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ESTIMATING THE DYNAMICS OF A MACHINE-TRACTOR ASSEMBLY CONSIDERING THE EFFECT OF THE SUPPORTING SURFACE PROFILE

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Results of theoretical studies of dynamics of the machine-tractor assembly taking into account the influence of a bearing surface profile were presented. It was established that in the course of operation, the machine-tractor assembly is exposed to a number of external factors leading to a change of vertical loads on the chassis and the engine. Mathematical models of dynamics of a tractor and a machine and a tractor unit consisting of a tractor of pivotally connected arrangement and a trailed sower were constructed. Such models make it possible to study dynamics and oscillatory processes of multi-element units. A mathematical model of tractor wheel dynamics was formed. Speeds and angles of orientation of elements of the machine-tractor assembly in space were determined. Influence of profile of the bearing surface on the unit elements when moving in the field prepared for sowing and the field after plowing was calculated. Theoretical studies of the influence of the bearing surface profile on dynamics of the machine-tractor assembly were performed on the example of KhTZ-242K tractor and Vega-8 Profi sower (Ukraine). When moving, the sower frame has a smaller amplitude of vibration accelerations than that of the tractor. Accordingly, the tractor has higher oscillation energy because it rests on the ground through its wheels having appropriate stiffness. The sower moves with its working bodies immersed into the soil which leads to a decrease in the amplitude of oscillations. The highest energy of amplitude of oscillation accelerations of the sower frame in the vertical direction was observed at frequencies of 15.9; 23.44; 35.3 and 42.87 Hz. It was found that the increase in working speeds of agricultural units leads to the fact that oscillations of all components reach significant values. This entails an increase in dynamic loads on soil and, as a consequence, its compaction

Keywords: machine-tractor assembly, oscillations of frame elements, mathematical model of the wheel, profile of bearing surface, amplitude of vibration accelerations, spectral density of profile height

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1. Introduction

The operating procedure of a machine-tractor assembly (MTA) is influenced by many external factors causing the action of extra vertical loads on the unit chassis and engine. For example, these factors include heterogeneity of physical and mechanical properties of the cultivated soil, unevenness of road surface, uneven traction resistance of a unitized agricultural machine. These impacts are random and described by random functions. In addition, the machine-tractor assemblies themselves, their engines, or transmission are also sources of oscillations.

Tractor oscillations cause soil over-compaction which complicates plant germination and reduces soil fertility.

Besides, oscillations lead to violation of agronomic requirements, creation of unfavorable conditions for growing plants (e.g., requirements to the depth of tillage and seed placement, etc.). Oscillations impair traction and soil grip properties of the tractor, worsen the working ability of the driver, have a detrimental effect on the operation of mechanisms causing their premature wear. Therefore, studies dedicated to the assessment of the impact of the bearing surface profile on dynamics of the machine-tractor assemblies are relevant.

2. Literature review and problem statement

The tractor design was studied in [1, 2] as an aggregate of a large number of inertial masses connected by shafts, links, elastic couplings, and other elastic elements with different tangential stiffness levels. These structural elements form complex inertial-elastic-dissipative oscillating systems interacting by means of elastic and dissipative elements [3].

Elastic elements have the ability to accumulate potential energy. All elastic elements of a machine and pneumatic wheel tires exposed to radial and tangential deformations have elastic properties.

Dissipative elements have the property of energy dissipation (scattering). Energy dissipation is caused by heat dissipation brought about by internal friction forces resulting from friction of surfaces of structural elements or hydraulic (viscous) friction of fluid against walls of the hydraulic system due to mechanical work.

It is a difficult task to construct the most complete dynamic model of a tractor or a unit with high reliability of results. Vertical accelerations and dynamic loads occurring in a "unit-tractor-transmission-engine-frame" system are calculated through the study of a simplified dynamic model of an idealized tractor or unit.

In this case, a dynamically equivalent design diagram (physical model) is adopted instead of a real MTA. The model includes inertial masses which replace individual moving masses, elastic elements, and elements that characterize the pliability of transmission parts [4].

For theoretical studies and oscillation calculations, the tractor and unit design elements are schematized and simplified taking into account the fact that the elements that have little effect on the behavior of the oscillating system are generally not considered [5]. It should be noted that the influence of various profiles of the bearing surface on oscillatory processes has not been studied.

MTA is a complex system for analysis, so to simplify it, all values of the stiffness of spring linkages, moments of inertia, and damping coefficients are reduced to one element, usually to the engine crankshaft [6]. However, spatial oscillations of MTA elements have not been studied in this work.

To determine damping coefficients in mechanical systems, the empirical results based on observation of damped oscillations are usually applied [7].

Mathematical models of technical objects should reflect the physical properties of the objects and the models should be as simple as possible. However, such mathematical models should be adequate. A model is considered adequate if it reflects results with acceptable accuracy or consistency of theoretical and experimental results.

However, there are unresolved issues connected with the development of a mathematical model of MTA which is inevitably associated with the idealization of the object under study.

Surface roughness is one of the main external impacts on the emergence of dynamic forces acting on the soil during the movement of a mobile unit [8]. An essential feature of soil surface irregularities is that they belong to the category of random irregularities in the probabilistic-statistical sense. Actual values and the nature of change can only be determined experimentally. The random nature of field surface irregularities is the main feature of conditions of movement of mobile units and, consequently, the dynamic forces transmitted by movers to the ground. Accordingly, dynamic forces acting on the soil will also be random. As the speed of units increases, the frequency of the disturbing effect of soil roughness increases [8]. Frequencies of the disturbing influences of soil irregularities and their influence on the MTA elements in one plane were studied.

Increasing the operating speeds of machine-tractor assemblies is one of the most important reserves for increasing labor productivity in agriculture. With an increase in the speed of machines in conditions of agricultural activities, their oscillations reach significant values, which entail an increase in dynamic loads on soil and hence an increase in its compaction [9].

The influence of field irregularities on the smoothness of agricultural machinery motion was determined in [10]. Statistical properties of field surface irregularity were calculated in [8]. It was established that values of statistical parameters and characteristics vary widely depending on the soil background and the cultivation direction [11].

However, previous studies did not take into account oscillations of the machine-tractor assembly elements in space, that is, oscillations were studied in only one plane.

Thus, studies of the MTA dynamics are promising with taking into account the influence of the bearing surface profile, oscillations in the three-dimensional space, and height of the bearing surface profile for an individual wheel.

3. The aim and objectives of the study

The study objective was to assess the dynamics of the machine-tractor assemblies by taking into account the influence of the bearing surface profile. This will make it possible to develop methods to reduce oscillations of the unit elements, improve controllability and stability.

To achieve this objective, it was necessary to perform the following tasks:

– to construct mathematical models of dynamics of a tractor and an MTA as a part of a tractor of a pivotally connected assembly;

– to construct a mathematical model of the tractor wheel taking into account the height of the bearing surface profile;

– to determine the influence of the height of the bearing surface profile on dynamics of the tractor and the unit;

– to confirm the adequacy of the developed mathematical model of the tractor and the unit by conducting experimental studies.

4. The materials and methods used in studying dynamics of the machine-tractor assembly taking into account the influence of the bearing surface profile

The study of dynamics of a tractor with a pivotally connected frame taking into account irregularities of the bearing surface requires the construction of kinematic diagrams, dynamic and mathematical models of the studied machine.

When constructing mathematical models of the tractor and the unit, the following assumptions were made: the MTA elements are absolutely rigid bodies and the whole unit is symmetrical about the longitudinal plane. Oscillatory processes in the unit elements are created by irregularities (profile) of the bearing surface and inhomogeneities of physical and mechanical properties of the soil.

The following were not taken into account: the processes that take place in the hydraulic steering drive, pressure losses in the hydraulic system, and physiological features of the operator's body.

Also, the processes that occur in the transmission and dynamic characteristics of the engine during acceleration and braking of the unit were not taken into account. The lateral forces acting on the tires are limited by the adhesion of the wheels to the bearing surface.

4. 1. Dynamic and mathematical model of a tractor

Solving the problem of the tractor and unit dynamics taking into account oscillations in the three-dimensional space requires the construction of a kinematic diagram (Fig. 1) and a dynamic model of a tractor with a pivotally connected frame (Fig. 2).

The following notations were used in the diagrams (Fig. 1, 2) and the mathematical model of the tractor and MTA dynamics: *XOYZ* for the global coordinate system; *xoyz* for the linked coordinate system; point *o* for the tractor mass center and point *O* for the center of the global coordinate system.

The angles of rotation of the tractor around the respective axes *x, y, z* are denoted as α, β, γ. Masses of the first and second half-frames of the tractor are denoted as m_1, m_2. The moments of inertia of the first and second half-frames of the tractor with respect to the respective axes are denoted as J_{1x}, J_{1y}, J_{1z}, J_{2x}, J_{2y}, J_{2z}, respectively. The translational speed of the tractor is denoted as v and the angle between the tractor half-frames is denoted as ψ. The height of the bearing surface profile is h_{11} for the front left wheel, h_{21} for the front right wheel, h_{21} for the rear left wheel, and h_{22} for the rear right wheel. Geometrical parameters of the tractor are as follows: distance from the center of mass to the axle of the front wheels: l_1, distance from the center of mass to the axle of the rear wheels: l_2. The front and rear axles of the tractor have tracks b_1 and b_2. Distances from the center of mass of the tractor to the axles of the front and rear wheels along the *z*-axis are hfa_z, hra_z, respectively. Radii of the front left wheel, front right wheel, rear left wheel, and rear right wheel: r_{11}, r_{12}, $r_{21,}$ and r_{22}, respectively. Tangential traction forces are denoted as P_{k11}, P_{k12}, P_{k21}, and P_{k22} for the front left wheel, front right wheel, rear left wheel, and rear right wheel, respectively. Torques on the front left wheel, front right wheel, rear left wheel, and rear right wheel are denoted as M_{k11}, M_{k12}, M_{k21}, and M_{k22}, respectively. Rolling resistance forces for the front left wheel, front right wheel, rear left wheel, and rear right wheel are denoted as P_{f11}, P_{f12}, $P_{f21,}$ and P_{f22}, respectively. The forces of lateral skid the front left wheel, front right wheel, rear left wheel, and rear right wheel are denoted as $P_{\delta 11}$, $P_{\delta 12}$, $P_{\delta 21}$, and $P_{\delta 12}$, respectively. Reduced rigidity of tires of the front left wheel, front right wheel, rear left wheel, and rear right wheel are denoted as C_{w11}, C_{w12}, C_{w21}, and C_{w22}, respectively. The reduced damping factor of the front left wheel tire, front right tire, rear left tire, and rear right tire is denoted as $k_{\mathrm{w}11}$, $k_{\mathrm{w}12}$, $k_{\mathrm{w}21}$, and k_{w22}, respectively.

The angle between the tractor half-frames ψ was taken as the control effect on the tractor.

The method of constructing mathematical models and drive connections of multi-element units is given in [7].

Excluding non-holonomic connections, the tractor frame, as a solid body, has six degrees of freedom and 6 independent speeds $\dot{X}$, $\dot{Y}$, $\dot{Z}$, ω_x, ω_y, ω_z. Taking into account the angle ψ and the nonholonomic connection as the instantaneous center of velocities in the *xy* plane, the number of independent velocities becomes four: $\dot{X}$, $\dot{Y}$, $\dot{Z}$, ω_y.

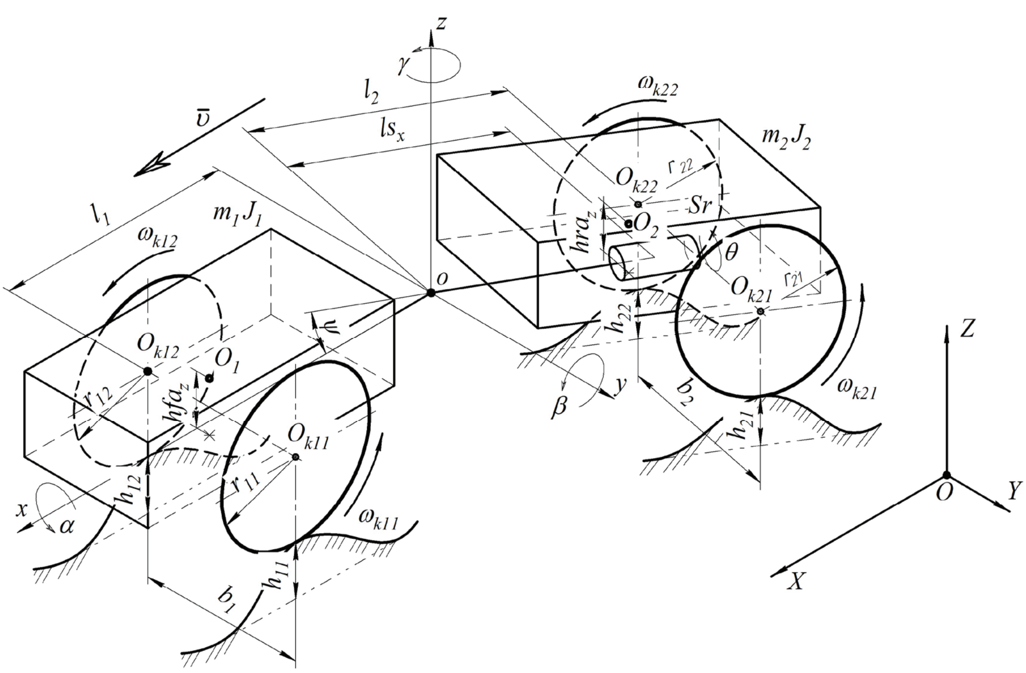


Fig. 1. Kinematic diagram of a tractor with a pivotally connected frame

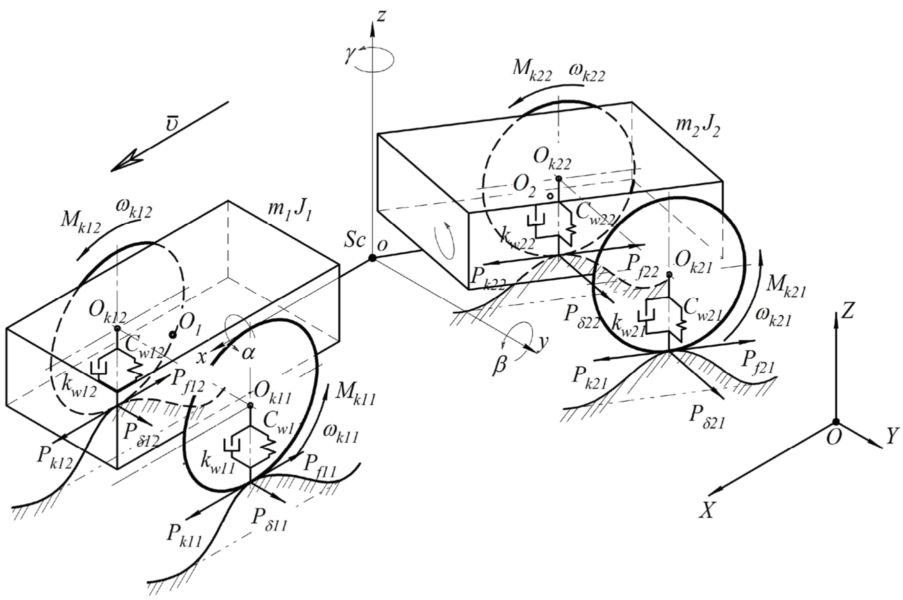


Fig. 2. Dynamic model of a tractor with a pivotally connected frame

Considering the case of the tractor movement without skidding and taking into account rotation of wheels, the number of degrees of freedom increases by another four. The formula for determining the angular speed of rotation of the wheels without taking into account skidding takes the form:

$$\omega_{ij} = \frac{v_{C_{ij}xy}}{Z_{C_{ij}}}, \tag{1}$$

where $Z_{C_{ij}}$ is the z-axis of the center of the wheel in the absolute coordinate system.

The movement of the tractor as a part of the machine-tractor assembly in the field during technological operations of crop production is accompanied by the skidding of the drive wheels [12]. Traction of the tractor wheels is in the range of 0 to 15 %, so it is necessary to take it into account when studying the tractor dynamics. Skidding is caused by many factors such as wheel load [13], torque [14], and tire pressure [15], so these factors must also be taken into account.

4. 2. Dynamic and mathematical model of the wheel

The dynamic model of the wheel was constructed. It takes into account the above factors. The model is shown in Fig. 3.

The wheel model can use a constant coefficient of rolling resistance or dependence on pressure and speed [16]. The force of rolling resistance is zero when the normal force acting on the road-wheel surface is less than or equal to zero [17].

For a dynamic wheel model that takes into account a constant coefficient of rolling resistance, dependence of the rolling resistance takes the form:

$$P_f = \mu F_z, \tag{2}$$

where μ is the coefficient of rolling resistance.

If the rolling resistance coefficient has a hyperbolic shape that excludes the gap at $\upsilon=0$, the rolling resistance coefficient is calculated from the expression:

$$\mu = \mu_0 \tanh\left(\frac{\upsilon}{\upsilon_{\max}}\right), \tag{3}$$

where μ_0 is the asymptotic coefficient of rolling resistance; $\upsilon_{\max}$ is the maximum speed of the wheel movement.

However, let us use a more complex wheel model that takes into account the dependence on pressure and speed. Then dependence of the rolling resistance will take the form:

$$P_f = \left(\frac{P}{P_0}\right)^{\alpha} \left(\frac{P_Z}{P_{Z0}}\right)^{\beta} P_{Z0} \times \times\left(A + B|\upsilon| + C\upsilon^2\right), \tag{4}$$

where P, P_0 are actual and nominal pressures in the tire; P_z, P_{z0} are actual and nominal loads on the wheel; α, β, A, B, C are approximating coefficients.

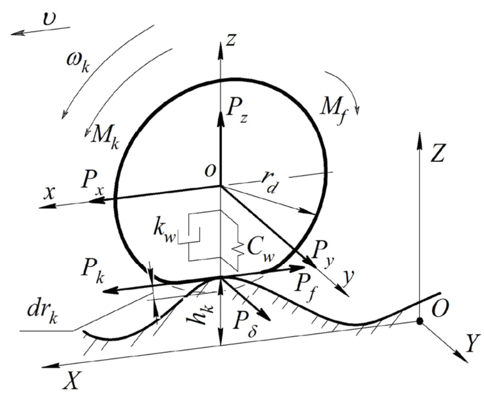


Fig. 3. Dynamic wheel model: $XOYZ$ is global (fixed) coordinate system; $xoyz$ is connected coordinate system; ω_k is the angular speed of rotation; υ is translation movement speed; h_k is the height of the soil profile; r_d is the dynamic radius of the wheel; dr_k is dynamic deformation of the wheel in the vertical direction; P_x, P_y, P_z are the forces acting on the wheel and applied to its center; P_k is the tangential traction force; M_k is the torque; P_f, M_f are the force and moment of rolling resistance, respectively; P_δ is the force of lateral skid; k_w and C_w are yield and rigidity of the tire in the vertical direction, respectively

A mathematical model of the wheel was constructed taking into account the formulas of rolling resistance (3), (4), and wheel rolling without skidding (1). It takes the form:

$$\begin{cases} J_{kij}\dot{\omega}_{kij} = M_{kij} - r_{dij}\left(\begin{array}{l} P_{kij} - \left(\dfrac{P_{ij}}{P_{0ij}}\right)^{\alpha}\left(\dfrac{P_{Zij}}{P_{Z0ij}}\right)^{\beta} \times \\ \times P_{Z0ij}\cdot\left(A + B\left|\upsilon_{Cij}\right| + C\upsilon_{Cij}^2\right) \end{array}\right), \\ \omega_{kij} = \dfrac{\upsilon_{C_{ij}xy}}{r_{dij}\delta_{kij}}, \\ r_{dij} = r_{kij} - dr_{kij}. \end{cases} \tag{5}$$

The dependence of wheel skidding on tangential thrust was obtained experimentally. It takes the form [18]:

$$\delta_{kij} = 0.5^{-17} P_{kij}^4 - 0.25^{-12} P_{kij}^3 + P_{kij}^2 - 0.5^{-4} P_{kij}. \tag{6}$$

The height of the bearing surface profile for the left and right sides of a wheeled tractor is set separately as a function of time:

$$h_{11} = h_l(t);$$

$$h_{12} = h_r(t);$$

$$h_{21} = h_l(t + \Delta t);$$

$$h_{21} = h_r(t + \Delta t);$$

$$\Delta t = \frac{l}{\upsilon}, \tag{7}$$

where $h_l(t)$, $h_r(t)$ are functions of the height of the bearing surface profile for the left and right sides of the tractor; l is the tractor base; υ is the speed of movement.

The proposed mathematical wheel model (5) takes into account skidding δ_k and its dependence on the tangential force P_k acting on the wheel as well as dr_k, dynamic deformation of the wheel in the vertical direction, which is calculated programmatically in study [5].

Geometric and kinematic properties of the dynamic model are described both as generalized coordinates and by generalized speeds:

$$\mathbf{q} = \{q_1, q_2, ..., q_s\}; \quad \dot{\mathbf{q}} = \{\dot{q}_1, \dot{q}_2, ..., \dot{q}_s\}, \tag{8}$$

as well as generalized coordinates and pseudo-velocities:

$$= \{\pi_1, \pi_2, ..., \pi_m\}; \; \dot{\pi} = \{\dot{\pi}_1, \dot{\pi}_2, ..., \dot{\pi}_m\}. \tag{9}$$

The tractor as a two-mass dynamic system (Fig. 1, 2) has a spatial motion of the elements, so the equation of dynamics is presented in the form of Apel equations [7]:

$$\mathbf{U} = \sum_{i=1}^{n} \left\{ \tilde{\mathbf{W}}_{C_i}^T m_i \vec{a}_{C_i} + \tilde{\mathbf{W}}_{\omega_i}^T \left(\left[\vec{J}_i \right] \cdot \vec{\varepsilon}_i + \vec{\omega}_i \times \left[\vec{J}_i \right] \cdot \vec{\omega}_i \right) \right\} -$$
$$- \tilde{\mathbf{W}}_P^T \mathbf{P} = 0, \tag{10}$$

where n is the number of solids in the studied model; m_i, $\left[\vec{J}_i\right]$, $\vec{a}_{C_i}$, $\vec{\omega}_i$, $\vec{\varepsilon}_i$ are the mass, inertia tensor, acceleration of the center of mass, angular velocity, and angular acceleration of the i-th body; $\tilde{\mathbf{W}}_{C_i}$, $\tilde{\mathbf{W}}_{\omega_i}$ are structural matrices of the radius vectors of the centers of mass and angular velocities of bodies, respectively (their formulas are given below); $\mathbf{W}_P$ is the structural matrix of power elements.

Vectors $\vec{a}_{C_i}$ are set in an absolute coordinate system, and vectors $\vec{\omega}_i$, $\vec{\varepsilon}_i$ are set in body-related coordinate systems (usually with the axes, principal axes of inertia). Structural matrices are formed through the matrix of geometric elements $\mathbf{G}$:

$$\tilde{\mathbf{W}}_{C_i}^u = \frac{\partial \vec{r}_{C_i}}{\partial} = \frac{\partial \vec{v}_{C_i}}{\partial \cdot} = \mathbf{W}_{C_i}^u \mathbf{G}, \tag{11}$$

$$\tilde{\mathbf{W}}_{\omega_i}^u = \frac{\partial \vec{\omega}_i^{(i)}}{\partial \cdot} = \mathbf{W}_{\omega_i}^u \mathbf{G}. \tag{12}$$

Also, kinematic parameters $\vec{a}_{C_i}$, $\vec{\omega}_i$, $\vec{\varepsilon}_i$ are programmatically formed according to geometric and differential structures:

$$\ddot{\vec{r}}_{C_i} = \frac{d}{dt}\left(\frac{\partial \vec{r}_{C_i}}{\partial \mathbf{q}} \dot{\mathbf{q}} + \frac{\partial \vec{r}_{C_i}}{\partial t} \right) =$$
$$= \frac{d}{dt}\left[\mathbf{W}_{C_i}^u (\mathbf{G}\dot{\pi} + \gamma) + \frac{\partial \vec{r}_{C_i}}{\partial t} \right] = \tilde{\mathbf{W}}_{C_i}^u \ddot{\pi} + \ldots, \tag{13}$$

$$\vec{\omega}_i^{(ii)} = \mathbf{W}_{\omega_i} \dot{q} = \tilde{\mathbf{W}}_{\omega_i} \dot{\pi} + \ldots, \tag{14}$$

$$\vec{\varepsilon}_i^{(ii)} = \frac{d}{dt}\left(\tilde{\mathbf{W}}_{\omega_i} \dot{\pi} + \ldots \right) = \tilde{\mathbf{W}}_{\omega_i} \ddot{\pi} + \ldots. \tag{15}$$

The matrix $\mathbf{G}$ in its transposed form is the general product term and accordingly for a case of spatial movement of the dynamic model of a tractor, the equation in the following form is obtained:

$$\mathbf{G}^T \left(\sum_{i=1}^{n} \left\{ \begin{matrix} \mathbf{W}_{C_i}^{u\,T} m_i \vec{a}_{C_i} + \\ + \mathbf{W}_{\omega_i}^{u\,T} \left(\begin{matrix} \left[\vec{J}_i^{(i)} \right] \cdot \vec{\varepsilon}_i^{(i)} + \\ + \vec{\omega}_i^{(i)} \times \left[\vec{J}_i^{(i)} \right] \cdot \vec{\omega}_i^{(i)} \end{matrix} \right) \end{matrix} \right\} - \mathbf{W}_P^T \mathbf{P} \right) = 0. \tag{16}$$

The equation of dynamics of a nonholonomic system is obtained by a linear combination of equations of holonomic system dynamics with coefficients taken from a linear form [7]. For numerical integration, the system of general differential equations (SGDE) (16) is transformed into a normal Cauchy form in pseudo coordinates $\mathbf{q}$ and $\mathbf{v}$:

$$\begin{cases} \dot{\mathbf{q}} = \mathbf{G}\mathbf{v} + \mathbf{g}, \\ \dot{\mathbf{v}} = \mathbf{M}^{-1}\mathbf{F}. \end{cases} \tag{17}$$

where $\mathbf{M} = \sum_{i=1}^{n} \left\{ \mathbf{W}_{C_i}^T m_i \mathbf{W}_{C_i} + \mathbf{W}_{\omega_i}^T \left[\vec{J}_i \right] \mathbf{W}_{\omega_i} \right\}$ is the matrix of system inertia; $\mathbf{F}$ is the vector-matrix of generalized forces of the system.

Values of generalized coordinates and independent generalized velocities (pseudo-velocities) at the initial moment are initial conditions for the system (17):

$$\mathbf{q}\big|_{t=0} = \mathbf{q}_0, \; \pi\big|_{t=0} = \pi_0.$$

The method of forming equations of dynamics of multi-element machines with an arbitrary arrangement of elements (8)–(17) has proven its effectiveness [5, 7].

4. 3. Dynamic and mathematical model of a machine-tractor assembly

To study the influence of the bearing surface profile on the dynamics of the machine-tractor assembly, a dynamic model of the sower was constructed. It is shown in Fig. 4.

The vast majority of notations in Fig. 4 coincide with those in Fig. 1 and Fig. 2, however, b_1, b_2 are the distances from the center of mass of the sower to the center of the wheels; hP_x, hP_y, hP_z are the distances from the center of mass to the resultant force of soil resistance; P_x, P_y, P_z are the projections of the force of soil resistance onto the respective axes.

The dynamic model of a unit consists of two dynamic models of the tractor (Fig. 1, 2) and the sower (Fig. 4) connected in series with the point Dhr of the tractor and the point Dhf of the sower.

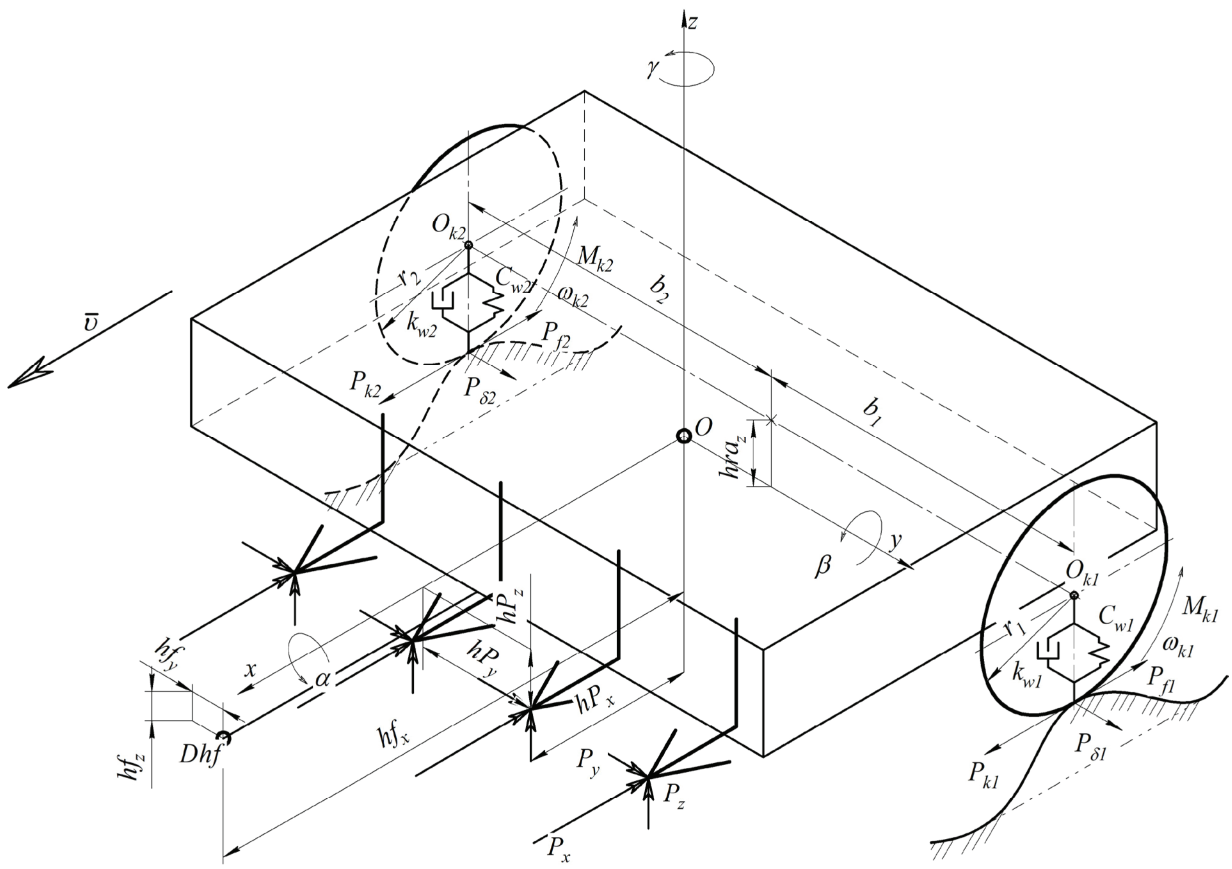


Fig. 4. Dynamic model of the sower

The mathematical model of the MTA has four generalized coordinates, that is, four degrees of freedom and six independent coordinates with dependent variations. The vector of generalized coordinates $\dot{\mathbf{q}}$ and the vector of independent coordinates with dependent variations in the Cauchy form are as follows:

$$\dot{\mathbf{q}} = \begin{bmatrix} X^{\mathrm{T}} \\ Y^{\mathrm{T}} \\ Z^{\mathrm{T}} \\ \beta^{\mathrm{T}} \\ \alpha^{\mathrm{C}} \\ \beta^{\mathrm{C}} \end{bmatrix}^{T}; \; \dot{\upsilon} = \begin{bmatrix} \alpha^{T} \\ \gamma^{T} \\ \omega_{k11}^{T} \\ \omega_{k12}^{T} \\ \omega_{k21}^{T} \\ \omega_{k22}^{T} \\ \gamma^{C} \\ \omega_{k1}^{C} \\ \omega_{k2}^{C} \end{bmatrix}^{T}, \qquad (18)$$

where X^{T}, Y^{T}, Z^{T} are longitudinal, transverse, and vertical coordinates of the center of mass of the tractor; α^{T}, β^{T}, γ^{T} are angles of rotation of the center of mass of the tractor around axes *X, Y* and *Z*; α^{C}, β^{C}, γ^{C} are angles of rotation of the center of mass of the sower around axes *X, Y* and *Z*; ω_{k11}^{T}, ω_{k12}^{T}, ω_{k21}^{T}, ω_{k22}^{T} are angular speeds of rotation of the tractor wheels (front left, front right, back left, back right); ω_{k1}^{C}, ω_{k1}^{C} are the angular speeds of rotation of the sower wheels (right and left).

The mathematical model of dynamics of a tractor with a pivotally connected frame and a sower which takes into account movement of the wheels along the profile of the bearing surface and oscillations of the frame elements in three-dimensional space takes the Cauchy form:

$$\begin{cases} \dot{X}^{T} = f_1(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}); \\ \dot{Y}^{T} = f_2(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}); \\ \dot{Z}^{T} = f_3(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}); \\ \beta^{T} = f_4(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}); \\ \alpha^{C} = f_5(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}); \\ \beta^{C} = f_6(\mathbf{G},\mathbf{g},\mathbf{M},\mathbf{F}); \\ \dot{\alpha}^{T} = \dfrac{a^{T}\dot{X}^{T} + b^{T}\dot{Y}^{T} + c\dot{Z}^{T} - \beta^{T}\left(d^{T}\cos\gamma^{T} + \sin\gamma^{T}\right)}{\cos\gamma^{T} - d^{T}\sin\gamma^{T}}; \\ \dot{\gamma}^{T} = \alpha^{T}\beta^{T} + \upsilon B_{x}^{T}\dfrac{tg\gamma^{T}}{l^{T}}; \\ \dot{\gamma}^{C} = \dfrac{\upsilon y A^{C}}{l_1^{C} - hf_x^{C}} + \alpha^{C}\beta^{C}; \\ J_{kij}\dot{\omega}_{kij} = M_{kij} - r_{dij}\left(\begin{array}{c} P_{kij} - \left(\dfrac{P_{ij}}{P_{0ij}}\right)^{\alpha}\left(\dfrac{P_{Zij}}{P_{Z0ij}}\right)^{\beta} \times \\ \times P_{Z0ij}\cdot\left(A + B\left|\upsilon_{Cij}\right| + C\upsilon_{Cij}^{2}\right) \end{array}\right); \\ \omega_{kij} = \dfrac{\upsilon_{C_{ij}xy}}{\left(r_{kij} - dr_{kij}\right)\delta_{kij}}; \\ h_{11} = h_l(t); \\ h_{12} = h_r(t); \\ h_{21} = h_l(t+\Delta t); \\ h_{21} = h_r(t+\Delta t), \end{cases} \qquad (19)$$

where f_i is functions from matrix vectors, **G, g, M, F** are the matrix vectors calculated from (16) and (17); i=1,..., 6 is the number of the generalized coordinate.

5. The results of the assessment of the influence of the bearing surface profile on dynamics of the machine-tractor assembly

According to the analysis of previous studies [8, 19, 20], the height of the bearing surface profile can be described by a polynomial of the form:

$$h_{ij} = \sum_{k=1}^{m} A_{ijk} \sin\left(\omega_{ijk} t + \varphi_{ijk}\right), \tag{20}$$

where k is the ordinal number of the harmonic; m is the number of harmonics; ij is the ordinal number of the wheel; A_{ijk} is the amplitude of the ij-th wheel of the k-th harmonic of the bearing surface profile; ω_{ijk} is the frequency of the ij-th wheel of the k-th harmonic of the bearing surface profile; φ_{ijk} is the phase of the ij-th wheel of the k-th harmonic of the bearing surface profile.

5. 1. Shape of the bearing surface profile

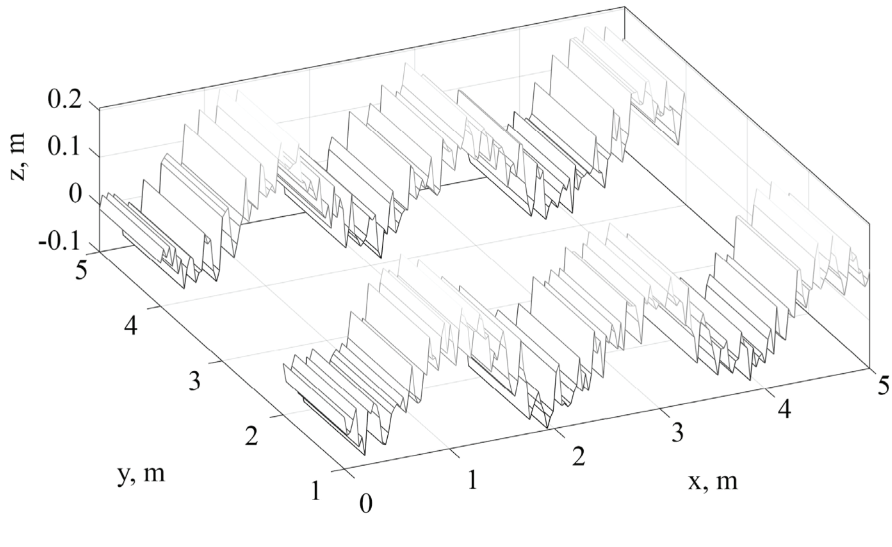
According to the results of experimental studies, the profile of the field surface after plowing described by polynomial (20), is shown in Fig. 5.

The shape of the bearing surface profile for individual sides of the tractor is shown in Fig. 5, *a*. Dependence of the bearing surface profile height h_{11}, h_{12}, h_{21}, h_{22} on time is shown in Fig. 5, *b* for the four tractor wheels. The spectral density of the bearing surface profile height was calculated (Fig. 6).

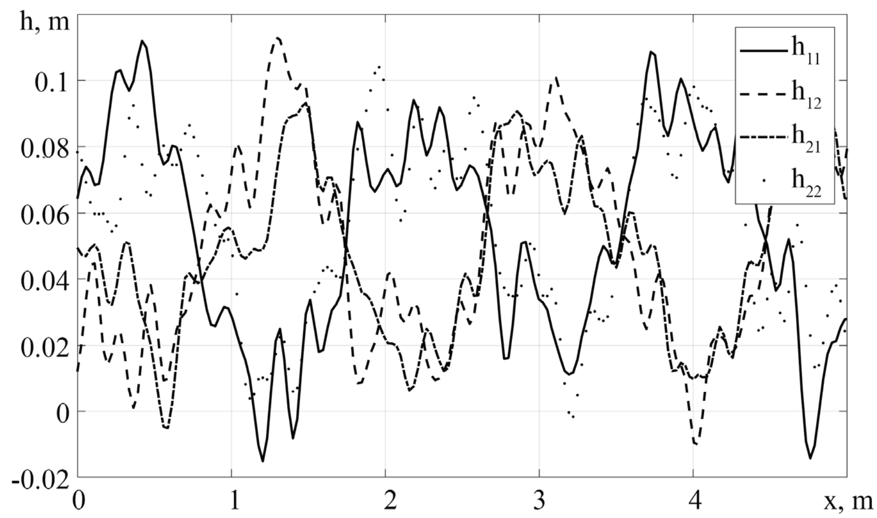
The amplitude of oscillations of the bearing surface profile height for the field after plowing is 0.135 m, the median is 0.0503 m, and the standard deviation is equal to $\bar{x} = 0.03$ (Fig. 5, *a*).

The spectral density of the bearing surface profile height has two harmonics: one with a value of 0.03 at a frequency of 1.5 Hz and the other with a value of 0.004 at a frequency of 15.6 Hz.

The profile of the field prepared for sowing which is mathematically described by polynomial (20) for each wheel of the tractor is shown in Fig. 7, *a*. The spectral density of the height of the bearing surface profile was also calculated for this profile (Fig. 8).



a



b

Fig. 5. Field surface after plowing: *a* — profile shape; *b* — dependence of the profile height on time

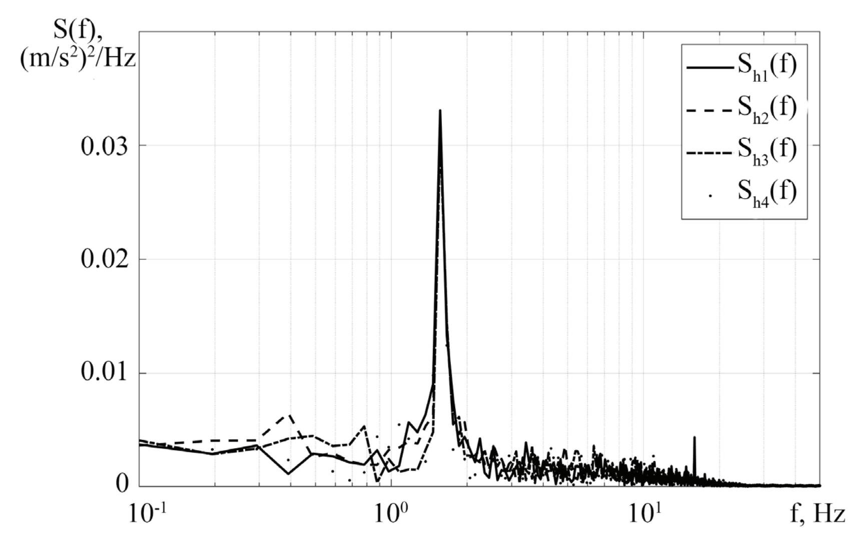
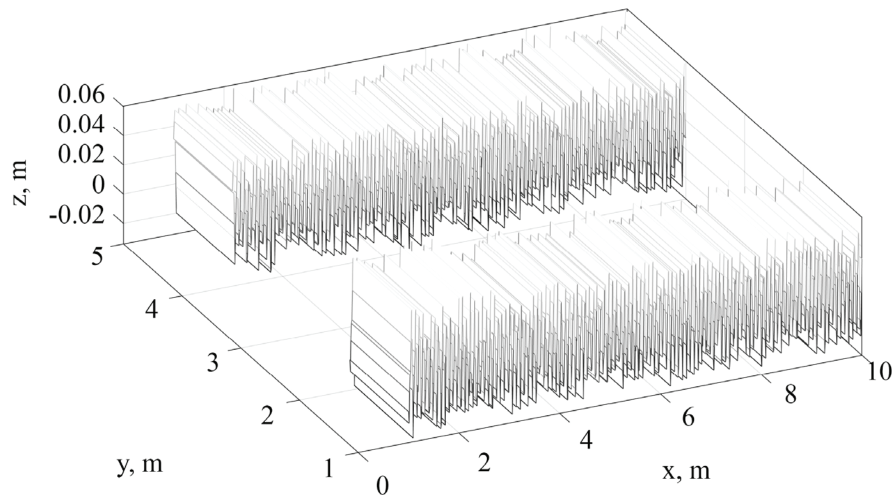
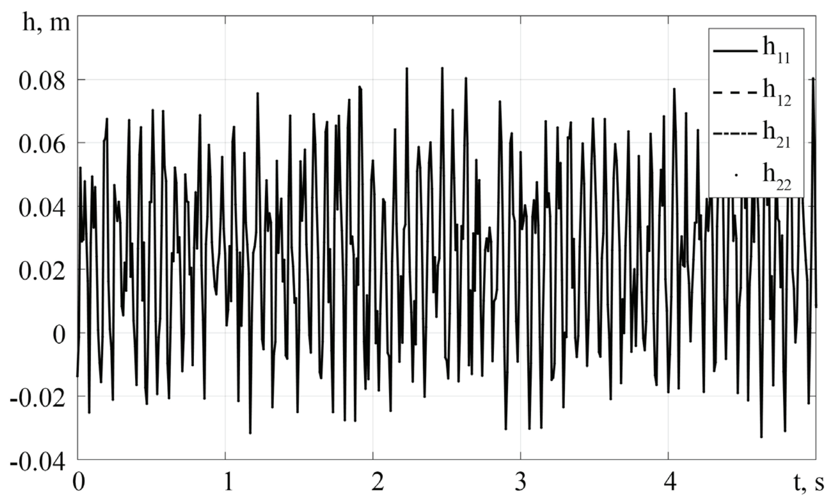


Fig. 6. Spectral density of the bearing surface profile height (field after plowing)



a



b

Fig. 7. Surface of the field prepared for sowing: *a* – profile shape; *b* – dependence of the profile height on time

Fig. 7 shows the amplitude of oscillations of the height of the bearing surface profile for the field prepared for sowing equal to 0.115 m, the median is 0.0252 m, and the standard deviation is equal to $\bar{x}=0.025$. Spectral density of height of the bearing surface profile has two harmonics: one is equal to 0.02 at a frequency of 12.7 Hz and the other is equal to 0.004 at a frequency of 24.8 Hz (Fig. 8).

5. 2. Influence of the bearing surface profile on tractor dynamics

The system of differential equations (18), (19) was solved in KiDiM CCKA and the results were additionally processed in MATLAB. The results of the theoretical study to assess the impact of the bearing surface profile on the tractor and unit dynamics are shown in Fig. 9–15.

The mathematical model of a tractor (without a unitized agricultural machine) (18), (19) was theoretically studied. Dependences of the angles of rotation (orientation) (Fig. 9, *a*) and projections of velocities of the center of mass of the tractor (Fig. 9, *b*) on the corresponding axes on time; dependences of speeds of the tractor wheel rotation (Fig. 10, *a*) and their dynamic radii (Fig. 10, *b*) on time; spectral density of amplitude of oscillation acceleration of the center of mass of the tractor in the vertical direction (Fig. 11) were calculated. The results were obtained for a tractor weight of 8,600 kg and speed of 2.8 m/s.

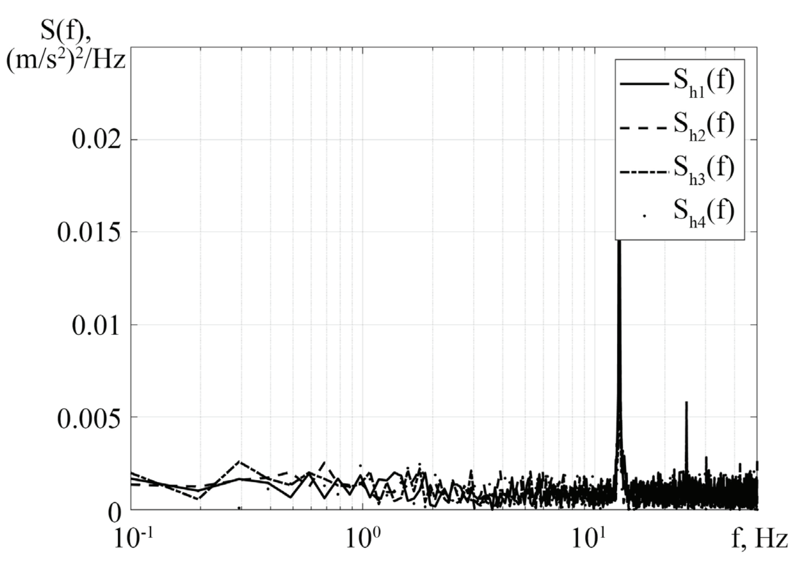


Fig. 8. Spectral density of height of the bearing surface profile (field prepared for sowing)

The tractor movement is accompanied by oscillations of the frame around the *x*-axis (Fig. 9, *a*). The amplitude of oscillations of the angle of rotation of the frame around the *x*-axis was 0.07 rad and the period was 0.63 s. Oscillations of the tractor frame around the *y*-axis had a damping form with an amplitude of 0.03 rad and a period of 0.66 s. In rectilinear motion, the angle of frame rotation around the *z*-axis was constant and equal to zero.

Speed of the tractor movement in the longitudinal direction (along the *x*-axis) corresponded to the agrotechnical speed of sowing when sowing cereals υ_x=2.8 m/s (Fig. 9, *b*). The amplitude of velocity oscillations in the vertical direction (along the *z*-axis) was much larger than in the transverse direction (along the *y*-axis) and was equal to 4.4 m/s and 0.41 m/s, respectively and their period was 0.63 s. It was determined that the shape of the bearing surface profile has the greatest influence on the speed of the tractor frame in the vertical direction.

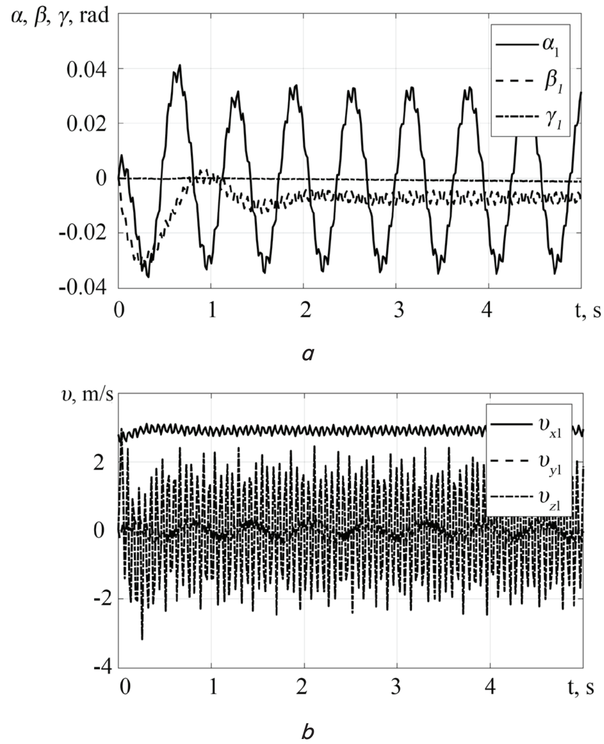


Fig. 9. Angles of orientation in space and projections of tractor frame speeds: *a* — dependence of angles of rotation (orientation) on time; *b* — dependence of velocity projections on time

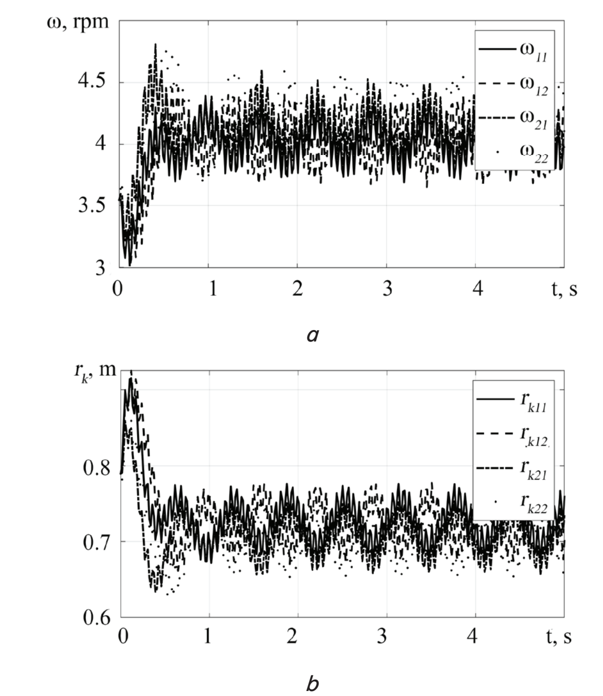


Fig. 10. Dynamics of tractor wheels: *a* —dependence of speed of rotation of wheels on time; *b* — dependence of the dynamic radii of wheels on time

It should be noted that the bearing surface profile has a direct effect on the tractor movers. Tractor wheels can be represented as dynamic models with elastic and damping elements (Fig. 3) which have a direct impact on dynamics. The average speed of rotation of the wheels was 4.15 rpm. The range of oscillations of the rotational speed of the wheels was 0.625 rpm (Fig. 10, *a*).

The tractor wheels are also subject to oscillation. The dynamic radius of the wheels had an amplitude of 0.11 m, the largest radius value was 0.75 m, and the smallest radius value was 0.68 m (Fig. 10, *b*).

The highest energy of amplitude of vibration accelerations of the tractor frame in the vertical direction was observed at a frequency of 15.9 Hz with a value of 2.188; second harmonic S_{az1} (23.44)=0.386; the third harmonic S_{az1} (35.3)=0.144 and the fourth harmonic S_{az1} (42.87)=0.24 (Fig. 11).

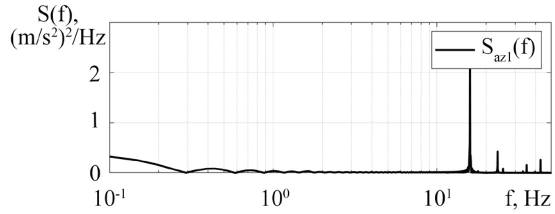


Fig. 11. Spectral density of amplitude of oscillation acceleration of the center of mass of the tractor in the vertical direction

5. 3. Influence of the bearing surface profile on dynamics of the machine-tractor assembly

The dynamics of the machine-tractor assembly were studied on the example of the KhTZ-242K tractor and the Vega-8 Profi sower. Dependences of angles of rotation (orientation) (Fig. 12, *a*) and projections of speeds of the center of mass (Fig. 12, *b*) around respective axes on time and spectral density of the amplitude of vibration acceleration of the center of the sower mass (Fig. 13) in vertical direction were determined.

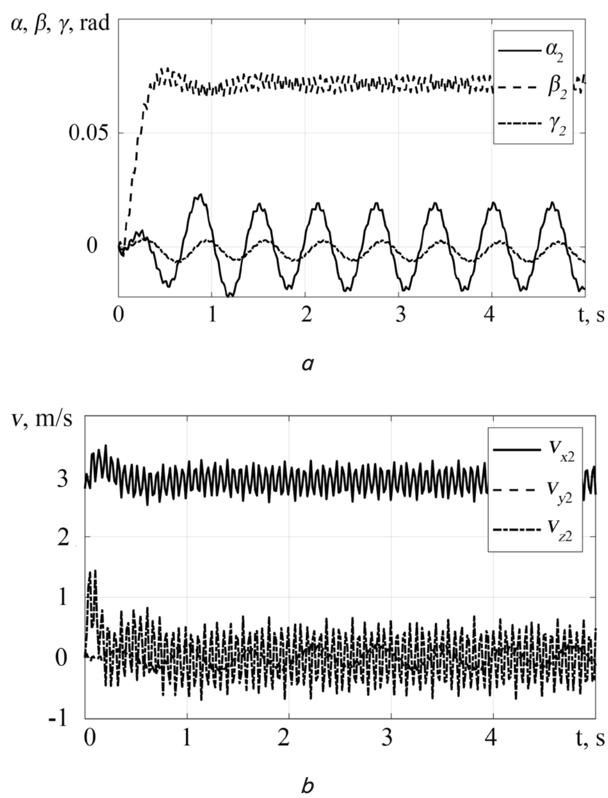


Fig. 12. Angles of space orientation and projection of speeds of the sower frame: *a* — dependences of angles of rotation (orientation) on time; *b* — dependence of velocity projections on time

As with the tractor, the angle of rotation of the sower frame around the *x*-axis had the form of harmonic oscillations with an amplitude of 0.03 rad (Fig. 12, *a*).

The translation movement velocity of the sower (longitudinal movement, *x*-axis) (Fig. 12, *b*) coincided with the tractor speed (Fig. 9, *b*), however, the speed and amplitude of their oscillations were much smaller in the sower. As in the tractor, periods of oscillation were equal to 0.63 s.

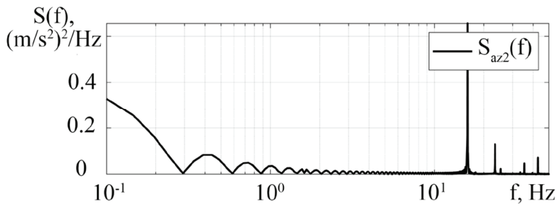


Fig. 13. Spectral density of the amplitude of vibration accelerations of the center of mass of the sower $S_{az2}(f)$ in the vertical direction

The highest energy of the amplitude of vibration accelerations of the sower frame in the vertical direction was observed at a frequency of 15.9 Hz with a value of 0.539; the second harmonic $S_{az2}(23.44)=0.107$; the third harmonic $S_{az2}(35.3)=0.031$ and the fourth harmonic $S_{az2}(42.87)=0.059$ (Fig. 13).

5. 4. The results of experimental studies and confirmation of the adequacy of the mathematical model of the tractor and unit

To verify the adequacy of the proposed mathematical model, a system for measuring the dynamics and energy of mobile machines [7] was used. Its block diagram is shown in Fig. 14.

The developed measuring system belongs to the technical means of diagnostics and operational control and can be used in the agriculture and machine-building industry. It is designed to determine the kinematic, dynamic, power, and energy characteristics of mobile machines and their elements in road, field, and bench tests.

The measuring system included inertial measuring devices (IMD) consisting of gyroscopes (G) and acceleration sensors (AS), a navigation receiver (NR), rotation speed sensors (RSS), electronic dynamometers (ED), analog (AI), and discrete (DI) inputs and fuel consumption sensors (Q). DASys PC Suite software responsible for storing information from sensors on internal or external media has been developed for the measuring system. Communication between the sensors and the computing module was realized via the CAN bus and the radio channel in the 2.4 GHz band.

Experimental studies were performed on an MTA including a KhTZ-242K tractor and a Vega-8 Profi sower which performed sowing of grain in the field prepared for sowing.

Adequacy of the developed mathematical model was verified by the method substantiated in [7]. It consisted of comparing spectral densities of amplitudes of vibration accelerations of the studied unit elements. Theoretical $S_{azt1}(f)$ and experimental $S_{aze1}(f)$ spectral densities of amplitudes of vibration accelerations of the tractor frame in vertical direction were compared (Fig. 15).

Frequencies of 15.9, 23.44, 35.3, and 42.87 Hz of maximum amplitudes of the vibration accelerations of the tractor frame in vertical direction coincided for theoretical and experimental studies. The difference between theoretical and experimental data of amplitudes of vibration accelerations of the tractor frame was 11%.

Similarly, theoretical $S_{azt1}(f)$ and experimental $S_{aze1}(f)$ spectral densities of the vibration acceleration amplitudes of the sower frame in vertical direction were compared (Fig. 16).

Frequencies of maximum amplitudes of the vibration accelerations of the sower frame in vertical direction also coincide for theoretical and experimental studies. The difference between the data of amplitude of vibration accelerations of the sower was 12 %.

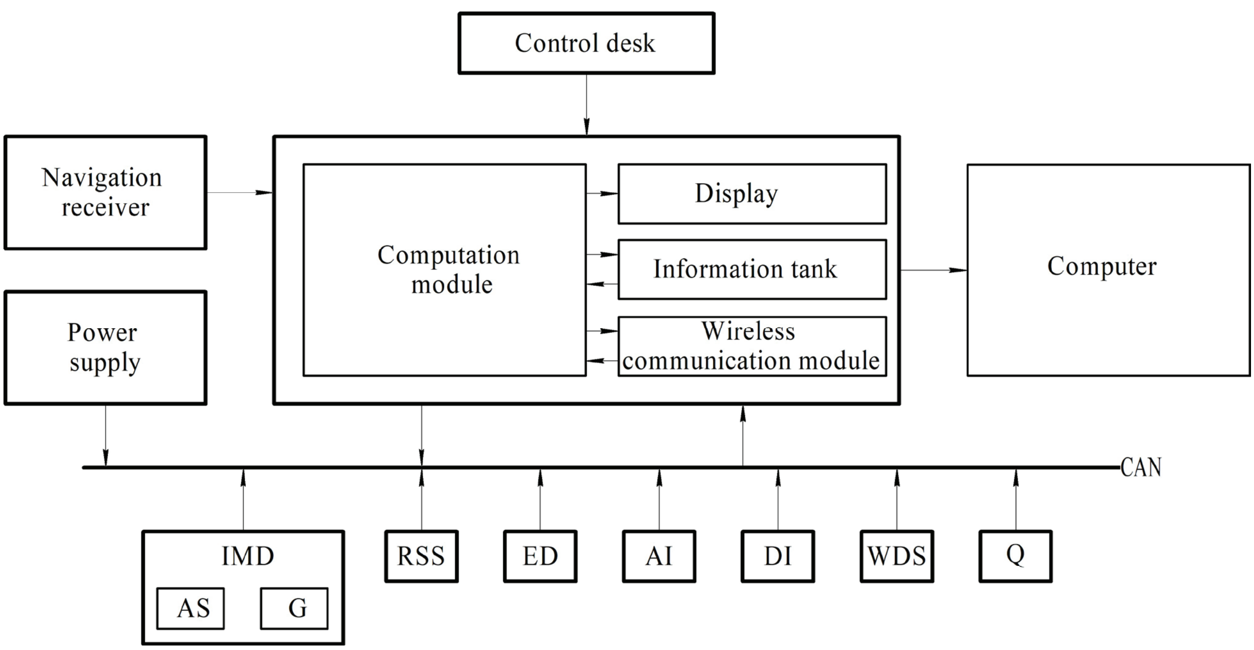


Fig. 14. Block diagram of the system for measuring dynamics and energetics of mobile machines

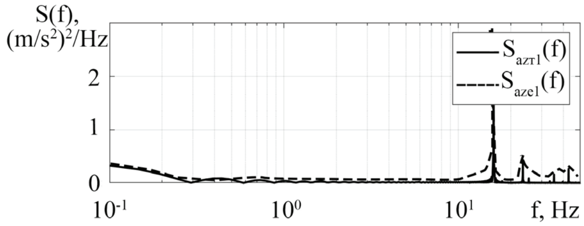


Fig. 15. Theoretical $S_{azt1}(f)$ and experimental $S_{aze1}(f)$ spectral densities of amplitudes of vibration accelerations of the tractor frame

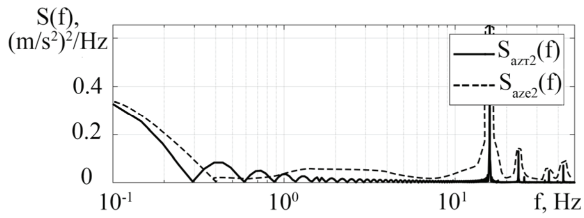


Fig. 16. Theoretical $S_{azt2}(f)$ and experimental $S_{aze2}(f)$ spectral densities of vibration acceleration amplitudes of the sower frame

Thus, the mathematical model proposed in this paper can be considered adequate.

6. Discussion of the results obtained in the study of dynamics of the machine-tractor assembly taking into account influence of bearing surface profile

Increasing the operating speeds of agricultural units leads to the fact that oscillations of all components reach significant values. This entails an increase in dynamic loads on soil and, as a consequence, an increase in compaction. It should be noted that irregularities of the soil surface belong to the category of random irregularities in the probabilistic-statistical sense.

Profile of the bearing surface significantly affects dynamics of the tractor and sower. The oscillating processes caused by the bearing surface are transmitted through the wheels to skeletons of the MTA elements. They mostly affect the rotation of the frames around the x and y axes and have almost no effect on the yaw (Fig. 9, *a*, 12, *a*).

The studies were limited to two profiles of the bearing surface (Fig. 5, 7). According to the agrotechnical requirements and operating conditions of the sowing machine-tractor assembly, these profiles of the supporting surface are the most common.

When moving, the seed sower frame has a smaller amplitude of vibration accelerations (Fig. 13) than that of the tractor (Fig. 11). Accordingly, the tractor has higher oscillation energy because it rests on the ground through its wheels that have an appropriate stiffness. The sower moves with its working bodies immersed into the soil which leads to a decrease in the amplitude of oscillations. Frequencies of the vibration acceleration harmonics for the tractor frame and the sower coincided (Fig. 11, 13).

Determining dynamic loads requires the construction of complex dynamic and mathematical models of a tractor or unit with high reliability. The method of automatic formation of equations of dynamics of multielement units was offered in the study. The method is resource-intensive, that is, requires considerable time for automatic formation of dynamics equations but is effective at the same time.

Dynamic parameters of the machine-tractor assembly for operating speed of 2.8 m/s (10 km/h) which is justified by agrotechnical requirements (Fig. 9, 12) were determined. Further improvement of productivity is possible through an increase in speed which requires additional theoretical studies.

The peculiarity of the solutions proposed in the study consists in the developed mathematical model of MTA (18), (19), which makes it possible to study dynamics of elements as separate models.

It should be noted that no studies of MTA dynamics were performed with simultaneous consideration of oscillations in three-dimensional space taking into account the bearing surface profile. Also, the oscillatory processes of multi-element MTAs connected through a hinge and a pin were not studied.

7. Conclusions

1. It was established that without taking into account non-holonomic connections, the tractor, as a solid body, has six degrees of freedom and four independent speeds. The mathematical model of the machine-tractor assembly has four generalized coordinates. There are four degrees of freedom and six independent coordinates with dependent variations. The dynamic model of the unit consists of two dynamic models of a tractor and a sower connected in series with each other at one point (rigging).

2. The dynamic model of the wheel takes into account the coefficient of rolling resistance and dependence on pressure and speed. It was determined that the tractor wheels are exposed to vibration. The dynamic radius of the wheels had an oscillation amplitude of 0.11 m, the largest value of the radius was 0.75 m and the smallest radius of 0.68 m. The period of oscillation was 0.63 s.

3. Variation of the height of the bearing surface profile for the field after plowing was 0.135 m, the median was 0.0503 m, and the standard deviation was $\overline{x} = 0.03$. Spectral density of height of the bearing surface profile had two harmonics: with a value of 0.03 at 1.5 Hz and 0.004 at 15.6 Hz. Accordingly, the variation range of height of the bearing surface profile for the field prepared for sowing was 0.115 m, the median was 0.0252 m, and the standard deviation was $\overline{x} = 0.025$. Spectral density of height of the bearing surface profile had two harmonics: with a value of 0.02 at a frequency of 12.7 Hz and 0.004 at a frequency of 24.8 Hz. It was found that the shape of the bearing surface profile has the greatest influence on the speed of the tractor frame in the vertical direction. Angles of rotation of the tractor and sower frames around the x-axis had the form of harmonic oscillations with an amplitude of 0.03 rad. The highest energy of the amplitude of oscillation accelerations of the sower frame in the vertical direction was observed with a value of 0.539 at a frequency of 15.9 Hz; the second harmonic $S_{az2}(23.44)=0.107$; the third harmonic $S_{az2}(35.3)=0.031$ and the fourth harmonic $S_{az2}(42.87)=0.059$.

4. The mathematical model proposed in the study can be considered adequate because spectral densities of amplitudes of oscillation accelerations of the tractor and sower frames obtained in theoretical and experimental studies coincided. The difference between the data of amplitudes of vibration accelerations did not exceed 12 %.

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