$ rojections \overline{R}_y onto the y axis of the perturbing force

Fig. 6 demonstrates that at the end of the range $(n_b^{(i)}, n_b^{(4)})$ the antiphase mode of platform motions appears in the most explicit fashion.

The results of integrating the differential motion equations, as well as the charts in Fig. 4, lead us to conclude that the vibratory machine under consideration can be used as an antiphase one in the range of rotor speeds of $(1, n_b^{(4)})$. That is, the rotor speed should exceed the highest resonance frequency of the vibratory machine.

The amplitudes of platform oscillations can be changed: – by changing the rotor speed;

– by a simultaneous increase in the number of loads and viscous resistance forces preventing the motion of loads.

6. Discussion of results of studying the resonance antiphase two-mass vibratory machine whose operation is based on the Sommerfeld effect

We have considered a two-mass vibratory machine with a vibration exciter in the form of a passive auto-balancer. In the vibratory machine, platforms 1 and 2 are viscoelastically attached to the stationary bed and are tied together viscoelastically. A passive auto-balancer is mounted on platform 2.

It has been established that the vibratory machine has two resonance frequencies and two corresponding forms of platform oscillations. In the absence of resistance forces in the system:

- we have found such a speed of load jamming (16) at which the antiphase mode of motion is executed, under which platforms 1 and 2 oscillate in opposite phases while the perturbing force acting on the bed from the elastic supports is zero;

- we have selected such values of support parameters (21) at which the frequency of platform oscillations under an antiphase mode coincides with the (second) resonance frequency.

It has been established that the antiphase mode is achieved due to the Sommerfeld effect when the loads get stuck in the vicinity of the second resonance frequency.

The laws of platform motion (25) have been found in the presence of viscous resistance in the system. It has been established that the antiphase mode of platform motions is not ideal. The perturbing force acting on the side of supports on the bed is not zero. The dynamic characteristics of the vibratory machine have been investigated by numerical methods. It has been established that in the case of small internal and external resistance forces:

there are five theoretically possible modes of load jamming (Fig. 2);

- the antiphase (second) form of platform oscillations is theoretically implemented under jamming modes 3 and 4;

 locally asymptotically stable is jamming mode 3 while mode 4 is unstable;

– for the loads to get stuck in the vicinity of the second resonance frequency, one needs to provide the vibratory machine with initial conditions close to jamming mode 3, or smoothly accelerate the rotor to the working frequency.

The antiphase mode of platform motion is more pronounced under the over-resonant rotor speeds (Fig. 4).

The amplitude of the antiphase oscillations of platforms can be increased (Fig. 4):

- by the increased rotor speed;

- by a simultaneous increase in the number of loads and viscous resistance forces preventing the motion of loads.

Our results are applicable for designing the above vibratory machines for general purposes.

It should be noted that the numerical studies have been conducted for specific values of the dimensionless parameters of some abstract vibratory machine. However, the devised procedure can be used to calculate the parameters of a specific vibratory machine for certain purposes.

In the future, it is planned to fabricate a prototype of the two-mass anti-phase resonance vibratory machine and experimentally investigate the dynamic characteristics of the vibratory machine.

7. Conclusions

1. It has been established that the two-mass vibratory machine under consideration has two resonance frequencies and two corresponding forms of platform oscillations. In the absence of resistance forces in the system:

- there is such a speed of load jamming at which the mode of motion is executed under which platforms 1 and 2 oscillate in opposite phases while the perturbing force acting on the bed from the elastic supports is zero;

 it is possible to choose such values for support parameters at which the frequency of platform oscillations under an anti-phase mode coincides with a higher resonance frequency.

In the synthesized vibratory machine, the antiphase mode would be achieved due to the Sommerfeld effect when loads get stuck in the vicinity of the second resonance frequency. However, the onset of the Sommerfeld effect requires the presence of the forces of viscous resistance.

2. In the presence of viscous resistance forces in the system, the antiphase mode of platform motion is not ideal. The perturbing force acting on the side of supports on the bed is not zero.

3. The dynamic characteristics of the vibratory machine have been investigated by numerical methods. It has been established that in the case of small internal and external resistance forces:

- there are five theoretically possible modes of load jamming;

- the antiphase (second) form of platform oscillation is theoretically implemented under jamming modes 3 and 4;

 locally asymptotically stable is jamming mode 3 while mode 4 is unstable;

– for the loads to get stuck in the vicinity of the second resonance frequency, one needs to provide the vibratory machine with the initial conditions close to jamming mode 3, or smoothly accelerate the rotor to the working frequency.

The antiphase mode of platform movements is more pronounced at the over-resonant speeds of rotor rotation. The amplitude of the antiphase platform oscillations can be increased:

by the increased rotor speed;

 by a simultaneous increase in the number of loads and viscous resistance forces preventing the motion of loads.

Acknowledgments

This paper is funded within the framework of scientific work No. 0119U001173 «Stabilization and stability of motion of an unbalanced rotating carrying body in a free or isolated mechanical system», which is carried out at the Central Ukrainian National Technical University and is financed by the state budget.

References

- 1. Kryukov, B. I. (1967). Dinamika vibratsionnyh mashin rezonansnogo tipa. Kyiv: Naukova dumka, 210.
- Gursky, V., Kuzio, I., Korendiy, V. (2018). Optimal Synthesis and Implementation of Resonant Vibratory Systems. Universal Journal of Mechanical Engineering, 6 (2), 38–46. doi: https://doi.org/10.13189/ujme.2018.060202

- Zhao, J., Liu, L., Song, M., Zhang, X. (2015). Influencing Factors of Anti-Resonant Inertial Resonant Machine Vibration Isolation System. 2015 3rd International Conference on Computer and Computing Science (COMCOMS). doi: https://doi.org/10.1109/ comcoms.2015.22
- 4. Li, X., Shen, T. (2016). Dynamic performance analysis of nonlinear anti-resonance vibrating machine with the fluctuation of material mass. Journal of Vibroengineering, 18 (2), 978–988. Available at: https://www.jvejournals.com/article/16559
- Zhao, C., He, B., Liu, J., Han, Y., Wen, B. (2017). Design method of dynamic parameters of a self-synchronization vibrating system with dual mass. Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-Body Dynamics, 232 (1), 3–20. doi: https://doi.org/10.1177/1464419316689643
- Shokhin, A. E., Panovko, G. Ya., Salamandra, K. B. (2016). On the choice of dynamic regimes for two-mass vibrating machine Vibroengineering Procedia, 8, 185–190. Available at: https://www.jvejournals.com/article/17720
- Sommerfeld, A. (1904). Beitrage zum dinamischen Ausbay der Festigkeislehre. Zeitschriff des Vereins Deutsher Jngeniere, 48 (18), 631–636.
- Yaroshevich, N., Puts, V., Yaroshevich, T., Herasymchuk, O. (2020). Slow oscillations in systems with inertial vibration exciters. Vibroengineering PROCEDIA, 32, 20–25. doi: https://doi.org/10.21595/vp.2020.21509
- Lanets, O., Shpak, Ya., Lozynskyi, I., Leonovych, P. (2013). Realizatsiya efektu Zommerfelda u vibratsiynomu maidanchyku z inertsiynym pryvodom. Avtomatyzatsiya vyrobnychykh protsesiv u mashynobuduvanni ta pryladobuduvanni, 47, 12–28. Available at: http://nbuv.gov.ua/UJRN/Avtomatyzac_2013_47_4
- Lanets, O. S., Hurskyi, V. M., Lanets, O. V., Shpak, Ya. V. (2014). Obgruntuvannia konstruktsiyi ta modeliuvannia roboty rezonansnoho dvomasovoho vibrostola z inertsiynym pryvodom. Visnyk Natsionalnoho universytetu «Lvivska politekhnika». Dynamika, mitsnist ta proektuvannia mashyn i pryladiv, 788, 28–36. Available at: http://ena.lp.edu.ua:8080/bitstream/ntb/24646/1/6-28-36.pdf
- 11. Kuzo, I. V., Lanets, O. V., Gurskyi, V. M. (2013). Synthesis of low-frequency resonance vibratory machines with an aeroinertia drive. Naukovyi visnyk Natsionalnoho hirnychoho universytetu, 2, 60–67. Available at: http://nbuv.gov.ua/UJRN/Nvngu_2013_2_11
- 12. Filimonikhin, G., Yatsun, V. (2015). Method of excitation of dual frequency vibrations by passive autobalancers. Eastern-European Journal of Enterprise Technologies, 4 (7 (76)), 9–14. doi: https://doi.org/10.15587/1729-4061.2015.47116
- Lu, C.-J., Tien, M.-H. (2012). Pure-rotary periodic motions of a planar two-ball auto-balancer system. Mechanical Systems and Signal Processing, 32, 251–268. doi: https://doi.org/10.1016/j.ymssp.2012.06.001
- Artyunin, A. I., Eliseyev, S. V. (2013). Effect of "Crawling" and Peculiarities of Motion of a Rotor with Pendular Self-Balancers. Applied Mechanics and Materials, 373-375, 38–42. doi: https://doi.org/10.4028/www.scientific.net/amm.373-375.38
- Jung, D., DeSmidt, H. (2017). Nonsynchronous Vibration of Planar Autobalancer/Rotor System With Asymmetric Bearing Support. Journal of Vibration and Acoustics, 139 (3). doi: https://doi.org/10.1115/1.4035814
- Artyunin, A. I., Barsukov, S. V., Sumenkov, O. Y. (2019). Peculiarities of Motion of Pendulum on Mechanical System Engine Rotating Shaft. Proceedings of the 5th International Conference on Industrial Engineering (ICIE 2019), 649–657. doi: https:// doi.org/10.1007/978-3-030-22041-9_70
- Yatsun, V., Filimonikhin, G., Dumenko, K., Nevdakha, A. (2017). Equations of motion of vibration machines with a translational motion of platforms and a vibration exciter in the form of a passive auto-balancer. Eastern-European Journal of Enterprise Technologies, 5 (1 (89)), 19–25. doi: https://doi.org/10.15587/1729-4061.2017.111216
- Yatsun, V., Filimonikhin, G., Dumenko, K., Nevdakha, A. (2017). Search for two-frequency motion modes of single-mass vibratory machine with vibration exciter in the form of passive auto-balancer. Eastern-European Journal of Enterprise Technologies, 6 (7 (90)), 58–66. doi: https://doi.org/10.15587/1729-4061.2017.117683
- Yatsun, V., Filimonikhin, G., Dumenko, K., Nevdakha, A. (2018). Search for the dualfrequency motion modes of a dualmass vibratory machine with a vibration exciter in the form of passive autobalancer. Eastern-European Journal of Enterprise Technologies, 1 (7 (91)), 47–54. doi: https://doi.org/10.15587/1729-4061.2018.121737
- Yatsun, V., Filimonikhin, G., Haleeva, A., Krivoblotsky, L., Machok, Y., Mezitis, M. et. al. (2020). Searching for the twofrequency motion modes of a threemass vibratory machine with a vibration exciter in the form of a passive autobalancer. Eastern-European Journal of Enterprise Technologies, 4 (7 (106)), 103–111. doi: https://doi.org/10.15587/1729-4061.2020.209269
- Yatsun, V., Filimonikhin, G., Pirogov, V., Amosov, V., Luzan, P. (2020). Research of antiresonance threemass vibratory machine with a vibration exciter in the form of a passive autobalancer. Eastern-European Journal of Enterprise Technologies, 5 (7 (107)), 89–97. doi: https://doi.org/10.15587/1729-4061.2020.213724