*A method has been developed to simulate propagation of train delays in branched railroad ranges using modified epidemiological SIR models. These models take into account the mutual influence of trains with different priorities in the flow. This makes it possible to study the heterogeneous dynamics in the propagation of delays among trains of different priorities. To consider the propagation of the primary delay in space and time, it is proposed to represent the topology of the railway network in the form of an undirected graph with reference to the edge of the graph in the mathematical system of differential equations of the SIR model. This unifies the process of constructing SIR models for each edge (section) of the network graph and reduces the dimension of the problem. To take into account the influence of the "network effect", it is proposed to determine the transit coefficient for each station of the section. This coefficient helps calculate the number of delayed trains for adjacent sections. To set SIR models, it is proposed to use empirical data on the propagation of the average delay in the standard traffic schedule in the corresponding section. For the sequential solution of SIR models corresponding to interconnected network sections, an algorithm is applied to turn the network graph into a directed tree the root of which is the station where the delay occurs. Tests on modelling the propagation of train delays in the railway network are carried out taking into account the mutual influence of different categories of trains in the flow and the built-in time reserves for the restoration of movement. The obtained simulation results have confirmed the adequacy of the solutions and helped quantify the influence of primary delays and the amount of time reserve in the schedules of trains of various categories on the reliability of the standard train schedule*

*Keywords: railway, network, train schedule, delay propagation, epidemiological model, SIR*

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# DEVELOPMENT OF A METHOD FOR MODELLING DELAY PROPAGATION IN RAILWAY NETWORKS USING EPIDEMIOLOGICAL SIR MODELS

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## 1. Introduction

For railway systems belonging to network industries with high fixed costs, it is important to increase the use of the network capacity to reduce the average cost of cargo and passenger transport services. The degree of using the capacity of the railway network sections or even the network overload depends on the system properties of the reliability and stability of the train schedule. In the conditions of daily train operations, exogenous train delays occur and cause the delay propagation among other trains in the network. Despite the importance of knowing the parameters of such delays' propagation in the railway network, methods to assess the vulnerability of the railway system are still insufficient. This problem is especially difficult for railway systems in which there is mixed movement of passenger and cargo trains in the same infrastructure and most cargo trains do not follow the schedule. This makes it impossible to provide detailed calculations of the delay propagation in the existing train schedules. That is why a study of complex dynamic processes of propagation of successive train delays at the macrolevel of the railway network is a relevant and very promising area of research.

## 2. Literature review and problem statement

There is much research on the problem of propagation of train delays in railway networks. Most scientific works are devoted to studying this problem for railway systems in compliance with the schedule and its qualitative analysis. This is primarily due to the possibility of detailed calculations of the impact of different types of delays in the planned train schedules [1]. There is a deterministic approach to calculating the propagation of the delay in the planned train schedule that was proposed in [2]. The suggested algorithm was tested on the schedules of Deutsche Bahn AG and proved its effectiveness in large computations. The deterministic approach was also included in [3]. This paper presents a model and an algorithm based on a linear system of max-plus algebra for calculating the propagation of primary delays according to the periodic train timetable. The dynamics of delay propagation are analysed depending on such properties of the schedule as realisability and stability.

The deterministic approach can very accurately estimate the network's response to delays. However, its application does not make it possible to realistically assess the dynamics of the propagation of delays for railway networks that completely or partially lack the planned train schedule. In such countries as Ukraine, Belarus, Kazakhstan, India, and similar, the railway systems are dominated by the movement of cargo trains in large volumes, but for such trains compliance with the schedule is impractical. The reason for this is the unpredictability in time and space of formation of cargo flows. As a result, cargo trains leave their stations of origin when ready. This technology provides that after all the necessary technical and commercial operations with the train at the sorting stations the dispatching staff decides to send the train. A train departs when the standard minimum interval between trains is observed and the nearest free time window for the train's running is available.

In such conditions, train flows follow the missing traffic plans in the network, which complicates the analysis of their deviations from the normative time of running in the network. Several studies have been aimed at researching the reliability of the train schedule in railway systems without following the schedule of cargo trains [4, 5].

A stochastic approach to modelling the propagation of delays was described in [6]. In [7], a stochastic mathematical model was proposed to simulate the propagation of passenger train delays taking into account passenger transfers in the German railway network. Such a model according to the detailed classification can be attributed to the microlevel approach where the modelling requires a planned train schedule. This does not allow its use for non-scheduled traffic systems, that is, to model the dynamics of the propagation of delays of cargo trains in the network without adherence to the schedule and when their movement is mixed with passenger trains in the line. The microlevel approach was included in [8]. More adequate in such conditions of a railway network functioning is application of the macrolevel approach to devising stochastic models of delay propagations. For example, in [9] a stochastic mathematical model based on the Markov chain was proposed to simulate the propagation of cargo train delays taking into account the ability of intermodal terminals to absorb delays occurring in the network.

Some studies have been devoted to researching the stability of train schedules on the basis of statistical analyses of real data on the implementation of train schedules. For example, work [10] was devoted to analysing the train schedules in the network of the infrastructure company DB Netze AG using the methods of Big Data analysis. Study [11] proposed a method for predicting train delays for the Beijing-Guangzhou railway line in China based on the gradient-boosted regression trees model (GBRT). In [12], methods for visualization and statistical processing of data on train delays in the railway network were proposed to accelerate the detection of problems in schedules. However, the statistical approach to studying the distribution of train delays allows exploring only the options that have emerged in historical retrospect. This does not make it possible to obtain a response from the railway network in the event of delays that were absent during the researched period.

Research aimed at factor analysis of the causes and parameters of the propagation of train delays is important. In [13], for the conditions of functioning of the US railway network, factor studies were described with reference to the impact of technical and operational indicators on the average running time of a train. An in-depth analysis of such areas of research was given in [14]. The authors have noted significant prospects for the application of analytical methods based on machine learning (ML).

Based on the conditions of the object under study, it is proposed to apply a macroscopic approach to modelling the propagation of train delays. This approach can more adequately describe the uncertainty of the parameters in the train traffic system without following the schedule in the network, taking into account the topological properties of the network and the impact of the so-called 'network effect' [15]. Based on the development of methods for analysing complex networks, there have been studies aimed at formalizing the dynamics of the propagation of train delays on topologies of large networks [16, 17]. Studies aimed at presenting the characteristics of the propagation of infectious diseases similar to the propagation of delays are effective here. As an example, we can cite a study in the field of air transport [18] where an epidemic model of the SIS type was used to simulate the propagation of delays in the airport network. Checks of the built models on these flights in Europe proved the realism of the obtained results. In [19], a model of the dynamics of delay propagation in complex air transport networks based on a modified epidemiological SIR model was applied. To increase the simulation accuracy, restrictions were introduced in the model as to the use of the air corridor capacity.

Given that, in addition to the aviation industry, models for the propagation of infections have been developed in the telecommunications industry to analyse the propagation of viruses, so their application in the field of rail transport is quite promising. Only a few studies have been carried out in the field of modelling the propagation of delays in the field of railway transport based on the infection propagation model. In [20], the authors used the complete software product LinTim without modifying the algorithm. The simulation results suggest that the propagation of train delay has a basis similar to the mechanism of propagation of infectious diseases. In [21], the SIR model was used to simulate the propagation of the delay over the entire Dutch transit network. A heuristic approach to determine the frequency of infection was proposed. As a result, the model adequately simulated the processes of large delays. However, the system of train traffic in the network was based on periodic schedules; it cannot be applied to networks like the Ukrainian one. In addition, this model does not take into account the limitations on the capacity of sections between stations in the network and their degree of congestion, which could increase the accuracy of modelling. In [22], the SIR model was used to study congestion in the urban passenger railway network and quantified the impact of various factors, including throughput on the speed of delay propagation.

The research findings prove that simulation methods based on epidemiological models facilitate calculations and take into account complex time dependencies. This suggests that in order to be able to accelerate complex calculations and take into account the dynamics of delays in the railway network with partial compliance with the train schedule, it is advisable to conduct research using a macrolevel approach. One of the promising areas is the use of modifications of epidemiological models.

## 3. The aim and objectives of the study

The aim of the study is to improve the quality of compiling normative train schedules based on the development of a method for simulating the propagation of train delays in railway networks on the basis of a macrolevel approach without adhering to the schedule.

To achieve this aim, the following research objectives were solved:

– to formalize the process of spreading train delays in branched railway domains using the epidemiological SIR model;

– to conduct experimental studies of the propagation of train delays in the railway network, taking into account the interaction of different categories of trains in the flow and the established time reserves for the resumption of traffic.

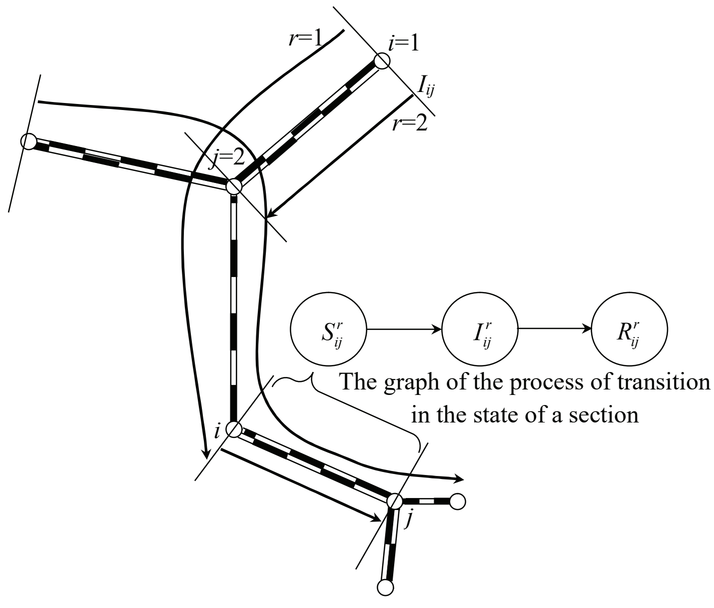


Fig. 1. The graph of the network G($P$,$E$) with the train flows and representation of the process of a section transition according to the SIR model

## 4. Development of a method for modelling the propagation of train delays in railway networks

**4. 1. Spatial representation of the problem of propagation of train delays in the railway system**

As part of solving the problem of modelling the propagation of train delays at the macrolevel of the railway network, it is proposed to present the topology of the railway network in the form of an undirected graph $G(P, E)$. The set of vertices of the graph $P$ corresponds to the railway stations that perform technical and technological operations for the processing of train flows. Accordingly, they are considered restrictive for the division of the network into sections with the same operating conditions, $i,j \in P$, where $i=\overline{1,n}$, $j=\overline{1,n}$; $E$ is the set of edges $e_{ij}$ connecting the corresponding vertices of the graph and corresponding to the railway sections between network stations, where $e_{ij} \in E$, $i,j=\overline{1,n}$ (Fig. 1). Accordingly, they are considered restrictive for the division of the network into sections with the same operating conditions, $i,j \in P$, where $i=\overline{1,n}$, $j=\overline{1,n}$; $E$ is the set of edges $e_{ij}$ connecting the corresponding vertices of the graph and corresponding to the railway sections between network stations, where $e_{ij} \in E$, $i,j=\overline{1,n}$ (Fig. 1).

Given the specifics of technological operations and the priority of traffic, it is important to take into account different types of train flows: passenger, commuter and cargo traffic. Therefore, we denote by $r$ the type of the train flow, $r$=1, 2, 3. According to the specified size of the movement of each train flow $r$ in the network there are specified routes. It is proposed to record the number of trains in the flow using the parameter $f^{st,r}_{ij}$, $s, t \in P$, where $s$ is the source station of the train flow, and $t$ is the terminal station for the train flow. Therefore, $f^{st,r}_{ij}$ is the flow of trains of the $r$-th type on the edge or section $e_{ij}$, which corresponds to the number of trains going from $s$ to $t$, $f^{st,r}_{ij}>0$. To simplify the representation, let us number each route $<s, t>$ with the parameter μ. Each route of the train flow $r$ from the point of departure $s$ to the point of arrival $t$ can be represented by an orderly set of edges: $\mu=(e_{ij},...,e_{ij})$, $\forall e_{ij} \in \mu$. Then $f^{t\mu,r}_{ij}$ is the number of trains of the $r$-th flow on the route μ passing through the section $e_{ij}$.

An important factor influencing the propagation of delays is the characteristics of the railway infrastructure, in particular the number of tracks. A variable function is introduced for each edge of the graph G to model different variants of the site infrastructure:

$$\delta_{ij}=\begin{cases}1, & \text{singletrack} \quad \text{section } ij,\\ 2, & \text{singletrack-doubletrack} \quad \text{section } ij,\\ 3, & \text{doubletrack} \quad \text{section } ij.\end{cases}$$

To describe the possibilities of letting a number of trains pass through the section, the edges are assigned a parameter $N^{\max}_{\delta ij}$, which corresponds to the available capacity of the section in the movement direction $ij$ [23, 24].

**4. 2. The epidemiological mathematical model of train delay propagation on the railway network graph**

At the macrolevel of the railway network, the dynamics of the process of propagation of train delays in the network can be represented as a continuous wave-like process that can develop with different amplitudes. Such dynamics can be described by mathematical models of the propagation of infectious diseases, which have long been used in medicine [25–27]. Of the classical mathematical models of epidemiology in this paper, it is proposed to use the SIR model (Susceptible-Infected-Recovered model), which describes an epidemic with recovery. When adapting this model to the problem of simulating the dynamics of train delay propagation, we can imagine the process of spreading infectious diseases as infection and recovery. In this case, the infection is the process of spreading train delays, and recovery can be seen as the resumption of movement after the delay. Thus, the total number of trains of different types in the area should be divided into susceptible, infected, and recovered. Thus, susceptible are $S(t)$ delay-sensitive trains that at time $t$ follow the schedule. Infected are $I(t)$ trains that follow with a delay and affect the course of other trains, transmitting the delay to them. Recovered $R(t)$ trains are those that during the journey through the railway section had a delay, absorbed it (performed overtaking), followed the schedule and arrived at the final station without affecting the occurrence of delays in further movement (Fig. 2). The term overtaking means the reduction of the delay time to the established standards of the current normative order of traffic, or the duration of movement of cargo trains along the section.

To improve the accuracy of the simulation, it is important to take into account the different effects of different types of trains on the speed of the delay propagation. For example, according to the normative document [28], in the railway network of Ukraine, which belongs to the networks without compliance with the schedule of cargo trains, the greatest influence in the railway network is produced by long-distance passenger and commuter trains. The reason for this is their priority and the importance of compliance with the regulatory schedule of trains. After passenger trains, cargo train flows, which run in the network without following the schedule, have a lower priority, but in case of deviation from the traffic plan devised by the control staff, they have a significant impact on the higher category trains. To account for the impact on the strength of the distribution of different groups of train flows in the study, it is proposed to take into account the heterogeneous epidemic dynamics of the classical SIR model in accordance with [29]. Under such conditions, it is assumed that the entire flow of trains in the section is divided into three classes according to their $r$ type. It should be noted that within one class it is possible to distinguish different categories of trains, for example, by speed, length, mass, and so on. However, in this study, it is assumed that all trains of the same class have similar characteristics of the processes of infection – transmission of delay and recovery – the restoration of movement. Migration across classes is not expected.

Therefore, let us consider a graph of transitions between the states of the proposed SIR model, which has three classes or types of train flows in the section $ij$.

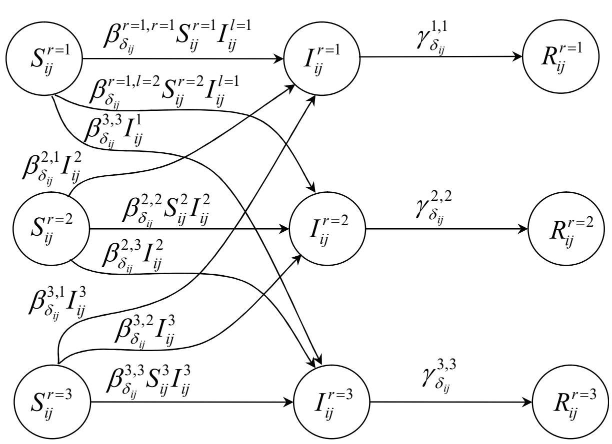

Fig. 2. The graph of transitions between states of the SIR model, which has separate groups of passenger, commuter and cargo train flows

According to the graph of transitions, which characterizes the state of the site, in column $G$ in Fig. 1 the rate of change of the number of trains susceptible to delay in the section $ij$ can be defined as

$$\frac{dS_{ij}^r(t)}{dt} = -\sum_l \beta_{\delta_{ij}}^{r,l} \cdot S_{ij}^r(t) \cdot I_{ij}^l(t), \tag{1}$$

where $S_{ij}^r$ is the number of susceptible trains of the class $r$ in the section $ij$; $\beta_{\delta_{ij}}^{r,l}$ is the rate of propagation of the delay from the train flow $r$ to the train flow $l$ in the section of type $\delta_{ij}$, where $r=l$, $r,l \in R$; $I_{ij}^l$ is the number of infected trains of the class $l$ in the section $ij$ of the trains; $t$ is the time or step of simulation, hour.

The rate of change in the number of trains passing through a delayed railway section can be recognized as

$$\frac{dI_{ij}^r(t)}{dt} = \sum_l \beta_{\delta_{ij}}^{r,l} \cdot S_{ij}^r(t) \cdot I_{ij}^l(t) - \gamma_{\delta_{ij}}^r I_{ij}^r, \tag{2}$$

where $I_{ij}^r$ is the number of detained trains in the class $r$ in the section $ij$; $\gamma_{\delta_{ij}}^r$ is the speed of restoring the train traffic $r$ in the section of the type $\delta_{ij}$.

Trains that ran and absorbed the delay pass to the group $R_{ij}^r(t)$ at a speed of $\gamma_{\delta_{ij}}^r I_{ij}^r$, so the speed of recovery can be defined as

$$\frac{dR_{ij}^r(t)}{dt} = -\gamma_{\delta_{ij}}^r I_{ij}^r. \tag{3}$$

All system parameters are positive. The system of equations (1)–(3) has a condition of normalization of the form $N_{ij}^r = S_{ij}^r(t) + I_{ij}^r(t) + R_{ij}^r$, where $N_{ij}^r$ – is the total number of trains of the class $r$ in the section $ij$, $N = \sum_r N^r = N = \sum_r f_{ij}^{t\mu,r}$, $N$ is the total number of trains of all classes in the section. The initial conditions for system (1)–(3) are equal to ($S_{ij,0}^r$, $I_{ij,0}^r$, 0), where $S_{ij,0}^r$, $I_{ij,0}^r$ denote the number of susceptible and delayed trains in the class $r$ of the section $ij$ with $t=0$, respectively.

Depending on the number of tracks in the section of type $\delta_{ij}$, it is proposed to link the $SIR_{\delta ij}$ model to the corresponding velocity coefficients. This unified the process of constructing SIR models for each edge $ij$ (of the section) of the graph and reduced the dimension of the problem.

To account for the transition of the number of detained trains to the adjacent section of the graph G, it is proposed to determine the number of detained trains that run as transit through the vertex $j$ by the transit coefficient. The transit ratio is determined by the expression

$$I_{jk,transf}^r = \frac{I_{ij}^r \sum N_{ik}^r}{N_{ij}^r}, \tag{4}$$

where $I_{jk,transf}^r$ is the number of detained trains of the class $r$ that are transit for the vertex $j$ and pass the section $jk$, $j=k$ of the trains; $\sum N_{ik}^r$ is the total flow of trains of the class $r$ running through the vertex $j$ in the direction of the vertex $k$; $k$ is the number of the vertex preceding the vertex $j$ on the edge of the graph G of the trains.

For each section with the characteristics of the infrastructure of type $\delta_{ij}$, it is proposed to configure the corresponding $SIR_{\delta ij}$ model with equations (1)–(3) by solving the optimization problem to find the coefficients of the delay propagation $\beta_{\delta_{ij}}^{r,l}$. To adjust the models, it is proposed to use data on the distribution of the average delay in the regulatory schedule. The velocity coefficients within the solution of the optimization problem were selected by a genetic algorithm based on the minimization of the mean absolute percentage error (MAPE) between the empirical data and the results of solving the system of equations (1)–(3). The solution of the system of differential equations (1)–(3) within the fitness function is proposed to be performed by the Runge-Kutta method of the 4th order.

An algorithm [30] was developed to sequentially solve $SIR_{\delta ij}$ models corresponding to each edge of the graph, which transforms the network graph into a directional tree, the root of which is the delay station. According to the built sequence, the $SIR_{\delta ij}$ model of the corresponding section was solved to determine the parameters of the delay propagation on the graph.

## 5. The results of modelling the propagation of train delays on the graph of the railway test site

Within the framework of experimental research, the results of the propagation of delays in the real sections of one of the railway domains of JSC Ukrzaliznytsia (Ukrainian Railway) were obtained. To implement the proposed method of modelling the propagation of train delays on the graph, a special software product was developed in the Matlab environment. Two $SIR_{\delta ij}$ models were set up for eight railway sections, of which two $SIR_{\delta ij=2}$ models correspond to the singletrack-doubletrack infrastructure of the section, and all other double-track sections correspond to the $SIR_{\delta ij=3}$ models.

The graph of the railway domain with given flows of trains of different classes $r$ is presented in Fig. 3.

Station A is taken as the initial vertex of the simulated delay propagation on the graph. To automatically determine the startup sequence of $SIR_{\delta ij}$ model systems in the direction of delay propagation, an algorithm was used to convert the domain graph into a directional graph tree with station A as the root vertex. Fig. 4 shows the transformed graph of the domain into a directional tree.

As part of the experiment, it is proposed to investigate the ability to resume traffic under the condition of delay of four passenger trains and five cargo trains departing from station A. The initial conditions of the section AB ($S_{ij}^{r}(t{=}0)$, $I_{ij}^{r}(t{=}0)$, 0) were as follows: $S^{r=1}(t{=}0){=}15$ passenger trains; $S^{r=2}(t{=}0){=}28$ commuter trains; $S^{r=3}(t{=}0){=}25$ cargo trains; $I^{r=1}(t{=}0){=}4$ trains; $I^{r=2}(t{=}0){=}0$ trains and $I^{r=3}(t{=}0){=}5$ trains. The following delay recovery coefficients were applied for the class $r$ of the trains in the section: $\gamma^{r=1}{=}3$ min=0.05 h; $\gamma^{r=2}{=}4$ min=0.0667 h; and $\gamma^{r=3}{=}6$ min= 0.10 h. The coefficients of the delay propagation speed $\beta_{\delta_{ij}}^{r,l}\in\{3\times 3\}$ were taken in accordance with the values of the solution of the optimization problem for setting the corresponding $SIR_{\delta ij}$ model.

The initial conditions and values of the states of trains of different classes $r$ in the sections of the railway domain under study are given in Table 1.

Visualization of the graphs of the dynamics of change in the states of the train flows in the conditions of restoring the movement of trains in the sections of the railway range at emergence of delays on departure from station A is given in Fig. 5.

According to the simulation results, the influence of the "network effect" on the reliability of train traffic in the branched railway domain was quantified. Of the four delayed passenger trains with a set recovery time of 3 minutes, it was possible to restore traffic on the line ABCEF for two to three passenger trains, respectively. However, at the exit of the domain at station F, there was still a delay of two trains. The reserve compensation time on the line ABCEF allowed five of the five cargo trains delayed at station A to resume their time before station F, and only one cargo train remained delayed. However, the set recovery time for trains in the section EF did not allow resuming the traffic time due to the delays of trains transferred from the previous section DE. This requires a revision of the time reserves in the direction of increase for the section EF or the adjacent DE to be able to absorb the initial delay at the railway site.

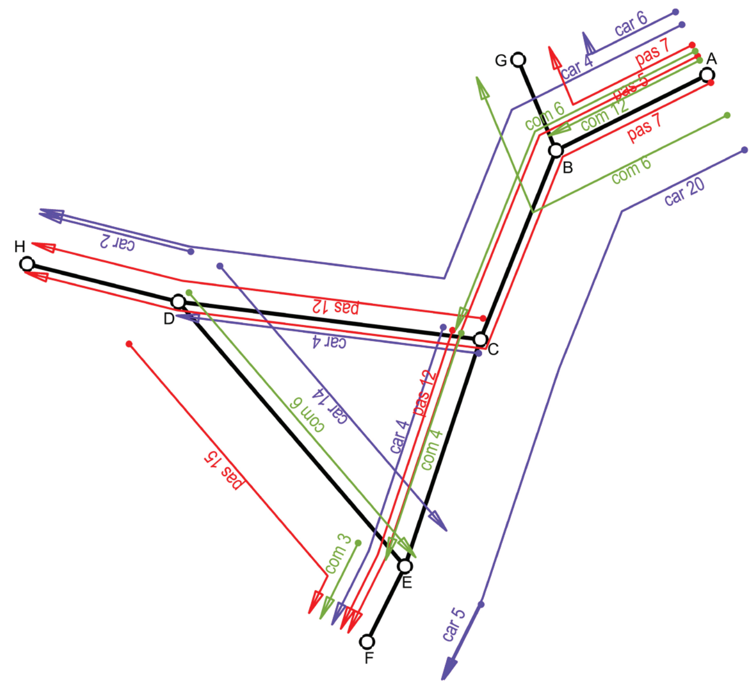


Fig. 3. The graph of the railway domain with visualization of the flows of trains of different classes

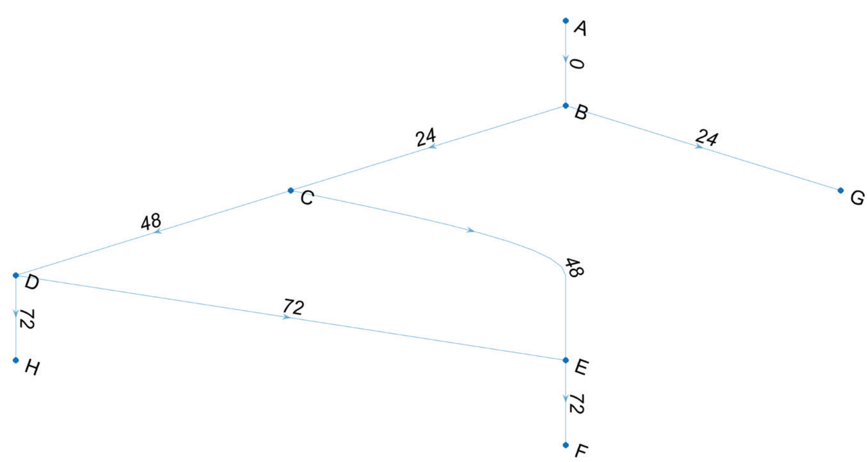


Fig. 4. The graph of a tree with a root at the vertex A for a sequential solution of the $SIR_{\delta ij}$ models

Table 1

The results of simulating the resumption of train traffic in the sections of the railway test site

| Section | Passenger trains, $r$=1 | Commuter trains, $r$=2 | Cargo trains, $r$=3 | Passenger trains, $r$=1 | Commuter trains, $r$=2 | Cargo trains, $r$=3 | Passenger trains, $r$=1 | Commuter trains, $r$=2 | Cargo trains, $r$=3 | Simulation time | Vector sign for SIR |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $S^r$ | $S^r$ | $S^r$ | $I^r$ | $I^r$ | $I^r$ | $R^r$ | $R^r$ | $R^r$ | $t$ | |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| AB | 15 | 28 | 25 | 4 | 0 | 5 | 0 | 0 | 0 | 0 | Entrance |
| AB | 18.22 | 0.00 | 16.42 | 2.34 | 3.96 | 3.50 | 9.45 | 15.04 | 8.07 | 24 | Exit |
| BC | 14.57 | 9.50 | 6.12 | 1.87 | 2.50 | 0.88 | 0.00 | 0.00 | 0.00 | 24 | Entrance |
| BC | 11.92 | 0.00 | 5.11 | 0.93 | 2.55 | 0.37 | 11.15 | 9.45 | 1.52 | 48 | Exit |
| CE | 23.07 | 13.39 | 4.42 | 0.93 | 3.61 | 0.25 | 0.00 | 0.00 | 0.00 | 48 | Entrance |
| CE | 17.79 | 0.00 | 3.18 | 1.59 | 3.65 | 0.37 | 4.62 | 13.35 | 1.12 | 72 | Exit |
| CD | 7.69 | 14.96 | 0.00 | 0.31 | 4.04 | 0.00 | 0.00 | 0.00 | 0.00 | 48 | Entrance |
| CD | 5.83 | 0.00 | 0.00 | 0.54 | 3.95 | 0.00 | 1.63 | 15.05 | 0.00 | 72 | Exit |
| DE | 13.06 | 11.88 | 0.00 | 0.94 | 3.12 | 0.00 | 0.00 | 0.00 | 0.00 | 72 | Entrance |
| DE | 10.44 | 0.00 | 0.00 | 0.82 | 3.15 | 0.00 | 2.74 | 11.85 | 0.00 | 96 | Exit |
| EF | 27.08 | 13.35 | 3.22 | 1.92 | 3.65 | 0.28 | 0.00 | 0.00 | 0.00 | 72 | Entrance |
| EF | 20.75 | 0,00 | 2.32 | 1.98 | 3.65 | 0.28 | 6.27 | 13.35 | 0.90 | 96 | Exit |
| BG | 5.53 | 5.54 | 6.12 | 0.47 | 1.46 | 0.88 | 0.00 | 0.00 | 0.00 | 24 | Entrance |
| BG | 4,99 | 0.00 | 5.24 | 0.20 | 1.55 | 0.34 | 0.80 | 5.45 | 1.42 | 48 | Exit |
| DH | 5.60 | 15.05 | 0.00 | 0.40 | 3.95 | 0.00 | 0.00 | 0.00 | 0.00 | 72 | Entrance |
| DH | 4.25 | 0.00 | 0.00 | 0.41 | 3.96 | 0.00 | 1.35 | 15.04 | 0.00 | 96 | Exit |

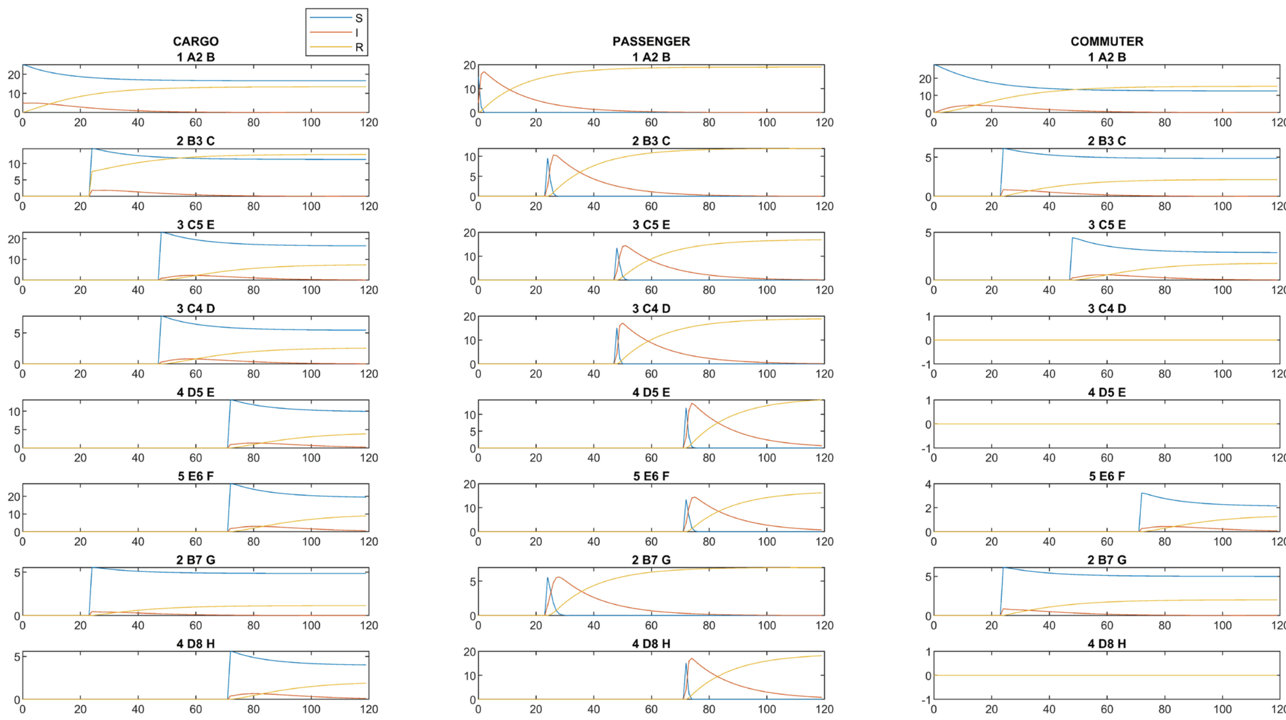


Fig. 5. The graphs of dynamics in the flows' change for trains when movement of the trains is restored in the sections of the railway domain if there are delays in departure at station A: *a* — passenger trains; *b* — commuter trains; *c* — cargo trains

The simulation results were used to estimate the effect of delay on the stability of the train flow with the deviation from the main line ABCEF at station C. With the deviation of the line CDH, primary delay resulted in one delayed passenger and four delayed commuter trains exiting the test site at station H. 58.4 % of the delayed passenger trains and 70.1 % of the delayed cargo trains resumed their running after the primary delay in the section AB. It was not enough to absorb the initial delays in the network. The most detrimental impact is on commuter trains' schedules, which can hardly be restored even throughout all sections of the test site if there are delays of passenger trains, which have a higher priority. The obtained simulation results confirmed the adequacy of the solutions based on the comparison of the real operating conditions of the sections during train delays with the simulated ones. The set delay recovery coefficients correspond to the normative ones in the traffic schedule and are insufficient for full-fledged traffic recovery. This proves that the current reserve values in the normative schedule need to be revised upwards.

One of the important advantages of this simulation method is its speed of calculations in the conditions of automation. The results were obtained in 52.411585 seconds of calculation with the parameters of computing power Intel(R) Core(TM)i5-8250U CPUv 1.60 GHz 1.80 GHz with 4 GB of RAM. For comparison, in study [6], where an optimization mathematical model was used to build a detailed schedule of trains, the calculation for only one section took up to 7 minutes. According to the manual method of plotting traffic schedules, which is the most accurate method, the delay check in only one section may take several working days.

## 6. Discussion of the results of the research on modelling the propagation of train delays at a branched railway test site

The obtained results of modelling the propagation of train delays at a branched railway test site indicate that the proposed method of simulation based on modified epidemiological models is quite accurate as the error between empirical and model results does not exceed 10 % under fast calculations. In addition, in contrast to the known approaches to the modelling of scheduled train traffic [21, 22] and micromodelling [6], this method helps predict the propagation of delays taking into account the set time reserves for resumption of traffic in railway systems without adherence to cargo trains. The results are confirmed by experimental studies of the propagation of delays in real sections of one of the railway domains of JSC Ukrzaliznytsia (Table 1).

According to the literature analysis, in a railway system with cargo trains without adhering to the schedule, it is difficult to predict the impact of different categories of trains in the flow. The proposed approach to the formalization of the heterogeneous dynamics of the propagation of delays in the flow of trains of different priorities has improved the accuracy of modelling. This was possible due to the constructed system of differential equations (1)–(3), which helped take into account the influence of the delay propagation force from different categories of train flows due to the coefficients of delay propagation. In the first stages of the study, the implementation of the modelling by using classical SIR models without dividing the flow of trains into three classes according to their category showed poor results: the average absolute error between the empirical and model results reached 35 %.

The proposed method of simulating the propagation of train delays on branched railway tracks requires further testing on larger networks. However, the obtained results of the speed of solving the systems of equations of SIR models indicate the sufficiency of the available computing power to increase the dimension of the problem. Without drawing up a real schedule of trains in a section, it is quite difficult to determine the impact of delays in one category of trains on another. In this study, it was possible to quantify at the macrolevel the interaction of trains of different priorities in the train schedule. This was ensured by the proposed procedure for adjusting the propagation velocity coefficients of SIR models. One of the disadvantages of the developed approach is the difficulty of obtaining real data on the propagation of delays in the railway network for the possibility of adjusting the speed coefficients. In addition, this approach has a limited threshold for improving accuracy compared to methods of accurate scheduling of trains.

The advantage of the method of modelling the propagation of train delays is the possibility of its use on branched railway tracks with acceptable speed and accuracy. This allows taking into account the impact of the ‘network effect’ [15]. The application of this approach will automate the complex process of finding rational values of compensation time in the threads of trains of different categories in a railway section and, as a consequence, increase the punctuality and reliability of regulatory train schedules.

## 7. Conclusions

1. The process of spreading train delays in branched railway domains with the help of modified epidemiological SIR models is formalized in the study. This makes it possible to speed up the accuracy of complex and lengthy calculations of the propagation of train delays, taking into account the impact of the "network effect". To increase accuracy, a mathematical model based on epidemiological models of the SIR type was developed to take into account the interaction of trains with different priority in the flow. To sequentially solve SIR models that correspond to interconnected sections of the network, it is proposed to convert a network graph into a directional tree whose root is a station or several stations where a delay has occurred.

2. The experimental tests of the propagation of train delays in the railway range, taking into account the interaction of different categories of trains in the flow and the set time reserves for the resumption of traffic, helped quantify the propagation of delays of five cargo and four passenger trains. It has been proven that a simultaneous primary delay in passenger and cargo trains at the beginning of movement from station A produces the most detrimental effect on commuter trains on all sites of the investigated range. The obtained quantitative results revealed shortcomings in establishing the values of time reserves in the section EF and the adjacent DE as the set reserve time for trains did not allow resuming traffic. The simulation results confirmed the adequacy of the obtained solutions. The proposed method of modelling the propagation of train delays can be used as a tool to study the impact of the set values of time reserves in the normative schedules of trains to improve them.

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