UDC 621.923
DOI: 10.15587/1729-4061.2021.226479

EFFECT OF SHAPE FORMATION ON THE ACCURACY OF GRINDING ENDS WHILE COMPENSATING FOR MACHINE TOOL ERRORS

Andrey Rudyk
PhD*
E-mail: andrei.rudik@gmail.com
Volodymyr Chupryna
Doctor of Technical Sciences*
E-mail: voldchu@ukr.net
Vladislav Rudyk
Researcher
E-mail: Var_911-91@mail.ru
*State Scientific Research Institute of Armament and Military Equipment Testing and Certification
Striletska str., 1, Chernihiv, Ukraine, 14033

There are several general methods for correcting errors related to positioning the machine tool structural units. The task to achieve optimal manufacturing accuracy can be resolved by using a compatible solution to vector equations, a variation of the shape formation function, or applying a matrix of transfer coefficients.

However, there is no mutual relationship between various calculation methods for the case of grinding flat surfaces. The methods should be simplified and tested for the elongated shape formation function while considering the links' dimensions.

This paper reports a study into the accuracy of grinding flat surfaces, determining and reducing the share of manufacturing errors. The content of variation matrices and transfer coefficients has been substantiated. The comparison of the orientation angles of the grinding machine headstock relative to the machine tool bed has demonstrated close results from all methods. These angles were taken as machine tool errors. The calculation error does not exceed 1.5 %. The experiments are consistent with the calculations.

Different signs of the transfer coefficients in the orientation angles of grinding machine headstocks in the matrix make it possible to mutually compensate for the overall impact. The calculations have shown that the accuracy of the side-end machining is largely affected by a change in the orientation angle in the vertical plane.

The effect exerted on the accuracy of individual mated parts by the machine tool structural units has been estimated. The calculations show that the error of positioning a part in the drum window acquires the highest absolute values and is random in nature, which requires a more accurate base positioning. The findings from both theoretical and experimental studies have been applied. The mathematical model makes it possible to determine the degree of scattering the end surface around the base plane via its variance.

The measured trajectory provides diagnostic information about the sources of error in the machine tool assembly. A task to calculate the accuracy of the end-grinding machine tool can be solved for other models of machine tools in the same way

Keywords: shape formation, grinding, end, correction, accuracy, error, variance, matrix, profilogram, structural unit

Received date 18.01.2020
Accepted date 25.01.2021
Published date 20.04.2021

***How to Cite:** Rudyk, A., Chupryna, V., Rudyk, V. (2021). Effect of shape formation on the accuracy of grinding ends while compensating for machine tool errors. Eastern-European Journal of Enterprise Technologies, 2 (1 (110)), 90–96. https://doi.org/10.15587/1729-4061.2021.226479*

1. Introduction

The responsible surfaces of most parts are finished by grinding operations. They predetermine the operational properties of an article. Among a series of requirements for surface quality, considerable attention is paid to the issues of ensuring roughness and the proper geometric shape. It is important to timely identify the manufacturing causes of errors at the stages of both design and technological development and during the fabrication and modernization of machine tools or their adjustment.

As regards side-end grinding, along with stringent requirements for machining accuracy, there additional requirements for high performance. Namely, for the ends of the rings, carbide plates, the requirements for the parallel position of the ends and permissible deviations relative to the base plane are important.

It is necessary to simultaneously resolve the issue of ensuring both the required quality and high performance. The task is solved by orienting grinding machine headstocks in the horizontal and vertical planes. That results in a technological error during machining at two-sided machine tools.

Reducing the impact of errors related to machine tool structural units' positioning, as well as their possible mutual correction, would decrease their contribution to the overall machining error, and determine the rational dimensions of the tool contact area.

Additional difficulties associated with ensuring the required accuracy arise from the issue related to that the abrasive tool's shape is compromised due to its wear. Such a tool needs adjustment. Consequently, the mutual correction of existing machine tool errors is a relevant task when machining the ends' flat surfaces that has not been resolved so far.

2. Literature review and problem statement

The issues of diagnosing the accuracy of machining and the impact exerted by errors in the position of machine tool structural units were addressed in work [1] using a lathe as an example. The machine tool has a relatively simple shape formation function (SF), which is composed of the product of only three conversion matrices of coordinate systems (CS). The authors derived a general dependence on the construction of the matrix of transfer coefficients between the k-th input (an error of the machine tool structural unit position) and the l-th output technological errors. An element of the matrix of transfer coefficients can be determined within the dimensions of a part's machined surface [1]:

$$w_{ik}=\frac{\iint\limits_{S} V_k(\beta,\rho)\cdot T_i(\beta,\rho)\cdot \mathrm{d}S}{\iint\limits_{S} T_i^2(\beta,\rho)\cdot \mathrm{d}S}, \qquad (1)$$

where β, ρ are the curvilinear machine tool coordinates that describe the required feed movements, or the feed and contact line on the tool, which are necessary to form a workpiece surface at the machine tool; V_k, T_l are, respectively, the k-th element of the vector, the input (machine tool) error, and the l-th element of the vector of technological (output) errors, composed of the X, Y, Z coordinates, which occur when adjusting or assembling a machine tool; $V_k \cdot T_l$ and $T_l \cdot T_l$ are the scalar products of the elements of machine tool and manufacturing errors, l, k is the number of vector elements: depends on the complexity of a task, S is the area of the machined surface, dS is the area element.

Machine tool errors include small angular and linear deviations of the position or size of machine tool structural units from the rated location. Such deviations are due to the quality of assembly, gaps in the mated parts, resizing due to thermal or force deformations. These errors are always present in machine tool structural units. The ordered set of input (machine tool) and output (technological) errors represents the corresponding vectors V and T.

Output, or technological, machining errors are deviations from the predefined shape of the machined surface (extreme SF link) or a change in manufacturing dimensions caused by a set of input errors.

In the equation, planes serve as a weight function to average the effect of errors within the area of the machined surface on the transfer coefficients. The total area and the area of individual elements are conveniently measured within the technological coordinate system (CS), which is connected to the machined part. An end surface manufacturing is described by the movements of feed within the machine-tool CS. The coordinates are related through the Jacobian transformation. The equation does not answer the question of how the shape formation affects the final accuracy of machining.

However, there remain unresolved issues related to taking into consideration the size of the machine tool structural units and the impact of force loads, and the resulting rigidity of the elastic system. In addition, it is proposed that the connection between the machine tool (input) and technological (output) parameters of the machined surface should involve the Jacobian transformation, the technique of finding which is not specified. Moreover, there is no explanation of the proposal on using the derived transfer coefficients between errors. This procedure requires simplification.

The task of establishing optimal accuracy can also be resolved by other methods, for example, by solving the vector equations jointly or by a variation of SF.

Paper [2] reports the results from a theoretical study of machine tool accuracy in order to compensate for failures using numerical control. The paper demonstrates a universal aspect. Errors are compensated by numerical control. The accuracy of the ends' machining was not considered separately.

Study [3] shows that Jacobian is used for multi-coordinate processing. The technique is presented in order to separate the geometric errors of machine tools from other sources. But the issue of accuracy when finishing flat surfaces for a specific type of machine tool remains unresolved.

Work [4] demonstrates that machining performance is affected by the position of the contact line (CL), which determines the shape formation by a tool at machine tool. The orientation angle v of grinding machine headstock in a vertical plane is necessary to improve performance when a tool's end surface is included in the rough shaving process. However, the issues of accuracy were not considered in the cited work.

Paper [5] addresses improving the effectiveness of two-sided grinding of the ends. One issue is the accuracy of the ends for the proposed tool dressing technique. The influence of mutual connection of the orientation angles of the grinding machine headstock is considered. The authors applied a method of solving vector equations, which implies sequentially finding the intersections of the surface of a circle with a family of the part's generatrix. The method makes it possible to find the optimal ratio between the orientation angles of the grinding machine headstock. This ratio is 1.56. When comparing the influence of orientation angles, it turned out that the error is more influenced (by approximately 1.57 times) by the orientation angle ν in the vertical plane. The angles' influence acquires different signs, so values can be adjusted.

Work [6] considers increasing the accuracy of shape formation of the ends of car parts during two-sided grinding. The calculation procedure is given only for the variational method and the predefined technique to dress the circle.

Paper [7] simulated the effect of emerged geometric and temperature errors in the case of multi-coordinated machining.

Work [8] reports an accuracy model for the case of machining ends at a two-sided machine tool. It is believed that shape formation is carried out by the edge of a circle of a larger radius. This is true only for flat-end machining. The results reported in [4] made it possible to determine the position of the contact line, which is not taken into consideration in the cited work. It is the position of the contact line that determines the resulting accuracy and makes it possible to take into consideration a change in the profile of a worn tool. The above accuracy model can be simplified, which affects the efficiency of calculations. The rational ratio between the orientation angles of grinding machine headstocks is determined.

Thus, the issues of shape formation and machining accuracy at a machine tool are considered separately. There is no connection between these characteristics of the grinding process.

3. The aim and objectives of the study

The aim of this study is to improve the accuracy of grinding the side ends based on establishing the effect of shape formation on the accuracy of machining at the machine tool of model 3342 ADO. That would make it possible to reduce and mutually compensate for the errors of mating the machine tool structural units in order to ensure an optimal ratio of angles.

To accomplish the aim, the following tasks have been set:

– to improve the SF of a machine tool by taking into consideration the CL equation;

– to investigate and compare calculations involving experimental data;

– to determine the optimal ratio for the orientation angles of machine tool structural units.

4. Materials and methods to study the influence of shape formation on the accuracy of side-end grinding

The study object is the influence of shape formation on the accuracy of two-sided grinding of the side ends of blanks of a round cross-section at the machine tool of model 3342 ADO (USSR) [4–6].

The advantages of choosing this particular machine tool and a workpiece are the following factors.

The machine tool (Fig. 1) belongs in the accuracy class A and is characterized by high performance; it is used in the finishing operations for two-sided grinding of parts' ends. The machine tool has an elongated SF where the shape of the tool is obtained during the dressing process; this tool then is used for finishing the shape formation of the surfaces of parts' ends.

The simplest round form of the cross-section of the machined surface, the area of which is determined as $S=\iint dS$, or $S=\pi \cdot r^2$, where r is the radius of a part (r=10).

The end surface of part 6 (Fig. 2) is characterized by a minimum number of errors – the total angular deviation Λ from the perpendicularity of the end. The unit vector of the normal to the rated surface of the part (ort) at the selected position of the coordinate systems is directed along the Z axis (Fig. 1), therefore:

$$\vec{n}=\begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}^T. \tag{2}$$

Calculation methods:

a) solving vector equations;

b) variation method;

c) using the matrices of the transfer coefficients.

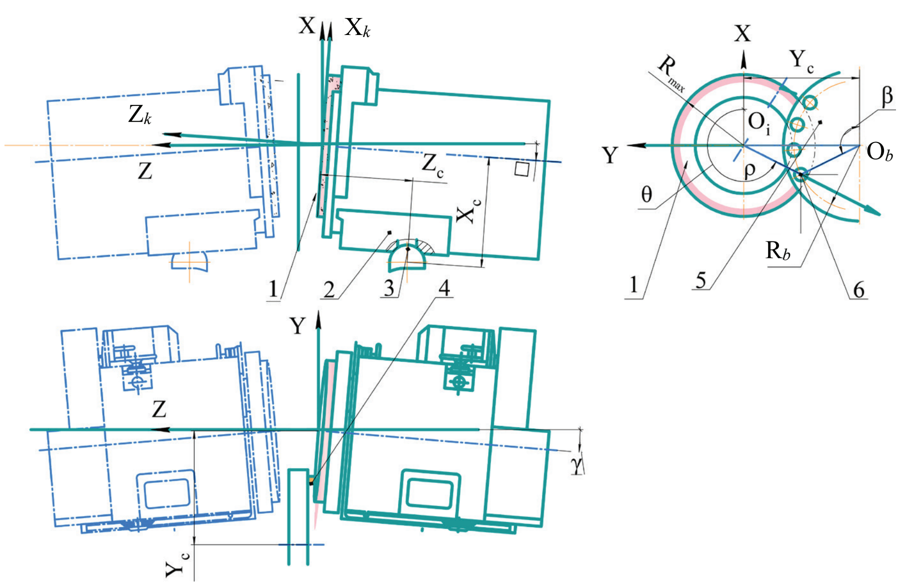

Fig. 1. Schematic adjusting of machine tool 3342 ADO for machining the ends of parts: 1 – grinding circle with a shape formation area; 2 – grinding machine headstock; 3 – spherical support; 4 – diamond pencil; 5 – drum to feed the parts with the place of installation of a diamond pencil; 6 – part

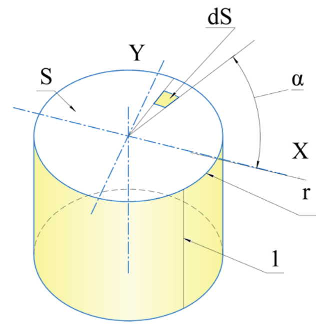

Fig. 2. Surface elements of part 6: generatrix 1, the element of area dS

5. Results of studying the effect of shape formation on the accuracy of grinding the ends

5. 1. Improving the machine tool SF by taking into consideration the CL equation

The input machine tool errors could be represented by the orientation angles of grinding machine headstocks in the vertical ν and horizontal γ planes used to efficiently machine the ends when the required accuracy is achieved at high performance.

The radius-vector of points on the end surface of part 6 machined at the machine tool 3342 ADO for SF when considering the CL position of grinding circle 2 and the size of the links is determined from equation [4]:

$$\begin{aligned}\vec{r}_0(\beta,\rho)=&A^1(-R_b)\cdot A^6(\beta)\cdot A^3(-Z_c)\cdot A^1(-X_c)\times\\ &\times A^2(-Y_c)\cdot A^4(-\gamma)\cdot A^5(-\nu)\times\\ &\times A^1(X_c)\cdot A^6(\theta(\rho))\cdot A^1(\rho)\cdot A^3(Z_{\max})\cdot\vec{e}^{\,4},\end{aligned} \tag{3}$$

where A^1, .., A^6 are the SC conversion matrices that simulate shifts and turns along and around axes [1]; θ, ρ are the parameters of the end surface of the part, predetermined by the position of the contact line θ(ρ) and the rotation of drum 5 that feed the parts that are responsible for the angular and radial position of an arbitrary point of the machined end; X_c, Y_c, Z_c are the dimensions that determine the position of the center of spherical support 3, around which the grinding machine headstock is oriented, relative to feed drum 5 [4, 5]; R_b is the distance between the axes of workpieces 6 and feed drum 5; $R_{\max}$, $Z_{\max}$ are, respectively, the radial and axial dimensions that characterize the position of the shape-forming edge of circle 1, formed by the movement of diamond pencil 4; $e^4=[0\ 0\ 0\ 1]^T$ is the radius-vector of the moving point.

Equation (3) describes SF and makes it possible to transfer the shape-forming points of the end surface of an abrasive tool to the CS of the machined surface of the part – a zero link. The transfer occurs sequentially through all the intermediate machine tool structural units taking into consideration their size.

The coordinates of the points of the machined surface are the functions of the machining parameters:

$$\vec{r}_0(\beta,\rho)=\left[X(\beta,\rho)\quad Y(\beta,\rho)\quad Z(\beta,\rho)\quad 1\right]^T. \tag{4}$$

A workpiece, as a zero or extreme SF link [1–3], characterizes technological (output) errors. To determine T_l of the technological (output) l-th error, one finds an SF vector error, which, in turn, consists of errors in the base $d\vec{r}_b$ and position $d\vec{r}_n$.

$$\Delta\vec{r}_0(\beta,\rho)=d\vec{r}_b(\beta,\rho)+d\vec{r}_n, \tag{5}$$

where dr_b is the error of surface basing relative to the rated location due to small shifts and turns; dr_n is the position error.

Positional errors $d\vec{r}_n$ characterize the processes of long or medium action time, which, for example, can be caused by thermal deformities, aging processes of links [9–11].

In turn, the base errors $\vec{r}_b(\beta,\rho)$ due to the rated location are divided into small linear and angular displacements. So, the vector error of the end surface relative to the rated location, both for a workpiece and any other link, is found as follows [1]:

$$\Delta\vec{r}_0(\beta,\rho)=\varepsilon_b\cdot\vec{r}_0(\beta,\rho)+d\vec{r}_b\approx\varepsilon_b\cdot\vec{r}_0(\beta,\rho), \tag{6}$$

where ε_b is the general matrix of the input (output) errors of the link, which takes into consideration unknown small shifts and angular orientations relative to the rated CS [1×3]:

$$\varepsilon_b=\begin{bmatrix}0 & -\gamma_b & \beta_b & \delta_{bE}\\ \gamma_b & 0 & -\alpha_b & \delta_{by}\\ -\beta_b & \alpha_b & 0 & \delta_{bz}\\ 0 & 0 & 0 & 0\end{bmatrix}. \tag{7}$$

Elements of the general matrix ε_b characterize the angular orientation and small shifts around and along the coordinate axes of the link [1, 3].

During the machining of flat surfaces, the elements δ_{xb}, δ_{yb} and γ_b of the general matrix ε_b, which determine the small shifts along the *X, Y* axes and the angular orientation around the *Z* axis, lead only to the slip of the plane «by itself». The element δ_{zb} can be adjusted, for example, by moving the tail spindle of the grinding machine headstock.

Disregarding the infinitely small of the second order, equation (5) for determining the vector error of the base surface relative to the rated plane of the end can be represented in the following form:

$$\begin{aligned}\Delta\vec{r}_b(\beta,\rho)&=\varepsilon_b^*\cdot\vec{r}(\beta,\rho)=\\&=\left\{\delta_{bz}\cdot D^3+\alpha_b\cdot D^4+\beta\cdot D^5\right\}\cdot\vec{r}_0(\beta,\rho),\end{aligned} \tag{8}$$

where D^1, ..., D^6 are the matrices of errors of the individual components of displacements relative to the coordinate axes [1–3].

The *T* vector of output technological errors can be represented in the following way:

$$T=\left[\delta_{bz}\quad \alpha_b\quad \beta_b\right]^T. \tag{9}$$

5. 2. Investigating the calculation and experimental data

The problem of improving the accuracy of grinding flat surfaces was analytically solved by three different methods [5, 6, 8]:

a) by solving the vector equations of the surface of the circle and the family of generatrix;

b) by a variation method;

c) by using the matrices of transfer coefficients.

Consider each method:

a) Work [5] gives the machine tool SF taking into consideration the size of the links. This SF made it possible to analyze the machining of end surfaces at the flat end of the circle. The final accuracy is obtained at the output of the mathematical model of a workpiece from the machining zone with a shape-forming section of the profiled abrasive tool.

b) In studies [1, 6], the optimal accuracy is found in a more expedient way – through the variation of the machine tool SF. Elements of the variance matrix are found from the results of measurement, or calculation.

With a rational value of the angle ν, which is predetermined by the productive rough shaving, the angle γ_0 is adjusted in the horizontal plane in order to improve accuracy. The initial value of angle γ_0 can be selected through the rational γ/ν ratio, followed by value clarification.

Consequently, the desired vector of input *V* machine tool adjustment errors includes the following as its elements: the angle $\Delta\gamma$ of adjusting the orientation of grinding machine headstocks and the size Δz of the axial position of their tailgate spindles $\delta=\left[\Delta\gamma\quad \Delta z\right]^T$.

The profilogram in Fig. 4 is used to construct, through equal angles, or based on the above calculations, a vector of length *p* (where $p<k$) for peripheral points from the errors Δ, measured in the direction of normal to the end.

The input errors caused by the orientation of grinding machine headstocks at a point with parameters β, ρ are determined as a variation of SF:

$$\begin{aligned}\vec{V}_{B\gamma}(\beta,\rho)&=A^1(-R_b)\cdot A^6(\beta)\cdot A^3(-Z_c)\cdot A^1(-X_c)\times\\&\times A^2(-Y_c)\cdot D^4\cdot A^4(-\gamma)\cdot A^5(-\nu)\times\\&\times A^1(X_c)\cdot A^6(\theta(\rho))\cdot A^1(\rho)\cdot A^3(Z_{\max})\cdot\vec{e}^{\,4},\end{aligned} \tag{10}$$

$$\begin{aligned}\vec{V}_{Bz}(\beta,\rho)&=A^1(-R_b)\cdot A^6(\beta)\cdot D^3\cdot A^3(-Z_c)\cdot A^1(-X_c)\times\\&\times A^2(-Y_c)\cdot A^4(-\gamma)\cdot A^5(-\nu)\times\\&\times A^1(X_c)\cdot A^6(\theta(\rho))\cdot A^1(\rho)\cdot A^3(Z_{\max})\cdot\vec{e}^{\,4},\end{aligned} \tag{11}$$

where D^4, D^3 are the matrices of input errors around the *X* axis, and along the *Z* axis.

The elements of matrix *M* the size of $p\times 2$, caused by the orientation of grinding machine headstocks at the point of a workpiece with parameters β, ρ, are determined by mapping a vector error onto the direction of normal:

$$M=\left[\vec{V}_\gamma\quad \vec{V}_z\right]\cdot\vec{n}. \tag{12}$$

Solve the matrix equation $M\cdot\delta=\Delta$ [1, 6] that produces the errors sought:

$$\delta=\left(M^T\cdot M\right)^{-1}\cdot M^T\cdot\Delta. \tag{13}$$

For initial angles $\nu=1/400$, $\gamma=1.37\cdot\nu$, they received an adjusting error value $\delta=\left[0.001\quad -0.098\right]^T$.

c) in work [8], the problem is solved using a matrix of transfer coefficients.

In the form of a matrix product [1], the relationship between machine tool errors (the grinding machine headstock orientation angles) and output technological errors, which characterize deviations from the perpendicularity of the end, takes the form $T=W\cdot\delta$, or:

$$\begin{bmatrix}\alpha_b\\ \beta_b\end{bmatrix}=\begin{bmatrix}w_{\alpha\nu} & w_{\alpha\gamma}\\ w_{\beta\nu} & w_{\beta\gamma}\end{bmatrix}\cdot\begin{bmatrix}\nu\\ \gamma\end{bmatrix}=\begin{bmatrix}w_{\alpha\nu} & w_{\alpha\gamma}\\ w_{\beta\nu} & w_{\beta\gamma}\end{bmatrix}\cdot\begin{bmatrix}1\\ \gamma/\nu\end{bmatrix}\cdot\nu. \quad (14)$$

They derived the following value:

$$W=\begin{bmatrix}-0.002 & -0.0014\\ -0.915 & 0.583\end{bmatrix}.$$

5. 3. Correction of machine tool errors

Consider the correction of angles for each method:

a) The optimal ratio of the headstocks' orientation angles [5] was determined stepwise by jointly solving the vector equations of the family of generatrices 6.1 of the billet to the surface of abrasive tool (Fig. 1–3). The comparison was carried out along the *Z* coordinate, which is responsible for the axial direction. Such a ratio of orientation angles was considered the best at which the difference in coordinates took the minimum value.

Changing the orientation of the angle γ reduces the error in the end's shape formation. It is this angle that determines the choice of the shape-forming section, which would provide the least value of the end machining error.

b) Find the correction of angle $\Delta\gamma$ for angles ν, γ_0. The following equation determines the optimal ratio of the headstocks' orientation $\gamma/\nu=1.57$ (Fig. 3):

$$\frac{\gamma_k}{\nu}=\frac{\gamma_0+\Delta\gamma}{\nu}\approx 1.58. \quad (15)$$

We acquired profilograms of adjusted surfaces (Fig. 4), the errors of which did not exceed the value of 2 µm for a wide range of change in the angles γ, ν.

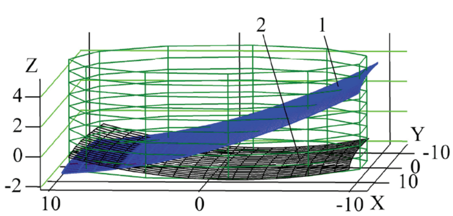

Fig. 3. Starting position 1 and the adjusted end 2

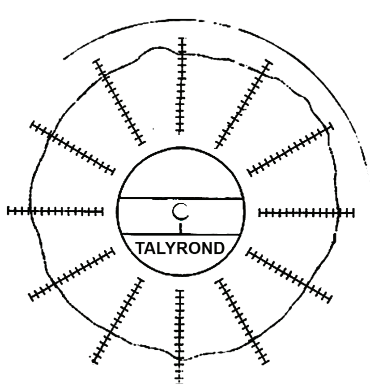

Fig. 4. Profilogram of the machined end surface

In addition to the errors caused by the perpendicularity of the position of the machined end, which is estimated by angle $\sqrt{\alpha_b^2+\beta_b^2}$, the mathematical model makes it possible to determine the degree of scattering around the base plane through the variance of σ^2 (Fig. 4):

$$\sigma^2=\frac{1}{S}\cdot\iint_s\left[\Delta r_n(\beta,\rho)-\Delta r_{b.n}(\beta,\rho)\right]^2 \mathrm{d}S, \quad (16)$$

where the projection $\Delta r_n(\beta,\rho)$ of the output vector error onto the normal for the measured points, as well as $\Delta r_{bn}(\beta,\rho)$ for the base plane, is determined at the end point; the ratio of areas $\mathrm{d}S/S$ can be considered as an element of probability [6].

c) Analyzing the obtained value $W=\begin{bmatrix}-0.002 & -0.0014\\ -0.915 & 0.583\end{bmatrix}$, [8], we can draw the following conclusions on the correction of angular errors:

1. The $W_{\alpha\nu}$ and $W_{\beta\nu}$ matrix element shows how strongly the angle of orientation ν of the grinding head (or γ) affects the value of the angular error of the position of the base plane α_b. The total effect of the angle ν on a change in the angles α_b, β_b, that is the deviation from the perpendicularity of the end face, is estimated as $\sqrt{W_{\alpha\nu}^2+W_{\beta\nu}^2}$.

2. The effect of the orientation angles γ, ν of the grinding machine headstocks on the output errors is opposite, so changing the value of one of these angles can be partially compensated for by the other.

3. The elements of a matrix of transfer coefficients make it possible to determine the ratio of the angles when the initial error of shape formation takes a minimum value. The sum of the output point squares, which determines the resultant when oriented around the center of the spherical finger, should be minimal $\alpha_b^2+\beta_b^2\to\min$.

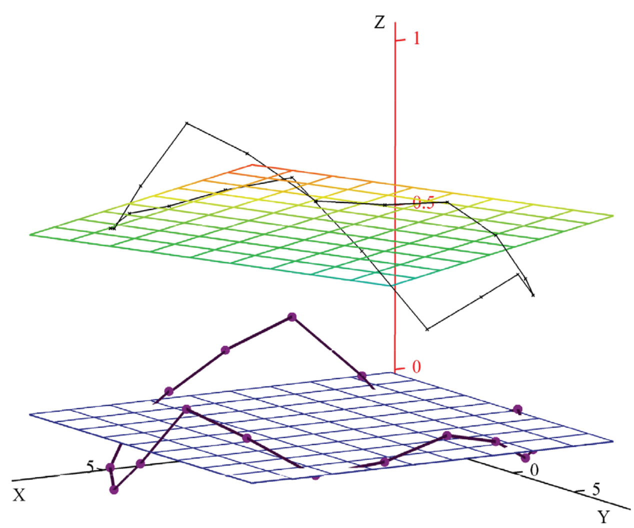

Fig. 5. The result of calculating the corrected position of the rms base plane

The total angular error of the shape formation of the end is determined from the following equation:

$$\Lambda(\gamma.\nu)^2=\left[w_{\alpha\nu}\cdot\nu+w_{\alpha\gamma}\cdot\gamma\right]^2+\left[w_{\beta\nu}\cdot\nu+w_{\beta\gamma}\cdot\gamma\right]^2\to\min. \quad (17)$$

The chart of this function is shown in Fig. 6.

Upon finding and making the derivative zero, one derives the optimal ratio of the orientation angles of grinding machine headstocks, which provides the best accuracy of the shape formation of ends:

$$\frac{\gamma}{\nu}=-\frac{w_{\alpha\nu}\cdot w_{\alpha\gamma}+w_{\beta\nu}\cdot w_{\beta\gamma}}{\left(w_{\alpha\gamma}\right)^2+\left(w_{\beta\gamma}\right)^2}=1.57. \quad (18)$$

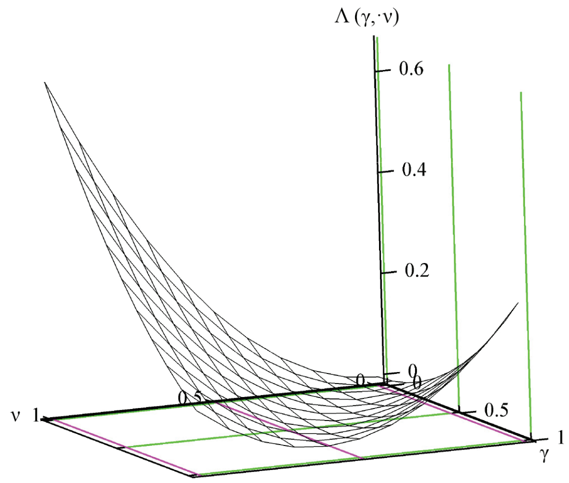

Fig. 6. Ratio of the orientation angles of a grinding machine headstock, which determines the minimum error of shape formation

Our study results are in good agreement with those reported in [4–6] and could be used to diagnose the grinding machine performance.

The resulting expressions can be generalized for other mated parts of machine tool structural units [10–12].

The input errors that occur when basing a part in the hole of the drum that feeds blanks are determined similarly:

$$\vec{V}_{D\gamma}(\beta,\rho) = D^4 \cdot A^1(-R\ ) \cdot A^6(\beta) \cdot A^3(-Z_c) \cdot A^1(-X_c) \times$$
$$\times A^2(-Y_c) \cdot A^4(-\gamma) \cdot A^5(-\nu) \times$$
$$\times A^1(X_c) \cdot A^6(\theta(\rho)) \cdot A^1(\rho) \cdot A^3(Z_{\max}) \cdot \vec{e}^{\,4}. \quad (19)$$

In a matrix form, the ratio involving the technological errors takes a similar form to equation (14).

The angular error is determined by the total transfer coefficient equal to 1.09, while these coefficients for the orientation angles of grinding machine headstocks ν and γ in the vertical and horizontal planes accepted lower values and could be partially compensated.

At a certain value of the angular error φ, the tangent of which is determined by the ratio of the gap to the length of the base in the drum of workpiece feed, the deviation from the perpendicularity of the end may be even greater. In addition, the presence of gaps can cause fluctuations in the movements of the blanks and lead to a deterioration in the quality of machining and a decrease in the stability of the tool.

This error is the largest of all the considered, which characterizes the deviation of the base surface from perpendicularity and cannot be corrected due to random nature. Therefore, reducing the error requires improving the base of blanks in the drum holes, for example, by improving the structure through force fixation, etc.

Errors in the machining of the end surface occur due to deviations in the position of the drum of the workpiece feed from the rated one. This can be caused by force exposure with insufficient spatial static and dynamic rigidity of the elastic system of the machine tool [13]. Such errors are determined in the same way as the above procedure [14].

This error, caused by inaccuracies in the assembly of the machine tool, may appear due to the force deformations of the feed drum, or gaps in its supports.

6. Discussion of results of studying the impact of shape formation on the accuracy of end grinding

The contact is limited by the position of the analytically defined contact line, rather than an extreme edge, as has been the case in previous studies. CL that is used to calculate the accuracy of the developed SF (3) θ(ρ), takes into consideration the techniques of tool dressing or tool wear. The developed SF leads to the change of equations (8), (11), (12), and others. In addition, for other machine tools, this can lead to a change in the area *S* of the machined surface area, taken into consideration by equation (1).

Several calculation methods have been tested, namely:

a) solving vector equations of the surface of a circle with a family of generatrix;

b) the variation of a shape formation function;

c) finding the matrix of transfer coefficients.

Each method has advantages. Thus, the variation of SF (3) makes it possible to take into consideration the variance of the machined surface while the transfer coefficients (14) demonstrate a relative simplicity of determining and the possibility to calculate other mated parts of machine tool structural units.

Experimental studies do not contradict our calculations. The peak in the profilogram likely corresponds to the final deformation of a workpiece through the fastening with a screw in the feed drum window (Fig. 4).

Mutual compensation for errors and mated parts in machine tool structural units has been achieved. The optimal ratio (15) and (18) between the orientation angles of grinding machine headstocks is ensured. The found error in the results from theoretical calculations by different methods does not exceed 1.5 %. The experimental findings do not contradict the estimated data (Fig. 3–5).

The content of the elements of matrices of variation and transfer coefficients has been substantiated. Our comparisons of the orientation angles of the grinding machine headstock relative to the drum of blank feed showed close results.

The constraints are the range of change in the angles γ, ν, which are due to the limit values of the allowance and, accordingly, the maximum performance.

The limitation of this study is the absence of unambiguous angular fixation of parts during the machining and measurement of the profile. This should be eliminated, which would lead to a more objective nature of the data obtained.

Our results could be used as diagnostic signals to find sources of machine tool errors. The findings could be generalized both for the mated parts in machine tool structural units and for other models of machine tools.

7. Conclusions

1. A shape formation function determines the final accuracy of machining. We have improved the shape formation function by refining the position of the contact line. The effect on accuracy is due to a change in the position of the contact line, which is predetermined by the orientation angles, or other machine tool errors. Another factor influencing accuracy is resizing the contact area.

2. Achieving the required accuracy of grinding flat surfaces can be checked using one of the following methods: by solving vector equations; through the variation of a shape

formation function; with the help of a matrix of transfer coefficients. The method of transfer coefficients has confirmed the relative simplicity, versatility, and convenience of correcting machine tool errors.

3. The optimal ratio between the orientation angles of the grinding machine headstock as a machine tool structural unit has been derived, which is equal to 1.57. The error of the estimation and experimental data is within 1.5 %.

References

1. Reshetov, D. N., Portman, V. T. (1986). Tochnost' metallorezhuschih stankov. Moscow: Mashinostroenie, 336.
2. Schwenke, H., Knapp, W., Haitjema, H., Weckenmann, A., Schmitt, R., Delbressine, F. (2008). Geometric error measurement and compensation of machines – An update. CIRP Annals, 57 (2), 660–675. doi: https://doi.org/10.1016/j.cirp.2008.09.008
3. Tian, W., Gao, W., Zhang, D., Huang, T. (2014). A general approach for error modeling of machine tools. International Journal of Machine Tools and Manufacture, 79, 17–23. doi: https://doi.org/10.1016/j.ijmachtools.2014.01.003
4. Rudyk, A. V., Rudyk, V. A. (2010). Produktyvnist obrobky tortsiv na tortsevoshlifuvalnykh verstatakh. Visnyk ChDTU, 45, 57–67.
5. Kalchenko, V. V. (2005). Modulne 3D modeliuvannia formoutvoriuiuchykh system shlifuvalnykh verstativ, instrumentiv ta obrobliuvanykh poverkhon. Visnyk Ternopilskoho derzhavnoho tekhnichnoho universytetu, 12 (2), 68–79.
6. Rudyk, A. V., Venzheha, V. I. (2008). Formoutvorennia tortsiv detalei avtomobiliv pry dvostoronnomu shlifuvanni. Visnyk ChDTU, 34, 80–88.
7. Srivastava, A. K., Veldhuis, S. C., Elbestawit, M. A. (1995). Modelling geometric and thermal errors in a five-axis cnc machine tool. International Journal of Machine Tools and Manufacture, 35 (9), 1321–1337. doi: https://doi.org/10.1016/0890-6955(94)00048-o
8. Rudyk, A. V. (2011). Vplyv pokhybok naladky verstatu modeli 3342 ADO na pokhybku formoutvorennia ploskykh poverkhon. Vestnik KhPI, 53, 94–106.
9. Kalchenko, V., Venzheha, V., Sliednikova, O., Kalchenko, D. (2016). Theoretical and experimental investigations process stock removal, wheels wear, forming accuracy and thermal voltage at grinding of ends part. Technical Sciences and Technologies, 4 (6), 25–34.
10. Ramesh, R., Mannan, M. A., Poo, A. N. (2000). Error compensation in machine tools – a review. Part I: geometric, cutting-force induced and fixture-dependent errors. International Journal of Machine Tools and Manufacture, 40 (9), 1235–1256. doi: https://doi.org/10.1016/s0890-6955(00)00009-2
11. Denis Ashok, S., Samuel, G. L. (2016). Regression method for identifying spindle radial errors of a miniaturized machine tool. Journal of Studies on Manufacturing, 1 (1), 26–33.
12. Chen, G., Liang, Y., Sun, Y., Chen, W., Wang, B. (2013). Volumetric error modeling and sensitivity analysis for designing a five-axis ultra-precision machine tool. The International Journal of Advanced Manufacturing Technology, 68 (9-12), 2525–2534. doi: https://doi.org/10.1007/s00170-013-4874-4
13. Chupryna, V. M. (2016). Tensor-mathematical modeling of machining accuracy in dynamic system of catting machine. Zbirnyk naukovykh prats Kharkivskoho universytetu Povitrianykh Syl, 1, 116–124.
14. Aurich, J. C., Biermann, D., Blum, H., Brecher, C., Carstensen, C., Denkena, B. et. al. (2008). Modelling and simulation of process: machine interaction in grinding. Production Engineering, 3 (1), 111–120. doi: https://doi.org/10.1007/s11740-008-0137-x