

Reduction of the complexity of production of articles from composite materials is largely ensured by the use of reinforcing semi-finished products in which fibers pre-form a framework. Among all the variety of reinforcing systems, woven sleeves (preforms) occupy a special place. The high degree of deformability in a nonimpregnated condition makes it possible to lay this reinforcement on any surface without folds and cuts that provide preservation of strand continuity. This advantage of woven sleeves is accompanied by a change in local reinforcement angles and, consequently, the variable nature of physical and mechanical characteristics of the curved part surface. A method for calculating physical and mechanical characteristics of the composite based on preforms at any point of the part depending on the pattern of laying strands on a curved surface has been developed. The possibility of application of the rod model of the composite to describe physical and mechanical characteristics of the composite material with a woven reinforcement was analyzed. The model essence consists in that the composite is modeled by a diamond-shaped rod system. The rhombus sides serve as fibers and the diagonals as the binder. To verify the theoretical results and substantiate practical recommendations, a series of experimental studies were performed based on the formation of material specimens from two types of woven sleeves with different reinforcement angles. The experimental study program included tensile, bending, and compression tests. A fairly good convergence of theoretical and experimental data was obtained. For example, a square of the correlation coefficient was not less than 0.95 for the modulus of elasticity, not less than 0.8 for the Poisson's ratio, and not less than 0.9 for tensile and compressive strengths. This is the rationale for using the rod model to describe the considered class of composites. The use of the developed procedure will make it possible to increase the perfection of the considered class of designs and obtain rational parameters of the process of their manufacture

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1. Introduction

The production of polymer composite materials (PCM) is one of the fastest-growing industries in the world. In both defense and civil industries, it is impossible to imagine the creation of new equipment samples without the use of PCM [1, 2].

The PCM advantage consists in that the material, production technology, and design are created simultaneously [3, 4]. This determines high innovation degree at all stages of the PCM life cycle: from obtaining raw materials to designing production and operation of products. These materials have high specific strength and rigidity and are able to work in such operating conditions where conventional structural materials cannot be used [5, 6].

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DETERMINING THE MECHANICAL CHARACTERISTICS OF COMPOSITE MATERIALS REINFORCED WITH WOVEN PREFORMS

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In recent decades, along with the development of new high-strength reinforcing materials for composites, new reinforcement technologies are being actively developed [7]. PCM production using woven reinforcement or premixes (preforms) [8] is one of them.

The woven preform is a composite semi-finished product in a form of a "soft sleeve", fabric or multilayer braiding of the mandrel if a variable diameter is necessary. Any high-strength carbon fibers, fiberglass, fibers of natural origin can be the material for the manufacture of such preforms. However, the application of this technology is the most effective when using continuous reinforcing fillers [8].

The use of preforms makes it possible to abandon rigging which significantly simplifies the technology of production

of products from PCM. The use of woven preforms in the production makes it possible to obtain a variety of geometric shapes and sizes and products with specified parameters (length, width, thickness) in one operation. The parts made of woven preforms are characterized by significant resistance to delamination and impact, high fatigue performance, and high strength near holes. The use of woven preforms simplifies the solution of the connection problem in prefabricated structures, provides the possibility of production automation, high speed, and efficiency of manufacturing PCM products. Preforms feature high mobility of the reinforcing material which makes it possible to create curved surfaces of complex shapes for which prepreg technology cannot be used [8].

However, the use of this type of material is not fully studied. Functional properties of woven preforms and physical and mechanical characteristics of structural materials based on them are determined by the nature of fibers from which the preforms are made as well as the angle of interweaving the fibers forming the material structure (Fig. 1).



Fig. 1. General view of fiber weaving in a sleeve

At present, the scope of application of structures based on woven reinforcement is limited because this type of reinforcing material is insufficiently studied, especially because of the lack of reliable procedures for calculating physical and mechanical characteristics of the PCM based on it.

In this regard, it seems topical to conduct studies on the development of scientifically substantiated methods of predicting physical and mechanical characteristics of PCM based on woven sleeves.

2. Literature review and problem statement

It was shown in [9, 10] that the physical and mechanical characteristics in longitudinal direction decrease with increasing weaving angle. When applying a load in the transverse direction, an increase in strength and modulus of elasticity in tension and compression is observed. The obtained results have made it possible to prepare practical recommendations for the manufacture of preforms having the reinforcement scheme optimally adapted to the load type.

Methods of computer modeling of structure and the deformation processes taking place in designing the preforms used in the production of aircraft and car parts were offered in [11]. However, the structure of the designed products is characterized not only by the weaving process parameters but also by the type of materials used, the reinforcement angle, and the shape and size of the preform surface. In addition

to the development of computer modeling of woven products and their designing process, compression, and tension properties of woven products with a weaving angle of 30°, 45°, 55°, 60° and 70° were experimentally studied in [12]. However, only the modulus of elasticity, compression, and tensile strength, and breakdown elongation were taken there as the main indicators.

A model of predicting elastic properties of woven composites was proposed in [13]. The model uses a geometric characteristic of the woven architecture in which the woven composite is considered as a set of three plates (plates with inclined and longitudinal fibers). The fiber curvature is taken into account in initial calculations, and the binder contribution is then considered on the basis of the rule of mixtures. As a result of the study, it was found that rigidity of the preform based PCM significantly depends on the amount of fiber bend and reinforcement angle.

A series of studies [14–16] report the results of studies of mechanical behavior of woven composites which make it possible to describe their stress-strain state based on complete modeling the shape of reinforcing material under elastic deformations.

However, no common analytical model of deformation of woven composites was provided in the literature so far. First of all, this is explained by the complexity of constructing discrete models of inhomogeneous woven structures taking into account curvature and interweaving of reinforcing material and computational difficulties in obtaining results of calculation of internal state parameters of the materials in a complex stressed state [17].

An analytical procedure of calculating rigidity characteristics of woven composite structures taking into account features in the fiber interweaving zones was developed in [18, 19]. The procedure involves the use of calculated mechanical characteristics of unidirectional and intertwined zones as initial parameters for further calculations by the finite element method. To verify the proposed methods, the results of experimental studies were presented in [19] which showed a satisfactory convergence.

Most of these studies used the classical theory of layered composites which is based on a recalculation of characteristics of any structure according to the known physical and mechanical characteristics of the monolayer [20, 21].

Analysis of possible approaches to modeling physical and mechanical characteristics of a composite reinforced with a woven sleeve was given in [22, 23], and the factors which do not make it possible to apply existing theoretical models of layered environments were revealed. Such factors include:

- the physical and mechanical properties of conditional monolayers of a woven sleeve modeled by the $\pm\phi$ structure depend on the angle between strands which is not observed according to the theory of layered composites;
- the size of the sleeves does not make it possible to determine physical and mechanical characteristics in different directions in strict accordance with standards because width does not make it possible to make specimens, e.g. determine the modulus of elasticity and shear strength;
- in many cases, the high mobility of strands in a sleeve does not make it possible to obtain the necessary thickness of specimens from layers with strictly identical reinforcement angles.

Longitudinal and transverse mechanical properties of woven tubular elements of carbon fibers were investigated in [24] using a basic model of theoretical micromechanics. The weaving architecture was modeled as a structure consisting of

threads with unidirectional layers. The model was described under the condition that the material properties were calculated by applying the principle of superposition of up to two sublayers. Results were obtained for longitudinal and transverse moduli of elasticity as well as for the Poisson's ratio.

The use of a rod model of symmetrically reinforced composites [22, 23, 25] may be of particular interest in predicting material properties.

Detailed results were given in a series of studies based on finite element modeling, for example, [26, 27]. The complexity of describing the weave structure is the main problem of modeling woven composites in numerical implementation. A complete description of the reinforcing material weave requires greater computational resources as well as a long time to be spent on solving the problems of deformation of composites. Simultaneous use of analytical and finite-element models of deformation of composites is the most rational in terms of optimizing requirements of calculation resources [6, 28, 29].

Results of experimental tensile and compression studies of woven cloths from carbon and glass threads were presented in [30]. Moduli of elasticity and tensile strength were taken as the main indicators of properties.

It was experimentally shown in [31] that the degree and nature of deformation of the sleeve in woven structures have a strong impact on axial rigidity and strength of the composite. Similar conclusions were obtained in [32] where results of mechanical tests of specimens prepared using preforms based on carbon filler and epoxy binders were presented.

Conducting experimental studies for each woven composite with different reinforcing materials is not advantageous because the results differ significantly for each type of microstructure of the woven PCM [33–35]. It should also be borne in mind that it is fundamentally impossible to implement unidirectional material in a reinforced composite sleeve composite ($\varphi=0^\circ$ and $\varphi=90^\circ$) and therefore it is impossible to experimentally determine basic physical and mechanical characteristics of PCM [22, 23, 36].

It can be concluded that no reliable calculation methods for determining physical and mechanical characteristics of the PCM based on woven preforms were sufficiently developed up to now. Availability of such methods would increase the perfection of the considered class of structures and enable obtaining of rational parameters of their production process.

3. The aim and objectives of the study

The study objective implied developing a method for calculating physical and mechanical characteristics of preform based composites depending on the degree and nature of deformation of the woven sleeve.

To achieve this objective, the following tasks were solved:

- analyze the possibility of applying a rod model of a composite to describe physical and mechanical characteristics of PCM made of a woven reinforcement and obtain the required calculated dependences to predict elastic and strength properties of the studied composites with any reinforcement angle;

- conduct an experimental study to verify the obtained theoretical results.

4. Materials and methods used in the study of physical and mechanical characteristics of a composite reinforced with a woven sleeve

To determine the physical and mechanical characteristics of a woven composite, a rod model of symmetrically reinforced PCM was used. The model essence consists in that the composite is modeled by a diamond-shaped rod system. The rhombus sides were taken as fibers and the diagonals as the binder. In this case, the rotary joint of the rods in nodes is essentially a condition for the deformation continuity. The fundamental difference between the adopted rod model of the woven PCM made from symmetrically reinforced composite is that length of the rhombus sides is considered constant. Besides, a variable volume of fiber content is permitted in this case.

The program of experimental studies included the preparation of specimens for mechanical tests. Sleeves of 120/4 and 96/33 grades based on woven carbon strands were impregnated with EDT-69N binder. Tensile, compressive, and bending test specimens with strand laying angles of $\pm 25^\circ$, $\pm 30^\circ$, $\pm 40^\circ$, $\pm 45^\circ$, $\pm 50^\circ$ and $\pm 60^\circ$ were prepared using each filler type. Tensile and bending tests were performed at the Instron tearing machine (USA) and compression tests on a 1932-U10 test machine (Russia). The results of mechanical tests were subjected to statistical processing.

Reliability of the proposed models of determining physical and mechanical characteristics of the composite reinforced with a woven sleeve is ensured by the correctness of the performed experiments and calculations which was confirmed by small values of divergence between the results of theoretical analysis and experimental study.

5. Modeling the physical and mechanical characteristics of the composite reinforced with a woven sleeve

As shown above, reinforcement of composite structures with woven sleeves generally leads to variable structural parameters and, consequently, to variable physical and mechanical characteristics of the PCM depending on the angle φ (Fig. 2) [22, 23].

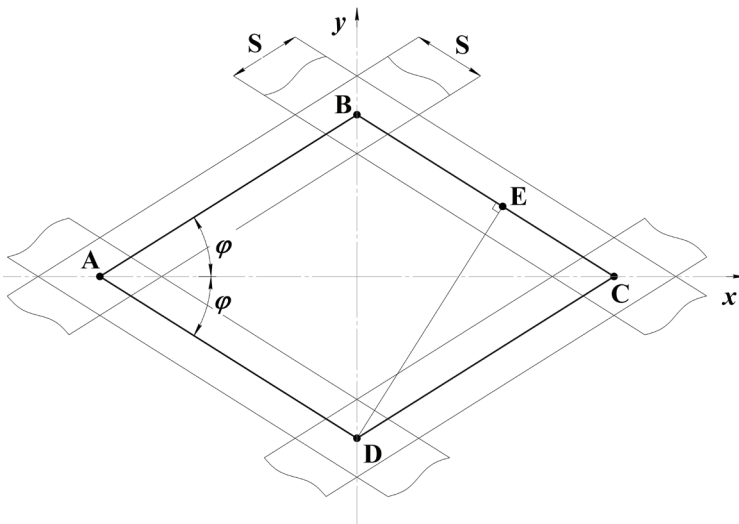


Fig. 2. Geometric model of a woven sleeve; S is the width of the reinforcing element

Analysis of the structure of the composite reinforced with a woven sleeve shows that a repetitive (representative) element can be distinguished in it (Fig. 3). This element is formed by straight lines parallel to axes x and y . These axes pass through the points of intersection of two pairs of adjacent strands. If all strands along the x -axis are concentrated in one rhombus diagonal and the strands along the y -axis are concentrated in the other diagonal, a rod system that is modeling the PCM of the considered structure is obtained [22, 25].

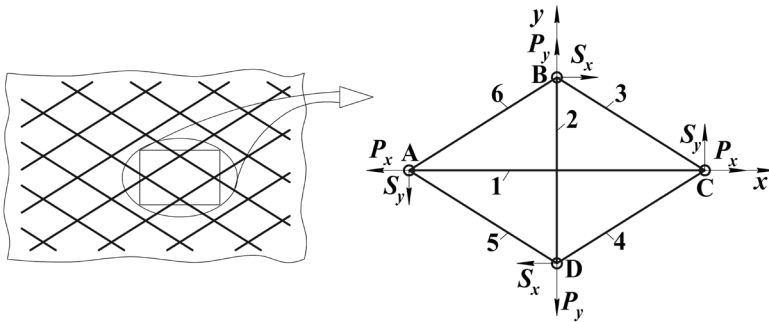


Fig. 3. Rod model of PCM

The peculiarity of the structures reinforced with woven sleeves consists in that the length t of the rhombus sides is considered constant. Besides, in this case, the variable volume content of fibers is first assumed. This assumption about the constancy of direction of fiber placing during the composite deformation is characteristic of the physical models used in the practice of calculation of thin-walled structures made of fibrous PCM reinforced with strands, tapes, or fabrics [22, 25, 29].

The model nodes (Fig. 3) are affected by the forces which for obvious reasons are determined by the following dependences:

$$\begin{aligned} P_x &= N_x \cdot \ell_2 = 2N_x t \sin \varphi; \\ P_y &= N_y \cdot \ell_1 = 2N_y t \cos \varphi; \\ S_x &= q_{xy} \cdot \ell_1 = 2q_{xy} t \cos \varphi; \\ S_y &= q_{xy} \cdot \ell_2 = 2q_{xy} t \sin \varphi. \end{aligned} \tag{1}$$

where

$$\begin{aligned} t &= AB = BC = CD = AD; \\ \ell_1 &= AC = 2t \cos \varphi; \\ \ell_2 &= BD = 2t \sin \varphi. \end{aligned}$$

Using the equation of equilibrium in the rod system nodes, static uncertainty, and the following relations for determining axial forces N_i in the model rods can be revealed (Fig. 3):

$$\begin{aligned} N_1 &= 2t \frac{2a_2 \sin^2 \varphi (N_x \sin^2 \varphi - N_y \cos^2 \varphi) + N_x a_3 \sin \varphi}{2a_1 \cos^3 \varphi + 2a_2 \sin^3 \varphi + a_3}; \\ N_2 &= 2t \frac{2a_1 \cos^2 \varphi (N_y \cos^2 \varphi - N_x \sin^2 \varphi) + N_y a_3 \cos \varphi}{2a_1 \cos^3 \varphi + 2a_2 \sin^3 \varphi + a_3}; \end{aligned}$$

$$\begin{aligned} N_3 &= N_5 = t \left[\frac{\sin 2\varphi (N_x a_1 \cos \varphi + N_y a_2 \sin \varphi)}{2a_1 \cos^3 \varphi + 2a_2 \sin^3 \varphi + a_3} + q_{xy} \right]; \\ N_4 &= N_6 = t \left[\frac{\sin 2\varphi (N_x a_1 \cos \varphi + N_y a_2 \sin \varphi)}{2a_1 \cos^3 \varphi + 2a_2 \sin^3 \varphi + a_3} - q_{xy} \right], \end{aligned} \tag{2}$$

where the a_1, a_2, a_3 are the parameters indicating pliability of the system rods.

Assigning rigidities of the layers along and across the fibers in part to certain rods, the following dependences are obtained to determine them:

$$\begin{aligned} a_1 &= \frac{1}{2t\delta E_2 \sin \varphi}; \\ a_2 &= \frac{1}{2t\delta E_2 \cos \varphi}; \\ a_3 &= a_4 = a_5 = a_6 = \frac{1}{t\delta(E_1 - E_2) \sin \varphi \cos \varphi}. \end{aligned} \tag{3}$$

In the above, E_1 and E_2 are the moduli of material elasticity which depend on the volume content of fibers and hence the reinforcement angle, so each value of the angle φ formally corresponds to the values of E_1 and E_2 ; δ is the total thickness of the PCM package.

Substitute (3) into (2) and perform a series of transformations to find that

$$\begin{aligned} N_1 &= 2t \sin \varphi \frac{A \sin^2 \varphi (N_x \sin^2 \varphi - N_y \cos^2 \varphi) + N_x}{A(\sin^4 \varphi + \cos^4 \varphi) + 1}; \\ N_2 &= 2t \cos \varphi \frac{A \cos^2 \varphi (N_y \cos^2 \varphi - N_x \sin^2 \varphi) + N_y}{A(\sin^4 \varphi + \cos^4 \varphi) + 1}; \\ N_3 &= N_5 = t \left[\frac{A \sin \varphi \cos \varphi (N_x \cos^2 \varphi + N_y \sin^2 \varphi)}{A(\sin^4 \varphi + \cos^4 \varphi) + 1} + q_{xy} \right]; \\ N_4 &= N_6 = t \left[\frac{A \sin \varphi \cos \varphi (N_x \cos^2 \varphi + N_y \sin^2 \varphi)}{A(\sin^4 \varphi + \cos^4 \varphi) + 1} - q_{xy} \right], \end{aligned} \tag{4}$$

where $A = \frac{E_1 - E_2}{E_2}$.

To determine the modulus of elasticity of the rod system in tension (or compression) along the x -axis, assume that $N_y = q_{xy} = 0$. Then

$$\epsilon_x = N_1 \cdot a_1 = \frac{\sigma_x}{E_2} \frac{A \sin^4 \varphi + 1}{A(\sin^4 \varphi + \cos^4 \varphi) + 1}, \tag{5}$$

hence

$$E_x = E_2 \frac{A(\sin^4 \varphi + \cos^4 \varphi) + 1}{A \sin^4 \varphi + 1}. \tag{6}$$

Solving a similar problem to determine the modulus of elasticity along the y -axis, the following is obtained:

$$\varepsilon_y = N_2 \cdot a_2 = \frac{\sigma_y}{E_2} \frac{A \cos^4 \varphi + 1}{A(\sin^4 \varphi + \cos^4 \varphi) + 1}; \quad (7)$$

$$E_y = E_2 \frac{A(\sin^4 \varphi + \cos^4 \varphi) + 1}{A \cos^4 \varphi + 1}. \quad (8)$$

The comparison of formulas (6) and (8) shows the identity of the equations

$$E_x(\varphi) = E_y(\pi/2 - \varphi)$$

and

$$E_x(\pi/4 - \varphi) = E_y(\pi/4 + \varphi),$$

as expected.

For the Poisson's ratio μ_{xy} the following is true:

$$\mu_{xy} = \frac{\varepsilon_y}{\varepsilon_x} = \frac{(E_1 - E_2) \sin^2 \varphi \cos^2 \varphi}{(E_1 - E_2) \sin^4 \varphi + E_2}. \quad (9)$$

Analysis of this formula shows that $\varphi=0^\circ$ and $\varphi=90^\circ$ for $\mu_{xy}=0$ which is not true. When entering in a similar way the correction factor found from the boundary conditions to [25], the following is obtained finally:

$$\mu_{xy} = \frac{A \sin^2 \varphi \cos^2 \varphi + \mu_{12}}{A \sin^4 \varphi + 1}, \quad (10)$$

where μ_{12} is the Poisson's ratio of the PCM.

Similarly, the Poisson's ratio μ_{yx} is found:

$$\mu_{yx} = \frac{A \sin^2 \varphi \cos^2 \varphi + \mu_{12}}{A \cos^4 \varphi + 1}. \quad (11)$$

To determine the shear modulus of the PCM, use the dependence

$$G_{xy} = \frac{q_{xy}}{\delta \gamma_{xy}}. \quad (12)$$

One part of the flow of forces q_{xy} is taken by compression and tension of the side rods (q_s) and the other part by actual displacement of the rest of the material (q_d). Accordingly, the following can be written:

$$q_{xy} = q_s + q_d. \quad (13)$$

The condition of continuity of deformations takes the form:

$$\gamma_{xy} = \gamma_s = \gamma_d. \quad (14)$$

Using geometric constructions, it is possible to show that the rhombus distortion is expressed by the dependence:

$$\gamma_s = \frac{\varepsilon_3}{\sin \varphi \cos \varphi} = \frac{a_3 N_3^*}{\sin \varphi \cos \varphi}, \quad (15)$$

where N_3^* is the part of the forces q_{xy} that deform only the side rods and are determined by

$$N_3^* = t \frac{q_s}{\sin 2\varphi}. \quad (16)$$

Then

$$\gamma_s = \frac{\varepsilon_3}{(E_1 - E_2) \delta \sin^2 2\varphi}, \quad (17)$$

$$\gamma_d = \frac{q_d}{\delta G_{12}}, \quad (18)$$

where G_{12} is the PCM shear modulus.

Finding the flows from (16), substituting them in (13) and (12) and performing corresponding transformations give the following:

$$G_{xy} = G_{12} + \frac{1}{4}(E_1 - E_2) \sin^2 2\varphi. \quad (19)$$

Let us consider the possibility of restoring the basic strength properties of the PCM based on the rod model.

Since each rod is formed of two conditional layers (with reinforcement angles 0° and 90° , deformation of the rod destruction is found as follows [22, 23, 25]:

$$\varepsilon = \begin{cases} \frac{F_1}{E_1}, & \text{if } \frac{F_1}{E_1} < \frac{F_2}{E_2}, \\ \frac{F_2}{E_2}, & \text{if } \frac{F_1}{E_1} > \frac{F_2}{E_2}. \end{cases} \quad (20)$$

To find the tensile strength along the x -axis in formulas (4), take $N_y = q_{xy} = 0$ and write the strength conditions

$$\begin{aligned} \varepsilon_1 &= N_1 a_1 = \\ &= \sigma_x \frac{(E_1 - E_2) \sin^4 \varphi + E_2}{E_2 [(E_1 - E_2) (\sin^4 \varphi + \cos^4 \varphi) + E_2]} \leq \frac{F_{1p}}{E_1} \vee \frac{F_{2p}}{E_2}; \end{aligned}$$

$$\begin{aligned} \varepsilon_2 &= N_2 a_2 = \\ &= \sigma_x \frac{(E_1 - E_2) \sin^2 \varphi \cos^2 \varphi}{E_2 [(E_1 - E_2) (\sin^4 \varphi + \cos^4 \varphi) + E_2]} \leq \frac{F_{1c}}{E_1} \vee \frac{F_{2c}}{E_2}; \end{aligned}$$

$$\begin{aligned} \varepsilon_3 &= N_3 a_3 = \\ &= \sigma_x \frac{E_2 \cos^2 \varphi}{E_2 [(E_1 - E_2) (\sin^4 \varphi + \cos^4 \varphi) + E_2]} \leq \frac{F_{1p}}{E_1} \vee \frac{F_{2p}}{E_2}, \end{aligned} \quad (21)$$

where F_{1p} , F_{2p} , F_{1c} , F_{2c} are tensile and compression strengths of PCM, respectively.

It is taken into account in inequalities (21) that rods 1, 3, 4, 5, 6 are stressed and rod 2 is compressed. Taking into account (20), the following is obtained finally:

$$F_{xp} = \min \left\{ \frac{F_{2p} B}{A \sin^4 \varphi + 1}; \frac{F_{2c} B}{A \sin^2 \varphi \cos^2 \varphi}; \frac{F_{1p} B}{(A+1) \cos^2 \varphi}; \frac{F_{1p}}{\cos^2 \varphi} \right\}, \quad (22)$$

where $B = A(\sin^4 \varphi + \cos^4 \varphi) + 1$.

For each specific laying angle φ , only one from the formulas in braces (22) is valid. For example, destruction of the PCM occurs from rupture of fibers at small values of the angle φ , from compression of the rod 2 at medium values of

this angle and from rupture of the rod 1 or PCM across fibers at the angles close to 90°.

When compressing the PCM along the x -axis, all rods except rod 1 are compressed. Then condition (22) will take the following form:

$$F_{xc} = \min \left\{ \begin{array}{l} \frac{F_{2c}B}{A \sin^4 \varphi + 1}; \frac{F_{2p}B}{A \sin^2 \varphi \cos^2 \varphi}; \\ \frac{F_{1c}B}{(A+1) \cos^2 \varphi}; \frac{F_{1c}}{\cos^2 \varphi} \end{array} \right\}. \quad (23)$$

Therefore, knowing experimental values of the ultimate strength (for example, F_{30} , F_{45} and F_{60} or for any other reinforcement angles), only part of the basic strength characteristics of the conditional monolayer can be restored.

Under the action of shear load, failure can occur from tensioning or compression of the side rods or only from shear.

Taking into account (13) and assuming that $N_x=N_y=0$ in formulas (4), strength conditions for the side rods can be obtained:

$$\begin{aligned} \varepsilon_3 = \varepsilon_5 = -\varepsilon_4 = -\varepsilon_6 = \\ = \tau_{xy} \frac{2 \sin 2\varphi + E_2}{(E_1 - E_2) \sin^2 2\varphi + 4G_{12}} \leq \frac{F_{1p}}{E_1} \sqrt{\frac{F_{1c}}{E_1}}, \end{aligned} \quad (24)$$

and the following is true for diagonal rods:

$$\gamma_{xy} = \tau_{xy} \frac{4}{(E_1 - E_2) \sin^2 2\varphi + 4G_{12}} \leq \frac{F_{12}}{G_{12}}, \quad (25)$$

where F_{12} is the shear strength of the PCM.

Hence, shear strength in the plane of laying the layers can be calculated:

$$\begin{aligned} F_{xy} = \frac{(E_1 - E_2) \sin^2 2\varphi + 4G_{12}}{2} \times \\ \times \min \left\{ \frac{F_{1p}}{E_1 \sin 2\varphi}; \frac{F_{1c}}{E_1 \sin 2\varphi}; \frac{F_{12}}{2G_{12}} \right\}. \end{aligned} \quad (26)$$

As can be seen from (26), destruction occurs only from shear at $2F_{1c}/E_1 > F_{12}/G_{12}$ and at $2F_{1c}/E_1 < F_{12}/G_{12}$ and reinforcement angles close to 45°, the side rods are first destroyed by compression or tension at $F_{1p} < F_{1c}$.

Obviously, it is theoretically impossible to estimate the value of F_{12} based on the rod model.

6. Experimental studies of physical and mechanical characteristics of a composite reinforced with a woven sleeve

In order to experimentally substantiate and verify the constructed model of predicting mechanical properties of composites reinforced with woven sleeves, sets of specimens of the PCM reinforced with different types of sleeves were prepared and tested. The specimens were formed and tested in the research department of composite materials of Antonov State Enterprise (Ukraine). Specimens were made by vacuum autoclave molding. The woven sleeves of 120/4 and 96/33 grades reinforced with carbon strands were impregnated with the EDT-69N (52 %) binder. Specimens with strand laying angles of $\pm 25^\circ$, $\pm 30^\circ$, $\pm 40^\circ$, $\pm 45^\circ$, $\pm 50^\circ$, and $\pm 60^\circ$ were made with the

use of each type of filler. The number of layers in the specimens varied depending on the reinforcement type as follows: two layers for a 120/4 sleeve type and one layer for a 96/33 sleeve type.

Specimens were cut from each panel for tensile, compression, and bending tests. Tensile and bending tests were performed at the Instron rupture machine and compression tests on the 1932-U10 test machine. The results of mechanical tests were subjected to statistical processing. Data obtained on the specimens that had deviations from normal fractures (fractures in grips, fractures not from shear during shear tests, etc.) were excluded from the test results.

The arithmetic mean \bar{X} of the test results was calculated from the formula:

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i, \quad (27)$$

where n is the number of tested specimens; X_i is the value of the characteristics found in the i -th specimen.

The magnitude of standard deviation S_n was calculated by the formula:

$$S_n = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2}, \quad (28)$$

where X_i is the value of the determined indicator X ; \bar{X} is the average value of the determined indicator X ; n is the number of values included in calculations.

The root-mean-square deviation \bar{S} was calculated by the formula:

$$\bar{S} = \frac{S_n}{\sqrt{n}}. \quad (29)$$

The coefficient of variation of the average value V was calculated by the formula:

$$V = \frac{S_n}{\bar{X}}. \quad (30)$$

Limits of the confidence interval ΔX were determined by the formulas:

$$\bar{X} - \Delta X, \quad (31)$$

for the lower limit and

$$\bar{X} + \Delta X, \quad (32)$$

for the upper limit where ΔX is the probable deviation of the indicator X determined from the obtained average value \bar{X} .

ΔX was calculated by the formula

$$\Delta X = \frac{t \cdot S}{\sqrt{N}} = t \cdot \bar{S} = Q \cdot S, \quad (33)$$

where $Q = \frac{t}{\sqrt{N}}$; t is the criterion of accuracy; N is the number of individual values included in calculations; $S = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (X_i - \bar{X})^2}$ and $\bar{S} = \frac{S}{\sqrt{N}}$ are values of the standard deviation.

Fig. 4–11 present a comparison of experimental data with theoretical values of physical and mechanical char-

acteristics of PCM calculated according to the proposed mathematical model.

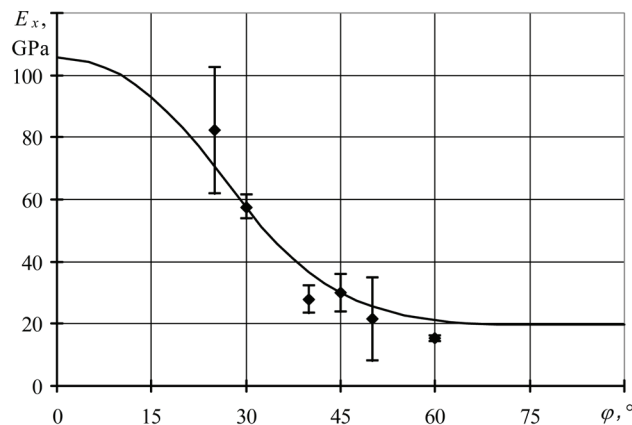


Fig. 4. Dependence of the modulus of elasticity on the angle of laying the strands in the sleeve 96/33: \blacklozenge – experimental data; — – obtained theoretical data

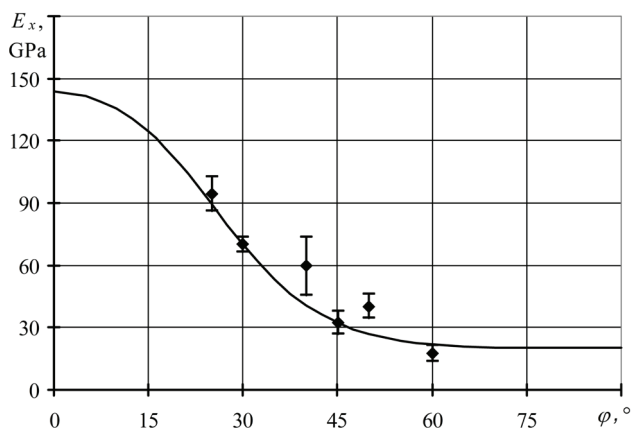


Fig. 5. Dependence of the modulus of elasticity on the angle of laying the strands in the sleeve 120/44; \blacklozenge – experimental data; — – obtained theoretical data

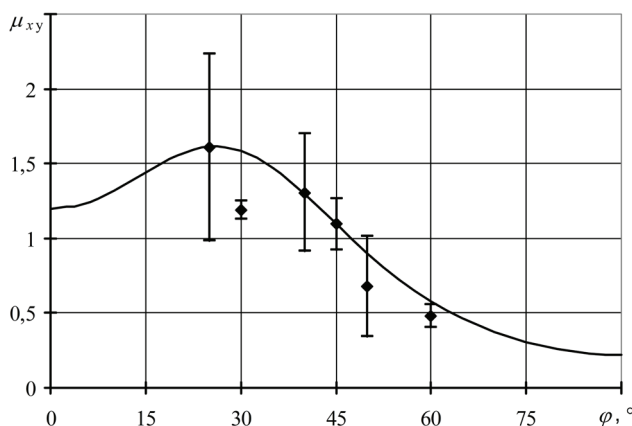


Fig. 6. Dependence of the Poisson's ratio on the angle of laying the strands in the sleeve 96/33: \blacklozenge – experimental data; — – obtained theoretical data

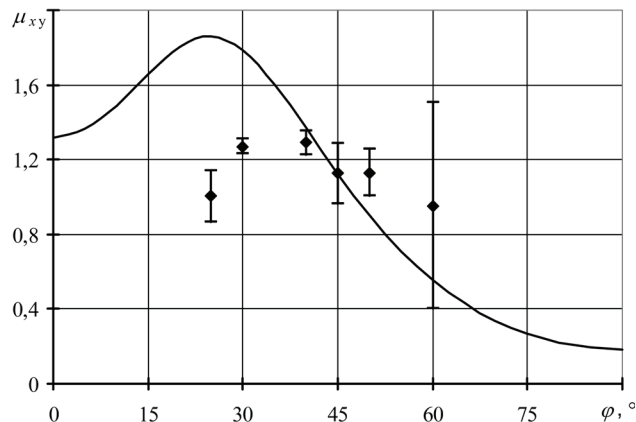


Fig. 7. Dependence of the Poisson's ratio on the angle of laying the strands in the sleeve 120/44: \blacklozenge – experimental data; — – obtained theoretical data

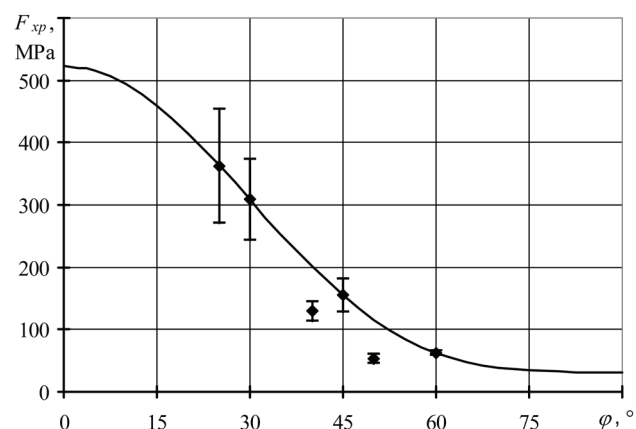


Fig. 8. Dependence of tensile strength on the angle of laying the strands in the sleeve 96/33: \blacklozenge – experimental data; — – obtained theoretical data

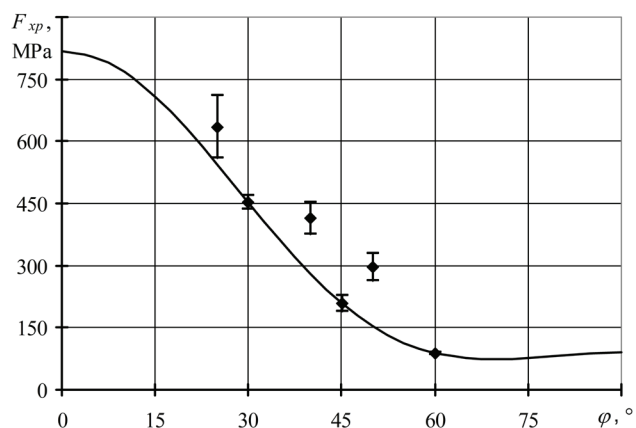


Fig. 9. Dependence of tensile strength on the angle of laying the strands in the sleeve 120/44: \blacklozenge – experimental data; — – obtained theoretical data

Thus, based on the experimental studies, verification of the obtained theoretical results was carried out.

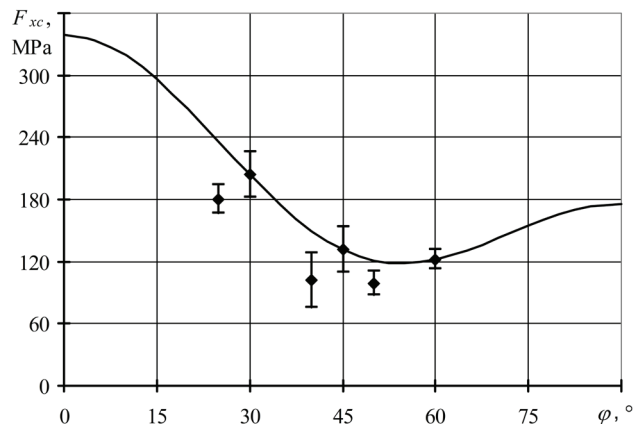


Fig. 10. Dependence of the compressive strength on the angle of laying the strands in the sleeve 96/33:

◆ – experimental data; — – obtained theoretical data

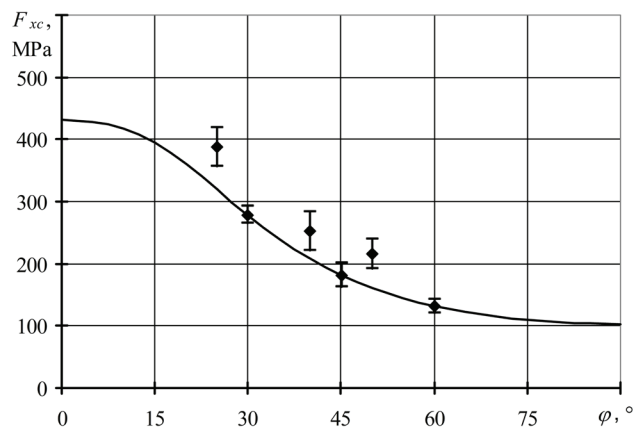


Fig. 11. Dependence of the compressive strength on the angle of laying the strands in the sleeve 120/44:

◆ – experimental data; — – obtained theoretical data

7. Discussion of the results of verification of the obtained theoretical results

Thus, complete mathematical support of calculation of physical and mechanical characteristics of PCM in any part point depending on the reinforcement angle and the strand laying scheme on a curved surface was synthesized.

In contrast to the use of the theory of layered media [17, 20], the application of the rod model makes it possible to reduce the number of required experimental elastic constants to four. For example, to describe the considered composites in accordance with the theory of layered media, it is necessary to experimentally determine five physical and mechanical characteristics. Parameters of the composite with reinforcement angle $\varphi=45^\circ$ and for the other two PCMs with laying angles that are subject to the condition $\varphi=\pi/4\pm\Delta\varphi$ (for example, $\varphi=30^\circ$ and $\varphi=60^\circ$) [22, 23, 25] will serve as such characteristics. When using the rod model, it is enough to know them for two PCMs: with laying angle $\varphi=45^\circ$ and any other, for example, $\varphi=30^\circ$. Analysis of the obtained formulas (6), (8), (10), (11), and (19) shows that to restore the elastic properties E_1 , E_2 , G_{12} and μ_{12} of a conditional PCM based on the rod model, less experimental data are required. However, the shear modulus G_{12} which appears

only in relation (19), it is fundamentally impossible to be established in absence of experimental values of G_{xy} for any reinforcement angle. This conclusion confirms the results of studies [22, 25].

Let the values of E_{30} , μ_{30} and E_{45} , μ_{45} were found according to the results of testing the specimens. Then to determine values of E_1 , E_2 i μ_{12} , the system of equations that follows from (6), (8), (10) can be used:

$$E_2 \frac{10A+16}{A+16} = E_{30}; \quad 2E_2 \frac{A+2}{A+4} = E_{45};$$

$$\frac{3A+16\mu_{12}}{A+16} = \mu_{30}; \quad \frac{A+4\mu_{12}}{A+4} = \mu_{45}. \tag{27}$$

The solution to the first two equations gives values of E_1 and E_2 and the solution of each of the last two gives the coefficient μ_{12} value.

The obtained formulas for calculating values of the physical and mechanical characteristics are similar in structure to those given in a number of studies for layered symmetrically reinforced composites [17, 20, 25]. This is explained by the interdependence of geometry, rigidity, and loads which maintain certain proportions.

A fairly good convergence of theoretical and experimental data was obtained. For example, a square of the correlation coefficient (determination coefficient) r^2 was not less than 0.95 for the modulus of elasticity and not less than 0.8 for the Poisson's ratio.

Both models (the model of symmetrically reinforced composite and the rod model) are based on the use of physical and mechanical characteristics of a unidirectional monolayer which cannot be considered the main structural unit of the PCM based on woven sleeves [22, 23]. In addition, properties of the considered material depend on the reinforcement angle as a result of changes in the volume content of fibers in the composite [8, 23]. Therefore, any theoretical dependence must include experimental points and the predicted values of the characteristics will not necessarily be true in the intervals between them. This is especially true for modeling strength properties. A fairly good convergence of theoretical and experimental data was obtained: the coefficient of determination r^2 was not less than 0.9. However, restoration of the monolayer strength according to the developed model at the obtained ratios between the tensile and compressive strengths F_{xp} and F_{xc} leads to inadequate results for different angles of laying. In this case, the strength limits of the monolayer are complex numbers. This requires additional studies to involve any of the many strength criteria [22, 23]. That is, it is desirable to know in advance which of them most adequately describes the behavior of a loaded real PCM.

Application of the developed procedure will make it possible to increase the perfection of the considered class of structures and obtain rational parameters of the process of their manufacture.

8. Conclusions

1. Based on the rod model of composite polymer material, a procedure was developed for determining physical and mechanical characteristics of the composites based on preforms at any point of the part depending on the reinforcement

angle and the pattern of laying strands on a curved surface. It was shown that the physical and mechanical properties of the conditional monolayers of the woven sleeve modeled by the structure $\pm\phi$ depend on the angle between strands which is not observed in the application of the theory of layered composites. In contrast to the application of the theory of layered media, the application of the rod model has reduced the number of required experimental constants.

2. To verify the obtained theoretical results, a series of experimental studies were performed on the basis of forming specimens of material from two types of woven sleeves with different reinforcement angles. A fairly good convergence of

theoretical and experimental data was obtained. For example, a square of the correlation coefficient (coefficient of determination) r^2 was not less than 0.95 for the modulus of elasticity, not less than 0.8 for the Poisson's ratio, and not less than 0.9 for tensile and compressive strengths. Taking into account the good convergence of theoretical and experimental data, it can be considered quite reasonable to use the rod model to describe the considered class of composites. However, it is desirable to limit its scope by the prediction of only deformation properties, especially if specifics of determining initial values for strength calculations (strength limits of the conditional PCM monolayer) are taken into account.

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