UDC 514.18
DOI: 10.15587/1729-4061.2021.227691

MODELING MATED SURFACES WITH THE REQUIRED PARAMETERS

Nelli Ismailova
Doctor of Technical Sciences, Professor*
E-mail: Nelly969@ukr.net
Valentyn Bogach
PhD, Associate Professor, Head of Department
Department Technology of Materials and Ship Repair
National University «Odessa Maritime Academy»
Didrichson road str., 8, Odessa, Ukraine, 65029
E-mail: valent57bog@gmail.com
Borys Lebedev
PhD, Associate Professor*
E-mail: prohojiygoogl@gmail.com
Nataliia Oliinyk
PhD
Department of Construction Production Technology
Odessa State Academy of Civil Engineering and Architecture
Didrichsona str., 4, Odessa, Ukraine, 65029
E-mail: natali.1727v@gmail.com
Serhii Manakov
PhD
Department of Information Technologies
State University of Intellectual Technologies and Communications
Kuznechna str., 1, Odessa, Ukraine, 65029
E-mail: manakov@onat.edu.ua
*Department of Engineering Mechanics
Odessa Military Academy
Fontanska doroha str., 10, Odessa, Ukraine, 65009

This paper has proposed improving the methods of circular and screw conversion, to be used in the design of cutting tools and toothing that include complex mated surfaces. Underlying the improvement of both methods is the construction of a mathematical base and the development of a computer subprogram, based on it, in the MATLAB system.

During the research, the original screw-type curved surface and the curvilinear generatrix axis were formed on the basis of improved methods, taking into consideration the exclusion of interference at the design stage.

A comprehensive solution to this problem is important for the manufacture of products by rolling. Given this, the original instrumental surface of the cutting tool takes into consideration the pairing condition between the article's and tool's points.

The result, when designing gears and cutting tools using the proposed improved methods, assigns the curvilinear surface parametrically, represented by two-dimensional arrays characterizing its coordinates. To avoid interference at the design stage, it is necessary to analyze the intersection of the axis of the curvilinear generatrix with horizontal planes. That would make it possible, when machining an article, to avoid cutting, jamming, as well as the dangerous concentration of stresses. The accuracy and reliability of a wide range of articles in machines and machinery and other kinematic pairs also improve.

The proposed improvement of circular and screw conversion methods to simulate curvilinear mated surfaces that exclude interference at the design stage is of practical interest in machine building

Keywords: mated surfaces, interference, toothing, geometric parameters, circular and screw methods

Received date 09.02.2021
Accepted date 06.04.2021
Published date 20.04.2021

How to Cite: *Ismailova, N., Bogach, V., Lebedev, B., Oliinyk, N., Manakov, S. (2021). Modeling mated surfaces with the required parameters. Eastern-European Journal of Enterprise Technologies, 2 (1 (110)), 21–26. doi: https://doi.org/10.15587/1729-4061.2021.227691*

1. Introduction

Resolving the tasks related to the design of cutting tools and toothing requires a comprehensive improvement in product quality and labor productivity in all sectors of the national economy. A special role belongs to machine building, which is intended to provide the national economy with modern machines and equipment.

One of the hallmarks of the modern stage of technological progress in mechanical engineering is the application of high-performance machining techniques. The high performance and precision of the cutting tool design allow it to be used to machine the parts with complex screw-class surfaces that are increasingly applied in mechanical engineering. The main step in creating a cutting tool is to get its profile (profiling) and eliminate interference when machining an article.

The development of the theoretical foundations of profiling is inextricably linked to two extensive applications of the envelope theory, namely profiling the metal cutting tool and the design of toothed gears. The nominal surface of a component and the producing surface of the tool are mutually conjugated. Mated surfaces are subject to a variety of requirements dictated by the envelope theory applications. One of the most important is the requirement for lack of interference; it is a prerequisite for profiling a metal cutting tool because it concerns not mathematical objects but actual bodies limited by conjugated surfaces.

2. Literature review and problem statement

The results of the interference studies of mated surfaces appeared a little later than the methods of profiling the metal cutting tool and the design of gears were constructed. It is shown that when developing a method of designing conjugated surfaces it was assumed that interference was not present when these surfaces came into contact.

In paper [1], the phenomenon of interference is presented as follows: «interference occurs when a part of the space is located within the volumes of two bodies at the same time». If one of the bodies is a cutting tool and the other is a part, the interference then leads to cutting (reduced accuracy, inability to machine). If these bodies are toothed wheels, the consequence of interference is jamming. Interference-related tasks have arisen from practice and resolving them is of great practical importance; moreover, it enriches the theory of conjugated surfaces, which is unthinkable without its extensive applications. The invariant method and algorithm for excluding the interference of conjugated non-linear surfaces are based on the theorem given in [2]. A new definition of interference was given: «interference is any improper touch of profiles outside the active areas of the toothing line, that is, the phenomenon when the trajectory of the edge of one tooth in its relative motion crosses the profile of the conjugated tooth». However, the definition of interference involved the construction of a large number of axoids, which makes the relevant studies impractical in actual use.

In study [3[, the author considered the problems on the curvilinear surface with special points. The position of a special point is at the surface with the edge of the return. At the same time, the issues of avoiding possible interference, where the required nominal surface of the part cannot be obtained completely, remained unresolved.

Work [4] reports the results of studying a model of digging working bodies when working the land. It is shown that the present interference was useful.

Of interest is work [5] that explores the possibility of no interference in the fabrication of silent and compact transmissions with large torque. However, the proposed notion of interference is generalized.

Of theoretical interest is study [6] whose authors use two-linear coordinates of the method of kinematic synthesis of spatial gears with sloping axes. This approach makes it possible to show that the basic screws of a system can be identified through the extreme values of the step, that the relevant research is impractical, the method is laborious for an ordinary design engineer.

Studies [7, 8] report the systems of toothed transmission, which monitor the occurrence of various types of nonlinear phenomena, such as bifurcation and chaotic reaction. Their authors describe a control system to eliminate chaotic behavior in the dynamic transmission system using external control excitation involving the Melnikov method. The method is used to develop a practical model of the toothed wheel system to control and eliminate chaotic behavior. To that end, non-linear dynamic model of a cylindrical toothed pair with time-changing rigidity and static transmission error was built. Using a non-feedback control method to eliminate chaos, additional control excitation is applied. Objective difficulties in the analytical approach to the elimination of chaos in the toothed system are eliminated using external control excitation involving the Melnikov method. Then the accuracy of theoretical forecasts, as well as the performance of the proposed control system, may contradict numerical modeling. The study was intended to test the validity of theoretical conclusions.

Interesting results about machining the non-circular gear wheels are reported in [9[. However, the method used has many limitations, including the impossibility to machine inner gears or non-round outer gears with a concave step curve, which gives a high probability of cutting when machining uncircular gears of greater curvature with fewer teeth.

The option of overcoming the difficulties in detecting interference may be the use of a method for determining the rigidity of the toothing, under load and contact stresses in the teeth, taking into consideration the accuracy of their profile [10]. The rigidity of one tooth is calculated by the method of potential energy. Contact tension is studied with the help of the classical theory of resilient contact by Hertz. Two cases are presented to test the model of rigidity of the gear transmission. The effect of the load on the leading crown of the tooth and the applied torque on the rigidity of the gear toothing is analyzed. The reason for this may be the rigidity of the mesh decreases, and the error of load transfer and maximum contact tension on the tooth grow with an increase in the relief of the leading crown of the tooth.

Paper [11] suggests the methodology for predicting the dynamic wear of parts, taking into consideration the dynamics of their operation. The worn surfaces are represented in the form of modulated grid excitation and are introduced into a dynamic model to study the effect of surface wear on the dynamic characteristics of the system. The forces of the dynamic gear meshing are converted into equivalent load using the Miner rule to detect the effect of dynamics on wear behavior. The simulation shows that surface wear and the gear dynamics are highly interconnected.

A noteworthy study reported in [12] considered the contact surface of kinematic pairs. The noted techniques of constructing mated surfaces most often focus on a certain range of tasks that are successfully resolved under certain limitations. Implementation of the enhanced technological requirements to improve the reliability of article operation can be achieved by developing new invariant techniques for designing kinematic pairs in order to eliminate interference.

3. The aim and objectives of the study

The aim of this study is to model conjugated curvilinear surfaces based on a condition for the lack of interference. This would allow one to design transmissions that could be better, more reliable, and longer in operation.

To accomplish the aim, the following tasks have been set:

– to perform a circular and screw conversion analysis to enable modeling the original curvilinear surface relative to the profile of the required curved mated shapes;

– to modernize the methods of circular and screw conversion, which would make it possible to eliminate interference at the design stage of conjugated kinematic pairs.

4. The study materials and methods

When modeling curvilinear mated surfaces, we consider modeling based on the circular and screw conversion methods as a basis for developing a computer subprogram in the MATLAB system.

It is proposed to modernize the methods of circular and screw conversion, to design the kinematic pairs of the required shape and the accuracy of cutting tool fabrication using a parametrically assigned curvilinear surface and an axis of the curvilinear generatrix.

Determining the shape of conjugated kinematic pairs' surfaces geometrically would make it possible to design curvilinear surfaces with greater accuracy and efficiency.

5. The analysis of circular and screw conversion methods using level lines

5. 1. The analysis of a circular conversion method

To form the original curved surface, one needs to build the original instrumental surface of the cutting tool relative to an article's profile. The study process represents a screw movement around a predefined axis and a translational motion along it, that is, the profile of an article must be shifted along the axis at a certain step while rotating at the predefined angle.

Using level lines to rotate the curvilinear surface around the curvilinear generatrix axis was very inconvenient given that one needs to derive the coordinates for the points of the rotated surface. When one creates lines, one can acquire the coordinates *X* and *Y*, which, however, is inconvenient because of the format of the *contEP* array whose dimensionality is <2×2772 *double*>. The levels' values are defined by the *hEP* array that does not include the explicit values for the *Z* coordinate.

In this case, one can still derive the *Z* coordinate using the array of levels ω*r*, but the values contained in it do not match those contained in the *Zp* array of the original curvilinear surface.

In addition, the resulting curvilinear surface is just a set of the level lines rotated around the curvilinear generatrix axis, which does not allow any modifications to the resulting curvilinear surface, which is not actually a surface.

Subsequently, the use of level lines would not make it possible to consider the intersection of the original and rotated surfaces required in the formation of a curvilinear surface.

5. 2. The analysis of a screw conversion method

The disadvantages of the level lines were detected in the circular conversion, which explicitly manifested themselves when devising a screw conversion method.

Since the *hEP* array does not explicitly contain the *Z* coordinate, it is not possible to change the *Z* coordinate at the assigned step of the screw conversion. In the method development, both the original curvilinear surface and the axis of the curvilinear generatrix around which the surface rotates should have been moved at the predefined step. This leads to more computation but does not affect the required result – the rotated surface remains a set of level lines, which, in turn, does not allow for any modifications with it.

6. Improving the circular and screw conversion methods

6. 1. A circular conversion method

We shall consider the resulting datasets for the built curvilinear surface, the system of equations (1), and the axis of the curvilinear generatrix (2):

$$\begin{cases} x = a\cdot\omega\cdot\cos\vartheta, \\ y = b\cdot\omega\cdot\sin\vartheta, \\ z = 0.5\cdot\omega^2, \end{cases} \tag{1}$$

where $0 \le \omega \le 0.5 \le \vartheta \le \pi$, $a=3$, $b=2$.

$$\begin{cases} x = x_0 + t\cdot\sin t, \\ y = y_0 + t\cdot\cos t, \\ z = z_0 + c\cdot t, \end{cases} \tag{2}$$

where $c = H/2\pi$; $0 \le t \le 2\pi$, $H=5$, $x_0=5$, $y_0=-25$, $z_0=-2$.

The parametric systems of the curvilinear surface equations are generally written in the following form:

$$\begin{cases} x = f_x(\omega,\vartheta), \\ y = f_y(\omega,\vartheta), \\ z = f_z(\omega,\vartheta), \end{cases} \tag{3}$$

where $\omega_{\min} \le \omega \le \omega_{\max}$, $\vartheta_{\min} \le \vartheta \le \vartheta_{\max}$.

The coordinates *x*, *y*, and *z* are two-dimensional arrays. At the same time, the number of lines in two-dimensional arrays coincides with the dimensionality of the ω array, and the number of columns – with the dimensionality of the ϑ array.

The following arrays are built when forming the surface of a paraboloid (1):

ω*p*=(0:0.05:5) dimensionality <101×1 *double*>;

ω*p*=[0:0.05*pi:pi] dimensionality <1×21 *double*>;

Xp=*ap**ω*p**cos(ϑ*p*) dimensionality <101×21 *double*>;

Yp=*bp**ω*p**sin(ϑ*p*) dimensionality <101×21 *double*>;

Zp=0.5*ω*p*.^2**ones*(*size*(ϑ*p*))
dimensionality <101×21 *double*>.

The values in the lines of the *Zp* array are equal, that is:

$$Z(\omega,\vartheta) = \begin{pmatrix} z_{11} & z_{12} & \dots & z_{1n} \\ z_{21} & z_{22} & \dots & z_{2n} \\ \dots & \dots & \dots & \dots \\ z_{m1} & z_{m2} & \dots & z_{mn} \end{pmatrix}, \tag{4}$$

where

$$z_{11} = z_{12} = \dots = z_{1n},$$
$$z_{21} = z_{22} = \dots = z_{2n},$$
$$\dots$$
$$z_{m1} = z_{m2} = \dots = z_{mn}.$$

Therefore, instead of an array of *ur* levels, one should use values of the lines in the *Zp* array.

```
leni=length(up);
for j=1:1
   for i=1:leni
      ur(i)=Zp(i,j)
   end
end
```

Instead of using the ω*r* level array, it is advisable to use values of the lines of the *Zp* array.

Using the ω*r* array, which contains the values of the *Zp* array, would make it possible to rotate the coordinate points (*Xp, Yp, Zp*) around the point of intersection of the curvilinear generatrix axis with a horizontal plane passing through the (0, 0, Zp) coordinates. In this case, one does not need to determine the minimum and maximum value of the Z coordinate of a curvilinear surface; rotating the points of the curvilinear surface is executed from the system of equations (3) (Fig. 1–3).

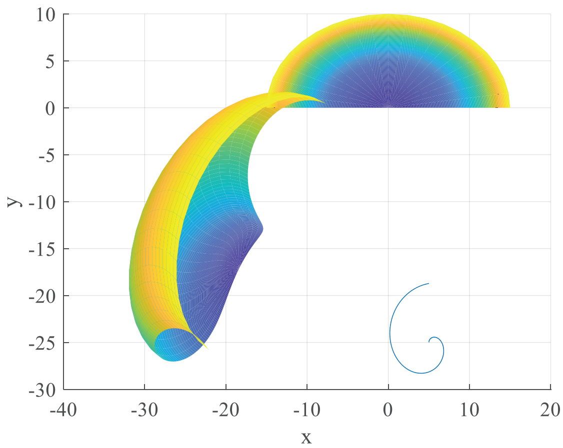

Fig. 1. Mapping onto the XY plane

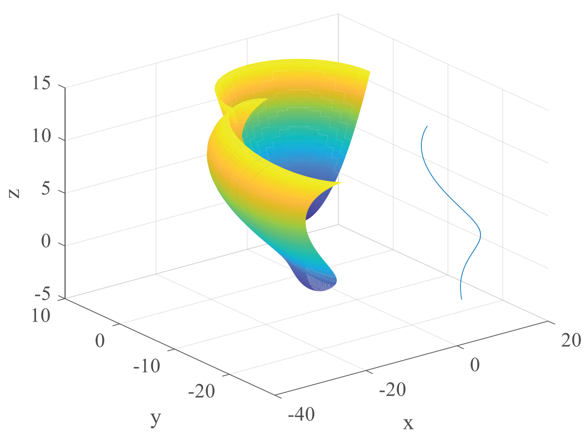

Fig. 2. An elliptical paraboloid rotated at 15° around point $C(-10, -15, 0)$

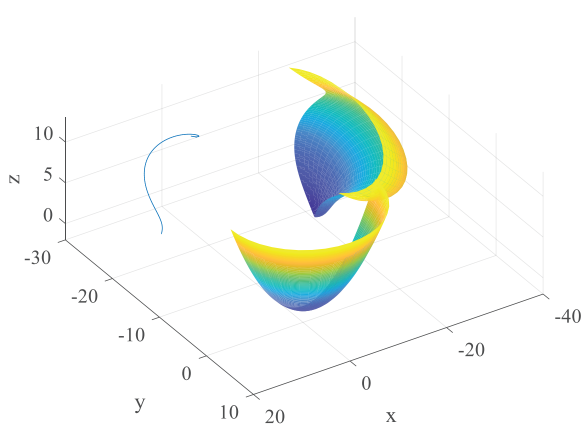

Fig. 3. An elliptical paraboloid rotated at 60° around point $C(-10, -15, 0)$

Given the confirmation from the system of equations (5) [12], and assuming that the curvilinear surface can be set by any other parametric equation, we modify the method of circular conversion in the following way.

Setting parameters of an elliptical paraboloid, rotated at 60° around the conical screw curve:

$$\begin{cases} x' = (a\omega\cos\vartheta - x_0)\cos\theta - (b\omega\sin\vartheta - y_0)\sin\theta + x_0, \\ y' = (a\omega\cos\vartheta - x_0)\sin\theta + (b\omega\sin\vartheta - y_0)\cos\theta + y_0, \\ z' = 0.5\omega^2. \end{cases} \quad (5)$$

```
Angle=pi/3;
Zpr=Zp;
for i=1:lenStr
    for j=1:lenStb
        x0=xpp(i);
        y0=ypp(i);
        Xpr(i,j)=(Xp(i,j)–x0)*cos(Angle)–
–(Yp(i,j)–y0)*sin(Angle)+x0;
        Ypr(i,j)=(Xp(i,j)–x0)*sin(Angle)+
+(Yp(i,j)–y0)*cos(Angle)+y0;
    end
end
```

The method of circular conversion in the specified manner makes it possible to obtain a result similar to the result shown in Fig. 1–3.

6. 2. A screw conversion method

Consider a screw conversion method based on the above improved circular conversion method.

In addition to the rotation angle, the screw conversion method sets a step of the shift. When the circular conversion method is implemented, the curvilinear surface can be obtained using the level lines, which makes it impossible to perform any operations involving them other than rotation.

Hence, based on the theorem given in [2], «Let the curvilinear surface Σ_1 be obtained by a generalized screw conversion of surface Φ_1 relative to the curvilinear line $m(u)$ with functions $\varphi(\sigma, \tau)$ and $h(\sigma, \tau)$. Then there is the surface Φ_2, which, in a generalized circular conversion around the same curvilinear line $m(u)$ with the same function $h(\sigma, \tau)$, forms the curvilinear surface Σ_2», we first move the original curvilinear surface at a predefined step and then perform a circular conversion resulting from the movement of the curvilinear surface. Under the sequence of operations described, the curvilinear generatrix axis should also be moved at a step. Let the curvilinear surface be rotated at an angle of 45° and shifted by 5 mm.

We shall change the code for implementing the circular conversion method as follows.

Setting parameters of an elliptical paraboloid, rotated at 45° around a conical screw curve with a step of 5 mm:

```
Angle=pi/4;
Sdvig=5;
Zpr=Zp+Sdvig;
for i=1:lenStr
    for j=1:lenStb
        x0=xpp(i);
        y0=ypp(i);
        Xpr(i,j)=(Xp(i,j)–x0)*cos(Angle)–
–(Yp(i,j)–y0)*sin(Angle)+x0;
        Ypr(i,j)=(Xp(i,j)–x0)*sin(Angle)+
+(Yp(i,j)–y0)*cos(Angle)+y0;
    end
end
```

We shall obtain the result of the construction shown in Fig. 4.

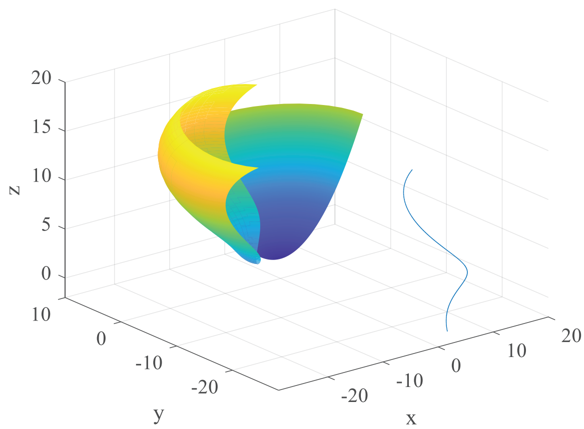

Fig. 4. An elliptical paraboloid rotated at 45° around a conical screw curve with a 5 mm increment

Improved methods have made it possible to analyze the intersections of the axis of the curvilinear generatrix with horizontal planes and move away from the use of lines of the level of a curvilinear surface and work with the values of its coordinates. This would eliminate interference at the design stage of conjugated curvilinear surfaces and improve the operational quality of kinematic pairs.

7. Discussion of results of studying the improved screw and circular conversion methods

Based on the improved circular and screw conversion methods, this study shows the possibility of determining a curvilinear surface using the coordinates of the original surface. We give the mathematical justifications for the rotation of the coordinates of the parametrically defined curvilinear surface and the curvilinear generatrix axis.

The calculation of the crossing points of the curvilinear generatrix axis with horizontal planes is carried out as follows:

```
[t0]=([ωr]-z0v)/cv;

xpp=x0ϑ+sin(t0).*t0;

ypp=y0ϑ+cos(t0).*t0.
```

The curvilinear surface points rotate according to formula (5). Setting parameters of an elliptical paraboloid, rotated at 15° around point *C*(–10, –15, 0):

```
Angle=pi/12;
x0=–10;
y0=–15;
Xpr=(ap*ωp*cos(ϑp)–x0)*cos(Angle)–
–(bp*ωp*sin(ϑp)–y0)*sin(Angle)+x0;
Ypr=(ap*ωp*cos(ϑp)–x0)*sin(Angle)+
+(bp*ωp*sin(ϑp)–y0)*cos(Angle)+y0;
Zpr=0.5*ωp.^2*ones(size(ϑp)).
```

Taking into consideration a change in the center of rotation at each level, one should modify the code above using two *for* loops.

Setting parameters of an elliptical paraboloid, rotated at 60° around a conical screw curve:

```
Angle=pi/3;
Zpr=0.5*ωp.^2*ones(size(ϑp));
for i=1:lenStr
  for j=1:lenStb
            x0=xpp(i);
            y0=ypp(i);
            Xpr(i,j)=(ap*ωp(i)*
*cos(ϑp(j))–x0)*cos(Angle)–(bp*ωp(i)*
*sin(ϑp(j))–y0)*sin(Angle)+x0;
            Ypr(i,j)=(ap*ωp(i)*
*cos(ϑp(j))–x0)*sin(Angle)+(bp*ωp(i)*
*sin(ϑp(j))–y0)*cos(Angle)+y0;
  end
end
```

We shall rotate each level line at an angle of 45° around the corresponding crossing point of the curvilinear axis relative to the *Z* axis. To this end, we shall use the *RotateDifferentCenter*(*lines, xC, yC, zC, length, angle*) function, created in the development of a circular conversion method, where *lines* is the set of level lines, *xC, yC, zC* are the coordinates of the crossing points of a curvilinear axis and the horizontal level plane, *length* is the array length, *angle* is the rotation angle.

```
function RotateDifferentCenter(lines, xC, yC, zC,
length, angle)
   for i=1:length
      rotate(lines(i), [0 0 1], angle, [xC(i) yC(i) zC(i)])
   end
end
```

The rotation function is requested in the Command Window.

```
RotateDifferentCenter(hEP,xpp,ypp,ωr,lenUr,45).
```

The result of the conversion is shown in Fig. 5.

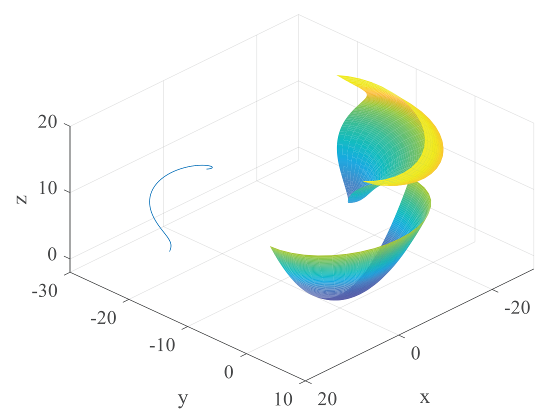

Fig. 5. An elliptical paraboloid rotated at 60° around a conical screw curve with a 5 mm increment

When determining the curvilinear surface by the coordinates of the original surface, it should be understood that the coordinates of the points of the projected curvilinear surface, owing to the use of the proposed arrays, would be in a certain ratio to the coordinates of the points of the original surface.

This eliminates interference and helps obtain the required shape of the projected surface of the cutting tool and toothing. It is important to note the following results:

– the levels' values are aligned with the *z* coordinate values of the original curvilinear surface, which has a positive effect on the dimensionality of the arrays;

– the coordinates of the curvilinear surface by the coordinates of the original surface are independent and can accept arbitrary values;

– all curvilinear surface parameters can be fixed except for one, and this parameter can be arbitrarily changed to adjust an article designed. The parameters of the changed surface are an attribute of the desired object.

To eliminate interference at the design stage of kinematic pairs, it is necessary that the parameters of the curvilinear surface should be within a certain ratio with the coordinates of the original surface points. Optimal profiling of mated curvilinear surfaces with the required parameters regarding the accuracy of their shapes would make it possible, when machining an article, to avoid cutting, jamming, the dangerous concentration of stresses, and, therefore, to improve the reliability and ensure high efficiency.

The proposed study should be developed for effective modeling of the toothing and cutting tool, where it is necessary to determine the curvilinear surface of mated pairs in relation to spatial mechanisms. The research can continue towards the automation and development of software for the manufacture of specific articles on machine tools with numerical control.

8. Conclusions

1. Calculating the crossing points of the curvilinear generatrix axis with horizontal planes would reveal interference, due to which it becomes possible to adjust the curvilinear surface of an article. The analysis data have been implemented in the form of an application program and are available for engineering work.

2. The proposed variant of the improvement of the methods of circular and screw conversion of the original curvilinear surface makes it possible to obtain coordinates of the curvilinear surface and the curvilinear generatrix axis without auxiliary curvilinear surfaces.

References

1. Podkorytov, A. M. (2000). Iteratsiynyi metod ta alhorytm vykliuchennia interferentsiyi skladnykh spriazhenykh poverkhon za napered zadanymy umovamy. Prykladna heometriya ta inzhenerna hrafika, 64, 109–113.
2. Podkorytov, A., Ismailova, N. (2016). General method for iterative interference exceptions connected surfaces kvazivintovyh. Suchasni problemy modeliuvannia, 5, 98–103. Available at: http://magazine.mdpu.org.ua/index.php/spm/article/view/1488
3. Havrylenko, Y., Kholodniak, Y., Vershkov, O., Naidysh, A. (2018). Development of the method for the formation of one-dimensional contours by the assigned interpolation accuracy. Eastern-European Journal of Enterprise Technologies, 1 (4 (91)), 76–82. doi: https://doi.org/10.15587/1729-4061.2018.123921
4. Yurchuk, V., Makhorin, Y. (2013). Construction of a knife screw auger beet tops cleaning machines. Konstruiuvannia, vyrobnytstvo ta ekspluatatsiya silskohospodarskykh mashyn, 43 (1), 208–211. Available at: http://nbuv.gov.ua/UJRN/Zmntz_2013_43%281%29__31
5. Jacob, D. V., Ramana, K. V., Rao, P. V. M. (2004). Automated manufacturability assessment of rotational parts by grinding. International Journal of Production Research, 42 (3), 505–519. doi: https://doi.org/10.1080/00207540310001613674
6. Abdel-Baky, R. A., Al-Ghefari, R. A. (2012). On the kinematic geometry of relative screw motions. Journal of Mechanical Science and Technology, 26 (8), 2497–2503. doi: https://doi.org/10.1007/s12206-012-0624-z
7. Saghafi, A., Farshidianfar, A. (2016). An analytical study of controlling chaotic dynamics in a spur gear system. Mechanism and Machine Theory, 96, 179–191. doi: https://doi.org/10.1016/j.mechmachtheory.2015.10.002
8. Farshidianfar, A., Saghafi, A. (2014). Identification and control of chaos in nonlinear gear dynamic systems using Melnikov analysis. Physics Letters A, 378 (46), 3457–3463. doi: https://doi.org/10.1016/j.physleta.2014.09.060
9. Zheng, F., Hua, L., Han, X., Li, B., Chen, D. (2016). Linkage model and manufacturing process of shaping non-circular gears. Mechanism and Machine Theory, 96, 192–212. doi: https://doi.org/10.1016/j.mechmachtheory.2015.09.010
10. Wang, Q., Hu, P., Zhang, Y., Wang, Y., Pang, X., Tong, C. (2014). A Model to Determine Mesh Characteristics in a Gear Pair with Tooth Profile Error. Advances in Mechanical Engineering. doi: https://doi.org/10.1155/2014/751476
11. Liu, X., Yang, Y., Zhang, J. (2016). Investigation on coupling effects between surface wear and dynamics in a spur gear system. Tribology International, 101, 383–394. doi: https://doi.org/10.1016/j.triboint.2016.05.006
12. Ismailova, N., Bogach, V., Lebedev, B. (2020). Development of a technique for the geometrical modeling of conjugated surfaces when determining the geometrical parameters of an engagement surface contact in kinematic pairs. Eastern-European Journal of Enterprise Technologies, 4 (1 (106)), 17–22. doi: https://doi.org/10.15587/1729-4061.2020.209108